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University of Nevada, Reno

Application of Adomian Method to Model Concentration Near the Surface of a Rotating Disk

A thesis submitted in partial fulfillment
of the requirements for the degree of

Bachelor of Science in Applied Mathematics
with Honors

by
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Abstract:

The Adomian decomposition method is the recurrent procedure to obtain the approximate solution in the form of the series. It is named after George Adomian, who developed an approximate analytical method for solving nonlinear differential equations, both ordinary and partial. The Adomian decomposition method represents a step toward unified theory for partial differential equations (PDEs).

My mentor is Dr. Aleksey Telyakovskiy, and my task is to solve the diffusion type equation using the Adomian decomposition method. The main idea of studying a diffusion equation is to understand how concentration gradients change over time and space. We first derived an expression for the velocity field of the fluid near the disk. It represents a complicated three-dimensional structure with the no-slip condition on the disk surface. Using this expression for the velocity field, we obtained the concentration equation. We then apply Adomian decomposition to construct an approximate solution for the concentration in the vicinity of the rotating disk. We analyze such problem, since in chemical engineering you often need to model processes when there are rotating parts and chemical reaction is happening. We consider infinite rotating disk to make mathematical analysis simpler. It is an approximation of more realistic disk of a finite radius; still we are able to get an idea of the key features of the process. As an example of the specific real-world system, we can use Ethanol as solvent and Sodium Ethoxide (NaC_2H_5O) and Methyl Iodide (CH_3I) as reactants. Moreover, we calculated the Peclet number that represents the ratio of advection to diffusion of a physical quantity for this specific system, and we calculated the diffusion coefficient.

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Chapter 1: Introduction

Understanding the effect of a volume reaction on a diffusion of substance near a rotating disk is an interesting area of study in hydrodynamics and mass and heat transfer in chemical engineering. Our task is to solve and analyze the diffusion type equation using the Adomian decomposition method. The main idea of studying diffusion equation is to understand how concentration gradients change over time and space.

We first derive the expression for the velocity field of the fluid near the rotating disk. It represents a complicated three-dimensional structure with the no-slip condition on the disk surface. Various mathematicians such as: Leonhard Euler, d'Alembert, Lagrange, Laplace, and Poisson, to name some, have studied velocity fields appearing in fluids [7], [9]. Moreover, more advances were made by Claude-Louis Navier and George Gabriel Stokes. In our work, we follow the approach of Cochran [2]. Using his expression for the velocity field, we obtained the form of the diffusion equation. After that, we apply the Adomian decomposition method to construct an approximate solution for the system in the vicinity of the rotating disk.

The Adomian decomposition method is a recurrent procedure to obtain approximate solution in the form of a series. Applying this method, we approximate the nonlinear reaction term by an Adomian polynomials. By constructing higher degree polynomials, we can obtain the term of chemical kinetic reaction for a particular system.

To undertake this problem involving the volume reaction, we study several approaches to approximate the system of governing equations. First, we obtain the velocity field to construct the flow in our particular system. The Navier-Stokes equation describes the flow. We have a boundary value problem for this system of PDEs. To obtain a specific expression for the velocity, we use the technique of Cochran provided in his paper, “The Flow Due to a Rotating Disk” [2]. Using his approach we express the system of differential equations that describe the steady-state motion of an incompressible viscous fluid due to an infinite rotating disk. Specifically, using the boundary conditions we can solve the system of differential equations. It the process when we needed to obtain coefficients in series expression, we used numerical integration. The procedure is effective, and we have an analytical expression for the velocity field. (For the mathematical details see Appendix A.)

Using the above solution of the system of equations, the diffusion on a rotating disk gave us an idea of the behavior of the concentration gradients in the system. (We have only dependence on space, since we analyze the steady-state.) Our work is based on the approach introduced by Polyanin et al [5]. In the paper, they express the reaction in terms of steady-state mass transfer at the surface of a disk rotating around with its axis at a constant angular velocity. By applying the boundary conditions, his expression can hold the approximation of the characteristic in diffusion. That is to say, the system of equations holds the expression of the motion in the fluid becoming dimensionless. In our system, particles began to concentrate at the surface of the rotating disc. Using this fact, we can rearrange the system of the equations what exponentially decrease from the rotating disk. (For the mathematical details see Appendix B.)

Finally, we obtained the governing equation that posesses the described characteristics. The equation now can express the diffusion of a substance to a rotating disk and moreover we have volume reaction in the fluid. One of the terms is the expression for the chemical kinetics, and as we mentioned at the beginning of this paper we applied the Adomian decomposition method to express the nonlinear reaction term with Adomian polynomial. That is to say, the Adomian method constructs a solution in the form of a series:

$$u(z,t) = \sum_{n=0}^{\infty} u_n(z,t) \ , \qquad (1.1)$$

where terms are found consequently. To apply the Adomian method, we represent a differential equation as:

$$L_n + R_n + N_n = c_n \ , \qquad (1.2)$$

where L_n is a linear term that is easily invertible, R_n is the remainder of the linear operator, N_n is the nonlinear term and c_n is the right-hand side. The nonlinear term is decomposed into a special series where each term is represented by Adomian polynomial:

$$N_n = \sum_{n=0}^{\infty} A_n \ , \qquad (1.3)$$

The method originated from the analysis of the stochastic differential equation.

We use the Adomian decomposition method that it involves Adomian polynomials. To generate the expression for the Adomian polynomials A_n, we used the approach introduced in [8]. The algorithm for calculating the Adomian polynomials for nonlinear operators is based on parameterization. As an example of the specific real-world system that can be modeled by our equation, we take Ethanol as a solvent and Sodium Ethoxide (NaC2H5O) and Methyl Iodide (CH3I) as solutes. Moreover, we calculated upper bound on Peclet number that represents the ratio of advection to the diffusion of a physical quantity for this specific system. Finally, we calculated the diffusion coefficient. (For the mathematical details see Appendix C.)

Chapter 2:
Formulation of Problem and Solution Procedure

We assumed that the process is accompanied by an irreversible volume chemical reaction with a rate:

$$W_v = K_v F_v(C), \qquad (2.1)$$

where K_v represents the rate for volume chemical reaction and $F_v(C)$ represents the kinetic function of the volume reaction. The Navier-Stokes equations describe the closed system of equations of motion for a viscous incompressible (in our case) Newtonian fluid consisting of the continuity equation which is written in the form:

$$\frac{\partial V_X}{\partial X} + \frac{\partial V_Y}{\partial Y} + \frac{\partial V_Z}{\partial Z} = 0,$$

and three momentum equations:

$$\frac{\partial V_X}{\partial t} + V_X \frac{\partial V_X}{\partial X} + V_Y \frac{\partial V_X}{\partial Y} + V_Z \frac{\partial V_X}{\partial Z} = -\frac{1}{\rho}\frac{\partial P}{\partial X} + \nu\left(\frac{\partial^2 V_X}{\partial X^2} + \frac{\partial^2 V_X}{\partial Y^2} + \frac{\partial^2 V_X}{\partial Z^2}\right) + g_X,$$

$$\frac{\partial V_Y}{\partial t} + V_X \frac{\partial V_Y}{\partial X} + V_Y \frac{\partial V_Y}{\partial Y} + V_Z \frac{\partial V_Y}{\partial Z} = -\frac{1}{\rho}\frac{\partial P}{\partial Y} + \nu\left(\frac{\partial^2 V_Y}{\partial X^2} + \frac{\partial^2 V_Y}{\partial Y^2} + \frac{\partial^2 V_Y}{\partial Z^2}\right) + g_Y,$$

$$\frac{\partial V_Z}{\partial t} + V_X \frac{\partial V_Z}{\partial X} + V_Y \frac{\partial V_Z}{\partial Y} + V_Z \frac{\partial V_Z}{\partial Z} = -\frac{1}{\rho}\frac{\partial P}{\partial Z} + \nu\left(\frac{\partial^2 V_Z}{\partial X^2} + \frac{\partial^2 V_Z}{\partial Y^2} + \frac{\partial^2 V_Z}{\partial Z^2}\right) + g_Z.$$

