Spectroscopic Measurements of Magnetic Field and Electron Density on Wire Array and Laser Plasmas

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Spectroscopic Measurements of Magnetic Field and Electron Density on Aluminum Wire Array and Laser Plasmas

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Magnetic field and electron density distributions within plasmas are crucial parameters in the study of high energy density laboratory plasmas. These measurements are challenging because often the magnetic fields undergo significant spatial and temporal fluctuations. Of the few measurement techniques possible, Zeeman splitting is the most promising option. However, this method has limitations when plasma conditions are such that line broadening due to the high plasma density and temperature surpasses the Zeeman splitting. The Zeeman broadening technique provides a solution to this field measurement problem by making simultaneous measurements of the widths of multiplet components. In this way, even if the splitting is not resolved, the difference in widths of multiplet lines provides an unambiguous field measurement. Additionally, through the Stark contribution to the line profile convolution, this technique offers estimates of the electron density.

We have implemented this technique in magnetized laser plasmas and magnetized exploding wire array plasmas.

We present spatially resolved magnetic field measurements from two different laboratory-produced plasmas using visible spectroscopy. We investigated the radial profiles of the magnetic field during the evolution of a plasma created by an intense laser pulse ($I > 10^{14} \text{ W cm}^{-2}$) that is allowed to expand in an azimuthal external magnetic field $B < 40 \text{ T}$. We observed that the external field completely diffuses into the plasma after nearly 100 ns, but the field profiles in the plasma develop modulations that differ from the external field profile as the plasma continues to evolve. In wire array plasmas driven by the Zebra pulsed power generator (delivering 1 MA in 90 ns), we measured the radial electron density profiles and magnetic field distribution around the wires during ablation and in the precursor plasma. In these measurements the Zeeman splitting was not resolved, but the magnetic field strength may be measured from the difference between the widths of the line profiles.

Space- and time-resolved measurements of magnetic field and electron density give insight into the dynamic behavior of these complicated systems. In this dissertation, we examine the strengths and limitations of this technique in different regimes and explore the feasibility of expanding this diagnostic beyond the plasma parameters observed in these experiments.
“We can measure the globula of matter and the distances between them, but Space plasm itself is incomputable.”

Ada or Ardor: A Family Chronicle (1969) by Vladimir Nabokov
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Chapter 1

Introduction

1.1 Motivation

Plasma and magnetic field interactions dictate much of the phenomena of the natural universe. Observed cosmic magnetic fields vary greatly in spatial scale and magnitude. The origin and amplification of these fields remain a contested issue. The fundamental question behind the investigation of galactic magnetic fields is whether the fields were amplified from a very weak \(10^{-20}\) G initial seed field (dynamo theory), or if a relatively strong \(10^{-10}\) G magnetic field was created before the galaxy was formed (primordial origin theory). Magnetic fields in the universe are responsible for the structure of the spiral arms of the galaxies, control how stars form and can direct and confine plasma jets and cosmic rays. Beyond the generation and coherent amplification of these ubiquitous fields, the question remains as to how these vast fields are maintained even in the rarefied intergalactic medium. Another astrophysically relevant phenomenon is collisionless shocks, in which the density is too sparse for the shock to be mediated by particle collisions so instead, magnetic fields are the mediator. The magnetic fields that drive collisionless shocks may already be present, or they may be self-generated via the
Weibel instability. [3, 4] This particular plasma instability helps to complete our current understanding of magnetic fields in the universe.

The investigation of matter under extreme temperatures and pressures, such as in astrophysical and stellar objects, are contained within the scope of high energy density (HED) physics. Laboratory experiments may be performed to study these types of systems. Two of the common methods to create a hot and dense plasma in the laboratory are z-pinches and laser ablation. A z-pinch refers to a particular configuration in which current is passed through a conducting electrode (wires, gas, or plasma) to generate an azimuthal magnetic field around the conductor. The high current ablates and ionizes the conductor and this plasma interacts with the self-generated magnetic field via the Lorentz force ($F_L = \mathbf{J} \times \mathbf{B}$) and is pushed radially inward, producing a column of radiating plasma with typical electron densities of $10^{17} - 10^{20}$ cm$^{-3}$ and electron temperatures of a few 100 eV. [5]

A laser beam is defined by its wavelength, focal spot, energy delivered by photons, and pulse duration. The latter three characteristics combine to define the laser intensity ($I = \frac{E_L}{\tau_L d_{fs}}$). A laser plume is produced by a laser beam that is incident on a target and deposits energy $E_L$ over a duration $\tau_L$ onto a cross-sectional area $d_{fs}$ which causes the surface material to evaporate off as a gas, or as a plasma if the beam is of sufficient intensity to create ionization. The depth to which the laser energy penetrates the target material is dependent on its wavelength. The physics of laser-produced plasmas are very rich, particularly because of the wide space of parameters that may be achieved by varying the laser pulse, target material, and experimental geometry. There have been many applications of interest to the field of plasma physics, including the investigation of the solar wind with Earth’s magnetosphere and inertial confinement fusion schemes.

Magnetohydrodynamics (MHD) is the study of the large-scale behavior of plasma.
The physical properties of a plasma may be understood in the context of approximating the plasma as a conducting fluid. The primary formalism is adapted from fluid mechanics with appropriate adaptations to the equations of state to account for the plasma’s interactions with electromagnetic fields.\[6\] The time scales of the fluid are considered much slower than the time scales of atomic processes, so the MHD treatment of a plasma is valid when the time scale of plasma evolution is slower than the plasma frequency ($\omega_p$), the ion gyro-frequency ($\omega_{c,i}$), and the collision frequency ($\nu_{ci}$). When considering plasma dynamics, the electron-ion collision frequency controls whether the plasma will be in the ideal or resistive regime. Ideal MHD is valid in the limit that the collision frequency approaches zero so that the conductivity of the plasma becomes infinite. This case is most applicable in astrophysical plasmas which are both weakly collisional and at high temperatures to aid conduction. Resistive MHD contains the most general formulation which allows the plasma collisions to become a relevant term. The complete set of MHD equations couple the Maxwell equations with the mass continuity equation, the momentum equation, Ohm’s law, and an equation of state. Two fundamental assumptions must be made about the plasma to render the Maxwell equations self-consistent within the model: the displacement current term $\epsilon_0 \partial \vec{E} / \partial t$ is considered small enough to neglect, and second, the plasma is considered electrically neutral, so the charge density is zero.

The crux of using MHD to fully describe macroscopic plasma behavior is encoded in writing a linear Ohm’s law, which gives the response of the current as a function of the applied electric field. Typically valid when the collisional mean-free path is short compared with the characteristic length scale of the system, generalized Ohm’s law (with the terms labeled) is

$$E + \frac{U \times B}{\sigma} \text{convection} - \frac{J}{\sigma} \text{diffusion} - \frac{1}{e n_e} J \times B \text{Hall} - \frac{1}{e n_e} \nabla \cdot P_e \text{electron pressure} + \frac{m_e}{n_e e^2} \frac{\partial J}{\partial t} + \nabla \cdot (JU + UJ) \text{electron inertia} = 0 \text{electron inertia}$$

(1.1)
**Chapter 1. Introduction**

\( \mathbf{U} \) is the center-of-mass fluid velocity, \( \sigma \) is the conductivity, \( e \) is the electric charge, \( n_e \) is the electron density, \( \mathbf{P} \) is the pressure tensor, and \( \mathbf{J} \) is the current density. Inserting the generalized Ohm’s law into Faraday’s law of induction \( \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \) and using Ampere’s law with the displacement current neglected (which is valid for frequencies lower than the plasma frequency) \( \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \), we obtain the time rate of change of the magnetic field in a plasma. Before we write the form \( \partial \mathbf{B} / \partial t \), it is convenient to make simplifications on the generalized Ohm’s law. The electron inertia term (named so due to its proportionality to the electron mass) may be ignored since \( m_e \ll m_i \). The electron pressure term derives what is called the Biermann battery term, which is proportional to \( \nabla T_e \times \nabla n_e \); it describes how a magnetic field may be generated from zero seed field. We neglect this term for the following discussion. Writing Faraday’s law with the remaining three terms:

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} - \nabla \times \left( \frac{\mathbf{J} \times \mathbf{B}}{en_e} \right). \tag{1.2}
\]

The relative importance of each of the terms of Ohm’s law dictates the plasma behavior. Important plasma parameters are derived from the ratios of the coefficients of the MHD equations. For example, from equation 1.2, the magnitude of the ratio of the convective term to the diffusive term gives a dimensionless number called the magnetic Reynolds number, \( R_m \). \( R_m \gg 1 \) indicates a highly conductive plasma, and likewise, \( R_m \ll 1 \) indicates a highly diffusive plasma. Approximating the \( \nabla \) operator as \( 1/L \), one obtains the simple expression \( R_m = \mu_0 \sigma U L \). From the momentum equation, a ratio between the plasma pressure term and the \( \mathbf{J} \times \mathbf{B} \) term defines the plasma \( \beta = P/(B^2/2\mu_0) \). The plasma pressure dominates, and the plasma pushes against the magnetic field when \( \beta \gg 1 \); the magnetic field pressure dominates when \( \beta \ll 1 \).

The ratio of the Hall term to the convective term is called the Hall coefficient \( H \propto \).
$c/\omega_{pi}L_n$ (see Appendix C) and it dominates the convective term when the electron density length scale $L_n \equiv n_e/\nabla n_e$ is much smaller than the ion inertial length, $c/\omega_{pi}$. This regime is termed Hall MHD and is distinguished from traditional MHD treatments because the Hall field facilitates a faster magnetic field diffusion than resistive diffusion alone in plasmas with low collisionality. \[7–10\] Since the Hall coefficient scales as $c/\omega_{pi}L$ and $\omega_{pi} \propto \sqrt{n_i}$, $H \sim n_e^{-1/2}$. The ratio of the diffusive coefficient to the convective coefficient is $1/R_m \propto n_e^{-1}$. From this relation, the effect of Hall diffusion is not only enhanced for lower densities but increases faster than the effect of resistive diffusion.

Manifestly, knowledge of the magnetic field within a plasma is crucial in understanding fundamental plasma phenomena. However, magnetic field measurements in these types of laboratory plasmas are elusive due to the turbulent and chaotic nature of their creation. Laser-produced plasmas and z-pinch plasmas experience evolution on nanosecond timescales and spatial variation on the order of micrometers. In these highly disruptive and explosive environments, the plasmas of interest exist within very high electric and magnetic fields that vary rapidly in magnitude and orientation throughout the plasma lifetime. To this end, characterizing the plasma parameters, and namely the magnetic fields, in laboratory experiments properly requires diagnostics that yield isolated space- and time-resolved data to high fidelity.

Standard methods for measuring magnetic fields are Faraday rotation, inductive probes, and atomic spectroscopy through the Zeeman effect. Faraday rotation is a technique that exploits an anisotropy induced in a dielectric medium in the presence of an external magnetic field. The indices of refraction for left- and right-circularly polarized light for plasma are affected differently in the presence of an external magnetic field. When linearly polarized light propagates through a plasma in an external magnetic field, the observed result is a rotation of the plane of polarization of the light. The degree of rotation is proportional to the
product of the integrated magnetic field and electron density along the line of sight of the probing beam. The high fluctuations in the magnetic field orientation along a particular line of sight make Faraday rotation challenging to observe. This technique is primarily used in astrophysical measurements since the rotation measure is proportional to $\lambda^2$, so using background radio sources produces a larger measurable effect. [11]

Another technique that has been successful in measuring magnetic fields is proton deflectometry. Magnetic fields alter particle trajectories so that charged particles are confined to orbit around field lines at the Larmor radius, $mv_\perp/qB$, and Larmor frequency, $\omega_L = eB/m$. By using a beam of protons to probe a plasma, the concentrations of the protons orbiting around high magnetic field regions reveals the structure of the magnetic field. However, setting up such a diagnostic requires the resources to produce a proton beam (via laser ablation) separately from using a driver to produce the experimental plasma, and with the timing precision to create the proton beam that will probe the experiment appropriately. Even though these capabilities are present at more advanced facilities, this method is restricted for use on plasmas that are sparse enough so that the proton beam may not only penetrate the plasma to be caught by the detector but also disturb the experiment minimally.

While the Zeeman splitting of atomic spectral lines is a direct result of the atom’s interaction with the magnetic field, electric fields and temperatures typical of HED plasmas smear out the splitting pattern, making a viable magnetic field measurement challenging. The goal of the present work is to implement a magnetic field diagnostic that gives the magnitude and orientation of the magnetic field within a plasma produced in laboratory experiments. This method is called the Zeeman broadening technique and was developed at the Weizmann Institute.[12, 13] This method can be applied even for cases when the field orientation fluctuates
over spatial scales much shorter than the plasma size. The basis for this technique is that upper and lower levels of multiplet components undergo different Zeeman splittings, and as a result have a different number of Zeeman transitions that are possible. With proper choice of atomic multiplet, the Stark and Doppler broadening contributions to the line profile are nearly identical for each of the Zeeman transitions. The difference in line broadening between each of the multiplet components is a direct function of the magnetic field magnitude. There are three diagnostic approaches based on the Zeeman effect that can be applied to HED plasmas: 1) Zeeman splitting, 2) polarization spectroscopy, and 3) Zeeman broadening. Table 1.1 summarizes past work done to make magnetic field measurements.

This dissertation is organized as follows: in Chapter 2 is an overview of the diagnostics used on experiments performed at the University of Nevada Reno’s Nevada Terawatt Facility, Chapter 3 describes the experimental platform on which this work was completed, Chapters 4 and 5 will detail two different experimental campaigns and their results, and Chapter 6 will detail conclusions and future work for this diagnostic.
### Table 1.1: Summary of past Zeeman spectroscopy work, including maximum electron temperature, magnetic field, and electron density for which the method is applicable, the atomic transition observed, the type of plasma and magnetic field that were probed, and the temporal and spatial resolution.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_e$ [eV]</td>
<td>5</td>
<td>&lt; 10</td>
<td>5-10</td>
<td>-</td>
</tr>
<tr>
<td>max $B$ [T]</td>
<td>&lt; 5</td>
<td>20</td>
<td>&lt; 5</td>
<td>500</td>
</tr>
<tr>
<td>max $n_e$ [cm(^{-3})]</td>
<td>$5 \times 10^{17}$</td>
<td>$1 \times 10^{18}$</td>
<td>$1 \times 10^{16}$</td>
<td>$5 \times 10^{17}$</td>
</tr>
<tr>
<td>Diagnostic Transition</td>
<td>O IV, O V</td>
<td>Al III</td>
<td>Al III</td>
<td>Na I (absorption)</td>
</tr>
<tr>
<td>Plasma generation</td>
<td>Gas puff</td>
<td>7 ns laser</td>
<td>6 ns laser</td>
<td>Pulsed power</td>
</tr>
<tr>
<td>B field generation</td>
<td>320 kA, 1.6 µs</td>
<td>270 kA, 1.7 µs</td>
<td>160 kA, 100 ns</td>
<td>27 MA, 100 ns</td>
</tr>
<tr>
<td>Recording method</td>
<td>Streaked</td>
<td>Time-gated, fiber coupling</td>
<td>100 ns time integration</td>
<td>Streaked</td>
</tr>
</tbody>
</table>
Chapter 2

Diagnostics

This chapter will detail the diagnostics used to study the plasmas discussed in the following chapters. The diagnostics can be divided into four categories: spectroscopic, pulsed laser probing, optical imaging, and electrical.

2.1 Spectroscopic Diagnostics

2.1.1 Magnetic Field Measurements

2.1.1.1 Zeeman Splitting

We consider the effect an external magnetic field has on the $ns$ and $np$ levels of an atomic system compared to the effects of fine structure energy perturbation. For the fine structure we consider only the spin-orbit coupling term and not the Darwin or kinetic contributions, as the spin-orbit term is the dominating term. The Hamiltonian is written as a sum of the unperturbed Hamiltonian of the system, the energy of the spin-orbit (also called LS) coupling $H_{so}$, and the energy of the
external magnetic field interaction with the magnetic moments of the atom $H_Z$:

$$\hat{H} = H_0 + H_{so} + H_Z$$

$$\hat{H} = A\vec{L} \cdot \vec{S} + \omega_e (\vec{L} + 2\vec{S}) \cdot \vec{B}$$

where $\omega_e$ is the electron’s Larmor frequency, given in terms of the electron mass $m_e$ and charge $q$ or the Bohr magneton $\mu_B$:

$$\omega_e = \frac{qB}{2m_e} = -\frac{\mu_B}{\hbar} B.$$ (2.3)

Following standard conventions, we take $\vec{B} = B_0 \hat{z}$ so that our perturbed Hamiltonian $H'$ becomes

$$H' = A\vec{L} \cdot \vec{S} + \omega_e (L_z + 2S_z).$$ (2.4)

It is necessary first to determine which of these two perturbations dominates the energy level structure in order to find the basis set which will diagonalize the Hamiltonian. The magnitudes of the fine structure and Zeeman energy contributions (ignoring numerical factors) are:

$$|H_{so}| = Ah^2$$ (2.5)

$$|H_Z| = \hbar \omega_e,$$ (2.6)

where $A$ is the magnitude of the spin-orbit coupling and is a complicated function of the atomic system.

The spin-orbit Hamiltonian is written in terms of the spin and orbital angular momentum operators and depends on the Coulomb interaction between the nucleus
and electrons

\[ H_{so} = \frac{\mu_B}{\hbar m_e c^2 r} \left( \frac{1}{r} \frac{\partial U(r)}{\partial r} \right) \mathbf{L} \cdot \mathbf{S}. \] (2.7)

For hydrogenic wavefunctions, the first-order shift due to spin-orbit coupling is a function of the quantum numbers \( j, l, n, s \) and is proportional to \( Z^4 \). In general, computing the first-order shift due to spin-orbit coupling is nontrivial due to the radial integral, \( \langle \psi \vert \frac{1}{r} \frac{\partial U(r)}{\partial r} \vert \psi \rangle \). However, the magnitude of the spin-orbit interaction for a given multiplet can be estimated by the difference in energy between the fine structure levels. Taking \( |H_{so}| \approx E_{u1} - E_{u2} \), where \( E_{u1} \) (\( E_{u2} \)) is the energy of the higher (lower) energy fine structure level, and equating \( |H_{so}| \) to \( |H_Z| = \hbar \omega_e = \mu_B B \), the magnitude of \( B \) that determines the Zeeman regime is found.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Upper level 1</th>
<th>( E_{u1} ) [eV]</th>
<th>Upper level 2</th>
<th>( E_{u2} ) [eV]</th>
<th>( \frac{(E_{u1} - E_{u2})}{\mu_B} ) [T]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na I</td>
<td>( 1s^22s^22p^63p ) ( 2P_{3/2} )</td>
<td>2.104492</td>
<td>( 1s^22s^22p^63p ) ( 2P_{3/2} )</td>
<td>2.102297</td>
<td>36.88</td>
</tr>
<tr>
<td>C IV</td>
<td>( 1s^22p ) ( 2P_{3/2} )</td>
<td>39.68507</td>
<td>( 1s^22p ) ( 2P_{1/2} )</td>
<td>39.69115</td>
<td>67.8</td>
</tr>
<tr>
<td>Al III</td>
<td>( 1s^22s^22p^64p ) ( 2P_{3/2} )</td>
<td>17.818203</td>
<td>( 1s^22s^22p^64p ) ( 2P_{1/2} )</td>
<td>39.69115</td>
<td>171.86</td>
</tr>
</tbody>
</table>

Table 2.1: Magnitudes of external magnetic fields for which the Zeeman perturbation becomes comparable to the spin-orbit coupling of the ion. For weak field Zeeman theory to apply to a given system, the external magnetic field must be much less than the tabulated values.

Table 2.1 lists magnetic fields for which the LS coupling becomes comparable to the Zeeman perturbation. For the optical doublet transitions from Al III and C IV being studied for this dissertation, the weak field approximation should be valid in magnetic fields less than 40 T created in experiment. For the Na I resonance lines, \( \Delta E/\mu_B = 37 \) T, so for \( B > 40 \) T the intermediate field considerations may be needed.

There are three cases to consider: (1) the weak field regime which occurs when \( \hbar \omega_e \ll \Delta \hbar^2 \); (2) the strong field (or Paschen-Back) regime which occurs when \( \hbar \omega_e \gg \Delta \hbar^2 \); and (3) the intermediate regime when \( \hbar \omega_e \sim \Delta \hbar^2 \). We solve the three cases for the \( ns \) and \( np \) sublevels.
1. Weak field limit

First-order Energy Shifts

When $\hbar \omega_e \ll A \hbar^2$, the magnetic field acts as a perturbation on the spin-orbit coupled states, which are the eigenstates of the $J^2$ and $J_z$ operators, where $J = L + S$ is the total angular momentum. These states are characterized by their quantum numbers in ket notation, $|n, l, s; j, m_j\rangle$ (abbreviated by the “good” quantum numbers $|j, m_j\rangle$). However, the Zeeman perturbation matrix is not diagonal in this basis since its eigenstates are the $|m_l, m_s\rangle$ states. The $ns - np$ manifold is spanned by 8 vectors. It becomes useful for the following derivations to write the $|j, m_j\rangle$ basis vectors in terms of the $|m_l, m_s\rangle$ basis vectors by using the Clebsh-Gordon coefficients.

\[
|n, l, s; j, m_j\rangle = |n, l, s; m_l, m_s\rangle
\]

- **s orbital**: $|n, l = 0, s = 1/2; 1/2, 1/2\rangle = |n, l = 0, s = 1/2; 0, 1/2\rangle \quad (2.8)

- **p orbital**: $|n, l = 1, s = 1/2; 1/2, 1/2\rangle = \sqrt{\frac{3}{2}}|1, -1/2\rangle - \sqrt{\frac{1}{2}}|0, 1/2\rangle \quad (2.9)

It is now straightforward to write the $L_z$ and $S_z$ matrices in the $|j, m_j\rangle$ basis by applying the operator to each $|j, m_j\rangle$ state written in its $|m_l, m_s\rangle$ basis.
representation. Using the basis ordering:

\[
\{|\frac{3}{2}, \frac{3}{2} \rangle, |\frac{3}{2}, -\frac{3}{2} \rangle, |\frac{3}{2}, \frac{1}{2} \rangle, |\frac{3}{2}, -\frac{1}{2} \rangle, |\frac{1}{2}, \frac{1}{2} \rangle, |\frac{1}{2}, -\frac{1}{2} \rangle \}\]

for the np manifold and \(\{|\frac{1}{2}, \frac{1}{2} \rangle, |\frac{1}{2}, -\frac{1}{2} \rangle\}\) for the ns manifold.

\[
S_z|_{\text{np}} = \frac{\hbar}{2} \times \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3} & 0 & -\frac{2 \sqrt{3}}{3} & 0 \\
0 & 0 & 0 & -\frac{1}{3} & 0 & -\frac{2 \sqrt{3}}{3} \\
0 & 0 & -\frac{2 \sqrt{3}}{3} & 0 & -\frac{1}{3} & 0 \\
0 & 0 & 0 & -\frac{2 \sqrt{3}}{3} & 0 & \frac{1}{3}
\end{pmatrix}
\]

(2.16)

\[
S_z|_{\text{ns}} = \frac{\hbar}{2} \times \begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\]

(2.17)

\[
L_z|_{\text{np}} = \hbar \times \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3} & 0 & \frac{\sqrt{3}}{3} & 0 \\
0 & 0 & 0 & -\frac{1}{3} & 0 & \frac{\sqrt{3}}{3} \\
0 & 0 & \frac{\sqrt{3}}{3} & 0 & \frac{2}{3} & 0 \\
0 & 0 & 0 & \frac{\sqrt{3}}{3} & 0 & -\frac{2}{3}
\end{pmatrix}
\]

(2.18)

\[
S_z|_{\text{ns}} = \hbar \times \begin{pmatrix}
0 & 0 \\
0 & 0
\end{pmatrix}
\]

(2.19)
So that the total Zeeman Hamiltonian in the $|j, m_j\rangle$ basis is

$$H_{Z_{\text{np}}} = \hbar \omega_c \times \begin{pmatrix}
2 & 0 & 0 & 0 & 0 \\
0 & -2 & 0 & 0 & 0 \\
0 & 0 & {\Delta^2 \over 3} & 0 & -{\sqrt{3} \Delta^2 \over 3} \\
0 & 0 & 0 & -{2 \over 3} & 0 & -{\sqrt{3} \over 3} \\
0 & 0 & -{\sqrt{3} \over 3} & 0 & {1 \over 3} & 0 \\
0 & 0 & 0 & -{\sqrt{3} \over 3} & 0 & -{1 \over 3}
\end{pmatrix}$$

(2.20)

$$H_{Z_{\text{ns}}} = \hbar \omega_c \times \begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}$$

(2.21)

Using first order (degenerate) perturbation theory, the energy corrections to each of the Zeeman sublevels may be read off as the diagonal elements of $H_Z$. The degeneracy is broken along each value of $j$ into $2j + 1$ sublevels. Alternatively, we may use the Wigner-Eckart projection theorem to obtain a formula for the energy shifts in terms of the quantum numbers. However, obtaining the full matrix representation in the alternate basis aids in the intermediate field calculations.