All of the physical variables are expressed in the orthogonal Cartesian system $X,\ Y$ and $Z,\ t$ represents time, g terms express the mass force density component, and the constant ν represents the kinetic viscosity of the fluid. Also $V_X,\ V_Y,\ V_Z$ are components of the velocity vector, and pressure is denoted by P. All of these quantities are unknown, except the kinetic viscosity. Consider the flow caused by an infinite plane disk rotating at a constant angular velocity ω around axis perpendicular to the surface of the disk. The flow has a three dimensional structure. Using the cylindrical system of coordinates $R,\ \phi,\ Z$ for the Navier-Stokes equations and Laplace operator Δ, we could describe the velocity field. Here we assume that there is no dependence on the angle ϕ, so the independent variables are $R,\ Z$ where R the distance from the axis of rotating, and Z is the distance from the surface of the rotating disk. (The detailed expression is in Appendix C). After transforming the system of equations into cylindrical coordinates, we obtain:

$$\frac{\partial V_R}{\partial R} + \frac{\partial V_Z}{\partial Z} + \frac{V_R}{R} = 0,$$

$$V_R \frac{\partial V_R}{\partial Z} + V_Z \frac{\partial V_R}{\partial Z} - V_\varphi^2 = -\frac{1}{\rho}\frac{\partial P}{\partial R} + \nu(\Delta V_R - \frac{V_R}{R^2}),$$

$$V_R \frac{\partial V_\phi}{\partial R} + V_Z \frac{\partial V_\phi}{\partial Z} + \frac{V_R V_\phi}{R} = \nu(\Delta V_\phi - \frac{V_\phi}{R^2}),$$

$$V_R \frac{\partial V_Z}{\partial R} + V_Z \frac{\partial V_Z}{\partial Z} = -\frac{1}{\rho}\frac{\partial P}{\partial Z} + \nu \cdot \Delta V_Z,$$

$$\Delta = \frac{1}{R}\frac{\partial}{\partial Z}(R\frac{\partial P}{\partial R}) + \frac{\partial^2}{\partial Z^2}. \qquad (2.2)$$

Let us describe the system that undergoes volume reaction. Here c is the ratio of the concentration C, and the particular concentration at the disk surface is C_s, or $c = C/C_s$. c represents the normalized concentration of substance in the fluid, z represents the normalized distance from the surface of the rotating disk, k_v represents the constant rate of particular volume reaction, and $f_v(c)$ represents the chemical kinetic function describing the rate of volume reaction. The expression $f_v(c) = F_V(C)/F_V(C_S)$ represents the normalized rate of chemical volume reaction at a particular distance from the rotating disk. For the high Peclet numbers, which represent the ratio of advection to diffusion of a physical quantity for this specific system, the distribution of the concentration of the substance in the fluid is described in the following form:

$$\frac{d^2c}{dz^2} + Pe \cdot z^2 \frac{dc}{dz} = k_v \cdot f_v(c) \ ,$$

with boundary conditions being:

$$c_0 = 1,\ c(\infty) = 0. \tag{2.3}$$

The dimensionless variables and parameters are introduced as follows: a is the characteristic length (This is defined in Appendix A), ω is the angular velocity and v is the kinetic viscosity of the fluid:

$$z = \frac{Z}{a},\ Pe = 0.51\frac{v}{D},\ a = \frac{\sqrt{v}}{\omega},$$

$$k_v = \frac{a^2 \cdot K_v \cdot F_v(C_S)}{D \cdot C_S}. \tag{2.4}$$

Moreover, we decided that the chemical kinetic function is in the form of a third degree polynomial. Such forms appear in real-world chemical reactions, and this is the nonlinear term to which we apply the Adomian decomposition method.

The Adomian decomposition method is one of the approximate solution techniques and it is often used for the solution of differential equations. We selected the Adomian decomposition method since it allows us to deal with nonlinearities in a very general form. The majority of the methods construct solutions numerically, but we are interested in analytical solutions since it allows us to better understand the structure of the solutions. We looked at other methods too. We applied the general procedure of a reduction of our differential equation to a quasilinear equation in the normal form. The transformed system of the solutions helps the simplification and classification of equations (Solution of the Wronskian). However, this type of strategy led to a very complex expression in terms of exponential functions, which cannot be integrated in the closed form. Although we used the Adomian decomposition method, there is more than one approach to obtaining the Adomian polynomials [1], [3], [8]. In our study, we applied the Adomian decomposition method of Zhu [8] to construct the Adomian polynomials to obtain the solution to the governing equation. The chemical kinetic function is nonlinear, and Zhu's technique is promising, since it can be applied to any type of equation, including nonlinear equations.

Using Zhu's technique, we are able to describe the term $f_v(c)$ in term of Adomian polynomials as:

$$f_v(c) = \sum_{n=0}^{\infty} A_n \, . \tag{2.5}$$

Applying Zhu's method [8] for calculating an Adomian polynomial, A_n, with the initial concentration and the initial rate of change in concentration c, we were able to re-construct the nonlinearly behaving concentration in the form of polynomial expression. Once we decided on the rate of change $c'(0)$, we were able to solve the initial value problems in terms of c. We could obtain Adomian polynomials for $f_v(c)$, in terms of earlier obtained approximations. It is easier to deal with polynomials than with the original expression for $f_v(c)$. It is hard to solve boundary value problems, so we solved the initial value problem. It resulted in an unknown $c'(0)$ that we vary in the solution process.

$$c_0 = 1, \; c'(0) = c_i . \tag{2.6}$$

We obtain the first four Adomian polynomials (Detailed calculations are in Appendix C):

$$\begin{aligned}
A_0 &= k_v \cdot c_0^3 \; , \\
A_1 &= 3 \cdot k_v \cdot c_0^2 \cdot c_1 \; , \\
A_2 &= 3 \cdot k_v \cdot c_0 \cdot c_1^2 + c_0^2 \cdot c_2 \; , \\
A_3 &= 3 \cdot k_v \cdot c_0^2 \cdot c_3 + 6 \cdot k_v \cdot c_0 \cdot c_1 \cdot c_2 + k_v \cdot c_1^3 \, .
\end{aligned} \tag{2.7}$$

A_0 is used to calculate c_1; A_1 is used to calculate c_2; etc... The accuracy of the solution is dependent on the number of iterations of the Adomian decomposition method. The calculation of Adomian polynomials is a work-intensive procedure. As a result, we represent our work to the first four Adomian polynomials for our specific nonlinear term. At this point, we represent our equation in the form of a general differential equation:

$$L_n c + R_n c = N_n c, \tag{2.8}$$

and the inverse transformation of the general differential equation:

$$L^{-1}Lc = L^{-1}(Rc) + L^{-1}(Nc).$$

Using the above expression, and applying it to equation (2.2) with variables (2.4) and (2.5), we construct the solution in the form:

$$c(y) = c(0) + c'(0)y + L^{-1}(Rc) + L^{-1}(Nc), \tag{2.9}$$

substituting Adomian polynomial (2.4) to (2.6):

$$c(y) = c(0) + c'(0)y + L^{-1}(Rc) + L^{-1}\left(\sum_{n=0}^{\infty} A_n\right). \tag{2.10}$$

Our task is to find an approximate solution to he equation (2.8). Later, using each term in (2.5) we will obtain each term of $c_n(y)$.