By virtue of the Wigner-Eckart projection theorem, we know that inside a subspace of constant total angular momentum $J = j$, any operator $A$ is proportional to $J$ through the formula:

$$\hat{A} = \langle \hat{A} \cdot \hat{J} \rangle_j \langle \hat{J^2} \rangle_j^{-1}$$

(2.22)

And as expected, the subspaces defined by $j = 3/2, 1/2$ (spanned by the first four, last two basis vectors for the for $l = 1$ matrix) and $j = 1/2$ for $l = 0$ are diagonal matrices. The energies for these subspaces are degenerate for a given $j$, so we must use degenerate perturbation theory to find a formula for the first order energy shifts (independent from having to do the lengthy basis
conversion done to create the full perturbation matrix). The energy shifts for degenerate levels are found by diagonalizing the perturbation matrix in the restriction of the degenerate subspace. The new diagonal elements are the energy shifts and the eigenkets which diagonalize the perturbation matrix are the zeroth order states to which the system reduces to as the perturbation becomes vanishingly small. Conveniently for the weak field Zeeman effect problem, the perturbation matrix in the restriction of each degenerate subspace is already diagonal. So the full analytic expression for the energy shifts of each state in the manifold may be found by evaluating the Wigner-Eckart projection coefficients.

Since \( J = L + S \) and \( J^2 = (L + S)^2 = L^2 + S^2 + 2L \cdot S \), we obtain an expression for \( L \cdot J \) by writing:

\[
(L + J)^2 = (2L + S)^2 \\
L^2 + J^2 + 2L \cdot J = 4L^2 + S^2 + 4L \cdot S \\
= 4L^2 + S^2 + 2(J^2 - L^2 - S^2) \\
L \cdot J = \frac{1}{2}(L^2 + J^2 - S^2) \tag{2.23}
\]

And similarly for projection of \( S \) onto \( J \),

\[
S \cdot J = \frac{1}{2}(S^2 + J^2 - L^2) \tag{2.24}
\]

The averaged value of these projections may be evaluated, for example, as

\[
\langle nls; j, m_j | J^2 | nls; jm_j \rangle = \langle nls; j, m_j | j(j+1) | nls; jm_j \rangle \\
= j(j+1) \langle nls; j, m_j | nls; jm_j \rangle \\
= j(j+1).
\]
So finally we obtain:

\[
\hat{L} = \frac{l(l + 1) + j(j + 1) - s(s + 1)}{2j(j + 1)} \hat{j}
\]

(2.25)

\[
\hat{S} = \frac{s(s + 1) + j(j + 1) - l(l + 1)}{2j(j + 1)} \hat{j}
\]

(2.26)

With the full Zeeman perturbation Hamiltonian as:

\[
H_Z = \frac{\mu_B}{\hbar} B J_z \left[ \frac{l(l + 1) + j(j + 1) - s(s + 1) + 2(s(s + 1) + j(j + 1) - l(l + 1))}{2j(j + 1)} \right]
\]

\[
= \frac{\mu_B}{\hbar} B J_z \left[ \frac{s(s + 1) - l(l + 1) + 3j(j + 1)}{2j(j + 1)} \right]
\]

\[
= \frac{\mu_B}{\hbar} B g_j J_z
\]

(2.27)

so that the energy shifts due to the Zeeman effect (\(\Delta E_Z\)) (within each degenerate subspace) are given by the inner product:

\[
\Delta E_Z = \langle j, m_j | \frac{\mu_B}{\hbar} B g_j J_z | j, m_j \rangle
\]

\[
= \langle j, m_j | \frac{\mu_B}{\hbar} B g_j K m_j | j, m_j \rangle
\]

\[
= \langle j, m_j | \mu_B B g_j m_j | j, m_j \rangle
\]

\[
\Delta E_Z = \mu_B B g_j m_j
\]

(2.28)

where \(g_j\) is the Landé g-factor and is constant for a given \(j, l, s\). Evidently, the energy shifts in the presence of an external magnetic field depend only on the so-called magnetic quantum number, \(m_j\). In this way we arrive at the same conclusion as before: the magnetic field breaks the degeneracy along \(J\).

A summary of the results for the energy shifts due to a weak magnetic field is listed in Table 2.2. Next we examine the dipole transitions between the \(l = 0\) and \(l = 1\) levels that arise due to the Zeeman-split energy levels. For dipole (E1) transitions, the selection rules dictate that \(\Delta J = 0, \pm 1, \text{ and} \)
\[ \Delta m_j = 0, \pm 1. \] We will henceforth be using standard spectroscopic notation of the form \[ ^{2s+1}L_j. \] From figure 2.1, we see that there are 4 transitions between the \(^2P_{1/2}\) level and the \(^2S_{1/2}\) level and 6 transitions between the \(^2P_{3/2}\) level and the \(^2S_{1/2}\) level. The shifted energies of these transitions are given in Table 2.3.

\[
\begin{array}{|c|c|c|c|}
\hline
\ell & \{j, m_j\} & B = 0 \text{ degeneracy} & g_j & \Delta E_Z \\
\hline
0 & |1/2, +1/2\rangle & 2 \text{ fold} & 2 & 1\mu_B B \\
0 & |1/2, -1/2\rangle & 2 \text{ fold} & 2 & -1\mu_B B \\
1 & |1/2, +1/2\rangle & 2 \text{ fold} & 2/3 & 1/3\mu_B B \\
1 & |1/2, -1/2\rangle & 2 \text{ fold} & 2/3 & -1/3\mu_B B \\
1 & |3/2, +3/2\rangle & 4 \text{ fold} & 4/3 & +2\mu_B B \\
1 & |3/2, -3/2\rangle & 4 \text{ fold} & 4/3 & -2\mu_B B \\
1 & |3/2, +1/2\rangle & 4 \text{ fold} & 4/3 & +2/3\mu_B B \\
1 & |3/2, -1/2\rangle & 4 \text{ fold} & 4/3 & -2/3\mu_B B \\
\hline
\end{array}
\]

Table 2.2: Summary of Zeeman energy shifts for the ns-np manifold. Note these results may be read directly off as the diagonal elements of the full perturbation matrix (Equations 2.20 and 2.21).

Figure 2.1: Schematic of the ten Zeeman components that arise in the \(^2S - ^2P\) doublet in the presence of an external magnetic field.
Table 2.3: $E_i$ refers to the energy of an upper level state (from one of the $^2P$ manifolds), and $E_f$ refers to the energy of the lower level state (from the $^2S$ manifold). The transition energies of the unperturbed doublet components as $E^{(2)}_0$ for $^2P_{3/2} - ^2S_{1/2}$ and $E^{(1)}_0$ for $^2P_{3/2} - ^2S_{1/2}$ ($E^{(1)}_0 > E^{(2)}_0$). The shifted transition energies ($\Delta E_{\text{transition}}$) of each of the 10 Zeeman components are given relative to their principal line center energy. The polarization of each line is given in the rightmost column, according to the $\Delta m$ of the transition.

<table>
<thead>
<tr>
<th>Manifold, energy</th>
<th>Transition</th>
<th>$\Delta E_{\text{transition}} = E_i - E_f$</th>
<th>Pol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^2P_{3/2} - ^2S_{1/2}$, $E^{(1)}_0$</td>
<td>$</td>
<td>^{3/2},^{3/2}\rangle \rightarrow</td>
<td>^{1/2},^{1/2}\rangle$</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>^{3/2},^{1/2}\rangle \rightarrow</td>
<td>^{1/2},^{1/2}\rangle$</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>^{3/2},^{1/2}\rangle \rightarrow</td>
<td>^{1/2},^{-1/2}\rangle$</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>^{3/2},^{-1/2}\rangle \rightarrow</td>
<td>^{1/2},^{1/2}\rangle$</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>^{3/2},^{-1/2}\rangle \rightarrow</td>
<td>^{1/2},^{-1/2}\rangle$</td>
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<td>$</td>
<td>^{3/2},^{-3/2}\rangle \rightarrow</td>
<td>^{1/2},^{-1/2}\rangle$</td>
</tr>
<tr>
<td>$^2P_{3/2} - ^2S_{1/2}$, $E^{(2)}_0$</td>
<td>$</td>
<td>^{1/2},^{1/2}\rangle \rightarrow</td>
<td>^{1/2},^{1/2}\rangle$</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>^{1/2},^{1/2}\rangle \rightarrow</td>
<td>^{1/2},^{-1/2}\rangle$</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>^{1/2},^{-1/2}\rangle \rightarrow</td>
<td>^{1/2},^{1/2}\rangle$</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>^{1/2},^{-1/2}\rangle \rightarrow</td>
<td>^{1/2},^{-1/2}\rangle$</td>
</tr>
</tbody>
</table>

2. Strong field limit In the limit of a strong magnetic field, the spin-orbit interaction acts as the perturbation, meaning that total angular momentum $J$ is no longer conserved. In this case, it is the states of definite orbital and spin angular momentum that form the basis set for the system. The $\{|m_l, m_s\}$ basis states diagonalize $H_0$ and $H_z$:

$$\omega_e B (L_z + 2S_z)|m_l, m_s\rangle = \mu_B B (m_l + 2m_s)|m_l, m_s\rangle$$ (2.29)

It remains to write the spin-orbit interaction in terms of this basis. To do so we use the ladder operators (e.g. for orbital angular momentum)

$$L_- = L_x + iL_y$$ (2.30)

$$L_+ = L_x - iL_y$$ (2.31)
and the spherical tensor basis to decompose the angular momenta operators as:

\[ \mathbf{\tilde{L}} = -\frac{1}{\sqrt{2}} L_+ \mathbf{\hat{e}}_+ + \frac{1}{\sqrt{2}} L_- \mathbf{\hat{e}}_- + L_z \mathbf{\hat{e}}_0 \]  
\[ \mathbf{\tilde{S}} = -\frac{1}{\sqrt{2}} S_+ \mathbf{\hat{e}}_+ + \frac{1}{\sqrt{2}} S_- \mathbf{\hat{e}}_- + S_z \mathbf{\hat{e}}_0, \]

which yields the inner product in terms of the \( \{ |m_l, m_s \rangle \} \) basis. Namely,

\[ \mathbf{\tilde{L}} \cdot \mathbf{\tilde{S}} = \frac{1}{2} L_+ S_+ + \frac{1}{2} L_- S_- + L_z S_z. \]

It is clear that in the \( l = 0 \) manifold (which is 2-fold degenerate), the spin-orbit perturbation matrix is diagonalized. However, the 6-fold degenerate \( l = 1 \) manifold will have cross terms between basis states. For example, we have for the \( |m_l = 1, m_s = \frac{1}{2} \rangle \) state:

\[ A \mathbf{\tilde{L}} \cdot \mathbf{\tilde{S}} |1, \frac{1}{2} \rangle = A \left( \frac{1}{2} L_+ S_+ + \frac{1}{2} L_- S_- + L_z S_z \right) |1, \frac{1}{2} \rangle \]
\[ = A \left( \frac{1}{2} L_+ S_+ |1, \frac{1}{2} \rangle + \frac{1}{2} L_- S_- |1, \frac{1}{2} \rangle + L_z S_z |1, \frac{1}{2} \rangle \right) \]
\[ = A \left( \frac{1}{\sqrt{2}} \hbar^2 |0, -\frac{1}{2} \rangle + \hbar^2 \frac{1}{2} |1, \frac{1}{2} \rangle \right) \]

There are two pairs of states that mix. Using the basis ordering
Chapter 2. Diagnostics

\{1, \frac{1}{2}, 1, -\frac{1}{2}, 0, \frac{1}{2}, 0, -\frac{1}{2}, -1, \frac{1}{2}, -1, -\frac{1}{2}\}\}, the spin-orbit perturbation matrix in the two manifolds becomes

\[
H_{so|\text{np}} = \frac{\mathcal{A} \hbar^2}{2} \times \begin{pmatrix}
1 & 0 & 0 & \sqrt{2} & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{2} \\
\sqrt{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & \sqrt{2} & 0 & 0 & 1
\end{pmatrix}
\] (2.35)

\[
H_{so|\text{ns}} = \frac{\mathcal{A} \hbar^2}{2} \times \begin{pmatrix}
0 & 0 \\
0 & 0
\end{pmatrix}.
\] (2.36)

| Orbital | \( |m_l, m_s\rangle \) | \( \Delta E [\mu_B B] \) |
|---------|-----------------|-----------------|
| ns      | \( |0, \frac{1}{2}\rangle \) | 1 |
|         | \( |0, -\frac{1}{2}\rangle \) | -1 |
| np      | \( |1, \frac{1}{2}\rangle \) | 2 |
|         | \( |1, -\frac{1}{2}\rangle \) | 0 |
|         | \( |0, \frac{1}{2}\rangle \) | 1 |
|         | \( |0, -\frac{1}{2}\rangle \) | -1 |
|         | \( |-1, \frac{1}{2}\rangle \) | 0 |
|         | \( |-1, -\frac{1}{2}\rangle \) | -2 |

Table 2.4: Energy shifts due to a strong magnetic field when fine-structure coupling is broken.

We see in Table 2.4 that the \( ns \) orbital is non-degenerate and the \( np \) orbital has one 2-fold degenerate state pair and the remaining four non-degenerate states in a strong magnetic field. Rearranging the basis to the following ordering:

\{1, \frac{1}{2}, -1, -\frac{1}{2}, 1, -\frac{1}{2}, -1, \frac{1}{2}, 0, \frac{1}{2}, 0, -\frac{1}{2}\}, we obtain for the
spin-orbit Hamiltonian,

$$H_{so}\big|_{nt} = \frac{Ah^2}{2} \times \begin{pmatrix} 1 & 0 & 0 & 0 & \sqrt{2} & \sqrt{2} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ \sqrt{2} & 0 & 0 & 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(2.37)

Evidently, within the degenerate subspace, the restriction on the perturbing Hamiltonian is diagonal. So we may write the first order energy shifts in the Paschen-Back regime as such:

| Orbital | $|m_l, m_s\rangle$ | $\Delta E$ |
|---------|-----------------|-----------|
| ns      | $|0, \frac{1}{2}\rangle$ | $1\mu_B B$ |
|         | $|0, -\frac{1}{2}\rangle$ | $-1\mu_B B$ |
| np      | $|1, \frac{1}{2}\rangle$ | $2\mu_B B + \frac{Ah^2}{2}$ |
|         | $|-1, -\frac{1}{2}\rangle$ | $-2\mu_B B + \frac{Ah^2}{2}$ |
|         | $|1, -\frac{1}{2}\rangle$ | $-\frac{Ah^2}{2}$ |
|         | $|-1, \frac{1}{2}\rangle$ | $-\frac{Ah^2}{2}$ |
|         | $|0, \frac{1}{2}\rangle$ | $+\mu_B B$ |
|         | $|0, -\frac{1}{2}\rangle$ | $-\mu_B B$ |

Table 2.5: Energy shifts due in a strong magnetic field with spin-orbit coupling as a small perturbation. We see that states of zero orbital angular momentum are unaffected, and the product $m_l m_s$ determines the sign of the linear shift due to the fine structure perturbation.
3. **Intermediate field.** The matrix \(2.35\) has the diagonalization \(P^{-1}QP\)

\[
Q = \frac{\hbar^2}{2} \times \begin{pmatrix}
-1 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 2 \\
\end{pmatrix}
\]

(2.38)

and

\[
P = \begin{pmatrix}
0 & 0 & -\frac{\sqrt{2}}{3} & 0 & 0 & \frac{1}{3} \\
0 & 0 & 0 & 0 & 1 & 0 \\
-\frac{\sqrt{2}}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\sqrt{2}}{3} & 0 & 0 & \frac{2}{3} \\
-\frac{\sqrt{2}}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\
\end{pmatrix}
\]

(2.39)

where the column vectors of \(P\) are the eigenvectors of linear combinations of the original basis kets that the system approaches as the perturbation goes to zero.

### 2.1.1.2 Polarization Spectroscopy

The individual line components of a transition between states \(|j', m'_j\rangle \rightarrow |j, m_j\rangle\) (the transitions between sublevels of the upper and lower states, where the prime denotes the upper level) are electric dipole transitions. These lines will be called the Zeeman components because these are the transitions that become non-degenerate in the presence of the external magnetic field. It then follows from the theory of electron dipole radiation that the allowable dipole transitions are polarized. For transitions with \(m_j - m'_j = +1(-1)\), a clockwise, \(\sigma^+\) (counterclockwise, \(\sigma^-\)) circularly polarized photon is emitted; for transitions with \(m_j - m'_j = 0\), a linearly
polarized, \( \pi \), photon is emitted. Here, clockwise refers to the direction the electric field rotates with respect to the direction of propagation. Table 2.3 shows the polarizations of the \( ^2S - ^2P \) doublet in the presence of an external (weak) magnetic field.

The \( \sigma \) components of a transition influenced by an external magnetic field experience a larger energy shift than the \( \pi \) components. In the weak field limit, the shifts of the polarized components are given as

\[
\Delta E_{\pi} = \mu_B B \{g_u - g_l\} m \tag{2.40}
\]

\[
\Delta E_{\sigma} = \mu_B B \{g_u m - g_l (m \pm 1)\} \tag{2.41}
\]

where \( m \) is the magnetic quantum number of the upper level, \( g_u \) and \( g_l \) are the Landé g factors for the upper and lower levels, given in equation 2.27.

### 2.1.1.3 Line Intensity Patterns

Likewise, the electric dipole operator also dictates the intensity patterns of the Zeeman components. The relative intensities of the Zeeman components within a given transition \( |j', m'_j\rangle \rightarrow |j, m_j\rangle \) are given by the Wigner 3\( j \) symbol [14]

\[
I \propto \begin{vmatrix}
  j & 1 & j' \\
  -m_j & m_j - m'_j & m'_j
\end{vmatrix}^2 \tag{2.42}
\]

These intensities assume an isotropic magnetic field orientation, but the intensity profiles observed parallel and perpendicular to the field can be obtained from them since the isotropic case is the average of the two orthogonal orientations. First, we obtain the intensity profiles for observations made along the field. These are the \( \sigma \) components. Next we can solve for the intensity profiles for observations
perpendicular to the field for each Zeeman component by the relation \( \frac{1}{2}(I_i^+ + I_i^-) = I_i^{\text{iso}} \). For the ns-np doublet, the intensity profiles are given in Table 2.6

<table>
<thead>
<tr>
<th>Transition</th>
<th>( ^2P_{3/2} \rightarrow ^2S_{1/2} )</th>
<th>( ^2P_{1/2} \rightarrow ^2S_{1/2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zeeman shifts</td>
<td>( \pm{\frac{1}{3}, 1, \frac{5}{3}}\mu_B B )</td>
<td>( \pm{\frac{2}{3}, \frac{4}{3}}\mu_B B )</td>
</tr>
<tr>
<td>Isotropic ( B ) intensities</td>
<td>2:3:1</td>
<td>1:2</td>
</tr>
<tr>
<td>( |B ) intensities</td>
<td>0:3:1</td>
<td>0:1</td>
</tr>
<tr>
<td>( \perp B ) intensities</td>
<td>4:3:1</td>
<td>1:1</td>
</tr>
<tr>
<td>( \pi )-component intensities</td>
<td>4:0:0</td>
<td>1:0</td>
</tr>
</tbody>
</table>

**Table 2.6:** Intensities of the Zeeman components relative to each other within a fine structure transition manifold for different orientations with respect to \( B \).

To obtain the absolute intensities of the transitions within a fine structure manifold (the ns-np transition for example), the full electric dipole line strength must be calculated. In the absence of opacity effects and under approximately pure LS coupling, the intensities of the multiplet lines relative to each other are determined by the line factor, \( D_{\text{line}} \), which contains the entire dependence of the electric dipole matrix element on the upper and lower levels’ \( J \). The relative intensity of lines within a multiplet arising from the transition \( |l', s\rangle \rightarrow |l, s\rangle \) is given by the Wigner \( 6j \) symbol [14]

\[
I \propto [j, j'] \left\{ \begin{array}{ccc} l & s & j \\ j' & 1 & l' \end{array} \right\}^2
\]

(2.43)

where \([j, j'] = (2j + 1)(2j' + 1)\) is the product of multiplicities of \( j, j' \). For the ns-np doublet, the intensity ratio of the lower energy transition to the higher energy transition is 2 : 1.
2.1.1.4 Zeeman Broadening

The method of Zeeman broadening to measure magnetic fields in regimes in which the Zeeman splitting cannot be resolved was developed at the Weizmann Institute of Science. [12] This method is based on the simultaneous observation of a multiplet transition of a particular ion well suited for the given plasma temperature, electron density, and external magnetic field magnitude. The upper and lower levels of a given total angular momentum $J$ corresponding to each component of a given multiplet transition undergo different Zeeman splittings. For the $ns - np$ doublet transition, figure 2.1 shows how the $2J + 1$ components of the upper and lower levels result in a difference of the line profiles of each doublet transition. As discussed earlier, these dipole transitions between the non-degenerate sublevels are the Zeeman components. The key to utilizing the Zeeman broadening method is that each of these Zeeman components will experience virtually identical Stark, Doppler, and instrumental broadening. Stark broadening is nearly identical due to the energy proximity between the two upper levels so that relevant matrix elements are close in magnitude. Doppler broadening is proportional to the wavelength, and for multiplet components, the wavelengths are nearly identical. Instrumental broadening is a function of the spectrometer slit width, grating, and detector pixel size. As long as the multiplet components are recorded simultaneously on the same instrument, the broadening will be identical for all Zeeman components. For this method to be applicable, the multiplet lines as broadened by contributions from Stark, Doppler, Zeeman, and instrumental effects, must be resolvable such that the full-width at half-maximum may still be measured. Therefore, under these conditions, a measurement of the width difference between the multiplet components is a direct measurement of the magnetic field strength and even orientation.

To demonstrate Zeeman broadening as a viable magnetic field diagnostic in any
field orientation, we consider the $^2S -^2 P$ doublet transition, as discussed in section 2.1.1.1. From Table 2.6, the Zeeman energy splittings of the $^2P_{3/2} -^2 S_{1/2}$ transition are $\pm\{\frac{1}{3}, \frac{5}{3}\}\mu_B B$ and the Zeeman energy splittings of the $^2P_{1/2} -^2 S_{1/2}$ transition are $\pm\{\frac{2}{3}, \frac{4}{3}\}\mu_B B$. Since each radiating atom or ion aligns with the external magnetic field, the nature of the dipole transition causes the line intensities to become a function of the line of sight with respect to the magnetic field. Parallel to the magnetic field, only the $\sigma$ components are radiated, and perpendicular to the magnetic field, both $\sigma$ and $\pi$ components are radiated. The relative intensity ratios (in order of increasing energy shift magnitude) for the $^2P_{3/2} -^2 S_{1/2}$ Zeeman components are 0:3:1 for longitudinal observation and 4:3:1 for transverse observation. Similarly, the relative intensity ratios for the $^2P_{1/2} -^2 S_{1/2}$ Zeeman components are 0:1 for longitudinal observation and 1:1 for transverse observation. For each doublet component, the Zeeman energy shifts and relative intensities of the Zeeman components can be combined to calculate a weighted average broadening, equation 2.44. The summations over the $i$ components of the $^2P_{3/2} -^2 S_{1/2}$, $^2P_{1/2} -^2 S_{1/2}$ (labeled by the $J$ of the upper level, $3/2, 1/2$, respectively) line performed for longitudinal ($||$), transverse ($\perp$), and $\pi$ component observations are calculated using equation 2.44 and are tabulated below.

\[
\langle \Delta E \rangle = \frac{\sum I_i |\Delta E_i|}{\sum I_i}.
\]

(2.44)

Because the line of sight observation of the doublet emission is some linear combination of the longitudinal and transverse projections, average broadening for the $^2P_{1/2} -^2 S_{1/2}$ component (henceforth labeled $w_{1/2}$) is always greater than the average broadening of the $^2P_{3/2} -^2 S_{1/2}$ component ($w_{3/2}$) in the presence of an external magnetic field by at least 14%. While the ratio $w_{1/2}/w_{3/2}$ establishes that there is an unambiguous anisotropy in the widths due to the magnetic field, it is not the most useful metric for measuring line width differences as it implicitly forces
a comparison relative to the smaller of the two widths. It is useful to consider instead the relative width difference between the two doublet lines. The relative line width difference is given by

\[ \Delta \bar{w} = \frac{2(w_{1/2} - w_{3/2})}{w_{1/2} + w_{3/2}}. \]  

(2.45)

The function of the two widths in the denominator (the average width) is arbitrary by definition, but is commonly used in spectral line width comparisons. In this application, we ensure that \( \Delta \bar{w} > 0 \) while still maintaining the generality to interpret the line width difference in the context of the average broadening of the two doublet lines which is necessary when incorporating other broadening mechanisms. To compare to table 2.7, \( \Delta \bar{w}_\parallel = 0.13 \), \( \Delta \bar{w}_\perp = 0.28 \), and \( \Delta \bar{w}_z = 0.66 \).