Specifically,

$$c_0 = 1 + c'(0)y,$$

For calculation purpose, $c'(0) \to c$ in the expressions below:

$$c_1 = \frac{1}{20}k_v \cdot c^3 \cdot y^5 + \frac{1}{4}K_v \cdot c^2 \cdot y^4 + \frac{1}{2}k_v \cdot c \cdot y^3 + \frac{1}{2}k_v \cdot y^2 - \frac{1}{12}Pe \cdot c \cdot y^4,$$

$$
\begin{aligned}
c_2 = {} & \frac{1}{480} k_v^2 \cdot c^5 \cdot y^9 + \frac{1}{8} (\frac{3}{70} \cdot k_v^2 \cdot c^4 + \frac{3}{7} \cdot k_v \cdot c^2 (-\frac{1}{12} Pe \cdot c + \frac{1}{4} \cdot k_v \cdot c^2)) y^8 \\
& + \frac{1}{7} (\frac{11}{40} k_v^2 \cdot c^3 + k_v \cdot c (-\frac{1}{12} Pe \cdot c + \frac{1}{4} \cdot k_v \cdot c^2)) y^7 \\
& + \frac{1}{6} (\frac{3}{5} \cdot k_v (-\frac{1}{12} Pe \cdot c + \frac{1}{4} \cdot K_v \cdot c^2) + \frac{9}{10} \cdot k_v^2 \cdot c^2) y^6 \\
& + \frac{9}{40} \cdot k_v^2 \cdot c y^5 + \frac{1}{8} \cdot k_v^2 \cdot y^4 - \frac{1}{224} \cdot Pe \cdot k_v \cdot c^3 \cdot y^8 \\
& - \frac{1}{42} \cdot Pe \cdot \left(-\frac{1}{3} \cdot Pe \cdot c + k_v \cdot c^2\right) y^7 \\
& - \frac{1}{20} \cdot Pe \cdot k_v c \cdot y^6 - \frac{1}{20} \cdot Pe \cdot k_v \cdot y^5.
\end{aligned}
$$

(2.11)

Similarly, c_3 is determined:

$$
\begin{aligned}
c_3 := {} & \frac{1}{20800} k_v^3 c^7 y^{13} + \frac{1}{1600} k_v^3 c^6 y^{12} + \frac{3}{800} k_v^3 c^5 y^{11} + \frac{11}{800} k_v^3 c^4 y^{10} + \frac{1}{9} (\frac{9}{80} k_v^3 c^3 + \frac{3}{8} k_v c (\\
& - \frac{1}{19200000} Pe c^4 k_v + \frac{1}{2} k_v^2 c^2)) y^9 + \frac{1}{8} (\frac{3}{7} k_v (- \frac{1}{19200000} Pe c^4 k_v + \frac{1}{2} k_v^2 c^2) + \frac{3}{7} k_v c (\\
& - \frac{1}{3840000} Pe k_v c^3 + \frac{1}{2} k_v^2 c)) y^8 + \frac{1}{7} (\frac{1}{2} k_v (- \frac{1}{3840000} Pe k_v c^3 + \frac{1}{2} k_v^2 c) + \frac{1}{2} k_v c (\\
& - \frac{1}{1920000} Pe c^2 k_v + \frac{1}{4} k_v^2)) y^7 + \frac{1}{6} (\frac{3}{5} k_v (- \frac{1}{1920000} Pe c^2 k_v + \frac{1}{4} k_v^2) - \frac{1}{3200000} k_v^2 c^2 Pe) y^6 \\
& + \frac{1}{5} (- \frac{1}{2560000} k_v^2 Pe c + \frac{1}{4915200000000} k_v c^3 Pe^2) y^5 + \frac{1}{4} (\frac{1}{3686400000000} k_v Pe^2 c^2 \\
& + \frac{1}{3} c^2 (\frac{1}{480} k_v^2 c^5 y^9 + \frac{1}{8} (\frac{3}{70} k_v^2 c^4 + \frac{3}{7} k_v c^2 (- \frac{1}{12} Pe c + \frac{1}{4} k_v c^2)) y^8 + \frac{1}{7} (\frac{11}{40} k_v^2 c^3 + k_v c (\\
& - \frac{1}{12} Pe c + \frac{1}{4} k_v c^2)) y^7 + \frac{1}{6} (\frac{3}{5} k_v (- \frac{1}{12} Pe c + \frac{1}{4} k_v c^2) + \frac{9}{10} k_v^2 c^2) y^6 + \frac{9}{40} k_v^2 c y^5 + \frac{1}{8} k_v^2 y^4 \\
& - \frac{1}{224} Pe k_v c^3 y^8 - \frac{1}{42} Pe (- \frac{1}{3} Pe c + k_v c^2) y^7 - \frac{1}{20} Pe k_v c y^6 - \frac{1}{20} Pe k_v y^5)) y^4 + \frac{1}{3} c (\frac{1}{480} \\
& k_v^2 c^5 y^9 + \frac{1}{8} (\frac{3}{70} k_v^2 c^4 + \frac{3}{7} k_v c^2 (- \frac{1}{12} Pe c + \frac{1}{4} k_v c^2)) y^8 + \frac{1}{7} (\frac{11}{40} k_v^2 c^3 + k_v c (- \frac{1}{12} Pe c \\
& + \frac{1}{4} k_v c^2)) y^7 + \frac{1}{6} (\frac{3}{5} k_v (- \frac{1}{12} Pe c + \frac{1}{4} k_v c^2) + \frac{9}{10} k_v^2 c^2) y^6 + \frac{9}{40} k_v^2 c y^5 + \frac{1}{8} k_v^2 y^4 \\
& - \frac{1}{224} Pe k_v c^3 y^8 - \frac{1}{42} Pe (- \frac{1}{3} Pe c + k_v c^2) y^7 - \frac{1}{20} Pe k_v c y^6 - \frac{1}{20} Pe k_v y^5) y^3 + \frac{1}{2} (\frac{1}{480} k_v^2 c^5 y^9 \\
& + \frac{1}{8} (\frac{3}{70} k_v^2 c^4 + \frac{3}{7} k_v c^2 (- \frac{1}{12} Pe c + \frac{1}{4} k_v c^2)) y^8 + \frac{1}{7} (\frac{11}{40} k_v^2 c^3 + k_v c (- \frac{1}{12} Pe c \\
& + \frac{1}{4} k_v c^2)) y^7 + \frac{1}{6} (\frac{3}{5} k_v (- \frac{1}{12} Pe c + \frac{1}{4} k_v c^2) + \frac{9}{10} k_v^2 c^2) y^6 + \frac{9}{40} k_v^2 c y^5 + \frac{1}{8} k_v^2 y^4 \\
& - \frac{1}{224} Pe k_v c^3 y^8 - \frac{1}{42} Pe (- \frac{1}{3} Pe c + k_v c^2) y^7 - \frac{1}{20} Pe k_v c y^6 - \frac{1}{20} Pe k_v y^5) y^2 + \frac{1}{2} (\frac{3}{160} k_v^2 c^5 y^8 \\
& + (\frac{3}{70} k_v^2 c^4 + \frac{3}{7} k_v c^2 (- \frac{1}{12} Pe c + \frac{1}{4} k_v c^2)) y^7 + (\frac{11}{40} k_v^2 c^3 + k_v c (- \frac{1}{12} Pe c + \frac{1}{4} k_v c^2)) y^6 \\
& + (\frac{3}{5} k_v (- \frac{1}{12} Pe c + \frac{1}{4} k_v c^2) + \frac{9}{10} k_v^2 c^2) y^5 + \frac{9}{8} k_v^2 c y^4 + \frac{1}{2} k_v^2 y^3 - \frac{1}{28} Pe k_v c^3 y^7 - \frac{1}{6} Pe (\\
& - \frac{1}{3} Pe c + k_v c^2) y^6 - \frac{3}{10} Pe k_v c y^5 - \frac{1}{4} Pe k_v y^4) y^2 :
\end{aligned}
$$