There are a number of optical transitions that are applicable to diagnose the magnetic field over a range of electron temperature, electron density, and magnetic field strength. Next, we explore applicability ranges for this diagnostic to determine upper bounds on electron density and magnetic field for a given multiplet. The first constraint on this diagnostic is the restriction of the Zeeman and Stark Hamiltonians to be perturbations to the fine structure Hamiltonian. In this limit, the line profile of the multiplet can be written as a convolution of a Zeeman-split pattern with the Stark, Doppler, and instrumental broadened profiles. In most cases, after the Zeeman perturbation, the Stark effect will be the dominating perturbation over thermal and instrumental effects. Additionally, the Griem semi-empirical Stark widths and shifts are loose functions of the electron temperature, making this diagnostic technique independently an electron density diagnostic. The Stark effect affects the line profile in two ways: first collisions from perturbing electrons shorten the lifetime of the transition which broadens the energy profile of the line and second, distant collisions cause a shift of the principal line centers. While in practice the total line profile is written as a sum of Voigt profiles resultant from the convolution of the Stark (Lorentz) and Doppler (Gauss)
Chapter 2. Diagnostics

and instrumental (Gauss) profiles, for the sake of producing simple limits on \( n_e \), we write the intensity profile of each doublet line as the sum of Lorentzians:

\[
    f(E) = \sum_i \frac{I_i \gamma(n_e, T_e)}{\pi (E - \Delta E_i - d(n_e, T_e))^2 + \gamma^2(n_e, T_e)}
\]  \hspace{1cm} (2.46)

Here \( I_i \) is the intensity of the \( i \)th Zeeman component, \( \gamma(n_e, T_e) \) is the Stark half-width half-maximum (HWHM) in the impact approximation, and \( d(n_e, T_e) \) is the Stark shift of the line center. Because \( d(n_e, T_e) \) is identical for each Zeeman component for a given fine structure transition, we omit it for the immediate treatment.

In order to determine limits on \( n_e \), we calculate the broadening of each doublet line when convolved with the Stark profiles. However, instead of using the traditional full-width at half-maximum (FWHM) to characterize the width of each doublet line, we opt for the full-width at half-area (FWHA) to avoid ambiguity in the case when the Stark broadening is much less than the Zeeman perturbation resulting in partial resolution of the Zeeman components. [12, 17] The FWHA is defined as the domain centered about the line center such that integral of the line profile is equal to half the total value, as illustrated in figure 2.2. The FWHA can be easily calculated by computing the normalized antiderivative function over the interval \((-\infty, x]\), evaluating at the points \( x = \frac{3}{4}, \frac{1}{4} \), and taking the difference.

\[
    \text{FWHA} = \Delta \lambda_{3/4} - \Delta \lambda_{1/4}
\]

\[
    A = \int_{-\infty}^{+\infty} I(\lambda) \, d\lambda
\]

\[
    \frac{1}{4}A = \int_{-\infty}^{\Delta \lambda_{1/4}} I(\lambda) \, d\lambda
\]

\[
    \frac{3}{4}A = \int_{-\infty}^{\Delta \lambda_{3/4}} I(\lambda) \, d\lambda
\]

**Figure 2.2:** Definition of full width half area (FWHA).
\[ F(x) := \frac{\int_{x}^{\infty} f(t) \, dt}{\int_{-\infty}^{x} f(t) \, dt} \]  

(2.47)

Inserting equation 2.46 (and dropping the Stark shift) into \( F(x) \) and invoking the integral of the Lorentzian function,

\[ F(E) = \sum_{i} \frac{1}{\pi} \tan^{-1} \left( \frac{E - \Delta E}{\gamma} \right) + \frac{1}{2} \]  

(2.48)

It is apparent that his summation cannot be solved analytically due to the inverse trigonometric functions. However, it can be readily solved numerically. The relative width differences calculated from the FWHA are computed over a grid of \( B \) and \( n_e \). Plotted are the average width differences as a function of relative line width difference of the two fine structure components for two orthogonal polarizations, inspired by the plots published in reference [12].

The second limit imposed by the Stark effect is the shifting of the doublet line centers. Similarly to the stark widths, they vary slightly for the two doublet components. The shifts decrease the energy spacing between the two line centers and combined with the broadening on the line, act to limit the resolution of the doublet lines. For the optical range, the typical shifts can be on the order of \( 0.5\mu_B B(0.8 \, \text{Å}) \), and it is most useful to evaluate shifts for a specified doublet. One can impose a limit on the maximum Stark shift for a given doublet by using the Raleigh criterion: in order for the two doublet components to be considered resolved, the peak-to-peak spacing must be larger than the Airy disk of the wider peak. The Airy disk of a line profile is defined as the distance from the line center to its first minimum. For our computations, we define the first minimum as the point at 10% of the maximum value.
Chapter 2. Diagnostics

Polarization: Transverse

(a) Average width as a function of relative width difference for observations transverse to $B$.

Polarization: Longitudinal

(b) Average width as a function of relative width difference for observations transverse to $B$.

Figure 2.3: Relative Zeeman width differences calculated from the FWHA over a range of magnetic field and electron density values for the Al III doublet transition.
(a) Average width as a function of relative width difference for observations transverse to $B$.

(b) Average width as a function of relative width difference for observations transverse to $B$.

Figure 2.4: Relative Zeeman width differences calculated from the FWHA over a range of magnetic field and electron density values for the Na I doublet transition.
2.1.2 Electron Density Measurements

2.1.2.1 Impact Approximation

Stark broadening of spectral lines is due to the presence of perturbing levels that interact with the upper and lower levels of the transition of interest. Collisions from perturbing electrons shorten the lifetime of the transition, causing a spread in energy states. The spectral distribution of the transition is such that the line shape, width, and shift are functions of the perturber (free electron) density. [16, 18] The theory for the broadening of spectral lines due to electron collisions is called the impact approximation and was developed by Baranger in 1958. [18–20] In the impact approximation, it is assumed that the various perturbers interact separately with the perturbed atom or ion and that only net changes in the perturbed system are significant. In 1962, Griem extended this theory using a semi-classical approximation for the Stark line widths of isolated neutral helium lines. [15] An isolated line is one for which the line width is much smaller than the energy separation to the nearest perturbing level. In Griem’s 1968 paper, Bethe’s dipole approximation was used to relate the inelastic cross section for collisional transitions to the product of the square of the radial matrix element and the Gaunt factor to calculate a semi-empirical Stark width and shift.

Implementation of these formulas requires a calculation of the Gaunt factor, which are not generally tabulated. Griem used the semi-empirical effective Gaunt factors proposed by Van Regemortor and Seaton. [21] The original semi-empirical formula for the Stark widths has been verified experimentally for neutral lines. It is accurate to within ±50% for singly ionized atoms, and to within ±100% for multiply ionized atoms. Experimental verifications to improve these accuracies are scarce, but improvements can be made by better approximations of the Gaunt factor.
2.1.3 Thermal Broadening

Doppler (thermal) broadening is due to the random motion of the radiator (ion) in random directions relative to line of sight of the observation. A particle moving toward or away from the observer with relative velocity \( v \) will cause the frequency of radiation emitted by the particle to be shifted up or down according to the Doppler effect,

\[
\nu = \nu_0 \left(1 + \frac{v}{c}\right)
\]

where \( \nu \) is new frequency as seen by the observer, \( \nu_0 \) is the rest frequency of radiation, and \( c \) is the speed to light. For a plasma composed of radiating ions of temperature \( T \) and mass \( m \), the velocity distribution in three dimensions will have a Maxwellian form if collisions are the dominant mechanism for particle interaction. Projected onto one axis (the line-of-sight axis) produces a Gaussian distribution

\[
f(v) \, dv = \frac{1}{\pi} \left(\frac{m}{2kT}\right)^{1/2} \exp \left[-\frac{mv^2}{2kT}\right] \, dv.
\]

with FWHM in velocity space,

\[
\Delta v_{1/2} = 2\sqrt{2 \ln 2} \, v_{th} \quad \text{where} \quad v_{th} = \sqrt{\frac{kT}{m}}
\]

is the thermal velocity of the ion along one axis. The velocity width can be transformed into frequency space via equation 2.49 since \( \Delta \nu = \nu_0 \Delta v/c \)

\[
\Delta \nu_{1/2} = 2\sqrt{2 \ln 2} \, \frac{\nu_0}{c} \, v_{th}.
\]

Similarly, the width in frequency units may be transformed to a width in wavelength units through the relation \( \Delta \nu/\nu = \Delta \lambda/\lambda \),

\[
\Delta \lambda_{1/2} = \frac{\lambda_0}{c} \, 2\sqrt{2 \ln 2} \, v_{th}.
\]
2.2 Optical Imaging

Self-emission from the plasma can be imaged to infer qualitative behavior of the luminosity within the spectral range of the detector which gives a spatial map that is a function of both electron density and temperature. The densest regions of plasma will produce the most intense radiation simply because more radiators are producing the signal. If the plasma is in thermal equilibrium, it may be approximated as a blackbody emitter. As temperature increases, the intensity of blackbody radiation increases for all frequencies. This implies that brighter emission corresponds to higher temperatures. However, if the plasma is not in thermal equilibrium, as is likely the case for laboratory plasmas, higher temperature plasmas may emit discretely over all frequencies. It may be the case that non-thermally equilibrated plasmas may emit primarily in the x-ray or UV range, resulting in minimal counts recorded by an optical detector. Assuming a blackbody radiator would lead to the incorrect conclusion that a plasma emitting less visible light is at a low temperature. Attempting to diagnose a plasma based solely on optical emission leads to ambiguous results. For this reason, this diagnostic does not provide quantitative information about the plasma radiation.

There are two limits on the usefulness of this diagnostic. The first occurs when the plasma becomes so hot that its radiated emission shifts from the visible range to lower wavelengths. Not only are the lenses not equipped to handle this light, but the quantum efficiency of the CCD detectors fall off steeply beyond the optical range. The second limit occurs at the other extreme in which the plasma is below a certain density and temperature threshold such that its emission falls below the sensitivity of the detector. Both of these effects factor into the recorded intensity maps and must be interpreted correctly. A more quantitative assessment of the plasma emission can be made by using narrow bandpass filters.
2.3 Inductive Probes

Three differential inductive ($\hat{B}$) probes measure the current delivered to the load. They yield a time series of the time rate of change of the magnetic flux from the load, measured from the anode plate about 14 cm away from the load region. Each $\hat{B}$ probe is constructed with two coils wound opposite each other. From Faraday’s law of induction, the varying magnetic flux from the load current induces a voltage in the loop that is proportional to $\frac{dB}{dt}$ that is read out to an oscilloscope. The oppositely wound coils provide a way of reducing the noise under the assumption that the coils are within close enough proximity to each other so that the electrical noise will be equal but opposite in magnitude. By averaging the two signals, integrating with respect to time, and applying the appropriate scaling factors, a current trace is obtained. The scaling factors are determined independently on a smaller pulser.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.5.png}
\caption{The voltage induced in $\hat{B}$ coils is proportional to the number of turns, the area of the loop, and the flux through the cross sectional area of the coil.}
\end{figure}
Table 2.7: Average broadenings of \(^2S \rightarrow ^2P\) doublet components in presence of a magnetic field. Note that \(\langle \Delta E^\parallel \rangle = \langle \Delta E^\sigma \rangle\) since only the \(\sigma\) components are emitted in parallel observation with respect to \(\vec{B}\).
Chapter 3

Experimental Apparatus

3.1 Zebra Pulsed Power Generator

The Nevada Terawatt Facility houses the two terawatt pulsed power generator Zebra, figure 3.1, which is used both as the z-pinch driver in wire array experiments and as the source of the external magnetic field in laser experiments. Zebra operates in two primary modes: the short pulse mode which delivers a peak current of approximately 1 MA in 100 nsec and the long pulse mode which delivers a peak current of approximately 0.6 MA in 200 nsec. The process begins by charging thirty-two 1.3 µF capacitors in parallel to 85 kV and discharging them in series. This configuration of capacitors is called the Marx capacitor bank. This results in a µs pulse of electrical energy that continues on to the second stage of power amplification, a coaxial 28 nF, 3.5 MV capacitor (intermediate storage). The intermediate storage is submerged in water which is a dielectric with high relative permittivity so that it acts as an insulator for the duration of the current pulse. An SF₆ gas rimfire switch can be set to allow self-breakdown or triggered breakdown which discharges the capacitor into the pulse-forming transmission line. At this stage, the µs pulse is compressed into the 200 ns long pulse, and if this is the
desired experimental configuration, this pulse bypasses the final stage of amplification and is delivered into the load region inside the vacuum chamber. If further pulse compression is required to operate in the 1 MA mode, the current continues through the vertical transmission line through the water spark-gaps, which halve the rise time and double the peak current. These water gaps are manually closed in order to bypass this amplification process.

3.2 Leopard Laser

The Leopard laser is a 200 TW, hybrid Ti:Sapphire/Nd:glass laser. The laser can be operated in short pulse mode delivering 10 J in 300 fsec, or in the long pulse mode delivering 25 J in 1 nsec. For the experiments conducted for the current study, the long pulse laser was utilized in conjunction with the Zebra generator. The facility at the NTF provides a unique platform in that the high intensity Leopard laser may be coupled with the pulse generator to create strong external magnetic fields. The full system allows for many degrees of freedom when investigating the parameter space of a particular experiment since the laser,
In the far left are the laser amplification and compression room. From there, the beam travels through an evacuated waveguide towards the Zebra vacuum experimental chamber where it is steered by mirrors and focused onto the load area. The beam remains in vacuum until approximately the final 4 meters before the target.

current pulse, and timing with respect to the two may be adjusted to the principal investigator’s discretion.

When coupling Zebra and Leopard, the relative timing of Zebra and the diagnostics are fixed with respect to the laser pulse with up to an accuracy of 5 ns. One second after the Marx bank is charged, a signal is sent via fiber optic to the Leopard amplifiers. Once the laser is primed to fire, a signal arrives via a second fiber optic to begin the discharge of the intermediate storage to fire Zebra and the other diagnostics. The timing of the discharge of Zebra can be adjusted so that the laser arrives at the target at various times on the current pulse. Careful timing calculations must be made to ensure that the time-sensitive diagnostics are always acquiring data during the desired laser-target interaction. In this way, it is reasonably straightforward to do time series with approximately 10 ns increments with reproducibility.
Chapter 4

Experimental Study 1: Laser Plasmas in an External Magnetic Field

4.1 Introduction

4.1.1 Experimental Set-Up

We operate Zebra in the long pulse mode (0.6 MA in \( \sim 200 \) ns) to create an azimuthal magnetic field by pulsing current through a cylindrical electrode with a diameter of 6 mm in the experimental region. This produces for a peak magnetic field of 40 T at the electrode surface. The 6 mm portion of the rod is whittled down from a larger diameter rod with a diameter of 20 mm so that the load may be shot tens of times before needing to be replaced.

To create the plasma, the Leopard laser in long pulse mode (\( \sim 20 \) J in 1 ns) enters the vacuum chamber through the south port (\(-\hat{x}\) in figure) and is focused onto the electrode surface. In this way, the plasma is initially magnetized as the plume
Figure 4.1: Schematic of the experimental chamber with lines of sight of the the optical diagnostics used.

Figure 4.2: Close up diagram of the geometry of the plasma, external magnetic field $B$. The rectangle depicts the projection of the plasma imaged onto the slit of the spectrometer.

is ablated from the target material. The timing between the current maximum and laser arrival can be chosen with $5 - 10$ ns precision before each shot to study a variety of plasma parameters.

Figures 4.1 and 4.2 set up the system of coordinates used to describe the experimental chamber. The $x - y$, or equivalently the $r - \varphi$, plane is defined as the plane of the azimuthal magnetic field. The line of sight of the spectroscopic diagnostic is along the $-\hat{z}$ direction and the projection onto the spectrometer slit can be oriented to measure $B$ either along radial lines or lines parallel to the $y$-axis (or parallel to the surface tangent to the electrode). The laser diagnostics have a field of view in the $x - z$ plane. Diagnostics implemented for these experiments include spectroscopy (Zeeman broadening), optical imaging, and pulsed laser probing (shadowgraphy, Faraday rotation, interferometry). The plasmas created remained cool enough, so they did not produce noticeable x-rays.
Figure 4.3: Optical system used to image the plasma onto the slit of a high-resolution imaging spectrometer and streak camera. Two 4\(f\) (where \(f\) is the focal length) relay systems were used to transport the image of the plume across 5.6 m. A fifth lens (\(f = 50\) cm) is used to image and demagnify the intermediate image at Image Plane 2 onto the slits of the imaging spectrometer and the streak camera.

Optical spectroscopy was performed by imaging the plasma onto the slit of a high-resolution spectrometer coupled to an iCCD camera. The optical system consisted of two lens relay systems and a final imaging lens that was \(f/#\)-matched to the entrance cone of the spectrometer, depicted in figure 4.3. Spatially resolved data were collected with three different imaging spectrometers: a 1-m Princeton Instruments Acton spectrometer\(^1\) providing 0.45 Å instrumental resolution, a 0.75-m Princeton Instruments Acton spectrometer\(^2\) providing 0.48 Å instrumental resolution, and a 0.5-m SPEX spectrometer providing 1.175 Å instrumental resolution.

To record the spectra, we coupled the spectrometer output to an Andor iStar ICCD to obtain a minimum 10 ns temporal gate or to a streak camera\(^2\) to obtain temporal resolution over a 400 ns window. When spectrograms are taken with the ICCD, this setup allows magnetic field measurements with spatial resolution to be made along the radius of the magnetic field. When the spectrometer was

\(^{1}\)on loan from Lawrence Livermore National Laboratory

\(^{2}\)on loan from National Security Technologies, LLC.
coupled to the streak camera, we can make magnetic field measurements of only a single region of the plasma, whose dimensions are limited by the slit size of the spectrometer and the slit size of the streak camera (200 µm). Typically, the spectrometer was operated with a slit size of $d_{\text{slit}} = 50$ µm.

### 4.1.2 Plasma Evolution in an External Magnetic Field

Without an external magnetic field present, a laser plume in vacuum expands hemispherically from the target surface. With a strong external field added, the plume dynamics undergo distinct phases as the plasma ions and electrons respond to the field, as shown in references [23], [24], [25]. The coarsest generalization of the plume’s evolution in the magnetic field can be subdivided into four stages: 1) the expansion as a diamagnetic cavity, 2) the deceleration and convergence of the plume onto its center axis, 3) the collimation and penetration of the plume across the magnetic field, and 4) the destabilization and turbulence of the collimated flow.

These stages of laser plasma evolution in an external magnetic field were experimentally investigated in [1]. The following summary and timescales are observations taken from [1] for a Carbon laser plasma expanding against a quasi-uniform external magnetic field. Immediately after the laser is incident on the target, the hot plasma begins a hemispherical expansion similar to the $B = 0$ case for approximately the first 5 ns. Shadowgraphy observations show that the plasma ram pressure is set up such that the plasma pressure along $\hat{r}$ is greater than the plasma pressure along $\hat{z}$, causing a preferential expansion along $\hat{r}$ and confinement along $\hat{z}$. Because the plasma is extremely conductive in this phase, the magnetic field is excluded from the bulk of the plume, and the plume expands as a diamagnetic cavity. This cavity disintegrates quickly as the plasma temperature drops and

\footnotesize{on loan from National Security Technologies, LLC.}
the plasma rarefies. This initiates the next phase, as magnetic diffusion becomes important within the plasma and the magnetic pressure begins to overwhelm the plasma pressure towards the tail end of the plume. At $t \approx 10$ ns, the plasma begins to get decelerated by the external field and a discontinuity region forms, as shown in reference [25]. As the plasma is decelerated by the magnetic field, the interactions at the plasma front have been shown to initiate flutelike instabilities (for example, via the magneto-Raleigh Taylor instability) in the $r - z$ plane.

**Figure 4.4:** Stages of plume evolution in the presence of an external magnetic field. This schematic is motivated by experimental evidence of this type of behavior which was documented in Reference [1].
The presence of these instabilities combined with the confinement of the plasma in the axial direction begins a redirection of the plume onto its center axis. The final phase, the formation of a directed flow across the magnetic field begins approximately at $t \approx 15$ ns. The mechanism for the magnetized flow to penetrate the magnetic field is the $\mathbf{E} \times \mathbf{B}$ drift observed by references [26] and [27]. In a magnetized plasma of narrow enough dimensions, the ion gyroradii may be larger than the size of the plasma so that they gyrate out of the bulk of the plasma. The magnetization of the electrons is much higher than that of the counterpart ions, so their trajectories prohibit an immediate neutralization of the positive charge created by the ions. This sets up a polarization field along $\hat{z}$, facilitating the flow of the plasma bulk along $\hat{r}$.

As the plasma flow becomes collimated across many mm, instabilities start to develop that cause structuring to develop. It has been shown in experiment and 3D MHD modeling in reference [28] that Kelvin-Helmholtz vortices begin to develop along the target normal in laser experiments very similar to those studied in the present context. These vortices develop very early in time, but they may develop into the large-scale, periodic density perturbations observed.
4.1.3 Evolution of Flutelike Instabilities

The dynamic way in which the plasma electrons and ions interact with the external magnetic field is responsible for varying degrees of magnetic field penetration within the plume. One important effect actuated by this interaction is charge separation within the plume. Various mechanisms can take place to produce charge separation, and it is possible that a combination of effects is observed. There are two consequences of maintaining charge separation along the axial boundaries of the plume: firstly, the local electric fields created perpendicular to $\mathbf{B}$ allow the plasma to penetrate the field through the $\mathbf{E} \times \mathbf{B}$ force, and second, the propulsion of the plasma across the field lines can in turn cause instabilities to become seeded or amplified if already present.

When a force acts perpendicular to a magnetic field, a charge-dependent drift velocity arises that causes particle motion mutually perpendicular to both the external force and the external magnetic field. The drift velocity for a force $\mathbf{F}$ perpendicular to $\mathbf{B}$ acting on a particle with charge $q$ is given by

$$V_f = \frac{1}{q} \frac{\mathbf{F} \times \mathbf{B}}{B^2}.$$  \hfill (4.1)

Figure 4.6 shows schematically how a radial force perpendicular to $\mathbf{B}$ in our experimental geometry causes ions and electrons to drift along $\pm \hat{z}$. This force can be external or due to interactions between the plasma and the magnetic field. If the plasma front is initially perturbed as it expands against the magnetic field, then these perturbations can grow into flutelike instabilities that allow the plasma to penetrate the magnetic field for large distances as an elongated structure. This phenomenon was first experimentally confirmed in Plechaty (2013). In general, the form of the radial force instigates different fluid or kinetic flutelike instabilities.
within the plasma. We will consider three cases for the radial force: a deceleration (which induces the fluid magnetic Raleigh-Taylor instability or MRTI), an electric field (which induces Large Larmor Radius Raleigh-Taylor Instability or LLRI), and a pressure gradient (responsible for the Lower Hybrid Drift Instability or LHDI).

When the plasma expands against the magnetic field, it experiences a deceleration due to the field. In this case, the drift velocity is called the gravitational drift velocity, due to the dependence of this acceleration on the particle mass. This drift velocity depends on the mass and charge of the species in the plasma,

$$v_g = \frac{m}{qB^2}g \times B.$$  \hspace{1cm} (4.2)

This interaction is conducive to perpetuating the fluid magnetic Rayleigh-Taylor instability (MRTI).

In plasmas dominated by the Hall term or so-called Hall plasmas, the ion gyroradius tends to be larger than the plasma length scale, implying that the ions are weakly unmagnetized (see Appendix C). If the electrons remain magnetized
so that they complete many gyrations about the field lines before experiencing a collision with an ion, then near the plasma boundaries local electric fields may form in the following manner. The ions can gyrate out of the bulk of the plasma, and since the electrons remain magnetized, they do not immediately follow the same trajectory of the ions.

If the ions gyrate out ahead of the plasma front, as depicted in figure 4.7, then the negative charge left behind produces an inward radial electric field. Since the electric force is proportional to the charge of the particle, both ions and electrons drift in the same direction, due to the $E \times B$ drift velocity. However, because the ions are much more massive than the electrons, they move slower, so there is an instantaneous charge separation produced along $\hat{z}$. This secondary charge separation then produces another electric field oriented along $\hat{z}$. This creates another $E \times B$ drift along $+\hat{r}$.

If the ions gyrate out of the $\pm \hat{z}$ boundaries, the same effect is achieved but without the need to produce an electric field that makes way for the charge separation (see figure 4.5). At the axial boundaries, the ions gyrating out of the plasma bulk creates a net charge separation along $\hat{z}$, so that the plasma may propagate along $+\hat{r}$ again through the $E \times B$ drift.

The Lower-Hybrid Drift Instability may also be driven in Hall plasmas, but the particle drift velocity is produced by gradients in pressure or magnetic fields. In our experimental set-up, the expanding plume experiences radial pressure gradients because it is expanding into a magnetic field with a radial gradient. Where the magnetic field is strongest, the plasma becomes compressed by the field. Pressure gradients $\nabla P$ formed by imbalances between the external magnetic pressure and the bulk plasma ram pressure cause the particles within the plasma to drift with
Mechanism that allows the growth of flute-like structures that penetrate the magnetic field with plasmas that have large Larmor radius ions. First, the ions gyrate out of the plume in the radial direction, setting up an electric field $E_1$. This electric field causes a secondary charge separation along $\hat{z}$ as a result of the ion inertia in response to the $E_1 \times B$ drift velocity. Finally, the lateral polarization electric field $E_p$ allows growth and penetration of the plume and any instabilities on the plume front to cross the magnetic field.

Figure adapted from Reference [2]

The diamagnetic drift velocity given by

$$v_{di} = -\frac{\nabla P \times B}{qnB^2}$$

(4.3)

where $q$ is the particle charge, and $n$ is the particle number density.

Figure 4.4 shows how the plasma ram pressure (which in the absence of shear forces is proportional to $v_\perp$) varies as a function of distance from the electrode due to the confinement of the external magnetic field. Non-uniformities along the plasma radius can magnify the radial pressure gradient felt by the plasma, further inducing charge separation along the plume’s axial boundaries. As depicted in figure 4.6, once the axial charge separation has been established, the electric field that is oriented along $\hat{z}$ allows the plasma to expand along $+\hat{r}$.

4.1.4 MHD Quantities

The effects of resistive and Hall diffusion within the plasma will be discussed using the framework established in Appendix C. A summary of relevant derived plasma quantities is presented.
From the generalized Ohm’s law we use the dimensionless quantities that quantify the relative importance of resistive diffusion or Hall diffusion over advection of the magnetic field,

\[
E \approx |\mathbf{v} \times \mathbf{B}| \left( \hat{e}_{\text{convection}} + \frac{1}{R_M} \hat{e}_{\text{diffusion}} + \frac{A_c}{L_B \omega_{pi}} \hat{e}_{\text{Hall}} + \frac{v_{e,\text{th}} \rho_{e,\text{th}}}{v_A L_n} \hat{e}_{\text{diagnmagnetic}} \right) \quad (4.4)
\]

where \( E \) is the electric field, \( \mathbf{v} \) is the plasma bulk velocity, \( R_M \) is the magnetic Reynolds number, \( A \) is the Alfvén velocity, \( \omega_{pi} \) is the ion plasma frequency, \( v_{e,\text{th}} \) is the electron thermal velocity, \( \rho_{e,\text{th}} \) is the electron thermal gyroradius, \( v_A \) is the Alfvén velocity, and \( L \) is the length scale of either \( B \) or \( n_e \) according to the subscript.

The diffusive coefficient, \( D = 1/R_M \) encapsulates the importance of resistive diffusion relative to the conduction term of the generalized Ohm’s law. In terms of the Spitzer resistivity \( \eta \), the plasma bulk velocity \( v_0 \), and the scale length of the magnetic field \( L_B \),

\[
D \equiv \frac{1}{R_M} = \frac{\eta}{v_0 L_B}. \quad (4.5)
\]

The Hall coefficient (equation C.13) gives the relative importance of the Hall term of the generalized Ohm’s law with respect to the conduction term. In terms of plasma parameters, the Hall coefficient is given by

\[
H = \frac{A_c}{L_B \omega_{pi}} \propto \frac{B}{n_e v_0 L_n}, \quad (4.6)
\]

where \( A = v_A/v_0 \) is the Alfvén number, \( \omega_{pi} \) is the plasma ion frequency, and \( L_n \) is the scale length of the electron density profile. When pressure equilibrium exists between the plasma and external magnetic field, the length scales have the relation \( L_B = 2L_n \).
Quantity & Formula

Coulomb logarithm, $\lambda$ & $23 - \log (\sqrt{n_e \cdot Z \cdot T_e^{-3/2}})$ (4.8a)

Electron collision rate, $\nu_e$ [s$^{-1}$] & $2.91 \times 10^{-6} n_e [\text{cm}^{-3}] \lambda T_e^{-3/2}$ (4.8b)

Electron collision time, $\tau_e$ [sec] & $\frac{1}{\nu_e}$ (4.8c)

Spitzer conductivity, $\sigma_0$ [s$^{-1}$] & $\frac{n_e e^2 [\text{statcoul}] \tau_e}{m_e [\text{g}]}$ (4.8d)

Magnetic Diffusivity, $\eta_{cgs}$ [cm$^2$ s$^{-1}$] & $\frac{1}{\mu_0 \sigma_0}$ (4.8e)

Magnetic Diffusivity, $\eta_{SI}$ [m$^2$ s$^{-1}$] & $\frac{\eta_{cgs}}{4\pi \varepsilon_0}$ (4.8f)

Alfven velocity, $V_A$ [cm s$^{-1}$] & $\frac{2.18 \times 10^{11} \cdot B [\text{Gauss}]}{\sqrt{n_i \cdot \mu}}$ (4.8g)

Alfven Number, $A$ [None] & $\frac{V_A}{v_0}$ (4.8h)

Ion Plasma Frequency, $\omega_{pi}$ [s$^{-1}$] & $1.32 \times 10^3 \cdot Z \cdot \sqrt{n_i/\mu}$ (4.8i)

Hall Coefficient, $H$ [None] & $\frac{A \epsilon}{l_0 \cdot \omega_{pi}}$ (4.8j)

Magnetic Reynolds Number, $R_M$ [None] & $\frac{v_0 l_0}{\eta_{SI}}$ (4.8k)

Diffusive Coefficient, $D$ [None] & $\frac{1}{R_M}$ (4.8l)

Hall Velocity, $V_H$ [m s$^{-1}$] & $\frac{B [\text{T}]}{2\mu_0 \cdot n_e [\text{m}^{-3}] \epsilon l_0 [\text{m}]}$ (4.8m)

Diffusive velocity, $V_D$ [m s$^{-1}$] & $\frac{\eta_{SI}}{l_0}$ (4.8n)

Hall penetration time, $\tau_H$ [ns] & $1 \times 10^9 \cdot l_0 / V_H$ (4.8o)

Diffusive penetration time, $\tau_D$ [ns] & $1 \times 10^9 \cdot l_0 / V_D$ (4.8p)

---

**Table 4.1:** Summary of plasma parameters. Formulas from NRL formulary, in cgs units, unless otherwise stated.

For the presented calculations, the length scale of a given quantity $A$ (along a given dimension $x$) is calculated by the formula

$$L_A = \left( \frac{d \ln A}{dx} \right)^{-1} = \frac{A}{\nabla A}.$$ (4.7)
4.2 Diagnostic Techniques

4.2.1 Spectroscopic Measurements

Two doublets were used to diagnose the magnetic field in plasma: the Al III 4s-4p doublet (569.6 & 572.2 nm) and the C IV 3s-3p doublet (580.1 & 581.1 nm). The former provides viable magnetic field information for for $B \leq 40$ T, $n_e \leq 1 \times 10^{18}$ cm$^{-3}$, and $T_e \leq 28$ eV and was used to make measurements late in the plume evolution ($t > 50$ ns). The latter provides viable magnetic field measurements for $B \leq 14$ T, $n_e \leq 2 \times 10^{18}$ cm$^{-3}$ and $T_e \leq 65$ eV and was used to make measurements in the mid-stage of plasma evolution ($30$ ns $< t < 50$ nsec). [12]

Each spectral component of the Zeeman splitting pattern is modeled as a convolution of the instrumental (approximated as a Gaussian profile), thermal Doppler (Gaussian profile), and Stark (Lorentzian profile). The convolution of three profiles for each of the 10 Zeeman components is computationally cumbersome, so instead a pseudo-Voigt profile is computed. An empirical formula for the width of a Voigt profile $w_V$ that is accurate to within 0.02% is given by [29]

$$w_V \approx 0.5346w_L + \sqrt{0.2166w_L^2 + w_G^2}$$

(4.9)

where $w_L$ is the width of the Lorentzian Stark profile, and $w_G$ is the convolution of the Doppler ($w_D$) and instrumental ($w_I$) Gaussian profiles given by [29]

$$w_G = \sqrt{w_D^2 + w_I^2}.$$  

(4.10)

Without an independent measurement of either $n_e$ or $T_e$, there exists a continuous range of pairs $(n_e, T_e)$ that will satisfy a given Voigt width for a given instrumental broadening. Coarse upper bounds on the electron temperature can be defined by the ionization potential for the ion responsible for the transition, $T_e^i$. Closer bounds
on the electron temperature can be placed if valid assumptions about the radiative
regime the plasma is in. By defining a $T_{e}^{\text{min, max}}$ such that the population of the
ion of interest remains above a certain threshold, the sensitivity of the electron
density to a fitted Voigt width can be analyzed over the range $T_{e}^{\text{min}} < T_{e} < T_{e}^{\text{max}}$.

4.2.1.1 Optical Depth

The plasmas observed for these measurements are all optically thin, as evidenced
by the fact that the line intensity ratio of the $2P - ^{2}S$ doublet is 2:1 for all
recorded data. Modeling done on PrismSPECT for Aluminum plasma parameters
that produce the transition of interest shows that the optical depth, $\tau < 0.15$
over a range of densities and temperatures typical to our experiments through a
plasma of width 0.2 mm. This plasma size is measured by interferometry images.
In the context of our system of coordinates, it is the size of the plasma in the $z-$
dimension, which is along the line of sight of the spectroscopic diagnostic.

4.2.1.2 LTE assumptions

Radiative properties of plasma are governed by the distribution of electrons among
ionic energy levels and the interactions between the ions and electrons. In general,
a full description of the plasma is made by a collisional radiative (CR) model,
in which all ionization, recombination, and decay rates are accounted for. These
rates describe level populations and characterize the species charge state distribu-
tion within the plasma, and consequently control the spectral emission features.
Solving a CR model requires solving a large number of equations, so it is advan-
tageous to be able to describe the system using only a few parameters under a
limiting case. Local thermodynamic equilibrium (LTE) is established in a plasma
system when collisions between electrons and ions are small compared to char-
acteristic times in the plasma so that an equilibrium is set between the electrons
and heavy particles. Because collisions are important, LTE is typically achieved at high electron densities and when temporal and spatial gradients within the plasma are negligible (the plasma is homogenous and stationary). [30] When a plasma system is in the LTE regime, locally it can be described by a single temperature, namely $T$, through the assumption that collisions have equilibrated ion and electron temperatures ($T_e \sim T_i$) and is described by its free electron density $n_e$ due to the assumption of localized gradients that allow equilibration. LTE is a valid description of a system when the Saha ionization equation adequately models the ion species population, and the plasma species follow a Maxwell distribution. A necessary but not sufficient criterion for validating the assumption of LTE within a system is the McWhirter criterion [31], which provides a minimum density above which LTE conditions may exist:

$$n_e \ [\text{cm}^{-3}] \geq 1.6 \times 10^{12} T_e^{1/2} \ [\text{K}] (\Delta E \ [\text{eV}])^3$$ (4.11)

where $\Delta E$ is the largest energy gap in the level scheme being considered. The primary reason the McWhirter criteria is not a strong enough condition to determine LTE on its own is that it was derived for time-independent and homogenous plasmas. Therefore, any radiative properties derived based on this assumption must be carefully evaluated.

A second limiting case of the CR model is the Corona model, which leads to Corona equilibrium (CE). CE is achieved at low densities when the collisional deexcitation rate is much smaller than the spontaneous decay rate. [32] In this model, the population of the ground state dominates of population of the excited states. Equilibrium is achieved between the rate of collisional excitation from the ground state with the rate of spontaneous radiative decay. The criterion for CE
to be valid is

\[ n_e [\text{cm}^{-3}] < 5.9 \times 10^{10} Z_N^6 \sqrt{T_e [\text{eV}]} \exp \left( \frac{0.1 Z_N^2}{T_e [\text{eV}]} \right) \] \tag{4.12} \]

where \( Z_N \) is the nuclear charge of the ion.

When neither of these conditions applies, or there is not enough evidence to support the CE or LTE assumptions, the plasma system is said to be in non-local thermodynamic equilibrium (NLTE) and can only be described by solving the full set of rate equations within the CR model. It has been shown that at low and high densities, the CR model converges to the CE and LTE results, respectively.

Without an atomic code to calculate the optical properties of our plasmas with the full CR model, we have resorted to assuming LTE. This assumption manifests namely in the assumption of \( T_i \sim T_e \) for the calculation of Doppler widths and in the estimation of temperature bounds from the Saha equation. We now assess the limits of this assumption and the sensitivity of the presented results to deviations from LTE.

Reference [32] publishes an extensive parameter-space characterization of CE, LTE, and NLTE regimes for Carbon plasmas. In this paper, average ionization as a function of electron density is calculated, as well as level population comparisons to classify a set of \( T_e \) and \( n_e \) in one of the three equilibrium regimes. Our Carbon plasmas had densities around \( 10^{17} \text{cm}^{-3} \) and the ion used for magnetic field measurements is C IV (\( Z=3 \)). From [32], at this density the average ionization, \( \bar{Z} \approx 4 \) for \( 10 \text{ eV} < T_e < 50 \text{ eV} \); \( \bar{Z} \approx 3 \) for \( 5 \text{ eV} < T_e < 10 \text{ eV} \). While this quantity does not give populations of individual species, it allows an inference that the highest population of C IV occurs around \( T_e \approx 10 \text{ eV} \) at this density.

From the Saha ionization calculations done on the NIST spectral database website [33], at \( n_e = 5 \times 10^{17} \), C IV consists of at least 1% of the total ion species for
electron temperatures $3 \, \text{eV} < T_e < 10 \, \text{eV}$. Table 4.2 shows the upper boundary for the Corona model and the lower boundary for the LTE condition for a few temperatures. The electron density boundaries and the calculations done in the reference suggest that our Carbon plasmas are in the NLTE regime. We fit all Carbon spectra with $T_e = 10 \, \text{eV}$ with the caveat that if the plasma is actually closer to LTE, the temperatures may be shifted lower. Figure 4.8 shows the sensitivity of a given Voigt width to variations in $T_e$ and $n_e$.

For example from the C IV chart, an average Voigt width of 1 Å can be gotten by the combinations ($T_e = 20 \, \text{eV}, n_e = 3 \times 10^{17} \, \text{cm}^{-3}$) or ($T_e = 40 \, \text{eV}, n_e = 2.5 \times 10^{17} \, \text{cm}^{-3}$). A difference of a factor of 2 in electron temperature results in a change in electron density by $5 \times 10^{16} \, \text{cm}^{-3}$, which is comparable to the uncertainty of the fitting procedure, about 20%. For this transition the Voigt widths are relatively insensitive to changes in electron temperature. This procedure also does not account for the errors intrinsic in the semi-empirical formula used to calculate the Stark widths, which are accurate to about 20%.

<table>
<thead>
<tr>
<th>$T_e$ [eV]</th>
<th>CE upper bound, $n_e$ [cm$^{-3}$]</th>
<th>LTE lower bound, $n_e$ [cm$^{-3}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$1.58 \times 10^{16}$</td>
<td>$6.73 \times 10^{19}$</td>
</tr>
<tr>
<td>7</td>
<td>$1.21 \times 10^{16}$</td>
<td>$1.02 \times 10^{20}$</td>
</tr>
<tr>
<td>10</td>
<td>$1.24 \times 10^{16}$</td>
<td>$1.22 \times 10^{20}$</td>
</tr>
</tbody>
</table>

**Table 4.2:** Evaluation of Corona equilibrium and McWhirter LTE criteria for Carbon ($Z_n = 6$), taking the largest energy gap from the level scheme as $1s^22s \rightarrow 1s^29p \approx 60 \, \text{eV}$.

The Aluminum plasma parameters suggest a closer fit to LTE assumptions, likely because these measurements are made 100s of ns after the laser pulse. From Table 4.3, for $n_e \approx 10^{18} \, \text{cm}^{-3}$, there is overlap between the CE and LTE model bounds on density. This overlap illustrates the loose nature of these bounds. Since LTE is not immediately ruled out given these plasma parameters, we continue to assume LTE for the late-time Aluminum laser plasmas and examine the sensitivity of this assumption. Under LTE, for $n_e = 10^{18} \, \text{cm}^{-3}$, the population of Al III
remains above 1% for $1.6 \text{ eV} < T_e < 6 \text{ eV}$. Without loss of generality, all fits to Aluminum data are done assuming $T_e = 3 \text{ eV}$. From figure 4.9, the sensitivity of the relationship between $T_e$ and $n_e$ can be charted.

**Table 4.3:** Evaluation of Corona equilibrium and McWhirter LTE criteria for Aluminum ($Z_n = 13$), taking the largest energy gap from the level scheme as $1s^22s^22p^63s \rightarrow 1s^22s^22p^87p \approx 25 \text{ eV}$.

<table>
<thead>
<tr>
<th>$T$ [eV]</th>
<th>CE upper bound, $n_e$ [cm$^{-3}$]</th>
<th>LTE lower bound, $n_e$ [cm$^{-3}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$1.38 \times 10^{20}$</td>
<td>$4.87 \times 10^{18}$</td>
</tr>
<tr>
<td>7</td>
<td>$8.42 \times 10^{18}$</td>
<td>$7.44 \times 10^{18}$</td>
</tr>
<tr>
<td>10</td>
<td>$4.88 \times 10^{18}$</td>
<td>$8.89 \times 10^{18}$</td>
</tr>
</tbody>
</table>

4.2.2 Optical Streak

The optical path that imaged the plasma onto the entrance slit of the spectrometer was also split via beamsplitter and directed onto an optical streak camera with a
480 ns window. In this way, the same spatial region that is spectrally analyzed is also optically streaked, equipment permitting (see figure 4.2). In certain shots, bandpass filters with FWHM = 40 nm centered about $\lambda = 580$ nm were used to filter the streaked emission. In the streaked emission images shown, the vertical axis is time, and the horizontal axis is the $x-$ axis from figure 4.2.

### 4.3 Experimental Observations

All spectroscopic measurements were made in the relatively late stages of plume evolution (30 ns < $t$ < 220 ns), when the plume has become collimated and continues to penetrate $B\hat{\varphi}$ along $\hat{r}$ as it eventually becomes unstable. The optical path sent into the spectrometer was also split and diverted into a streak camera.
with 400 ns window. From the streaked emission, plasma luminosity front velocity as a function of time over the plume lifetime is obtained.

All measurements made for 30 ns < t < 220 ns show that indeed the plasma remains collimated along +\( \hat{y} \) when there is an external magnetic field present. Figure 4.10 shows a comparison of two plasmas taken at 130 ns < t < 140 ns. In shot 4053 there is a magnetic field present, but in 4088 there is almost no magnetic field. The plasma with the external field is almost twice as large in the radial dimension than the plasma with no field. This indicates that the magnetic field confines the plasma to a large density within a larger localized region to be bright enough for the camera to detect. Spectroscopic measurements of radial magnetic field profiles within the plume will examine how the azimuthal magnetic field penetrates the plume during the collimation phase. The stability and dynamics are inferred from the magnetic field and density profiles.
Chapter 4. Laser Plasma Experiments

Figure 4.11: Examples of streaked optical emission of laser plasmas with various time settings between the ablation laser and maximum current peak of Zebra, taken along the same line of sight as the spectroscopy diagnostic. The vertical axis is the temporal axis and the horizontal axis is spatial resolution along the radius of $B_0$, in the plane of $B_0 \hat{z}$. The target materials and specific timing settings are noted in each subfigure, the streak window is approximately 480 ns.
4.3.1 Streaked Emission

Some key features of the plume response to the external magnetic field are inferred from the streaked optical emission images. As one would expect, the timescales on which these dynamics occur vary depending on the mass of the ion being ejected from the target surface. We observed both the streaked radial expansion of both Carbon ($m_C \approx 12$ u) and Aluminum ($m_{Al} \approx 26$ u) plasmas. Both plasmas exhibit similar trends, but the Carbon plasma evolution occurs at nearly twice the rate of the Aluminum plasmas. The presence of the magnetic field heavily affected the emission of the Carbon plasmas. Our data for streaked Carbon emission falls under two categories: shots in which the current peak occurs roughly 100 ns after the arrival of the laser pulse (labeled Case 1) and shots in which the current peak occurs roughly 250 ns after the arrival of the laser pulse (labeled Case 2). For Case 1, the front of the plasma emission expands at a rate of about $10^4$ m s$^{-1}$ for about 50 ns, coinciding roughly with the maximum current (see figure 4.12). Just as peak current begins the emission drops, suggesting that the plasma has become too hot to emit in the visible spectrum. As the current drops, recombination within the plasma occurs, and another bright emission is observed that nearly uniformly lights up several mm across r. For Case 2 wherein the peak current occurs about 250 ns after the laser pulse, the streaked emission was filtered by bandpass filters centered about 600 nm with a width of 40 nm. In this range, the initial plume expansion does not emit, and it is not until the start of the 200 ns current pulse that the plume begins to radiate uniformly.

Streaked data for the Aluminum plumes also exhibit behavior highly dependent on the timing of the laser pulse with respect to the current pulse. Again we observe a dichotomy form in the data. They will be referred to similarly as before: Case 1 describes shots in which the current maximum occurred soon after the laser pulse, and Case 2 describes shots in which the current maximum happened after some
threshold time after (or before) the laser pulse. The exact threshold times will be different from those defined for the Carbon plasmas.

When the current maximum occurs within about 50 ns after the laser pulse arrival (Case 1), the initial plume expands until the peak current occurs. At this time, a deceleration occurs which lasts for about 100-150 ns, and then the emission burns out into the higher frequency range outside the scope of the streak camera. After 20 ns of minimal visible emission, bright and uniform recombination is seen that extends many mm. A possible explanation is that as the current begins to decrease, so does the $E \times B$ drift velocity. As the electrons become less magnetized, they can move more freely within the bulk plasma and can neutralize the ions, partially eliminating the electric field. The plasma towards the tail end of the plume becomes less and less dense, resulting in emission below the threshold of the detector.

The majority of Aluminum spectra was recorded for Case 1 plasmas. The $t = 150$ ns boundary which defines the switch from emission to recombination emission by the plume in Case 1 plasmas will motivate the discussion of the measured magnetic field results.

The Case 2 observation made for Aluminum plasmas occurred when either the current peak was before or nearly coincident with the laser pulse or when the current maximum was nearly 150 ns after the laser pulse. Here, the laser plume emission front expands at a nearly constant velocity of about $10^4$ m s$^{-1}$ until about 100 ns, when a sudden deceleration of the front occurs that is independent of the current peak (see figure 4.13). After the acceleration and recession, there is no recombination later in time characteristic of the case when the current peak happens roughly 50 ns into the plasma lifetime. Without a strong external field to force the plasma dynamics, there appears to be an equilibrium condition achieved.
4.3.2 Early Carbon Spectroscopic Measurements

The Carbon laser plume was created by affixing a 2 mm thick polyethylene tubing over the 6 mm diameter electrode. In all Carbon spectrograms, the electrode surface is defined at 5 mm from the center of the electrode. The C IV transition is accessible at higher electron temperatures, so B measurements made in carbon plasmas give snapshots of the plume evolution in the 30 ns < t < 70 ns window. Our measurements in Carbon laser plasmas are from two different regimes: one in which the plasma is initially ablated around 200 ns before the current peak so that the plume expands in a low (< 10 T) magnetic field (see figure 4.11a), and
Figure 4.13: Position and velocity traces for shot 4828, which is an example of a Case 2 Aluminum plasma. Trace of the emission front that shows the extent of optical emission as a function of time (top figure). Velocity of plasma luminosity front gotten by differentiating the position trace (bottom figure).

the other case is when the laser arrives on target during the rising slope of the current pulse (see figure 4.11b).