(2.12)

and the total concentration is expressed in the form of the series:

$$C = c_0 + c_1 + c_2 + c_3 + \ldots.,$$

C represents the total concentration for a specific distance y from the surface of the disk.

So the approximate solution is:

$$C = \sum_{i=0}^{n} c_i \,. \tag{2.13}$$

This result is our approximate solution for the concentration of substance near the rotating disk when the substance undergoes the irreversible volume chemical reaction.

Chapter 3:
Results and Discussion

We obtained the following approximation for the concentration:

$$
\begin{aligned}
& C := 1 + \frac{9}{40} k_v^2 c y^5 + \frac{1}{480} k_v^2 c^5 y^9 + \frac{1}{8} k_v^2 y^4 + \frac{1}{3} c \left(\frac{1}{480} k_v^2 c^5 y^9 + \frac{1}{8} \left(\frac{3}{70} k_v^2 c^4 + \frac{3}{7} k_v c^2 \left(-\frac{1}{12} Pe\, c \right.\right.\right. \\
& \left.\left. + \frac{1}{4} k_v c^2 \right) \right) y^8 + \frac{1}{7} \left(\frac{11}{40} k_v^2 c^3 + k_v c \left(-\frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2 \right) \right) y^7 + \frac{1}{6} \left(\frac{3}{5} k_v \left(-\frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2 \right) \right. \\
& \left. + \frac{9}{10} k_v^2 c^2 \right) y^6 + \frac{9}{40} k_v^2 c y^5 + \frac{1}{8} k_v^2 y^4 - \frac{1}{224} Pe\, k_v c^3 y^8 - \frac{1}{42} Pe \left(-\frac{1}{3} Pe\, c + k_v c^2 \right) y^7 \\
& \left. - \frac{1}{20} Pe\, k_v c y^6 - \frac{1}{20} Pe\, k_v y^5 \right) y^3 - \frac{1}{20} Pe\, k_v y^5 + \frac{1}{1600} k_v^3 c^6 y^{12} + \frac{11}{800} k_v^3 c^4 y^{10} + \frac{1}{9} \left(\frac{9}{80} k_v^3 c^3 \right. \\
& \left. + \frac{3}{8} k_v c \left(-\frac{1}{19200000} Pe\, c^4 k_v + \frac{1}{2} k_v^2 c^2 \right) \right) y^9 + \frac{1}{8} \left(\frac{3}{7} k_v \left(-\frac{1}{19200000} Pe\, c^4 k_v + \frac{1}{2} k_v^2 c^2 \right) \right. \\
& \left. + \frac{3}{7} k_v c \left(-\frac{1}{3840000} Pe\, k_v c^3 + \frac{1}{2} k_v^2 c \right) \right) y^8 + \frac{1}{7} \left(\frac{1}{2} k_v \left(-\frac{1}{3840000} Pe\, k_v c^3 + \frac{1}{2} k_v^2 c \right) + \frac{1}{2} k_v c \left(\right.\right. \\
& \left.\left. -\frac{1}{1920000} Pe\, c^2 k_v + \frac{1}{4} k_v^2 \right) \right) y^7 + \frac{1}{6} \left(\frac{3}{5} k_v \left(-\frac{1}{1920000} Pe\, c^2 k_v + \frac{1}{4} k_v^2 \right) - \frac{1}{3200000} k_v^2 c^2 Pe \right) y^6 \\
& + \frac{1}{5} \left(-\frac{1}{2560000} k_v^2 Pe\, c + \frac{1}{4915200000000} k_v c^3 Pe^2 \right) y^5 - \frac{1}{224} Pe\, k_v c^3 y^8 - \frac{1}{12} Pe\, c y^4 + \frac{1}{2} k_v c y^3 \\
& - \frac{1}{20} Pe\, k_v c y^6 + \frac{1}{2} k_v y^2 + \frac{1}{20} k_v c^3 y^5 + \frac{1}{20800} k_v^3 c^7 y^{13} + \frac{1}{8} \left(\frac{3}{70} k_v^2 c^4 + \frac{3}{7} k_v c^2 \left(-\frac{1}{12} Pe\, c \right.\right. \\
& \left.\left. + \frac{1}{4} k_v c^2 \right) \right) y^8 + \frac{1}{7} \left(\frac{11}{40} k_v^2 c^3 + k_v c \left(-\frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2 \right) \right) y^7 + \frac{1}{6} \left(\frac{3}{5} k_v \left(-\frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2 \right) \right. \\
& \left. + \frac{9}{10} k_v^2 c^2 \right) y^6 + \frac{1}{4} \left(\frac{1}{3686400000000} k_v Pe^2 c^2 + \frac{1}{3} c^2 \left(\frac{1}{480} k_v^2 c^5 y^9 + \frac{1}{8} \left(\frac{3}{70} k_v^2 c^4 + \frac{3}{7} k_v c^2 \left(\right.\right.\right.\right. \\
& \left.\left. -\frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2 \right) \right) y^8 + \frac{1}{7} \left(\frac{11}{40} k_v^2 c^3 + k_v c \left(-\frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2 \right) \right) y^7 + \frac{1}{6} \left(\frac{3}{5} k_v \left(-\frac{1}{12} Pe\, c \right.\right. \\
& \left. \left. + \frac{1}{4} k_v c^2 \right) + \frac{9}{10} k_v^2 c^2 \right) y^6 + \frac{9}{40} k_v^2 c y^5 + \frac{1}{8} k_v^2 y^4 - \frac{1}{224} Pe\, k_v c^3 y^8 - \frac{1}{42} Pe \left(-\frac{1}{3} Pe\, c + k_v c^2 \right) y^7 \\
& \left.\left. - \frac{1}{20} Pe\, k_v c y^6 - \frac{1}{20} Pe\, k_v y^5 \right) \right) y^4 + \frac{1}{2} \left(\frac{1}{480} k_v^2 c^5 y^9 + \frac{1}{8} \left(\frac{3}{70} k_v^2 c^4 + \frac{3}{7} k_v c^2 \left(-\frac{1}{12} Pe\, c \right.\right.\right. \\
& \left.\left. + \frac{1}{4} k_v c^2 \right) \right) y^8 + \frac{1}{7} \left(\frac{11}{40} k_v^2 c^3 + k_v c \left(-\frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2 \right) \right) y^7 + \frac{1}{6} \left(\frac{3}{5} k_v \left(-\frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2 \right) \right. \\
& \left. + \frac{9}{10} k_v^2 c^2 \right) y^6 + \frac{9}{40} k_v^2 c y^5 + \frac{1}{8} k_v^2 y^4 - \frac{1}{224} Pe\, k_v c^3 y^8 - \frac{1}{42} Pe \left(-\frac{1}{3} Pe\, c + k_v c^2 \right) y^7 \\
& \left. - \frac{1}{20} Pe\, k_v c y^6 - \frac{1}{20} Pe\, k_v y^5 \right) y^2 + \frac{1}{2} \left(\frac{3}{160} k_v^2 c^5 y^8 + \left(\frac{3}{70} k_v^2 c^4 + \frac{3}{7} k_v c^2 \left(-\frac{1}{12} Pe\, c \right.\right.\right. \\
& \left.\left. + \frac{1}{4} k_v c^2 \right) \right) y^7 + \left(\frac{11}{40} k_v^2 c^3 + k_v c \left(-\frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2 \right) \right) y^6 + \left(\frac{3}{5} k_v \left(-\frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2 \right) + \frac{9}{10} \right. \\
& \left. k_v^2 c^2 \right) y^5 + \frac{9}{8} k_v^2 c y^4 + \frac{1}{2} k_v^2 y^3 - \frac{1}{28} Pe\, k_v c^3 y^7 - \frac{1}{6} Pe \left(-\frac{1}{3} Pe\, c + k_v c^2 \right) y^6 - \frac{3}{10} Pe\, k_v c y^5 \\
& \left. - \frac{1}{4} Pe\, k_v y^4 \right) y^2 + \frac{3}{800} k_v^3 c^5 y^{11} - \frac{1}{42} Pe \left(-\frac{1}{3} Pe\, c + k_v c^2 \right) y^7 + \frac{1}{4} k_v c^2 y^4 :
\end{aligned}
\tag{3.1}
$$