One example of the first case is found in shot 4839 (figure 4.15), in which the spectrum is taken 34 ns after the laser pulse and 195 ns before the current peak. Based on the discussion in Section 4.2.1, we fit the following Carbon spectra under an LTE assumption $T_e \approx T_i = 10$ eV. Figure 4.8 shows the sensitivity of the convolved Voigt widths to electron temperature. An error $\pm 50\%$ in temperature at 10 eV results in an error in density no more than $10\%$ at $4 \times 10^{17}$ cm$^{-3}$.

The first feature to note is that for $r < 7$ mm, the electron densities are nearly twice the values for $r > 7$ mm. In this region, the magnetic field is much lower than the radial profile for larger $r$. These values are not accurate, because as is shown in figure 4.14, there is no positive width difference that corresponds to a magnetic field measurement. The spectra from this region demonstrate a shortcoming of this diagnostic where the Zeeman splitting is not larger than the line broadening
Figure 4.14: Widths of the two doublet components fit independently from each other. The dotted red line is the relative difference in widths in percents.

due to Stark, Doppler, and instrumental contributions. When this is the case, the magnetic field remains unknown. The reason for the negative percent difference found in certain lineouts might be due to errors in background subtraction or a poor quality in the signal to noise ratio.

Aside from this artifact in the data, it may be possible that the field in the plume for this region may be excluded due to the high electron density. When the laser was incident on the target, the azimuthal field from the current drive was about 1 T. This field may have been sufficient to magnetize the expanding plume in order to create a diamagnetic cavity. The reduced field could be an indication that the cavity has not fully dissipated by the time of measurement. The high electron density also supports the fact that the field may continue to be excluded from the bulk of the plasma 30 ns into its evolution. If this were the case, it is likely that the plasma temperature is higher due to the high conductivity necessary to shield the field. From the sensitivity plot (figure 4.8), a 4-fold increase in temperature does not see much variation in density.

In the region $7 < r < 10$ mm, the measured magnetic field approaches the vacuum field values. These measurements coincide with a reduction in the electron density
by a factor of 3 in the span of roughly 1 mm. In this region, the Hall coefficients are between 3-10 times larger than the resistive diffusion coefficients. In this region, Hall diffusion may be responsible for the magnetic field diffusion. As seen in figure 4.14, in this region, significant width differences are measured between the profile widths. Without a definitive measurement of the field for $r < 7$ mm, it remains to be shown whether the field for larger $r$ has diffused rapidly due to density gradients or if the measurements were made possible due to the reduced Stark broadening contribution.

In shot 4840 (figure 4.16), the widths of the Zeeman components were nearly twice as large as the Zeeman component widths for shot 4839. Since the temperature was held constant at 10 eV for both shots, the larger widths correspond to higher densities. Because shot 4840 was taken about 10 ns later than shot 4839, the actual temperature may actually be even lower, resulting in a slightly higher density. Another indication of lower temperature is the lower signal to noise ratio for this shot. Spectral data could only be fit for $5.8 \text{ mm} < r < 6.8 \text{ mm}$. The low signal may be from the temperature too high or too low to excite an appreciable number of electrons to the excited state for this transition. However, having a higher electron density in a plasma that is 10 ns more progressed into its evolution seems contradictory to observations where the plasma expands and rarefies in time. This dramatic difference in line broadening in shot 4840, despite it being taken 13 ns after shot 4839 is attributed to the higher magnetic field present for shot 4840. The gate of shot 4840 was taken 30 ns later with respect to the current maximum than the gate in shot 4839, so the plasma of shot 4840 was created with a higher external field present. These two sets of data highlight the sensitivity of the plume dynamics to the external magnetic field.

Both 4839 and 4840 are taken in the case when the current maximum happens much later than the laser pulse (Case 2). Recall from the streaked emission images, the initial laser deposition is not imaged, but the plasma ionization by the presence
of the increasing current is evidenced by the sustained 200 ns brightness during the current ramp. Since 4840 is 30 ns later in the increasing current ramp than shot 4839, the broader lines and the smaller radial extent is a sign that this transition is burning out. The increase in brightness corresponds to higher densities, and the higher density is confirmed in the broadening of the spectral lines.

First, we consider the role of magnetic field diffusion for shot 4839. For $r < 7 \text{ mm}$, the reduced field in the plasma implies that $\beta > 1$ in this region. Since the plasma pressure is greater than the magnetic pressure here, the plasma fluid motion advects the magnetic field lines. This is evidenced by the fact that the measured field lines immediately outside of this region are magnified two-fold. As expected, in the region $r < 7 \text{ mm}$, the Hall coefficient is insignificant, while the diffusive coefficient is of roughly the same order as the conductive term. The relative importance of conduction implies that the plasma is experiencing pushing due to the $\mathbf{J} \times \mathbf{B}$ force. The simultaneous expansion (due to the high plasma pressure) and the compressing (due to the magnetic field) could be the cause of more turbulent motion that is seen later in time.

For larger radii, $7 < r < 10 \text{ mm}$, increased magnetic field penetration is observed. Along the length of the plume, the ion inertial length $c/\omega_{pi} < 1.5 \text{ mm}$. For $7.8 \text{ mm} < r < 10.5 \text{ mm}$, the electron density length scale is smaller than the ion inertial length, $L_n c/\omega_{pi} < 1$ so that the Hall parameter becomes significant. In this region, the Hall coefficients are greater than 1, whereas the diffusion coefficient remains low. However, resistive diffusion may be more significant than this analysis shows due to our treatment of the temperature. Our assumption of an electron temperature was based on an assumption of probability, i.e., that it was most likely that measurable signal was recorded when the C IV ion was in high population. Since the diffusion coefficient is inversely proportional to the electron temperature, the true diffusion coefficients may be larger than cited in the graphs. If the plasma
(a) C IV doublet spectrogram (color inverted) for shot 4839. The $^2S_{1/2} - ^2P_{3/2}$ component (580.133 nm) on the left and the $^2S_{1/2} - ^2P_{1/2}$ component (581.198 nm) on the right.

(b) Measured $B$ and $n_e$ trends taken along the plume radius with $2\sigma$ uncertainty in the fits.

(c) Hall and diffusive coefficients for shot 4839, calculated with $n_e$ values obtained assuming $T_e = 10$ eV.

**Figure 4.15:** Data from shot 4839, a polyethylene laser plasma taken with a 10 ns gate 34 ns after the laser pulse and 195 ns before the current peak.
(a) C IV doublet spectrogram (color inverted) for shot 4840. The $^2S_{1/2} \rightarrow ^2P_{3/2}$ component (580.133 nm) on the left and the $^2S_{1/2} \rightarrow ^2P_{1/2}$ component (581.198 nm) on the right.

(b) Measured $B$ and $n_e$ trends taken along the plume radius with 2σ uncertainty in the fits.

(c) Hall and diffusive coefficients for shot 4840, calculated with $n_e$ values obtained assuming $T_e = 10$ eV.

Figure 4.16: Data from shot 4840, a polyethylene laser plasma taken with a 10 ns gate 47 ns after the laser pulse and 161 ns before the current peak.
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(a) C IV doublet spectrogram (color inverted) for shot 4835. The $^{2}S_{1/2} - ^{2}P_{3/2}$ component (580.133 nm) on the left and the $^{2}S_{1/2} - ^{2}P_{1/2}$ component (581.198 nm) on the right.

(b) Measured $B$ and $n_e$ trends taken along the plume radius with $2\sigma$ uncertainty in the fits.

(c) Hall and diffusive coefficients for shot 4839, calculated with $n_e$ values obtained assuming $T_e = 10$ eV.

Figure 4.17: Data from shot 4835, a polyethylene laser plasma taken with a 10 ns gate 66 ns after the laser pulse and 66 ns after the current peak.
were cooler than 10 eV, then the diffusion term of the induction equation becomes more relevant.

Additionally, in the temperature sensitivity analysis, the inverse relationship between $T_e$ and $n_e$ in the widths of the Zeeman components must be considered. The diffusive coefficient $D \propto T_e^{-3/2}$, reduction in $T_e$ by a factor of two increases the diffusive coefficient by a factor of 2.82. Reducing the electron temperature reduces the Doppler width contribution to the Voigt width, increasing $n_e$ typically by about 30%. An increase in $n_e$ subsequently decreases the Hall coefficient, since $H \propto n_e^{-1}$. One last thing to consider when estimating the magnitudes of the Hall and diffusive coefficients are the dependencies on the local length scales. The diffusion coefficient depends on the characteristic length scale of the magnetic field, whereas the Hall coefficient depends on the length scale of changes in electron density. Because the external field is azimuthal, it is unlikely that integration over the axial dimension of the plume will result in much changes in $L_B$. $L_n$, on the other hand, may experience significant deviations along $\hat{z}$ compared to deviations along $\hat{r}$. If $L_n$ were smaller than the values calculated from radial $n_e$ profiles, this might compensate for the reduction in $H$ due to a larger $n_e$.

For $r > 7$ mm, the ratio $B/n_e$ is nearly constant, which provides further evidence for Hall diffusion. So it may be the case that the temperature here is greater than 10 eV, which would enhance the Hall coefficient. The plasma $\beta$ is reduced, so that magnetic pressure dominates plasma pressure. This causes further axial gradients due to the pinching of the plasma axially. Large gradients in $n_e$ facilitate Hall diffusion.

To compare shot 4839 with shot 4840, the electron density values in 4840 for the region $7 \text{ mm} < r < 10 \text{ mm}$ are about one order of magnitude larger than the density values for the same region in shot 4839. The Hall and diffusive coefficients
are both much less than 1, indicating that the magnetic field diffusion is complete. The field values match well with the vacuum field values.

The plasma of shot 4835, however, is formed about 50 ns before peak current. This shot is an example of a Case 2 plasma. With the timing between Zebra and Leopard as such, we observe the plasma immediately expand rapidly as a very bright plume that burns out around the time of peak current. For this particular shot, the spectrum was taken when the plasma was in the recombination stage. To contrast with the other shots, spectral data could only be extracted for $r > 9.5$ mm because the plasma at smaller radii was too hot to emit this transition, as seen by the continuum in the spectrogram (figure 4.17a). Since the plasma is in the process of cooling in this spectrogram, the plasma is transitioning from a more conductive state to a more resistive state. $H \sim 1$ for this region. The plasma electron density length scales are much larger than the ion inertial lengths, indicating that Hall diffusion is not important for this shot.

### 4.3.3 Mid Time Aluminum Spectroscopic Measurements

The next epoch in plasma evolution we consider is between $70 \text{ ns} < t < 150 \text{ ns}$ in Aluminum plumes. Earlier than 70 ns, continuum levels were too high in the Aluminum plasmas to be able to observe the Al III doublet. We have 20 viable spectrograms from this period that have been analyzed. All data shown except for shot 4828 were from shots that had the relative timing between Leopard and Zebra such that the current peak was typically no more than 50 ns after the arrival of the laser pulse. This means that the emission pattern from these data is typical of the Case 1 Aluminum emission (figure 4.11c). Shot 4828 falls under the Case 2 emission and will be discussed separately. The $t = 150 \text{ ns}$ boundary for this section is motivated by the switch from emission to recombination emission seen
in the streaked images. The results from the spectra from this period are marked by the closest match between the measured B and the vacuum B.

The three data sets that exemplify typical plume behavior from this time period are shown in figures 4.18, 4.19, and 4.20. All three of these spectra are taken between 120-150 ns after the laser pulse. Shots 4824 and 4827 are Case 1 laser plumes (see figure 4.11c for the streaked emission from shot 4824) and shot 4828 is a Case 2 laser plume (see figure 4.11d for the streaked emission from this shot).

The first observation to note about these three spectra is that the magnetic field profiles decrease monotonically with \( r \), except for a 1 mm region nearest the target. In the region closest to the target, the density sees a spike in comparison to the behavior in the rest of the plume and the magnetic field is reduced. This suggests that during this phase in the plasma evolution, the density remains high enough closest to the target that the magnetic diffusion has not yet fully penetrated the plume following the dissipation of the diamagnetic cavity.

The primary difference between the Case 1 and Case 2 magnetic field profiles is that there is a larger deviation from the vacuum magnetic field in the Case 2 plasma, shot 4828. This might be because the plasma of shot 4828 was able to expand without as strong of a magnetic field as the plasmas of the Case 1 plasmas. The lack of the stabilizing magnetic field might have introduced irregular density gradients which may be responsible for the partial shielding of the external field. Evidence of the effect the lack of the strong magnetic field had on 4828 is seen in the spectrograms. The spectrogram for shot 4828 (figure 4.20a) shows obvious differences in line broadening along the length of the plume, with an exaggerated drop in line width occurring around \( r = 5 \) mm. To contrast with the spectrograms for shots 4824 and 4827 (figures 4.18a and 4.19a) which show a smooth tapering in the line broadening as a function of \( r \).

For these mid-time shots, the agreement between the vacuum field and measured
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Figure 4.18: Data from shot 4824, an Aluminum laser plasma taken with a 10 ns gate 80 ns after the laser pulse and 40 ns after the current peak. The spectrogram shows the high level of continuum for $r < 4$ mm that did not allow the spectra to be fit uniquely to determine $B$. 

(A) Al III doublet spectrogram (color inverted) for shot 4824. The $^2S_{1/2} - ^2P_{3/2}$ component (569.6603 nm) on the left and the $^2S_{1/2} - ^2P_{1/2}$ component (572.2273 nm) on the right.

(B) Measured $B$ and $n_e$ trends taken along the plume radius with $2\sigma$ uncertainty in the fits.
(A) Al III doublet spectrogram (color inverted) for shot 4827. The $^2S_{1/2} - ^2P_{3/2}$ component (569.6603 nm) on the left and the $^2S_{1/2} - ^2P_{1/2}$ component (572.2273 nm) on the right.

4827 Optimized $B$ and $n_e$ with 2σ uncertainty

(B) Measured $B$ and $n_e$ trends taken along the plume radius with 2σ uncertainty in the fits.

Figure 4.19: Data from shot 4827, an Aluminum laser plasma taken with a 10 ns gate 120 ns after the laser pulse and 75 ns after the current peak. In the spectrogram, the Zeeman-split pattern of the $^2S_{1/2} - ^2P_{1/2}$ component is resolved due to a combination of high external field and lower electron density.
(a) Al III doublet spectrogram (color inverted) for shot 4828. The $^2S_{1/2} - ^2P_{3/2}$ component (569.6603 nm) on the left and the $^2S_{1/2} - ^2P_{1/2}$ component (572.2273 nm) on the right.

(b) Measured $B$ and $n_e$ trends taken along the plume radius with 2σ uncertainty in the fits.

**Figure 4.20:** Data from shot 4828, an Aluminum laser plasma taken with a 10 ns gate 141 ns after the laser pulse and 30 ns before the current peak. In the spectrogram, the Zeeman-split pattern of the $^2S_{1/2} - ^2P_{1/2}$ component is resolved due to a combination of high external field and lower electron density.
field values indicate that magnetic field diffusion is nearly complete. Accordingly, the magnitudes of the diffusive and Hall coefficients are approximately of order unity, within uncertainties in the electron temperature and densities. In figures 4.21a and 4.21b, the Hall coefficients are again calculated with the upper bounds on \( n_e \), which accounts for the consistently larger values of \( H \) compared to \( D \). Despite the apparent importance of the Hall coefficient over the diffusive coefficient, radial Hall diffusion is unlikely a dominant process since \( c/L_n \omega_{pi} \approx 4 \) for these shots. However, in contrast with the spread of \((n_e, T_e) -\) parameter space for the C IV transition due to the higher ionization potential of that particular ion, the Al III transition observed has an ionization potential of about 20 eV. This means that a variation in \( T_e \) over the full range of possible temperatures results in a smaller spread of possible \( n_e \) values. In fact, for this transition, varying \( T_e \) by about 10 eV results in lower bounds on \( n_e \) between 15 – 20% of the upper bound values, which is within the error margin of the semi-empirical Stark width formula.

### 4.3.4 Late time Aluminum Spectroscopic Measurements

The final range of times we discuss is between 150 ns < \( t < 200 \) ns, with the lower temporal boundary inspired by the start of the recombination phase of Case 1 plasmas. We have 16 viable spectrograms from these times, all from Aluminum emission; the data shown have a relative timing between the laser and current maximum typical of the Case 1 plasma emission. Magnetic field profiles from this range showed the most significant deviation on average from the vacuum magnetic fields. These plasmas show a marked departure from the well-behaved, smoothly decreasing \( B \) and \( n_e \) profiles characteristic of the plumes during the expansion phase. Because the plasmas during this time are emitting visible light from the recombination phase of the plume evolution, it is possible that the plasma is undergoing turbulent motion caused by instabilities in the rapidly cooling plasma. Instabilities in the form of density perturbations may have been seeded early in the
(a) Hall and diffusive coefficients calculated from the measured $B$ and $n_e$ values along the plume radius for shot 4827.

(b) Hall and diffusive coefficients calculated from the measured $B$ and $n_e$ values along the plume radius for shot 4828.

Figure 4.21: Derived values from mid-time Aluminum measurements.
plasma evolution. It has been shown in similar experiments that Kelvin Helmholtz vortices may form early in the plume creation.

Figure 4.22 shows spectra from a plasma taken 190 ns after the laser ablation and 130 ns after peak current. Spectra from this plasma reveal that the magnetic field profile maintains a near-constant value about three times less than the vacuum field at the target surface. At around $r = 9$, there is a sudden spike in electron density that leads to a drop in magnetic field value locally. Figure 4.23 shows spectral results from shot 3135, taken 200 ns after the laser pulse and 140 ns after the peak current. The magnetic field profile of this plasma is very different from that of shot 4081. The magnetic field drops as a function of distance from the electrode, however, the opposite behavior is observed: the shaded region shows where there is a spike in $B$ field penetration and by a drop in $n_e$. Both effects of magnetic field enhancement and exclusion are indications that there are perturbations to the density profile sufficient to affect the field diffusion on these timescales.

These plasmas take approximately 200 ns to exhibit this turbulent behavior. The instabilities present within the plasma plume alter the density and magnetic field profiles. The density length scales do not suggest that magnetic diffusion is occurring in the radial direction, so the field must be transported through either axial or azimuthal directions.

In shot 4081 (figure 4.22), the latter half of the plume has density gradients sufficiently small to allow $c/L_n\omega_{pi} < 1$. In shot 3135, nowhere along the radial profile is the length scale smaller than the ion inertial length. Despite these inconsistencies between shots, the diffusive and Hall coefficients for both sets of data are very low, as shown in figure 4.24. This indicates that neither types of diffusion are particularly relevant given these plasma parameters. However, the field profiles in this time period exhibits the largest deviation from the smoothly behaved profiles from the plasmas probed nearly 100 ns earlier.
In the measured radial profiles, there are defined regions of perturbations in the magnetic field and electron density profiles. In the case of shot 4081, some of these density length scales are indeed small enough for the Hall term to become dominant, but the magnitude of the Hall parameter remains small. In the case of shot 3135, none of the radial length scales are small enough to warrant consideration of Hall diffusion. This anomaly provides evidence of plasma instabilities that have grown late in the plume evolution from some seed instigated by the plasma turbulence early on. The fact that there is locally enhanced field compression and penetration within the plasma means there should be inhomogeneity within the plasma density profile. Since the radial $L_n$ measured do not appear to be small enough to warrant radial Hall diffusion, we look to axial or azimuthal instabilities that can cause this behavior. It may be the case that there exist density perturbations such that there are large axial or azimuthal density gradients. Since this diagnostic line of sight is integrating over the axial dimension of the plume, axial gradients could be the cause of the structure seen in the magnetic field profiles. The case for azimuthal diffusion is more challenging since the external field is already azimuthal; without kinks in the plasma that distort the field azimuthally, there will be no diffusion along this coordinate. Axial diffusion is more viable, considering many instabilities are known to be present within the plume that can lead to large density gradients, and subsequently, enhanced magnetic diffusion or shielding. The fact that conduction may be relevant in these plasmas means that the plasma dynamics are more dominated by the field. The constricted motion of the plasma within the external field may be one source of instabilities.

4.4 Discussion

Over the time series from which the magnetic field measurements were made, magnetic diffusion was seen in various stages. For the plasma measurements taken
(a) Al III doublet spectrogram (color inverted) for shot 4081. The \( ^2S_{1/2} - ^2P_{3/2} \) component (569.6603 nm) on the left and the \( ^2S_{1/2} - ^2P_{1/2} \) component (572.2273 nm) on the right.

(b) Measured \( B \) and \( n_e \) trends taken along the plume radius with 2\( \sigma \) uncertainty in the fits.

**Figure 4.22**: Data from shot 4081, an Aluminum laser plasma taken with a 10 ns gate 190 ns after the laser pulse and 130 ns after the current peak. Each lineout is integrated over 0.5 mm.

At \( t \approx 30 \) ns, evidence of the diamagnetic cavity for small \( r \) is caught, whereas the magnetic field diffusion process has begun in the cooler regions of the plume. For \( 70 < t < 150 \) ns, the embedded magnetic field profiles most closely match the external field profile, and the density profiles are smoothly decreasing. In the late stage of \( 150 < t < 220 \) ns, the plume enters a turbulent stage marked by localized regions of compressed magnetic field and sudden changes in the radial density profiles.
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(a) Al III doublet spectrogram (color inverted) for shot 3135. The $^2S_{1/2} - ^2P_{3/2}$ component (569.6603 nm) on the left and the $^2S_{1/2} - ^2P_{1/2}$ component (572.2273 nm) on the right.

(b) Measured $B$ and $n_e$ trends taken along the plume radius with $2\sigma$ uncertainty in the fits.

**Figure 4.23:** Data from shot 3135, an Aluminum laser plasma taken with a 10 ns gate 200 ns after the laser pulse and 140 ns after the current peak.

However, the measured radial profiles do not indicate that either resistive or Hall diffusion took place. The first conclusion to be made about the plasmas in the late stage of evolution is that since radial density profiles cannot explain the increased penetration of the magnetic field in specific regions along the plume radius, there must be either axial or azimuthal gradients steep enough to facilitate diffusion. Many instabilities may have evolved so that they become dominating late in time that can cause these gradients.

The diffusion which manifests as unpredictable behavior in the radial density and magnetic field profiles may be an indication of slowly developing instabilities that
(a) Hall and diffusive coefficients calculated from the measured $B$ and $n_e$ values along the plume radius for shot 4081.

(b) Hall and diffusive coefficients calculated from the measured $B$ and $n_e$ values along the plume radius for shot 3135.

**Figure 4.24:** Derived values from mid-time Aluminum measurements.
are periodic in structure forming along the plume length. These instabilities might have been seeded early in the plume ablation phase and only late in time begin to exhibit perturbations in the field and density profiles macroscopically. It might also be the case that our spatial resolution is not fine enough to detect the earlier growth of an instability that results in radial density modulations. Without measurements made in the azimuthal or axial planes, it is not possible to say with certainty the exact nature of the instabilities in the plume. We explore a few different possibilities that may explain the density modulation and enhanced field penetration through large density gradients.

The first proposed way to maintain axial density gradients along the plume is due to the axial electric fields that are formed in response to an external radial force (see section 4.1.3). Due to charge-dependent drift velocities of the species in the plasma, gradients in electron density will exist along \( \hat{z} \). If the plume is compressed by the magnetic field along \( \hat{z} \) enough, then these density gradients may vary on scales short enough to facilitate Hall diffusion. From interferometric images, the axial dimensions of the plumes around 60 ns into evolution are on the order of 1 mm. Charge separation due to guiding center forces may be a viable explanation for large axial gradients. The modulations along \( \hat{r} \) are only explained if the radial force itself is nonuniform. As seen in section 4.1.3, the radial force may of the form of a deceleration, pressure gradients, or electric fields produced by large Larmor radius ions.

Pressure imbalances between the plasma ram pressure and magnetic field pressure along the plume radius can also cause axial pinching of the plume. The pinching of the plume may explain the plume density modulation along \( \hat{r} \). The axial component of the velocity may overcome the magnetic pressure causing expansion along \( \hat{z} \), or the magnetic pressure may overcome the axial velocity causing necking. The plasma is expanding along \( -\nabla B \) so that the effect may become more prominent for larger \( r \). For these plasmas, length scales shorter than 1-1.5 mm will
be sufficient to have diffusion time scales of less than 100 ns. If turbulent motion is present along \( \hat{z} \), these numbers suggest it is likely to have regions of increased magnetic field within the plasma. MHD modeling of an experiment with similar conditions shows the formation of periodic Kelvin-Helmholtz vortices along the target normal. [28] The structures formed early in the creation of the laser plasma may be the seed for the instabilities observed in these measurements.

The firehose instability is another possible way to introduce radial density perturbations; however, its application in this geometry is at least a two-step process. First, an azimuthal perturbation must exist so that the field is not continuous along \( \phi \). This is because the firehose instability requires pressure imbalances parallel and perpendicular to the field. The pressure imbalances perpendicular to the field have already been discussed. Pressure imbalances parallel to the field in this geometry require an introduction by an initial perturbation in the plume. The firehose instability creates large density gradients along \( \phi \) that may look like radial perturbations due to their asymmetry. When anisotropies in thermal pressures parallel and perpendicular to the magnetic field exist, local structure in the magnetic field and density of the plasma may arise. If \( \beta_\parallel - \beta_\perp > 2 \) (where \( \beta \) is the ratio of thermal pressure to magnetic pressure), the firehose instability drives bending of the field lines resulting in periodic structuring to form along the field lines. [34–36] Bending of the field lines creates nonuniform magnetic flux along \( \phi \), allowing magnetic diffusion to occur.

Lastly, we consider the flutelike structures that form along the magnetic field lines in the azimuthal plane (e.g., laser Schlieren done in the \( r - \varphi \) plane in reference [37]). These flutelike structures become less prominent as the field is decreased. These finger structures might be evolving into structures that do not only affect the plume along the fringe edges but contribute to the electron distribution within the centroid of the plume, near the probing region.
For the case of cylindrical geometry, the Hall term of the induction equation from MHD may be expressed as [38]

\[
\left( \frac{\partial B}{\partial t} \right)_{\text{Hall}} = \nabla \times \frac{J \times B}{n_e e} = \frac{m_i c}{e} \nabla \times \frac{dV_i}{dt}. \quad (4.13)
\]

If the plasma acceleration is primarily radial, then the Hall term expressed in cylindrical geometry dictates that magnetic flux will penetrate either axially or azimuthally. If \( \frac{d\hat{v}_i}{dt} = A(r, \phi, z) \hat{r} \), then

\[
\nabla \times \frac{d\hat{v}_i}{dt} = \frac{\partial A}{\partial z} \hat{\varphi} - \frac{1}{r} \frac{\partial A}{\partial \varphi} \hat{z}. \quad (4.14)
\]

In general, the ions may experience acceleration with \( \hat{r}, \hat{z}, \) and \( \hat{\varphi} \) components. However, knowing that the plasma spreads in the \( r - \varphi \) plane and is constricted in the \( r - z \) plane due to the magnetic pressure, the assumption of a dominating radial acceleration is not misguided. This analysis shows that the presence of axial or azimuthal gradients are both conducive to Hall diffusion occurring in regions with large enough local density gradients.
Chapter 5

Experimental Study 2: Magnetic Field Measurements in Exploding Wire Arrays

In this section, B field measurements made on wire arrays are detailed. We make measurements during wire ablation phase and along the axial precursor.

5.1 Experimental Setup

The Zebra generator (in short pulse mode) was used to drive 1 MA in 90 ns through wire array loads composed of 8 \times 14 \mu m Al wires arranged in conical or cylindrical geometry or 4 \times 25 \mu m Al wires arranged in the x-pinch configuration.

Figure 5.1 shows the coordinate system deployed for this chapter: \( \hat{x} \) points in the direction of the laser diagnostics; \( \hat{y} \) points towards the line-of-sight of the imaging spectroscopic diagnostic; \( \hat{z} \) points to the top of the load.
The load region was imaged along $\hat{y}$ using a 1-m optical relay lens system and a 10 cm diameter $f = 50$ cm imaging lens to control the magnification and $f/\#$ match to the spectrometer, shown in figure 5.2. We split this optical line so that one leg is analyzed spectroscopically and the other leg is sent to a streak camera. Using mirror rotations, we direct either a radial or axial projection of the imaged load onto the slit of the spectrometer and streak camera. With the radial view, the Zeeman broadening diagnostic can measure the B fields locally around each wire early in the ablation phase, and the streak camera can measure the implosion time of the load. In the axial view, the Zeeman diagnostic can measure the spatial structure of the B fields along the precursor of the pinch and the streak camera can observe hot spot dynamics in time. The Zeeman broadening diagnostic has been implemented in two different configurations and will be described in subsections 5.2.1.1 and 5.2.2. A schematic representation of the coordinate system and the two different orientations of the field of view of the Zeeman and streak diagnostics are shown in figure 5.1.

The field of view of the laser diagnostics is the $y - z$ plane. Two-frame shadowgraphy was implemented with a pulsed probing laser ($\lambda_{\text{probe}} = 532$ nm; $E \approx 500$ mJ; $\tau_{\text{probe}} \approx 1$ ns) which was split before the experimental chamber with a variable delay between the two frames.
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Figure 5.1: Two different alignments of the spectroscopic and optical streak diagnostics with respect to the wire array (in this example, a conical) load.

Figure 5.2: Optical set up to image the load plasma on to the slit of the spectrometer. The imaging system consists of a relay system (two 1-m lenses) and one imaging lens (diameter 10 cm, \( f = 50 \) or \( f = 40 \) cm).
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5.2 Diagnostic Techniques

5.2.1 Emission Spectroscopy

5.2.1.1 Experimental Implementation

At the time these measurements were made, the current sent through the wires is relatively low, roughly 50 kA, resulting in $B \sim 1 - 3$ T. For the line broadening expected due to the electron densities of about $5 \times 10^{16} - 5 \times 10^{17}$ cm$^{-3}$, 1 T is close to the lower limit of the magnetic field induced line width difference resolvable for the Al III doublet. As discussed in section 2.1.1.2, a measurement of the $\sigma$ components of the doublet lines can yield more sensitive magnetic field measurements than the $\pi$ components in the presence of equal line broadening.

In principle, we may exploit the geometry of the experimental configuration to both verify this difference in sensitivity as well as obtain $B$ field measurements in the coronal plasma around each wire that have higher accuracy. Because the magnetic field maintains an azimuthal geometry around each wire, a side observation of the wire will result in the line of sight of the diagnostic to be perpendicular to the magnetic field directly in front of the wire, as depicted in figure 5.3. Parallel to the magnetic field, only the $\sigma$ components of the doublet are emitted, and perpendicular to the magnetic field both $\sigma$ and $\pi$ components are emitted.

However, in early implementations of this diagnostic, the signal-to-noise (SNR) ratio was not sufficient to allow subdivision of the signal emitted from each wire location. This prohibited polarization sensitive measurements for those sets of data. In practice, it is also important to realize that the error introduced by performing such a subdivision would be difficult to characterize. Instead, for these types of data with low SNR, lineouts were obtained which integrated over the approximate 1.5 mm plasma region around each wire. The lineouts were then fit
Figure 5.3: Projection of the local azimuthal magnetic field along the line of sight of the spectroscopic diagnostic. Directly in front of the wire core, the magnetic field is aligned perpendicular to the line of sight of the diagnostic, so that both $\sigma$ and $\pi$ components of the doublet are emitted. Along the sides of the wires, a combination of parallel and perpendicular projections of the field along the line of sight are observed.

twice, assuming a line of sight purely perpendicular and purely parallel to $B$, to obtain an upper and lower bound on $B$.

5.2.1.2 Polarized Spectroscopy

In subsequent implementations of this diagnostic on wire array loads, polarizers were installed to observe the azimuthal structure of the magnetic field around each wire and obtain more accurate $B$ measurements from the line profile shape itself. In front of the entrance slit to the spectrometer, two orthogonal linear polarizers (Thorlabs, extinction ratio > 1000:1) were placed on the upper and lower halves of the slit. The polarizers were placed so that the boundary between them coincided very closely with the $z$-axis. The spatial resolution provided by the imaging quality of the spectrometer results in a spectrogram in which one half of the load is recorded in one linear polarization and the other half is recorded in the orthogonal linear polarization.
5.2.1.3 Considerations for Polarization Sensitive Measurements

The σ components of the Zeeman profile are circularly polarized in the plane perpendicular to the magnetic field’s axis. That is to say, in observations parallel to $B$ the σ lines are circularly polarized and in observations perpendicular to $B$ the σ lines appear linearly polarized in the direction perpendicular to $B$.

\[ E_0 = E_x \hat{x} + E_y e^{i\phi} \hat{y}, \]  

(5.1)

where $E_x$ and $E_y$ are the amplitudes of the $x$ and $y$ components of the electric field and $\phi$ is the relative phase difference between them. Linear polarization corresponds to $\phi = 0$ and circular polarization refers to the case $E_x = E_y, \phi = \pm \pi/2$.

The direction of circular polarization of the Zeeman components depends on whether for the transition $\Delta m = \pm 1$. Right-handed circularly polarized light
(\(\sigma^+\)) corresponds to \(\phi = -\pi/2\) and left-handed circularly polarized light (\(\sigma^-\)) corresponds to \(\phi = \pi/2\). Linearly polarized light corresponds to \(\Delta m = 0\). The addition of \(\sigma^+\) and \(\sigma^-\) light results in linearly polarized light since

\[
E_{0+} + E_{0+} = 2E_x \hat{x} + E_y e^{-i\pi/2} \hat{y} + E_y e^{i\pi/2} \hat{y} \\
= 2E_x \hat{x} + i(-E_y + E_y) \hat{y} \\
= 2E_x \hat{x}.
\]

(5.2) 
(5.3) 
(5.4)

Therefore, for our observations that were made in the plane perpendicular to the azimuthal field, all \(\sigma\) lines are linearly polarized perpendicular to the field, and as expected, the \(\pi\) components are linearly polarized along the field.

### 5.2.2 Absorption Spectroscopy

The optical system is identical to the one used for emission spectroscopy (figure 5.2). In this section, we describe the preparation of the load chamber with NaCl precipitate to obtain absorption measurements during the rising slope of the current pulse.

We observe the Na I doublet in absorption during the positive current ramp. A sodium chloride solution was coated onto the current return posts in the chamber. During the chamber evacuation, the sodium chloride molecules diffused into the chamber. The current return posts are a 5 cm away from the wires, so we expect sodium chloride vapor to be present near the wires. During the wire ablation, the NaCl molecules dissociate, producing neutral Na that absorb at the sodium resonance lines observed. We observed the Na doublet lines during the rising slope of the current pulse. However, since the volume and distribution of the absorbing Na present around the magnetized plasma region is unknown, interpretation of this data is left for future work. Using a 10 ns gate on the iCCD camera, a time
series of absorption spectra was obtained on x-pinches. Shown in figure 5.20 are lineouts from a typical spectrogram. The line intensity ratios vary greatly from the expected 2:1 ratio, so the optical depth of the sodium cloud must be calculated to infer estimates on the magnetic field and density from the spectra.

5.2.3 Optical Emission

5.2.3.1 Streaked and Time-Gated Emission

The optical path for the spectroscopy diagnostic is split via a 50/50 beamsplitter and directed into an optical streak camera with approximately 480 ns window. The streak camera is aligned so that the projection of the image of the load into the slit of the streak camera’s photocathode is identical to the projection imaged through the spectrometer.

When the orientation of the slit with respect to the load is radial, implosion velocities of material being accelerated onto the z-axis are obtained. When the orientation of the slit with respect to the load is axial, a history of the hot spot dynamics is obtained.

5.2.4 Spectroscopic Measurements 1: Emission Data from Early Ablation

5.2.4.1 Unpolarized Spectroscopy

The transition used for the diagnostic was the Al III 4s-4p doublet. See section 4.2.1 for more details about using this transition as a magnetic field diagnostic. The plasma created around each wire early in the current pulse had an external magnetic field low enough and electron density high enough so that the widths of
Figure 5.5: Streaked image of imploding conical wire array load with slit of the streak camera aligned along the $z$–axis. 10 ns gated emission images are shown at various times along the current pulse. All images are taken from separate shots.

Figure 5.6: Streaked image of imploding x-pinch wire array load with slit of the streak camera aligned along the $z$–axis. 10 ns gated emission images are shown at various times along the current pulse. All images are taken from separate shots.
the individual Zeeman components were of the same order as the widths of the full fine structure components. This made spectroscopic measurements challenging with the given level of noise. However, the advantages of implementing crossed polarizers to isolate the line profile components were demonstrated under these conditions.

Two examples of radial spectrograms recorded without polarizers are shots 4366 (figure 5.7) and 4494 (figure 5.8). The azimuthal field around each wire has both parallel and perpendicular projections along the line of sight of the detector. Because of low SNR, the lineout was obtained by averaging over the emission region for each wire. The lineout was then fit assuming both a longitudinal observation and a transverse observation profile since the averaged profile is a linear combination of these two profiles. The lineout fit according to the longitudinal profile will provide a lower bound estimate on $B$, while the lineout fit to the transverse profile will provide an upper bound estimate on $B$. In the results for shot 4366, there is essentially no difference between the transverse and longitudinal profiles. The magnetic field values from the two different profiles are within 0.2 T of each other. Figure 5.7c shows the widths of the Zeeman components (labeled with subscript $V$), the average widths of the fine structure components with upper level $^2P_{3/2}$ (labeled $w_{3/2}$ and $^2P_{1/2}$ (labeled $w_{1/2}$), and the relative width difference between the fine structure components in percents. As shown in the figure for shot 4366, the average widths of the composite fine structure peaks are barely 10% larger than the widths of the Zeeman components. Also from this plot, the percent difference between fine structure widths ranges between 0 and 10%. The fact that fits to the lineouts by each of the two profiles show 0% difference while still registering a magnetic field value is likely a machine roundoff error and illustrates the importance of imposing at least a 10% difference between profiles. There is an increasing trend for these widths that seems to be dominated by an increased electron density for both longitudinal and transverse profiles. From $\dot{B}$ measurements at this time,
the field should have a strength of 0.5 T at a distance \( r = 0.5 \) mm from the wire center. This discrepancy between field values will be discussed later.

In contrast to shot 4366 which had Voigt profile widths \(< 1\ \text{Å}\) and produced \( B \sim 1\ \text{T}\), shot 4494, figure 5.8, shows the shortcomings of this technique on profiles with large widths compared to the Zeeman splitting. This shot was taken 10 ns before the start of the main current ramp so that the field generated by the precursor around each wire should be approximately 1 T at \( r = 1\ \text{mm}\) from the wire center. However, the field was not large enough to produce a measurable difference between component widths. The solutions from the fitting method showed \( B = 0\), with the largest errors at the largest component widths.

Figure 5.9 shows the minimum \( B\) that is detectable under the constraint that there must be at least 10% width difference between the fine structure components as a function of average fine structure component widths. Figure 5.11 shows the average widths of the doublet components as a function of field magnitude. These calculations were done by modeling the Zeeman components as Lorentz profiles, instead of the Voigt profiles used in the actual data fitting. The differences are negligible for this discussion since we are interested in quantifying the profile widths as opposed to a detailed line shape analysis. These plots can be used to assess the feasibility of using this technique for a set of plasma parameters. For example, the average doublet component widths of shot 4366 are approximately 0.8 Å. From figure 5.9, the full transverse and longitudinal profiles both have traces that extend beyond the domain printed, \( B_{\text{min}} > 2.5\ \text{T}\). Given the current at the time of gating, fitting the spectra with these two profiles is not a sensitive enough technique to measure fields below 2.5 T.

Figure 5.9 also shows that a profile composed only of the \( \pi \) components shows the greatest ability to access low field measurements with at least 10% width difference. This can be better understood by looking at the average widths of the doublet
components as a function of magnetic field for the three different profiles (figure 5.11). The profile composed of the sums of only the $\sigma$ components have the largest average widths, and the profile composed of only sums of the $\pi$ components have the smallest average widths, with the transverse profile (both $\sigma$ and $\pi$) in between. This is due to the fact that the $\sigma$ components experience the largest splitting from the Zeeman effect. When the line broadening is comparable to the Zeeman shifts, then the relative width difference becomes harder to estimate. For the case of the $\pi$-only profile, because the Zeeman shifts are smaller than the component widths, variation in $B$ produces a measurable width difference between components.

### 5.2.4.2 Polarized Spectra

The results from the high density and low magnetic field conditions typical of the ablation plasmas around the wires taken without polarizers highlight the limitations of using profiles dominated by the $\sigma$ components. Next, we implemented two crossed polarizers so that the $\pi$ components could be recorded separately to gain more sensitive measurements.

The three shots with polarizers implemented (figures 5.12, 5.13, and 5.14) were all taken before the start of the current ramp so that the current through the wires is from the prepulse. Assuming that the current is flowing along the outer boundary defined by the radial extent of the $\sigma$ component emission, around each wire $B < 1$ T. Unfortunately, the average widths of the doublet components for all three shots $\approx 0.7 – 0.8$ Å, meaning that the minimum magnetic field (as fit by the $\pi$ profile) is between 1.75 and 2 T. Therefore, magnetic field measurements under these conditions will be larger than the true magnetic field if the current is actually flowing on the outer edge of the plasma.

The immediate benefit of implementing polarizers is that we can visually see the structure of the magnetic field around the wires. The magnetic field around a wire
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(A) Al III doublet spectrogram (false colored) for shot 4366. The $^2S_{1/2} - ^2P_{3/2}$ component (569.6603 nm) on the left and the $^2S_{1/2} - ^2P_{1/2}$ component (572.2273 nm) on the right. Each distinct emission region along the vertical axis (labeled by regions) is the projection of the coronal emission around each wire and spans approximately 2 mm.

(B) Measured $B$ and $n_e$ trends taken along the plume radius with $\sigma$ uncertainty in the fits.

(C) Fitted widths of the Voigt profiles that model each Zeeman component (blue) and fitted widths of the average doublet widths (green) for both parallel and perpendicular observations with respect to the magnetic field. The widths are calculated according to the FWHA of the components assuming $T_e \approx 3$ eV.

Figure 5.7: Radial emission spectral data from shot 4366, an Aluminum cylindrical wire array taken with a 10 ns gate 130 ns before the current peak (no polarizers).
(A) Al III doublet spectrogram (false colored) for shot 4494. The $^2S_{1/2} \rightarrow ^2P_{3/2}$ component (569.6603 nm) on the left and the $^2S_{1/2} \rightarrow ^2P_{1/2}$ component (572.2273 nm) on the right. Each distinct emission region along the vertical axis (labeled by regions) is the projection of the coronal emission around each wire and spans approximately 2 mm. Region 3 is from the precursor plasma.

(b) Measured $B$ and $n_e$ trends taken along the plume radius with $1\sigma$ uncertainty in the fits.

(c) Fitted widths of the Voigt profiles that model each Zeeman component (blue) and fitted widths of the average doublet widths (green) for both parallel and perpendicular observations with respect to the magnetic field. The widths are calculated according to the FWHA of the components assuming $T_e \approx 3$ eV.

**Figure 5.8**: Radial emission spectral data from shot 4494, an Aluminum cylindrical wire array taken with a 10 ns gate 100 ns before the current peak (no polarizers).
FIGURE 5.9: Minimum $B$ measurements possible calculated by taking the Zeeman components with Lorentz profiles with the constraint that there must be at least a 10% difference between the $^2S_{1/2} - ^2P_{1/2}$ and $^2S_{1/2} - ^2P_{3/2}$ components.

FIGURE 5.10: The range of $B$ such that the relative width difference between the $^2S_{1/2} - ^2P_{1/2}$ and $^2S_{1/2} - ^2P_{3/2}$ components is less than 10%.
is azimuthal and the polarized projections of the azimuthal field are verified by the spatial extent of the $\sigma$ and $\pi$ emission. In the projection of the line of sight of the diagnostic which sees the diameter of the wire, the $\sigma$ components are emitted at all azimuthal angles, or $2\pi r$, where $r$ is the radius of the emitting plasma boundary.

The intensity of the $\pi$ components is proportional to $|\sin \alpha|$, where $\alpha$ is the angle made off of the $x-$ axis. Integrating $r|\sin \alpha|$ over $2\pi$, yields the projected intensity to be proportional to $4r$. Hence, the ratio of the $\pi$ intensity to the $\sigma$ intensity projected along the diameter of the plasma in an azimuthal field is $2/\pi \approx 0.6$.

In the polarized spectrograms, the spatial extents of the polarized emission follow this ratio. The azimuthal field structure is expected for this experimental setup, but this technique can be invaluable in experiments with unknown field geometry.

Under these conditions (high density and low field), the advantage of using the polarizer is seen in the $\pi$ component profile fits. Using the $\pi$ components only allows for an upper bound limit on the local field. This is because the $\pi$ components are minimally spread by the external magnetic field, and there is greater variation in the profile width so that a solution may be found. This feature of the profile
widths is illustrated in Figure 5.10, which shows that at low fields, the $\pi$ component profile has the most substantial percent difference.

### 5.2.5 Spectroscopic Measurements 2: Emission Data from Precursor Plasma

In figure 5.15, axial emission from the precursor of an x-pinch is shown. In the spectrogram, the emission from the plasma jets emanating from either side of the hot crossing point are imaged in the Al III doublet wavelengths. From the lineouts, the plasma furthest away from the crossing point appears to have the highest magnetic field measurements, with nearly zero field effects being measured in the plasma from the center of the pinch.

In shot 4357 (figure 5.16), the conical array, a decreasing trend in the magnetic field is seen with the highest field measured at the larger diameter of the array. In shot 4358, the inverse conical array, the decreasing trend in $B$ is measured with the highest fields from the smaller diameter of the array. The results of these two shots appear to be in contrast with each other because it would be expected that the higher fields would occur in the same locations of the load, whether at the smaller or larger diameter. However, an explanation for this behavior may be found in the streaked axial emission images. In figure 5.18a, at the time from which the spectrogram was recorded, the axis is uniformly illuminated by the plasma. In figure 5.18b, the uniformity of the plasma emission is broken closest to the cathode side of the load, which corresponds to the larger diameter of the array. It is possible that in this region there is an instability that exists to increase current density so that the emission shifts to higher photon energies. This shift to higher photon energies is supported by the fact that the spectrogram closer to the cathode shows that the spectral lines become wider. In shot 4357, the reason why the higher fields are found in the larger diameter of the array might be because
the precursor plasma from the smaller diameter of the array is much hotter so the plasma excludes the ambient field.

The larger field values that were measured were gotten from doublet component width differences of 10% or greater. It is interesting to note that the line profiles fit by the transverse intensity profiles gave the largest width differences. This is justified since the field is likely transverse to the line of sight of observation.

5.2.6 Spectroscopic Measurements 3: Absorption Data from Precursor Plasma

Spectrograms from the absorption series of spectral data have the spatial dimension defined along the diameter of the array. For these data, the array that created the plasma is an Aluminum x-pinch composed of 2 50 µm wires. A series of 10 ns gated spectrograms were taken along the rising slope of the current pulse. These spectrograms show qualitatively how the magnetic field increases in the plasma; however, due to opacity effects the 2:1 ratio between the doublet lines was not preserved. This made field extractions from the spectral lineouts impossible. An example of the quality of lineouts with unknown continuum profiles is shown in figure 5.20.

The fine structure splitting for the Na I 3p-3s doublet lines becomes comparable to the magnitude of the Zeeman splitting around 40 T. From this value, a rough estimate on the field produced in the neighborhood of the wires can be obtained.

At 40 ns before current peak, there appears to be a rapid change in the plasma conditions. At -38 ns, the doublet components are broadened but still resolved separately. At -40 ns, the doublet lines completely overlap and broaden to a width of about 3 nm. Although every spectrogram shown in figure 5.19 are from separate shots, there are shots taken with repeat timing that show that the behavior captured in the shown spectrograms is reproducible. However, it is also possible
that the dramatic differences between shots 4834 and 4835 could be due to isolated events rather than a general trend in the pinch evolution. The broadening of these lines is likely an indication of the increasing temperature and density of the coronal plasma around each wire.

5.3 Discussion

In this chapter, the spatially resolved magnetic field diagnostic was implemented to measure the field around each wire during the ablation phase and in the plasma that accumulates on axis during the plasma streaming phase. In the measurements taken around each wire, the use of polarizers demonstrated the advantages of considering the polarized emission for the case of width limited profiles. In these profiles, the field was low enough and the electron density high enough that differences in widths of the doublet profiles were difficult to measure. By isolating the \( \sigma \) and \( \pi \) components of the profile, the width differences between doublet components were more exaggerated, facilitating a width difference measurement. Also, the polarizers show the geometry of the field structure around each wire, providing a useful technique for determining the field structure in other experiments.
(A) Al III doublet spectrogram (false colored) for shot 4515. The $^2S_{1/2} \rightarrow ^2P_{3/2}$ component (569.6603 nm) on the left and the $^2S_{1/2} \rightarrow ^2P_{1/2}$ component (572.2273 nm) on the right. Each distinct emission region along the vertical axis (labeled by regions) is the projection of the coronal emission around each wire and spans approximately 2 mm.

(B) Measured $B$ and $n_e$ trends taken along the plume radius with $1\sigma$ uncertainty in the fits.

(C) Fitted widths of the Voigt profiles that model each Zeeman component (blue) and fitted widths of the average doublet widths (green) for both parallel and perpendicular observations with respect to the magnetic field. The widths are calculated according to the FWHA of the components assuming $T_e \approx 3$ eV.