The above expression is the final formula that describes the concentration at the particular distance y, with a dimensionless constant rate for a volume reaction k_v, and for a variable Peclet number Pe [6]. Moreover, c is the rate of change in the solvent at the surface of the disk, and it is arbitrary. (The calculation of k_v and Pe for our example are in Appendix D).

The following figure shows the values of parameters that we used to analyze the Total Concentration equation.

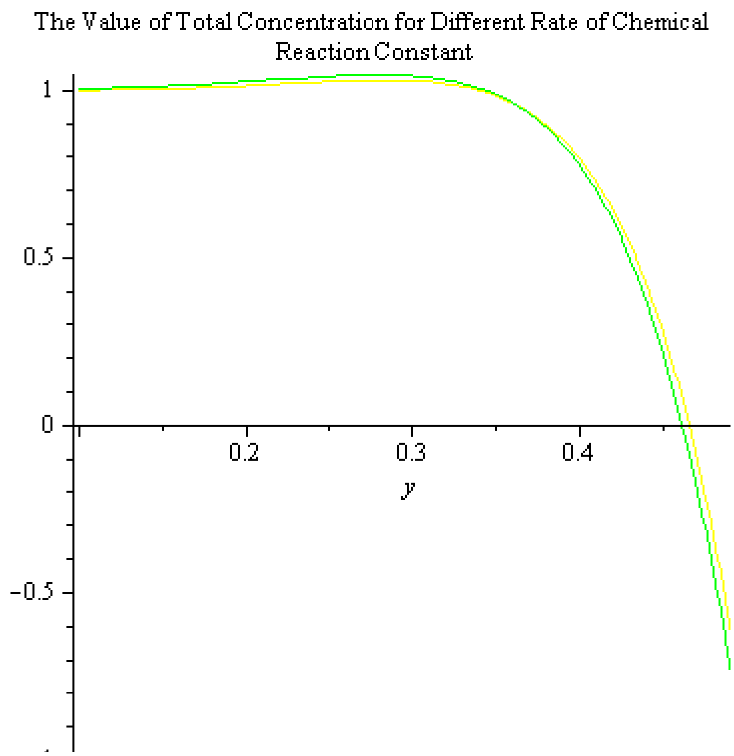
(1)Table for the Volume Reaction for Selected Values of *Kv*:

Terms	**Values**
Diffusion Coefficient: D	8.0×10^{-9} $[cm^2 \cdot s^{-1}]$
Arrhenius Parameter: A	2.42×10^{11} $[dm^3 \cdot mol^{-1} \cdot s^{-1}]$
Activating Energy: Ea	81.6 $[kJ \cdot mol^{-1}]$
Gas Constant: R	8.134 $[J \cdot K^{-1} \cdot mol^{-1}]$
Room Temperature:	285.15 $[K]$
Volume Reaction at the Surface of Rotating Disk: $Fv(Cs)$	2.726092×10^{-4} $[dm^3 \cdot mol^{-1} \cdot s^{-1}]$
Rate of Change in Concentration: c	-1 $[concentration \cdot s^{-1}]$
Peclet Number: Pe	120
Concentration at Surface of Disk: Cs	10

[Table 1]. Table of Values to Calculate the Total Concentration for Selected Kv

Some of the values in the table are taken from [10], [12]. The corresponding graph is constructed using Maple for the following values of K_v:

$$K_v = \{1\times10^{-10}, 1\times10^{-8}, 1\times10^{-4}, 1\times10^{-3}, 1\times10^{-0}\},$$



[Figure 1]. The Value of Total Concentration for Different Rate of Chemical Reaction Constant

The figure above shows the total concentration with respect to the distance. We selected various rate constants of the volume reaction K_v, and we substituted them into the equation. Each curve on the graph corresponds to one value of K_v. As we take different values of K_v, it does not seem to make a significant difference in the results. In addition, our expression does not produce physically meaningful results for large values of y, i.e. from the surface of the rotating disk.

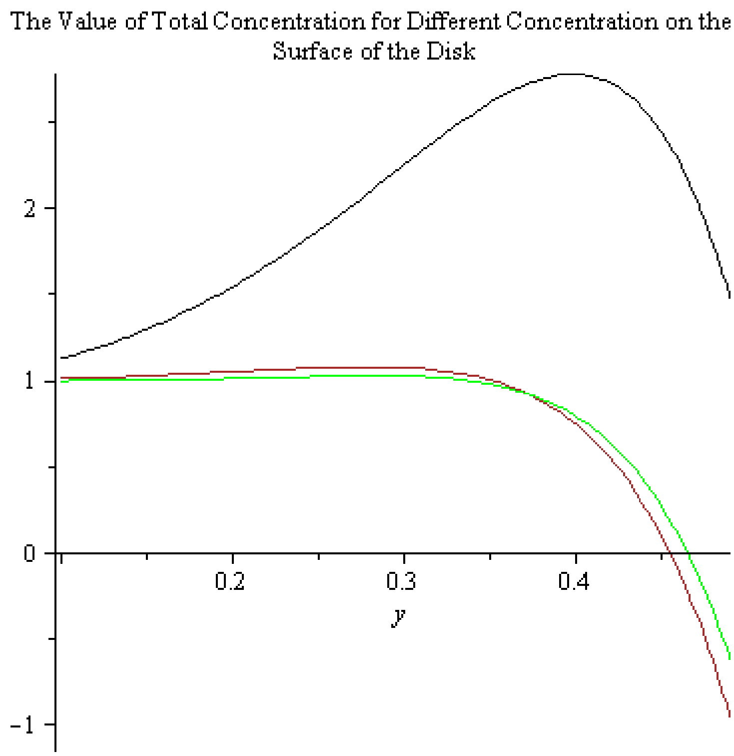
(2) Table for the Volume Reaction for Selected Values of *Cs*:

Terms	**Values**
Diffusion Coefficient: D	8.0×10^{-9} $[m^3 \cdot s^{-1}]$
Arrhenius Parameter: A	2.42×10^{11} $[dm^3 \cdot mol^{-1} \cdot s^{-1}]$
Activating Energy: Ea	81.6 $[kJ \cdot mol^{-1}]$
Gas Constant: R	8.134 $[J \cdot K^{-1} \cdot mol^{-1}]$
Room Temperature:	285.15 $[K]$
Volume Reaction at the Surface of Rotating Disk: $Fv(Cs)$	2.726092×10^{-4} $[dm^3 \cdot mol^{-1} \cdot s^{-1}]$
Rate of Change in Concentration: c	-1 $[concentration \cdot s^{-1}]$
Peclet Number: Pe	120
Rate of Constant for Volume Reaction	3×10^{-4}

[Table 2]. Table of Values to Calculate the Total Concentration for Selected *Cs*

The corresponding graph is constructed using Maple:

$Cs = \{0.0001, 0.001, 10, 50, 100, 150, 1000\}$,



[Figure 2]. The Value of Total Concentration for Different Concentration on the Surface of the Disk

The figure above shows the total concentration with respect to the distance. We selected various concentrations Cs on the surface of the rotating disk, and we substituted them into the resulting expression. Each curve on the graph corresponds to one value of Cs. As **[Figure 2]** and **[Table 2]** expressing the relationship of the results by selecting Cs, the volume reaction make a significant difference within the result, especially when we choose a small concentration of the surface of the rotating disk. In addition, if we take the distance y relatively far away from the rotating disk, we are not able to obtain physically meaningful solutions. Our concentration becomes negative.

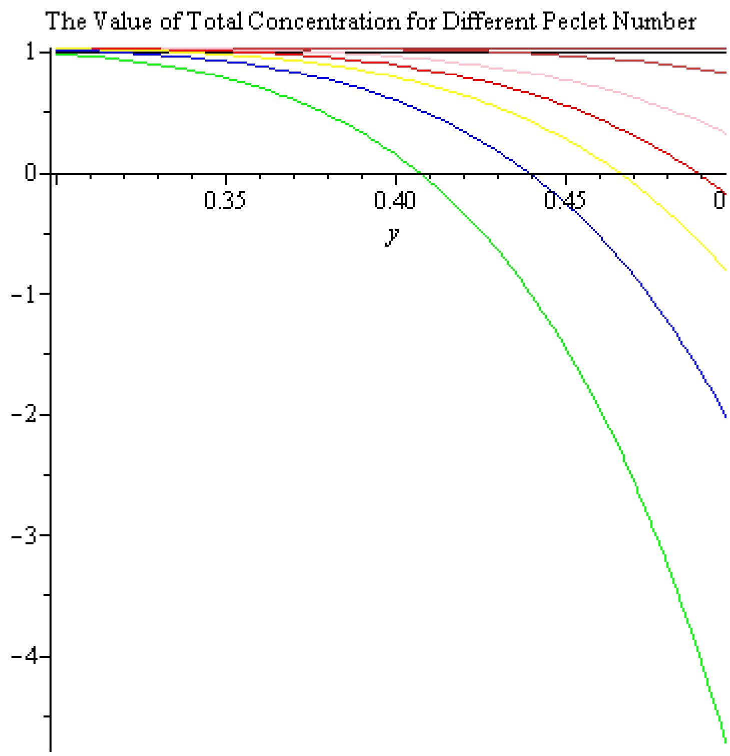
<u>(3) Table for the Volume Reaction for Selected Values of *Pe*:</u>

Terms	**Values**
Diffusion Coefficient: D	8.0×10^{-9} $[m^3 \cdot s^{-1}]$
Arrhenius Parameter: A	2.42×10^{11} $[dm^3 \cdot mol^{-1} \cdot s^{-1}]$
Activating Energy: Ea	81.6 $[kJ \cdot mol^{-1}]$
Gas Constant: R	8.134 $[J \cdot K^{-1} \cdot mol^{-1}]$
Room Temperature:	285.15 $[K]$
Volume Reaction at the Surface of Rotating Disk: $Fv(Cs)$	2.726092×10^{-4} $[dm^3 \cdot mol^{-1} \cdot s^{-1}]$
Rate of Change in Concentration: c	-1 $[concentration \cdot s^{-1}]$
Concentration at Surface of Disk: Cs	10
Rate of Constant for Volume Reaction	3×10^{-4}

[Table 3]. Table of Values to Calculate the Total Concentration for Selected Pe

Some of the valuables on the table are taken from [10], [12]. The corresponding graph is constructed using Maple:

$Pe = \{1,20,50,80,100,120,150,200\}$,



[Figure 3]. The Value of Total Concentration for Different Peclet Number

The figure above shows the total concentration with respect to the distance. We took various Peclet numbers, and we substituted them in the expressions for the solution. Each curve on the graph corresponds with each variable of Pe. **[Figure 3]** and **[Table 3]** express the relationship of the results by selecting Pe. The expression of the total concentration decreases by selecting higher Peclet number. In addition, if we take the distance y large, i.e. relatively far away from the rotating disk, we are not able to express the physical meaningful solutions.

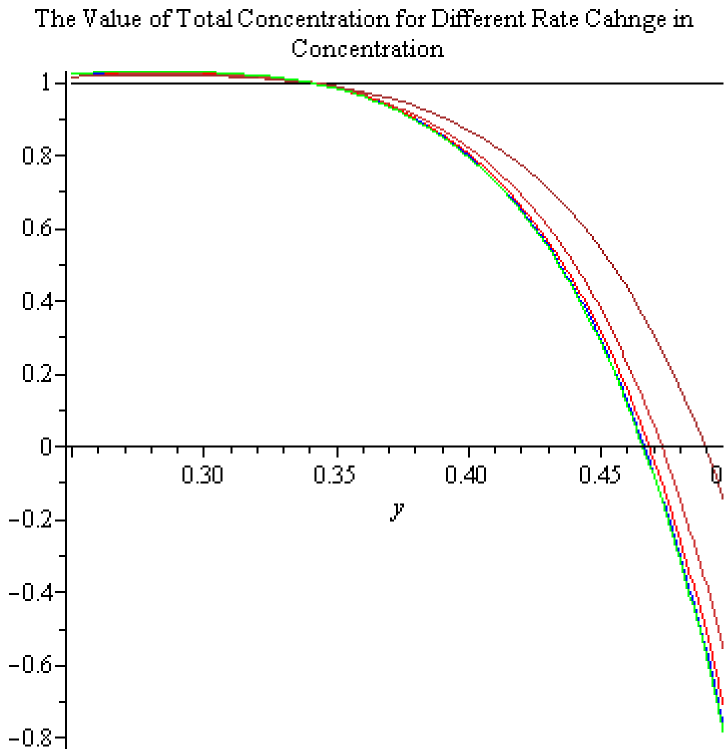
(4) Table for the Volume Reaction for Selected Values of *c*:

Terms	**Values**
Diffusion Coefficient: D	8.0×10^{-9} $[m^3\cdot s^{-1}]$
Arrhenius Parameter: A	2.42×10^{11} $[dm^3\cdot mol^{-1}\cdot s^{-1}]$
Activating Energy: Ea	81.6 $[kJ\cdot mol^{-1}]$
Gas Constant: R	8.134 $[J\cdot K^{-1}\cdot mol^{-1}]$
Room Temperature:	285.15 $[K]$
Volume Reaction at the Surface of Rotating Disk: $Fv(Cs)$	2.726092×10^{-4} $[dm^3\cdot mol^{-1}\cdot s^{-1}]$
Peclet Number: Pe	120
Concentration at Surface of Disk: Cs	10
Rate of Constant for Volume Reaction	3×10^{-4}

[Table 4]. Table of Values to Calculate the Total Concentration for Selected Rate of Change for Concentration

Some of the valuables in the table are taken from [10], [12]. The corresponding graph is constructed using Maple:

$$c=\{-1+e^{0},-1+e^{-1},-1+e^{-2},-1+e^{-3},-1+e^{-4},-1+e^{-5}\},$$



[Figure 4]. The Value of Total Concentration for Different rate Change in Concentration

The figure above illustrates the total concentration with respect to the distance. We selected various rates of change, and we substituted them into the resulting expression. Each curve on the graph corresponds to one value of c. All of the curves show the increases of the total concentration with take distance. As **[Figure 4]** and **[Table 4]** express the result as selecting c, although we took various values of c and construct the graph, the tendency stay same. In addition, if we take the distance y relatively far away from the rotating disk, we are not able to obtain the physically realistic solution.

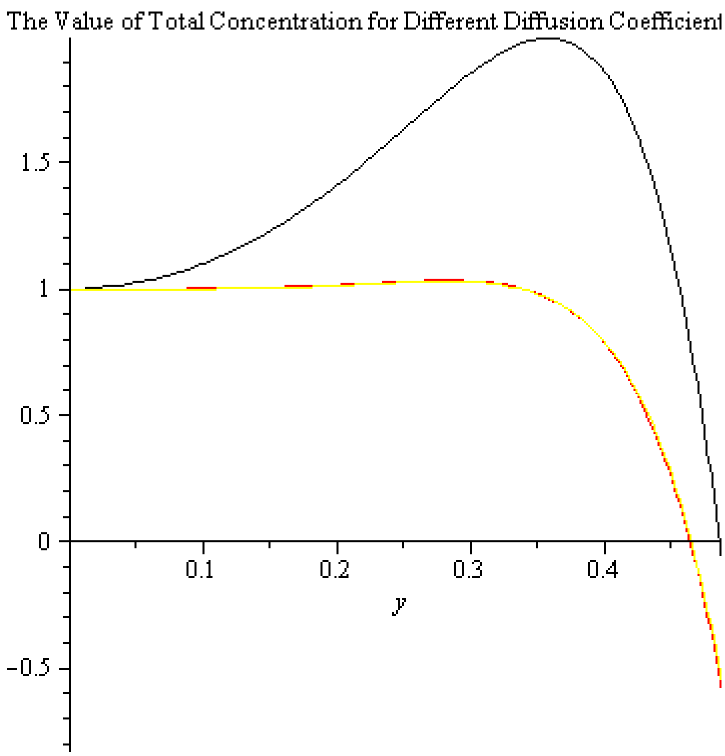
(5) Table for the Volume Reaction for Selected Values of Diffusion Coefficient:

Terms	Variables
Arrhenius Parameter: A	2.42×10^{11} $[dm^3 \cdot mol^{-1} \cdot s^{-1}]$
Activating Energy: Ea	81.6 $[kJ \cdot mol^{-1}]$
Gas Constant: R	8.134 $[J \cdot K^{-1} \cdot mol^{-1}]$
Room Temperature:	285.15 $[K]$
Volume Reaction at the Surface of Rotating Disk: $Fv(Cs)$	2.726092×10^{-4} $[dm^3 \cdot mol^{-1} \cdot s^{-1}]$
Peclet Number: Pe	120
Concentration at Surface of Disk: Cs	10
Rate of Constant for Volume Reaction	3×10^{-4}

[Table 5]. Table of Values to Calculate the Total Concentration for Selected Diffusion Coefficient

Some of the valuables on the table are given in [10], [12]. The corresponding graph is constructed using Maple for:

$$D = \{1\times10^{-10}, 1\times10^{-8}, 1\times10^{-6}, 1\times10^{-4}, 1\times10^{-3}\},$$



[Figure 5]. The Value of Total Concentration for Different Diffusion Coefficient

The figure above illustrates the total concentration with respect to the distance. We selected a diffusion coefficient, and we substituted it into the resulting expression. Each curve on the graph corresponds to one values of D. **[Figure 5]** and **[Table 5]** shows that the result by selecting diffusion if the diffusion coefficient were significantly smaller than the denominator term, the equation of the total concentration express increasing tendency as the distance increases. In addition, if we take the distance *y* relatively far away from the rotating disk, we are not able to obtain realistic solution.

Chapter 4:
Conclusions and Future Research

Our results provide an analytical expression for the total concentration with respect to the distance. We conducted a number of simulations for reasonable values of parameters. We construct the expression for concentration near the surface of the rotating disk. If we take a distance too far away from the surface of the disk, it results in physically unreasonable solutions. The collection of graphs illustrates these problems for total concentration.

Our analysis was conducted for the case of time-independent problem. As the next future step it would be interesting to see the effect that time variables can have on the results. So we think on considering time-dependent problem.

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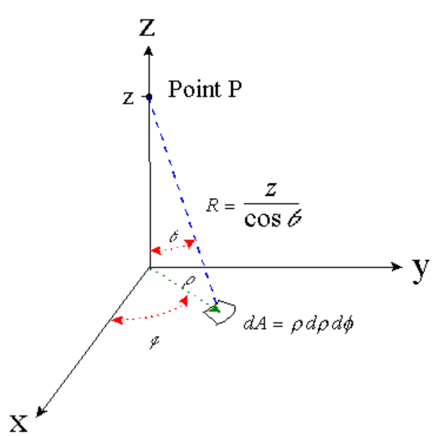
Appendix A:
Calculating the Velocity Field

Nonlinear Boundary Value Problem for the Navier-Stokes Equations

We consider the flow caused by an infinite planner disk rotating at a constant angular velocity ω. Three-dimensional motion is modeled around the rotating axis. Using the cylindrical system of coordinates $R,\ \phi,\ Z$ to describe the Navier-Stokes equations with the Laplace operator Δ, we can provide a schematic picture. Disk is in XY-plane [5]:



[Figure 6]. Three Dimensional Illustrate on Conversion of Cartesian Coordinates to Cylindrical Coordinates [4]

Using the following equations with the no-slip condition on the disk surface and condition of non-perturbed radial and angular motions and pressure remote from the disk, we can write [5]:

$$\frac{\partial V_R}{\partial R} + \frac{\partial V_Z}{\partial Z} + \frac{V_R}{R} = 0,$$

$$V_R \frac{\partial V_R}{\partial Z} + V_Z \frac{\partial V_R}{\partial Z} - V_\varphi^2 = -\frac{1}{\rho}\frac{\partial P}{\partial R} + \nu(\Delta V_R - \frac{V_R}{R^2}),$$

$$V_R \frac{\partial V_\phi}{\partial R} + V_Z \frac{\partial V_\phi}{\partial Z} + \frac{V_R V_\phi}{R} = \nu(\Delta V_\phi - \frac{V_\phi}{R^2}),$$

$$V_R \frac{\partial V_Z}{\partial R} + V_Z \frac{\partial V_Z}{\partial Z} = -\frac{1}{\rho}\frac{\partial P}{\partial Z} + \nu \Delta V_Z,$$

$$\Delta = \frac{1}{R}\frac{\partial}{\partial Z}(R\frac{\partial P}{\partial R}) + \frac{\partial^2}{\partial Z^2}. \qquad \text{(A1)}$$