Figure 5.12: Radial emission spectral data from shot 4515, an Aluminum cylindrical wire array taken with a 10 ns gate 130 ns before the current peak.
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(A) Al III doublet spectrogram (false colored) for shot 4511. The $^2S_{1/2} - ^2P_{3/2}$ component (569.6603 nm) on the left and the $^2S_{1/2} - ^2P_{1/2}$ component (572.2273 nm) on the right. Each distinct emission region along the vertical axis (labeled by regions) is the projection of the coronal emission around each wire and spans approximately 2 mm.

(B) Measured $B$ and $n_e$ trends taken along the plume radius with $1\sigma$ uncertainty in the fits.

(c) Fitted widths of the Voigt profiles that model each Zeeman component (blue) and fitted widths of the average doublet widths (green) for both parallel and perpendicular observations with respect to the magnetic field. The widths are calculated according to the FWHA of the components assuming $T_e \approx 3$ eV.

Figure 5.13: Radial emission spectral data from shot 4511, an Aluminum cylindrical wire array taken with a 10 ns gate 130 ns before the current peak.
(A) Al III doublet spectrogram (false colored) for shot 4506. The $^{2}S_{1/2} \rightarrow ^{2}P_{3/2}$ component (569.6603 nm) on the left and the $^{2}S_{1/2} \rightarrow ^{2}P_{1/2}$ component (572.2273 nm) on the right. Each distinct emission region along the vertical axis (labeled by regions) is the projection of the coronal emission around each wire and spans approximately 2 mm.

(B) Measured $B$ and $n_e$ trends taken along the plume radius with 1σ uncertainty in the fits.

(C) Fitted widths of the Voigt profiles that model each Zeeman component (blue) and fitted widths of the average doublet widths (green) for both parallel and perpendicular observations with respect to the magnetic field. The widths are calculated according to the FWHA of the components assuming $T_e \approx 3$ eV.

Figure 5.14: Radial emission spectral data from shot 4506, an Aluminum cylindrical wire array taken with a 10 ns gate 130 ns before the current peak.
(a) Al III doublet spectrogram (false colored) for shot 4350. The $^2S_{1/2} - ^2P_{3/2}$ component (569.6603 nm) on the left and the $^2S_{1/2} - ^2P_{1/2}$ component (572.2273 nm) on the right.

(b) Measured $B$ and $n_e$ trends taken along the plume radius with 1σ uncertainty in the fits.

(c) Fitted widths of the Voigt profiles that model each Zeeman component (blue) and fitted widths of the average doublet widths (green) for both parallel and perpendicular observations with respect to the magnetic field. The widths are calculated according to the FWHA of the components assuming $T_e \approx 3$ eV.

**Figure 5.15:** Axial emission spectral data from shot 4350, an Aluminum conical wire array taken with a 10 ns gate 140 ns before the current peak.
(A) Al III doublet spectrogram (false colored) for shot 4357. The $^2S_{1/2} - ^2P_{3/2}$ component (569.6003 nm) on the left and the $^2S_{1/2} - ^2P_{1/2}$ component (572.2273 nm) on the right.

(B) Measured $B$ and $n_e$ trends taken along the plume radius with 1σ uncertainty in the fits.

(C) Fitted widths of the Voigt profiles that model each Zeeman component (blue) and fitted widths of the average doublet widths (green) for both parallel and perpendicular observations with respect to the magnetic field. The widths are calculated according to the FWHA of the components assuming $T_e \approx 3$ eV.

**Figure 5.16:** Axial emission spectral data from shot 4357, an Aluminum conical wire array taken with a 10 ns gate 106 ns before the current peak.
(A) Al III doublet spectrogram (false colored) for shot 4358. The $^2S_{1/2} - ^2P_{3/2}$ component (569.6603 nm) on the left and the $^2S_{1/2} - ^2P_{1/2}$ component (572.2273 nm) on the right.

(B) Measured $B$ and $n_e$ trends taken along the plume radius with 1σ uncertainty in the fits.

(C) Fitted widths of the Voigt profiles that model each Zeeman component (blue) and fitted widths of the average doublet widths (green) for both parallel and perpendicular observations with respect to the magnetic field. The widths are calculated according to the FWHA of the components assuming $T_e \approx 3$ eV.

Figure 5.17: Axial emission spectral data from shot 4358, an Aluminum conical wire array taken with a 10 ns gate 110 ns before the current peak.
(a) Shot 4357: Streaked image taken with spatial resolution along the axis of the conical wire array.

(b) Shot 4358: Streaked image taken with spatial resolution along the axis of the inverse conical wire array.
Figure 5.19: Raw spectrograms taken from a time series along the rising slope of the current pulse from Na I doublet lines in absorption. Spectrograms are taken with a 10 ns gate.
Figure 5.20: Raw spectrograms taken from shot 4834 (taken 38 ns before current peak) and lineouts taken along the diameter. The horizontal axis shows spectral resolution and the vertical axis shows spatial resolution.
Chapter 6

Future Work and Conclusions

6.1 Future Work

The progression of this diagnostic technique lies in applying to plasmas with higher densities, temperatures, and field strengths. The Zeeman broadening technique performed in the visible range has many advantages. First, the relative splitting of the Zeeman effect is a larger effect for lower photon energies since $\Delta \lambda \propto \lambda^2$. Thus, performing spectroscopy on hotter plasmas requires much higher resolution instruments because the radiation from these plasmas will be in the UV or x-ray range and those transitions will experience a much smaller relative splitting. Second, the ability to use optical elements to transport, image, and manipulate polarizations play key roles in the creation of the data in this dissertation. In order to have a similar degree of spatial resolution and polarization analysis, more sophisticated techniques must be used. For example, a single-crystal polarization splitting x-ray spectrometer with spatial resolution has been developed and proves to be a viable way to carry out these measurements with sensitivity to polarization for higher photon energy transitions.[39] Finally, lower energy transitions are usually emitted by low Z ions in lower temperature plasmas. The advantage here lies in
the fact that the for lower Z ions LS coupling is a good approximation so that the relative line intensities of the multiplets being used as the diagnostic lines are easy to calculate. In order to maintain lower energy transitions at higher temperature plasmas, high Z ions must be used as dopants. In these ions, the relative intensities are difficult to calculate and in general, are not easily known. This requires modeling of the radiation from the plasma to implement this technique correctly.

Given these challenges, there are several applications for which measurements of fields in high density and temperature plasmas are highly desired. One example is in the MagLIF concept at Sandia National Laboratories. In this experiment, measurements of the magnetic field during the pinch stagnation are unknown. Current knowledge of the experiment relies on modeling to estimate the fields, so a direct measurement is critical for the success of the experiment.\cite{40–42} This experiment has three phases: pre-magnetization of the fuel by an external axial field, pre-heat of the fuel by a laser, and compression of the magnetized fuel by the imploding liner driven by the Z current. As shown in Chapter 4, an external magnetic field does not immediately diffuse uniformly into a laser plasma, but instead is excluded by the diamagnetic cavity that forms due to the high conductivity of the plasma. Therefore, the assumption that the magnetic field will be compressed uniformly within the fuel is faulty. The behavior of the plasma at stagnation is largely unknown, and presently no direct measurements of the magnetic field in the fuel exist.

There is an added difficulty in performing measurements on this experiment which lies in the thick, absorbing liner used to implode the fuel. The liner is necessarily thick to minimize instabilities and non-uniformity in the compression, but the thick liner absorbs lower energy radiation from the plasma inside. From published data in reference\cite{43}, radiation below 2 keV is absorbed by the liner, and typical radiation is near 8 keV. This makes the magnetic field measurement difficult because the Zeeman splitting will be small. This problem might be worked around
if a window were cut into the liner that allowed lower energy photons to pass.

Even with photon energies around 2 keV, the resolution required to measure $\Delta E \approx \mu_B B$ is extremely high.

$$\frac{E}{\Delta E} \approx \frac{10^5 E \text{ [keV]}}{B \text{ [kT]}}$$  \tag{6.1}

In order to have instrument $E/\Delta E \sim 10^4$, $B \approx 20 \text{ kT}$ is required for approximately 2 keV transitions. These fields are theoretically feasible in the MagLIF experiments, indicating the possibility of making these measurements.

An alternative technique to extracting magnetic field information from spectra depends on the ability to perform polarization analysis. This technique relies on the feasibility of an instrument that can measure orthogonal polarizations of x-rays separately. This technique may be applied to any line since the relative intensities of the Zeeman components for any given line depend only on the quantum numbers $j$ and $m_j$, which are always good quantum numbers. The benefit here is that these intensities should be unaffected by departures from LS coupling. If the polarized components can be distinguished, then a measurement of the $\sigma$ and $\pi$ component profiles of a single line will show differences that depend only on the magnetic field.
Appendix A

Data Fitting

A.1 Parameter Estimation

Given a set of \( m \) experimentally measured data points \( D_m = \{(x_i, y_i)\}_{i=1}^m \), one can use statistical regression analysis to infer the underlying physical parameters that gave rise to the observed data. [44, 45] The most common way to approach this problem is by using a model based optimization, or fitting, procedure. A function \( f(x_i; \theta_1, \ldots, \theta_n) \) whose \( n \) independent variables \( \theta_i \) represent physical parameters must be chosen to simulate the observed data as accurately as possible. The objective of the fitting procedure is to minimize the difference (according to some metric) between the data set and the model by varying the set of parameters \( \theta \).

If certain assumptions about the data are met, a useful metric to quantify the difference between the data and the model is the chi-square function, given by equation A.1

\[
\chi^2 = \sum_{i=1}^{m} \frac{(y_i - f(x_i; \theta))^2}{\sigma_i^2}.
\]  
(A.1)
The use of this particular objective function is motivated by the concept of the likelihood function. The likelihood function characterizes the probability that a particular measurement outcome subject to random noise was observed, given a set of parameters into a fixed model. Thus, the procedure of adjusting the parameters of a model to best match the experimental data minimizes the residuals between the data and the model, or equivalently, maximizes the likelihood function. The exact form of equation A.1 is derived assuming the observation of \( m \) independent data points all sampled from a Gaussian distribution with variance \( \sigma^2_i \). Only when all observations are Gaussian can chi-square fitting and the built-in properties of the \( \chi^2 \)-distribution be applied to assess goodness-of-fit of the model to the data. In most cases, the number of observations is large enough that the statistical behavior of the data points can be approximated by a Gaussian distribution. [45]

In principle, the optimization of model parameters by minimizing the square residuals between the data and model does not require that the observations are Gaussian or even that the sampling distribution is known. These assumptions aid in making statistical claims about the uncertainty of the fits. A less stringent way to implement a regression model onto data is through the ordinary least squares (OLS) or the weighted least squares (WLS) method. We will see that both the chi-square fitting and OLS are special cases of the more general WLS. The objective function to minimize for WLS is

\[
\min \left\{ \sum_{i=1}^{m} w_i (y_i - f(x_i; \theta))^2 \right\}. \tag{A.2}
\]

The weight factor \( w_i \) is related to the measurement error \( \sigma_i \) of the data point \( y_i \). It is common to choose \( w_i = 1/\sigma_i^2 \) so that data points with higher variance are given less weight in the objective function. For the case when the data errors are Gaussian, the chi-square objective function is recovered.

If all of the data errors are the same, WLS simplifies to OLS. These objective
Appendix 1. *Data Fitting*

functions all require that the experimenter know the data errors before beginning data analysis. In most cases, this is not possible, so instead the errors may be estimated by examining the post-fit residual. By setting all weights to 1 and proceeding with the calculation of the optimal parameters, one can then take the residuals of the optimized fit and use its distribution to infer the variance of the data.

$$\sigma_{res} = \sqrt{\frac{\sum_{i=1}^{m}(y_i - f(x_i; \theta))^2}{m - n}}.$$  \hspace{1cm} (A.3)

The OLS and WLS have analytic solutions when the model is a linear function of the parameters. When the model is nonlinear in the parameters, the solution must be found iteratively. This is the case for all of our data since our spectra were formed by a convolution of ten Voigt profiles. The outline of the method may be compared to a multi-dimensional Newton’s method for finding roots of an equation since the objective function is minimized by solving for the set of parameters such that the first derivative of the objective function is zero. First, the Jacobian matrix, $A$, of first partial derivatives of $f$ is computed and evaluated at each $x_i$. Explicitly, $A_{ij} = \left( \frac{\partial f(x_i; \theta)}{\partial \theta_j} \right)$. The Jacobian matrix is used to advance the initial input for the parameters incrementally until the function is minimized to yield the best estimate of the parameters. One of the fundamental assumptions in employing the least squares minimization procedure is based on the central limit theorem. In the limit the residuals are well-behaved functions of the parameters, any likelihood function (or least squares objective function, equivalently) approaches a Gaussian distribution. This is important as it establishes that a global minimum will be found with this procedure. The well-behaved assumption is subject to breaking if, for example, the observed data is noisy or if the functional model is not a good representation of the data. \[45\]
A.2 Uncertainty Analysis

Once the objective function has been minimized, the formal variances and covariances of the parameter estimates can be computed via the covariance (or variance-covariance) matrix. The covariance matrix has the same dimension as the parameter vector along both axes (for this discussion, $n \times n$). Diagonal elements are the standard variances for the corresponding parameter in $\theta$, and off-diagonal matrix elements are covariances between parameters. The variance of a random variable quantifies the expectation value of the squared deviation from its mean, and also represents a one standard deviation, or 68%, confidence bound. The covariance is the variability between two random variables and tells if the variables are correlated. The covariance matrix is calculated from the Jacobian matrix, evaluated at the final fit parameters, where the matrix $W = \text{diag}\{w_i\}$,

$$C = (A^T W A)^{-1}.$$  \hfill (A.4)

For data with constant data errors, the weights factor out so the weight matrix need not be carried through calculations. For $w = 1/\sigma_0^2$, $C = \sigma_0^2 (A^T A)^{-1}$. If the measurement errors are not normally distributed, then the covariance matrix $C$ provides the formal error for the parameter estimates, assuming the parameters were sampled from a normal distribution. If the measurement errors are normally distributed, then the machinery of the $\chi^2$ distribution may be used to assess uncertainties between parameters further. The chi-squared function evaluated across a $k$-dimensional parameter space follows a chi-squared distribution with $k$ degrees of freedom. Often it is justifiable to forgo the normality assumption if the number of data points is much larger than the number of parameters. Restricting the uncertainty analysis to only one parameter essentially means taking the projection of the covariance matrix onto the basis vector corresponding to that parameter, which is exactly the formal standard error given by the diagonal elements of $C$. 
A.2.1 Confidence Intervals

We wish to formulate joint confidence intervals between the parameters of our model. This is done by performing a brute-force computation of $\chi^2$ across a grid of parameter space centered about the optimal parameters that yield the minimal chi-squared value, $\chi^2_{\text{min}}$. Varying the optimal parameters in any direction will cause $\chi^2$ to increase. The confidence boundary on the joint parameters is defined by the contour of constant $\chi^2$ greater than $\chi^2_{\text{min}}$ by a constant amount, $\Delta \chi^2$. The confidence interval, or variation from $\chi^2_{\text{min}}$, $\Delta \chi^2$ is chosen for a certain confidence level $p$ and is given by the chi-square distribution of $k$ degrees of freedom. [44]

In one dimension, the confidence interval is given by the standard deviation of a Gaussian distribution, in two dimensions, the confidence region is an ellipse, and in three dimensions the joint confidence region is an ellipsoid. Confidence regions greater than three dimensions are impossible to plot graphically, but by taking subsets of the parameters, one can still obtain bounds on the confidence of the estimate. Table A.1 shows maximum deviation from $\chi^2_{\text{min}}$ that stays within the $p$ percentile for a given number of parameters $k$. The contour line that is formed by varying $\chi^2$ by the amount specified for the table at the desired number of parameters and confidence level defines the confidence interval for each of the parameters at that confidence level. It is the projection of the confidence region onto the axis corresponding to a parameter that is the desired joint confidence interval for that parameter. From table A.1, the one parameter confidence intervals are given by the standard normal errors from the covariance matrix, as expected. For two parameters, the $1 - \sigma$ joint confidence ellipse is defined by the contour $\chi^2 = \chi^2_{\text{min}} + 2.30$. The projection of this region will be slightly larger than the one-dimensional confidence interval because we are now allowing a greater uncertainty due to the additional degree of freedom introduced by the other parameter. Essentially, the one-dimension projection of the uncertainty implicitly assumes that
all other parameters are fixed at their true value, discounting random error.

Again, if the data errors are not Gaussian, then the chi-squared objective function
does not follow its namesake probability distribution. For non-normal errors, one
can still vary the values of the chi-squared objective function, but the constant
contours must be determined via Monte Carlo methods if statistical significance
is to be assigned to the confidence regions. \[44\]

<table>
<thead>
<tr>
<th>Std. Dev.</th>
<th>p</th>
<th>$k = 1$</th>
<th>$k = 2$</th>
<th>$k = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1\sigma$</td>
<td>68.3%</td>
<td>1.0</td>
<td>2.30</td>
<td>3.53</td>
</tr>
<tr>
<td>$1.64\sigma$</td>
<td>90%</td>
<td>2.71</td>
<td>4.61</td>
<td>6.25</td>
</tr>
<tr>
<td>$2\sigma$</td>
<td>95.4%</td>
<td>4.0</td>
<td>6.17</td>
<td>8.02</td>
</tr>
<tr>
<td>$3\sigma$</td>
<td>99.7%</td>
<td>9.0</td>
<td>11.8</td>
<td>14.2</td>
</tr>
</tbody>
</table>

**Table A.1:** $\Delta \chi^2$ given by number of parameters (degrees of freedom of the
distribution) and confidence level.

### A.2.2 Estimation of data errors for optical photons

The physical process of measuring $n$ photons for a given pixel can be modeled by
a Poisson distribution. The Poisson distribution gives the probability of a number
of events occurring within a discrete interval in time and space, assuming the
events are independent of one another, they occur a constant rate, and an average
number of occurrences $\mu$ is known. Since the Poisson distribution measures the
probability of occurrence, $\mu > 0$. The probability of measuring $n$ photons in a
pixel given that the average value of photons expected in that pixel is $\mu$ is

$$Pr(n|\mu) = \frac{e^{-\mu} \mu^n}{n!}.$$ \hspace{1cm} (A.5)

In the limit of large $n$ and large $\mu$, the Poisson distribution tends towards the
Gaussian distribution with $\sigma^2 = \mu$. For $\mu > 10$, the Poisson distribution begins to
closely resemble the Gaussian distribution. We can interpret this result as meaning
that for a high signal count, the measured counts in a pixel, say $\mu^*$, was sampled from a distribution with mean $\mu = \mu^*$ and standard deviation $\sigma = \sqrt{\mu^*}$. Hence, the standard error of measurement for each pixel is simply the square root of the measured signal. We say this approximation is valid for the optical regime since there is a usually a large flux of photons. For cases when the photon count could fall below 10, more careful consideration of the signal standard deviation must be done.

A.2.3 Computational Method for Determining Uncertainties in Fitting Parameters

Typical spectra obtained for our experimental results may be fit via three parameters if the wavelength scale is fixed beforehand. The free parameters are the magnetic field magnitude, electron density, and amplitude (assuming opacity effects are negligible). We are most interested in the error estimates on $B$ and $n_e$, as these are the plasma quantities we wish to measure. To determine a confidence interval for $B$ and $n_e$ jointly, we fix the amplitude at the optimized value and compute a 2D grid of $\chi^2$ values about the neighborhood of the fitted values. This produces an ellipse of constant $\chi^2$, chosen to reflect a given accuracy level. This ellipse is projected onto the $(B, n_e)$ axes to get the confidence interval.

As discussed earlier, the basis for performing the search for the minima of the WLS objective function is that by the central limit theorem, the objective function tends towards a multivariate Gaussian, so a minimum does exist. However, for data with sufficient random noise or a low enough signal-to-noise ratio, the well-behaved normality feature of the objective function may not be upheld. This results in an objective function that is multi-modal, rather than being singly peaked to yield a single best estimate with definite, localized confidence regions. Multi-modal objective functions may indeed have a single global minimum that the least squares
algorithm may converge on to, but potentially also have many local minima. These local minima may, in fact, be the “true” model values but due to the quality of the data, they do not globally minimize the residuals. The only way to determine if the objective function has a single global minimum is to do a full parameter-space search by computing $\chi^2$ on a 3D grid.

The method we used to quickly find secondary minima was to take 2D slices of the parameter space, along various values of fixed $B$. This creates contours of constant $\chi^2$ in the $n_e - A$ plane. Rather than analyze many such slices arbitrarily, two operations we performed on the slices. The minimum $\chi^2$ value along an axis of fixed $n_e$ or $A$ was recorded, along with its associated parameter values. In this way, the 2D sets of values are flattened to a 1D list, whose elements are the minima from their row (or column). In addition to storing these values for each constant $B$, the array of the projections of the minima across an axis may be stacked and displayed as an image to get a quick visual interpretation of the distribution of the least squares. To get a quantitative assessment of the spread of the optimized parameter values, the script returns a sorted value of $\chi^2$ along with its associated $\theta$.

If the objective function were indeed a Gaussian, then the tuples of parameters associated with the minimum $\chi^2$ function values would be the same for the fixed $n_e$ or fixed amplitude projections, and they will all be bounded within a single convex region of parameter space. For multi-modal distributions, not only will the lists of $\chi^2$ be different depending on the projection, but the parameters associated with the smallest objective value will not be contained within a single region of parameter space.

Evidently, for a multi-modal objective function, any assumptions about the statistical nature of its distribution must be left behind. Rather than making strict
statistical claims about the confidence region, we provide maximum range of parameters as an error bar, while loosely following the $\Delta \chi^2$ defined by the distribution. For example, in our three parameter fit, $\Delta \chi^2 \approx 15$ was used to determine the cut off range.
Appendix B

Electron Density Calculations

The Cowan Atomic Code outputs all possible pairs of transitions given an input of level populations of a neutral atom or ion. The full output of the code consists of the ordered energy levels of the fine structure states with the ground state energy configuration at 0, the reduced electric dipole matrix element between allowed dipole transitions, and the energy of the transition in units of eV, km$^{-1}$, Å.

We use Gaunt’s semiempirical impact approximation formula for the Stark half-width (FWHM), together with the semiempirical Gaunt factor.[16, 21, 46] This formulation is accurate for singly ionized atom to within ±50%.

\[
W = N \frac{8\pi^2\hbar^2}{3\sqrt{3}m^2} \left( \frac{2m}{\pi kT} \right)^{1/2} \left[ \sum_{i'} \left| \langle i' | r | i \rangle \right|^2 g \left( \frac{E}{\Delta E_{ir}} \Delta E_{ri} \right) + \sum_{i'} \left| \langle i' | r | i \rangle \right|^2 g \left( \frac{E}{\Delta E_{ir}} \right) \right] 
\]

(B.1)

where $E = 3kT/2$ is the energy of the perturbing electron, $E_H$ is the energy of the ground state of Hydrogen, $\Delta E_{j'j}$ is the difference in energy between states $j$ and $j'$, and the summation is performed over the perturbing states ($i'$, $f'$) to the initial ($i$) or final ($f$) level.
The semiempirical Gaunt factor from the van Regemorter formula is

\[
g(x) = \begin{cases} 
0.2 & \text{if } x < 2 \\
(0.33 - 0.3x^{-1} + 0.8x^{-2}) \log x & \text{if } x > 2 \text{ and } \Delta n = 0 \\
(0.276 - 0.18x^{-1}) \log x & \text{if } x > 2 \text{ and } \Delta n \neq 0 
\end{cases}
\]  

(B.2)

where \( x = \frac{3}{2} kT/|E_i - E_f| \). According to Griem, if the energy of the perturbing electron is small compared to the energy difference between the level and the perturbing level, the gaunt factor is a constant.

To calculate the Stark broadening a doublet pair, first the user specifies the labels of the three states (two upper, one lower) according to the labels made by the output of the Cowan code. For each of these initial (i) or final (f) states, the script finds all perturbing states (i', f'), i.e. any states that have a non-zero radial matrix element with either the upper or lower levels (\( \langle i' | r | i \rangle = \langle f' | r | f \rangle = 0 \)). Once the perturbing states are found, the summation in the brackets of Equation B.1 is calculated in the following steps. For each pair \((j, j_0)\), first the square of the coordinate operator matrix element is obtained from the reduced electric dipole moment, then using \( \Delta E_{j', j} \) and \( \Delta n_{j', j} \), the semiempirical gaunt factor is calculated using Equation B.2.

In terms of the spherical tensor operator of rank \( k = 1 \), the square of the coordinate operator matrix element between states \( j \) and \( j' \) is expressed as

\[
|\langle j' | r | j \rangle|^2 = \langle j' | T_{1-1}^1 | j \rangle^2 + \langle j' | T_{00}^1 | j \rangle^2 + \langle j' | T_{11}^1 | j \rangle^2.
\]  

(B.3)

A result from the Wigner-Eckart Theorem is the following representation of any matrix element in which all angular momentum projection information is factored into one term,
\[ \langle jm | T^k_q | j'm' \rangle = (-1)^{j-m} \begin{pmatrix} j & k & j' \\ -m & q & m' \end{pmatrix} \langle j | T^k | j' \rangle, \]  

(B.4)

where \( q \) are the \( 2k + 1 \) components of the rank \( k \) tensor operator, the \( 3 \times 2 \) matrix is called the Wigner \( 3 - j \) symbol, and the inner product is called the reduced matrix element. Cowan’s code calculates the reduced electric dipole moment, the product of the electric charge and reduced radial matrix element. Each component of the coordinate operator matrix element was summed over the substates of the level’s angular momentum \( j \) and averaged over the substates of the perturbing state’s angular momentum \( j' \).
Appendix C

Summary of Relevant Plasma Parameters

In this appendix, a summary of the relevant plasma parameters is collected and discussed. In-depth derivations are beyond the scope of this section; it primarily serves as a reference of frequently used terms and formulae.

C.1 Collisions and Transport Processes

A plasma is a collection of charged particles that maintains quasi-neutrality, meaning that in the absence of external forces, the net electric charge of the plasma is zero. Locally within the plasma bulk, microscopic space charge fields may exist due to charge separation. The Debye length, $\lambda_D$, is the characteristic length over which the electrostatic potential of a charged species is shielded by a factor of $1/e$ (in this context, $e$ is Euler’s number, in following formulas, it will signify the elementary charge)

$$\lambda_D = \sqrt{\frac{\epsilon_0 T_e \ [eV]}{n_e e^2}}. \quad (C.1)$$
Within $\lambda_D$, charged particles may interact with one another. The particles are free to move about the plasma, and they neutralize regions of excess space charge, shielding the electrostatic fields of other particles to approximately $\lambda_D$. Effectively, particles only interact with each other if they are contained within the same Debye sphere.

One of the fundamental properties of the plasmas is the collective behavior of the charged species. When the plasma experiences a perturbation, for example, a small charge separation within the plasma, the resulting electric field will accelerate the electrons to neutralize the space charge. Through the attractive Coulomb force exerted on the electrons by the ions, the electrons undergo periodic oscillations about the relatively stationary ions with a natural frequency of oscillation called the plasma frequency. The period of an oscillation provides a characteristic time scale of the plasma on which various other processes occur. The electron plasma frequency, $\omega_p e$, is inversely proportional to the electron mass. The plasma frequency of ions will be approximately $50\sqrt{m_i}$ times smaller

$$
\omega_{pe} = \sqrt{\frac{n_e e^2}{m_e \epsilon_0}}, \quad \text{(C.2)}
$$

$$
\omega_{pi} = \sqrt{\frac{n_i Z^2 e^2}{m_i \epsilon_0}}, \quad \text{(C.3)}
$$

The primary mechanism that acts to dampen the plasma oscillations are plasma collisions. The collisionality of the plasma determines transport processes. Charged species within a plasma scatter off each other through Coulomb collisions, as opposed to classical ‘hard’ scattering. Particles only interact with other particles within one Debye sphere, so the task of determining the collision frequency of electrons and ions is to determine the collective effects of the Coulomb collisions (scattering) from all other particles within the Debye sphere. Because there are a large number of particles within a Debye sphere, the net effect of motion that
a single charged particle experience is random. The way the effects of trajectory altering electrostatic potentials are accounted for is by first considering limiting cases of particle deflection off of a force center. The maximum impact parameter that will cause an effect on the particle’s trajectory will be \( \lambda_D \). The Coulomb logarithm, \( \lambda \equiv \ln \Lambda \), is the term that cumulatively takes into account all effects of small angle Coulomb scattering events up to the maximum impact parameter. Its value is often an approximation due to the random nature of the particle interactions. However, for most plasmas, \( \ln \Lambda \gg 1 \), and variations in the accuracy of the formula typically varies by \( 5 - 10\% \).

Through the Coulomb collisions, there is a net loss of momentum from the deflected particle. This effect is called the Coulomb collision frictional force, and gives context for defining a collision frequency. For an electron moving through the plasma sampled from the Maxwellian distribution centered about a velocity \( v \) less than its thermal speed, \( v_{th} = \sqrt{2T_e/m_e} \), the average momentum relaxation rate for the Maxwellian distribution of electrons through the fixed ions is called the electron-ion collision frequency,

\[
\nu_e \left[ \text{s}^{-1} \right] = \frac{5 \times 10^{-11} n_e \left[ \text{m}^{-3} \right] Z}{T_e^{3/2} \left[ \text{eV} \right]} \left( \frac{\ln \Lambda}{17} \right),
\]

where \( Z \) is the ionization number.

The collisions between electrons and ions within a plasma can be thought of as a source of friction. In the presence of a macroscopic electric field \( \mathbf{E} \), the electrons in the plasma will be accelerated and create a current. The magnitude of the current will be proportional to the strength of the electric field accelerating the electrons but limited by the resistive effects of collisions between particles. In this way, the plasma electrical conductivity will be defined. A particle-ensemble analog to Ohm’s law is found by taking the Maxwellian average of the electron momentum density equation that results when considering the flow of electrons relative to
the stationary ions. The equation for the electron momentum density due to the external electric field and the relative inertia of the electron flow to the ion flow is given by

$$m_e n_e \frac{d\mathbf{v}_e}{dt} = -e n_e E - m_e n_e \nu_e (\mathbf{v}_e - \mathbf{v}_i). \quad (C.5)$$

Taking the steady state solution, i.e. for times much longer than the collision period, we obtain Ohm’s law,

$$\mathbf{J} = -n_e e (\mathbf{v}_e - \mathbf{v}_i) = \sigma_0 \mathbf{E}. \quad (C.6)$$

In this equation, the plasma conductivity is $\sigma_0$ and the plasma diffusivity is given by the inverse, $\eta$:

$$\sigma_0 = \frac{n_e e^2}{m_e \nu_e} \equiv \frac{1}{\mu_0 \eta}. \quad (C.7)$$

The formulas for conductivity and diffusivity are the same in both cgs and SI units. However, care must be taken when converting between the two conventions.

**Remark C.1.** Converting diffusivity from cgs to SI:

$$\sigma_0 \text{ [cgs]} = \frac{n_e \, [\text{cm}^{-3}] \, e^2 \, [\text{statCoul}]^2}{m_e \, [\text{g}] \nu_e \, [s^{-1}]} \quad \text{and}$$

$$\sigma_0 \text{ [SI]} = \frac{n_e \, [\text{m}^{-3}] \, e^2 \, [\text{Coul}]^2}{m_e \, [\text{kg}] \nu_e \, [s^{-1}]}$$

$$= \frac{10^6 n_e \, [\text{cm}^{-3}] \, e^2 \, [\text{Coul}]^2}{10^3 m_e \, [\text{g}] \nu_e \, [s^{-1}]} \cdot \left(\frac{10c}{10c}\right)^2$$

$$= \frac{10^9 n_e \, [\text{cm}^{-3}] \, e^2 \, [\text{statCoul}]^2}{m_e \, [\text{g}] \nu_e \, [s^{-1}]}$$

$$= \frac{10^7}{c^2} \sigma \text{ [cgs]}$$
Magnetic diffusivity, $\eta$, may be calculated by the following formula which holds true in SI or cgs units by conversion through a factor of $4\pi\epsilon_0$:

$$
\eta \text{ [SI]} = \frac{1}{\mu_0 \sigma_0 \text{ [SI]}}
$$

$$
= \frac{1}{\mu_0 10^7 \sigma_0 \text{ [cgs]}}
$$

$$
= \frac{1}{4\pi \times 10^{-7} 16\pi \sigma_0 \text{ [cgs]}}
$$

since $\mu_0 \equiv 4\pi \times 10^{-7}$

$$
= \frac{1}{4\pi \mu_0 \epsilon_0 \sigma_0 \text{ [cgs]}}
$$

$$
= \frac{1}{4\pi\epsilon_0 \mu_0 \sigma_0 \text{ [cgs]}}
$$

$$
= \frac{1}{4\pi\epsilon_0 \mu_0 \sigma_0 \text{ [cgs]}} = \frac{1}{4\pi\epsilon_0} \eta \text{ [cgs]}
$$

C.2 Magnetic Field Interactions

In the presence of an external magnetic field, the charged particles experience uniform circular motion about the field lines. The radius of the orbit is called the gyroradius, or Larmor radius and the frequency of precession is called the gyrofrequency, or Larmor frequency. The Lorentz force $F_L = q(v \times B)$ acts only on the component of the velocity perpendicular to the magnetic field. So the gyroradii and gyrofrequencies of the electrons and ions only depend on $v_\perp$. The charged particles continue to move along field lines with $v_\parallel = v \cdot B$.

The electron and ion Larmor frequencies are given by

$$
\Omega_{ce} = \frac{eB}{m_e}
$$

(C.8)

$$
\Omega_{ci} = \frac{ZeB}{m_i}
$$

(C.9)
The electron and ion Larmor radii are given by

$$\rho_{ce} = \frac{v_{Te}}{\omega_{ce}}$$ \hspace{1cm} (C.10)

$$\rho_{ci} = \frac{v_{Ti}}{\omega_{ci}}$$ \hspace{1cm} (C.11)

### C.3 Derived MHD Quantities

Magnetohydrodynamics (MHD) describes plasma behavior that evolves slowly compared to the three fundamental plasma time scales: the plasma frequency, the electron-ion collision frequency, and the ion gyrofrequency.

For example, from the momentum equation, taking the ratio of the plasma pressure force to the magnetic pressure, the parameter called the plasma $\beta$ is defined as

$$\beta = \frac{P}{B^2/2\mu_0}$$ \hspace{1cm} (C.12)

For a plasma with $\beta \gg 1$, the plasma pressure dominates; for a plasma with $\beta \ll 1$, the magnetic pressure dominates the plasma dynamics.

To gain insight to how a magnetic field interacts with a plasma, consider the generalized Ohm’s law: [47]

$$E = -v \times B + \eta J + \frac{1}{n_e e}(J \times B - \nabla P_e),$$ \hspace{1cm} (C.13)

where $v$ is the ion velocity, $B$ is the magnetic field, $\eta$ is the magnetic diffusivity, $J$ is the current density, and $P_e$ is the electron pressure tensor. The generalized Ohm’s law, together with Faraday’s law, $\nabla \times E = -\partial B/\partial t$, gives the induction equation of MHD. The relative importance of the various terms of the generalized Ohm’s law (or the induction equation) gives information on which effects dominate the
plasma system. To obtain a simplified version of the induction equation, we neglect the electron inertia (pressure) term and substitute equation C.13 into Faraday’s law:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \eta \nabla \times \mathbf{J} - \nabla \times \left( \frac{\mathbf{J} \times \mathbf{B}}{n_e e} \right). \quad (C.14)$$

We use Ampere’s law (while neglecting the displacement current), $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$, to eliminate $\mathbf{J}$. We also invoke the vector identity $\nabla \times (\nabla \times \mathbf{a}) = \nabla (\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a}$ and the fact that the magnetic field has zero divergence ($\nabla \cdot \mathbf{B} = 0$) to obtain the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} - \nabla \times \left( \frac{\mathbf{J} \times \mathbf{B}}{n_e e} \right). \quad (C.15)$$

The first term on the right hand side describes the conduction of the magnetic field. The second term describes the diffusion of the magnetic field; its coefficient $\eta$ is the magnetic diffusivity introduced earlier. The third term is the Hall term.

### C.3.1 The Diffusive Coefficient and Resistive Diffusion

First consider the ratio of the conduction term of equation C.15 to the diffusive term. We approximate $\nabla \sim 1/L_x$, where the subscript denotes the characteristic length scale of the variable that is being spatially differentiated

$$\frac{|\nabla \times (\mathbf{v} \times \mathbf{B})|}{|\eta \nabla^2 \mathbf{B}|} \sim \frac{vB/L_B}{\eta B/L_B^2} = \frac{vL_B}{\eta} \equiv R_M. \quad (C.16)$$

This dimensionless parameter is called the magnetic Reynolds number ($R_M$) and provides a measure of how conductive or resistive the plasma is. Essential conclusions about plasma dynamics can be made from the magnitude of $R_M$. Alternatively, the inverse ratio is called the diffusive coefficient, $D = 1/R_M$. 
When particle collisions are the dominating process within the plasma, magnetic field lines can diffuse into the plasma through resistive diffusion when a density or temperature gradients exist. \cite{48} The diffusivity of the plasma is aptly expressed in the induction equation as the coefficient of the diffusive term,

\[
\frac{\partial B}{\partial t} = \eta \nabla^2 B.
\] (C.17)

The magnetic diffusivity of the plasma, $\eta$ (in cgs units), can be written in terms of the plasma conductivity, $\sigma_0$. The conductivity $\sigma_0$ in cgs is given by

\[
\sigma_0 \text{ [s}^{-1}] = \frac{n_e e^2 \text{ [statcoul]} \tau_e}{m_e \text{ [g]}}.
\] (C.18)

The magnetic diffusivity, in SI units, from the cgs conductivity is given by

\[
\eta \text{ [m}^2 \text{s}^{-1}] = \frac{1}{4\pi \epsilon_0 \mu_0} \frac{1}{\sigma_0}.
\] (C.19)

From the diffusivity (or equivalently, the conductivity), the diffusive velocity $v_D$ and diffusive penetration time can be calculated. The velocity with which the magnetic field penetrates a plasma with characteristic length scale $L_B$ over which $B$ varies is found from the diffusivity,

\[
v_D \text{ [m s}^{-1}] = \frac{\eta \text{ [m}^2 \text{s}^{-1}]}{L_B}.
\] (C.20)

The characteristic time scale over which the magnetic field penetrates the plasma through resistive diffusion is found by a similar manipulation:

\[
\tau_D \text{ [s]} = \frac{L_B}{v_D} = \frac{L_B^2}{\eta}.
\] (C.21)
For the case in which $R_M \ll 1 (D \gg 1)$, the evolution of the magnetic field is dominated by the diffusive term of the induction equation. In the diffusive regime, the plasma can be said to be in a more resistive state, or dominated by collisions. The magnetic field lines are allowed to move independently of the bulk plasma flow and dissipate.

When $R_M \ll 1 (D \gg 1)$, advection of the magnetic field dominates the dynamics, and the plasma behaves conductively. In this case, the magnetic field does not penetrate the plasma, as it would not in an ideal conductor. Instead, the magnetic field lines are 'frozen' into the plasma flow and move with it.

Recall that $\eta \propto \sigma_0^{-1} \propto \nu_e \propto T_e^{-3/2}$. So for higher $T_e$, the diffusivity $\eta$ drops, leading both a lower $v_D$ and higher $\tau_D$. For higher temperatures, the plasma is more conductive ($R_M \gg 1$) and effectively shields the magnetic field from penetrating the plasma. Conversely, when $T_e$ is lower, that leads to faster penetration velocities and subsequently faster penetration times over the same length scale, i.e. the plasma behaves resistively ($R_M \ll 1$).

### C.3.2 The Hall Coefficient

The Hall coefficient is defined as the ratio of the Hall term to the conductive term of the generalized Ohm’s law. Taking the ratio of the Hall term to the conductive term of the Ohms’ law:

\[
H = \frac{|\mathbf{J} \times \mathbf{B}|/n_e e}{|\mathbf{v} \times \mathbf{B}|} = \frac{|(\nabla \times \mathbf{B}) \times \mathbf{B}|/\mu_0 n_e e}{|\mathbf{v} \times \mathbf{B}|} = \frac{1}{\mu_0 n_e e} \frac{B^2/L_B}{vB} = \frac{1}{\mu_0 n_e e vL_B},
\]

(C.22)
and using the following quantities,

\[ v_A = \frac{B}{\sqrt{\mu_0 m_i n_i}} \quad \text{Alfven velocity} \quad (C.23) \]

\[ n_e = Z n_i \quad \text{Plasma neutrality enforcement} \quad (C.24) \]

\[ \omega_{pi} = \sqrt{\frac{e^2 c^2 \mu_0 n_i Z^2}{m_i}} \quad \text{Plasma ion frequency} \quad (C.25) \]

the Hall term can be written as

\[ H \equiv \frac{c}{\omega_{pi}} \frac{v_A}{L_B} v. \quad (C.26) \]

Note that the Hall coefficient depends on \( L_B \). However, in most literature \( L_n \) is a more useful quantity because it is both easier to measure through traditional methods (interferometry) and because of observed dependence of Hall diffusion on density gradient length scales. There are two ways to justify the importance of \( L_n \) over \( L_B \) in the Hall coefficient. The first stems from a pressure equilibrium argument. If there is equilibrium between the thermal and magnetic pressure, \( n k_B T = B^2 / 2 \mu_0 \), then \( L_n \) and \( L_B \) become related by taking the divergence of both sides of this equation and approximating the gradient operator as \( \nabla \sim 1/L \), as follows.

\[ \nabla \cdot n k_B T = \nabla \cdot \left( \frac{B^2}{2 \mu_0} \right) \quad (C.27) \]

\[ \frac{\mu k_B T}{L_n} = \frac{2 B}{2 \mu_0 L_B} \quad \text{From equilibrium condition} \quad (C.28) \]

\[ L_B = 2 L_n \quad (C.29) \]

A second argument for the consideration of \( L_n \) for Hall diffusion applies only for Cartesian geometry. First we apply the product rule to the Hall term of equation
Appendix 3. Plasma Parameters

C.15 because in general the plasma will not be homogenous (i.e. $\nabla n_e \neq 0$),

$$\nabla \times \left[ \frac{1}{n_e} \mathbf{J} \times \mathbf{B} \right] = \frac{1}{n_e} \nabla \times (\mathbf{J} \times \mathbf{B}) - \frac{1}{e} (\mathbf{J} \times \mathbf{B}) \times \nabla \left( \frac{1}{n_e} \right)$$

(C.30)

$$= \frac{1}{n_e} \nabla \times (\mathbf{J} \times \mathbf{B}) - \frac{1}{n_e} \nabla n_e \times (\mathbf{J} \times \mathbf{B})$$

(C.31)

Next we use Ampere’s Law as before to eliminate the current density from equation C.31 and show that the first term of the expanded Hall term is always zero in Cartesian coordinates [38],

$$\frac{1}{n_e \mu_0} \left[ \nabla \times ((\nabla \times \mathbf{B}) \times \mathbf{B}) - \frac{1}{n_e} \nabla n_e \times ((\nabla \times \mathbf{B}) \times \mathbf{B}) \right]$$

(C.32)

$$= \frac{1}{n_e \mu_0} \left[ \left( (\nabla \times \mathbf{B}) (\nabla \cdot \mathbf{B}) - \mathbf{B} (\nabla \cdot (\nabla \times \mathbf{B})) \right)_{=0} - \frac{1}{n_e} \nabla n_e \times ((\nabla \times \mathbf{B}) \times \mathbf{B}) \right]$$

(C.33)

$$= -\frac{1}{n_e^2 \mu_0} \nabla n_e \times ((\nabla \times \mathbf{B}) \times \mathbf{B}).$$

(C.34)

where we have used the vector identity $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$. To obtain the Hall coefficient, we take the ratio of the simplified Hall term (equation C.34) with the convection term of the induction equation:

$$H_e = \frac{\frac{1}{n_e^2 \mu_0} \nabla n_e \times ((\nabla \times \mathbf{B}) \times \mathbf{B})}{|\nabla \times (\mathbf{v} \times \mathbf{B})|}$$

(C.35)

$$\sim \frac{1}{n_e \mu_0} \frac{n_e/L_n B^2/L_B}{v B/L_B} = \frac{B}{n_e \mu_0 v L_n}$$

(C.36)

$$= \frac{c/\omega_{pi}}{L_n} \frac{v_A}{v}.$$  

(C.37)

We are motivated to explicitly use $L_n$ in our discussion of Hall diffusion because of its overarching presence in literature and because spatial $n_e$ profiles are obtained by our experimental techniques. Next, we examine orders of magnitude of the
terms of the Hall coefficient (equation C.26) using $L_n$, but it must be noted that in many instances the length scale must be estimated so the small errors introduced by an imprecise definition of the length scale may be within the margin of error of the system itself.

$H$ becomes large when $L_n \ll c/\omega_{pi}$, or when $v_A \gg v$. The quantity $c/\omega_{pi}$ is called the ion skin depth. It is generally accepted in the literature that the Hall term becomes relevant for spatial scales smaller than the ion skin depth, or for time scales shorter than the ion gyroperiod, $\Omega_{ci}^{-1}$. The relevance of the Alfvén velocity being larger than the bulk plasma velocity is that the Hall effect is an effect instigated by an external magnetic field. This is contrasted by the magnetic Reynolds number which only depends on the electron-ion collision rate, which is primarily a function of $T_e$.

The Hall magnetic field penetration velocity [7] is given in SI units by

$$v_H [\text{m s}^{-1}] = \frac{B [T]}{2\mu_0 n_e [\text{m}^{-3}] eL_n [\text{m}]}.$$  \hfill (C.38)

This velocity is gotten by invoking the equilibrium condition $L_B = 2L_n$ to eliminate $L_B$ in favor of the more relevant length scale.

The time it takes for magnetic flux to penetrate the plasma by the characteristic length scale via the Hall effect is called the Hall penetration time,

$$\tau_H [\text{s}] = \frac{L_n}{v_H}.$$

(C.39)

From these equations, it is shown that for plasmas with slowly varying density gradients, the Hall penetration is very slow. Hall diffusion is enhanced by larger density gradients.
C.3.3 Alfvén Waves

In MHD, Alfvén waves are a small-amplitude, low-frequency wave that oscillates the ions and magnetic field. This wave propagates along the direction of the magnetic field at the Alfvén velocity (in SI units),

\[ v_A = \frac{B}{\sqrt{\mu_0 \rho}}, \]  

(C.40)

where \( \rho \) is the mass density of the ions. Alfvén waves are a transverse mode within the plasma, meaning that the perturbation of the magnetic field and the motion of the ions are perpendicular to the direction of the field. In a low \( \beta \) plasma the Alfvén velocity is the characteristic wave velocity. [38]

The Alfvén number is the ratio, \( A = v_A/v \).

C.3.4 Hall Diffusion

The Hall term of the induction equation may be written as [38]

\[ \nabla \times \left( \frac{\mathbf{J} \times \mathbf{B}}{n_e e} \right) = \frac{m_i e}{e} \nabla \times \frac{d\mathbf{v}_i}{dt}. \]  

(C.41)

If the ion acceleration, \( d\mathbf{v}_i/dt \equiv \mathbf{a}_i = \mathbf{a}_r \hat{r} + \mathbf{a}_\varphi \hat{\varphi} + \mathbf{a}_z \hat{z} \), then the curl in cylindrical coordinates may be used to expand the Hall term as follows,

\[ \frac{m_i e}{e} \nabla \times \frac{d\mathbf{v}_i}{dt} = \frac{m_i e}{e} \left[ \left( \frac{\partial a_z}{r} - \frac{\partial a_\varphi}{\partial r} \right) \hat{r} + \left( \frac{\partial a_r}{\partial \varphi} - \frac{\partial a_\varphi}{\partial z} \right) \hat{\varphi} + \frac{1}{r} \left( \frac{\partial a_r}{\partial r} - \frac{\partial a_\varphi}{\partial \varphi} \right) \hat{z} \right]. \]  

(C.42)

Under the presence of the azimuthal magnetic field, \( \mathbf{B} = B(r) \hat{\varphi} \), the ions will gyrate about the field lines in the \( r - z \) plane. The ions also have trajectories along the field lines, forming highly periodic filamentation in the plane of the magnetic field.
field. These structures were observed experimentally by Mostovych et al. in a Barium laser plasma expanding against a transverse magnetic field. [26] From the form of equation C.42, axial penetration due to the Hall effect is diminished at larger $r$, due to the $1/r$ dependence.

Along $\phi$, diffusion of the field into the plasma can only be understood if some perturbation in the field topology exists so that the field lines were kinked out of the azimuthal plane. If the field was purely azimuthal, then no diffusion would occur in this direction. Such a perturbation can come about from the initial expansion of the plume, in which the plasma expands as a conductive sheath so that initially the magnetic field lines are frozen into the plasma. In this way, as the hot plasma expands and excludes the field, the field lines get concentrated at the boundaries of the diamagnetic cavity. As the plasma cools, the field can begin diffusion back into the volume of the plume. From the Hall term expansion, azimuthal Hall diffusion occurs if $\partial a_r/\partial z - \partial a_z/\partial r \neq 0$. This form of the Hall term also suggests that plasma anisotropies out of the azimuthal plane are necessary to have diffusion along $\phi$. 
Bibliography