We obtained the boundary conditions of the form:

$$V_R = 0, \quad V_\phi = R \cdot \omega, \quad V_Z = 0, \quad at \quad Z = 0,$$

$$V_R \to 0, \quad V_\phi = 0, \quad P \to P_{initial}, \quad at \quad Z \to \infty, \qquad \text{(A2)}$$

and we look for a solution to the problem following the von Karman transformation as:

$$V_R = \omega \cdot R \cdot \mu_1(z), \quad V_\phi = \omega \cdot R \cdot \mu_2(z), \quad V_Z = \sqrt{\nu \cdot \omega} \cdot \nu(z),$$

$$P = P_{initial} + \rho \cdot \nu \cdot \omega \cdot p \cdot z, \quad \text{where} \quad z = \frac{\sqrt{\omega}}{\nu} Z. \qquad \text{(A3)}$$

Substituting expressions from (A3) into (A1), we generated a system of ordinary differential equations instead of a system of partial differential equation. So the Navier-Stokes equations are transformed into the system of ordinary differential equations:

$$\mu_1^{(2)}(z) = \mu'_1 \cdot \nu(z) + \mu_1^2(z) - \mu_2^2(z),$$

$$\mu_2^{(2)}(z) = \mu'_2 \cdot \nu(z) + 2\mu_1(z)\mu_2(z),$$

$$\nu^{(2)}(z) = \nu'(z) \cdot \nu(z) + p',$$

$$\mu'(z) = -2\mu_1(z). \qquad \text{(A4)}$$

Substituting (A3) into (A2), we also obtained transformed boundary conditions:

$$\mu_1 = 0, \ \mu_2 = 1, \ \nu = 0, \ \text{at} \ z = 0,$$

$$\mu_1 \to 0, \ \mu_2 \to 1, \ p \to 0, \ \text{at} \ z \to 0. \qquad \text{(A5)}$$

The equation for the pressure is not coupled with other equations. We can obtain the distribution of pressure knowing the distribution of the transverse velocity:

$$p(z) = -\nu'(\infty) + \nu'(z) + \frac{\nu^2(\infty)}{2} - \frac{\nu^2(z)}{2}, \qquad \text{(A6)}$$

So we obtained pressure distribution in terms of velocity.

Boundary condition at z = 0

In order to find the velocity distribution, we take the solution in the form of a series near the disk when z approaches zero and we take the solution in the form of series when z approaches infinity, and merged these series. So near $z = 0$, we look for a solution to the system of the differential equations in the form of the Taylor series:

$$\mu_1(z) = \mu_1(0) + \mu_{1,1}z + \frac{\mu_{1,2}}{2!}z^2 + \frac{\mu_{1,3}}{3!}z^3 + \ldots.,$$

$$\mu_2(z) = \mu_2(0) + \mu_{2,1}z + \frac{\mu_{2,2}}{2!}z^2 + \frac{\mu_{2,3}}{3!}z^3 + \ldots.,$$

$$\nu(z) = \nu_1(0) + \nu_{1,1}z + \frac{\nu_{1,2}}{2!}z^2 + \frac{\nu_{1,3}}{3!}z^3 + \ldots.. \quad \text{(A7)}$$

Using the general form of equations, we used (A7) to obtain the derivative terms:

$$\mu_1(z), \quad \mu'_1(z), \quad \mu_1^{(2)}(z), \quad \mu_2(z), \quad \mu'_2(z), \quad \mu_2^{(2)}(z), \quad \nu(z), \quad \nu'(z), \quad \nu^{(2)}z. \quad \text{(A8)}$$

We then substitute the series representation of each term in the governing equations and combine terms with the same powers of z. This expansion gives us the behavior of the flow, modeled by the system (A4), near the surface of the disk. In other words, looking at the same power of z, we can obtain coefficients of the series in system (A7).

The obtained results as z approaches zero are:

$$\mu_{1,2} = -1, \quad \mu_{1,3} = \nu\mu_{1,2} - 2\mu_{2,1},$$

$$\mu_{2,2} = 0, \quad \mu_{2,3} = 2\mu_{1,1} + \nu_1\mu_{2,1},$$

$$\nu_1 = 0, \quad \nu_2 = -2\mu_{1,1}.$$

Boundary condition as $z \to \infty$

In order to find all of these coefficients in the series expansion we needed to analyze the behavior as $z$approaches infinity. We note that not all coefficients can be found explicitly. We look for the solution in the form of a series as $z$approaches infinity, which is expressed in terms of exponential functions:

$$\mu_1(z) = U_{1,0} + U_{1,1}e^{-cz} + U_{1,2}e^{-2cz} + U_{1,3}e^{-3cz} + \ldots.,$$
$$\mu_2(z) = U_{2,0} + U_{2,1}e^{-cz} + U_{2,2}e^{-2cz} + U_{2,3}e^{-3cz} + \ldots.,$$
$$\nu(z) = V_0 + V_1e^{-cz} + V_2e^{-2cz} + V_3e^{-3cz} + \ldots.. \qquad \text{(A9)}$$

Using the above series expansion, we write derivative terms of relevant orders appearing in (A4):

$$\mu_1(z),\quad \mu'_1(z),\quad \mu_1^{(2)}(z),\quad \mu_2(z),\quad \mu'_2(z),\quad \mu_2^{(2)}(z),\quad \nu(z),\quad \nu'(z). \qquad \text{(A10)}$$

We then substitute the series representation of each term in the governing equations, and combine terms with the same exponents. This expansion gives the behavior of the flow, modeled by the system (A4), as distance z approaches infinity. In other words, looking at the exponential terms, we obtain unknown coefficients in the system (A9).

The obtained results as $z$approaches infinity are:

$$\mu_1 = U_{1,1}e^{-cz} - \frac{U_{1,1}^2 + U_{2,1}^2}{2c^2} + U_{1,2}e^{-3cz} + \cdots,$$
$$\mu_2 = U_{2,1}e^{-cz} + U_{2,3}e^{-3cz} + \ldots,$$
$$\nu_1 = -c + \frac{2U_{1,1}}{c}e^{-cz} - \frac{U_{1,1}^2 + U_{2,1}^2}{2c^3}e^{-2cz} + \ldots.$$

The above expressions with respect to z give a characterization of the velocity field. Unfortunately, there are unknown coefficients which cannot be determined using Taylor expansion even if we take higher degree expression. However using expressions at z = 0 and as $z \to \infty$ we are able to obtain the whole velocity field as outlined below.

Application of the technique from [2]

Cochran's technique [2] allows us to model the steady motion of an incompressible viscous fluid caused by an infinite rotating disk in cylindrical coordinates [4]. We use the system of ordinary differential equations (A4) with boundary conditions (A5) [9]. We reanalyzed the system of equations following his method. The equations with boundary conditions are:

$$\mu_1^{(2)}(z) = \mu'_1(z)\nu(z) + \mu_1^2(z) - \mu_2^2(z),$$
$$\mu_2^{(2)}(z) = \mu'_2(z)\nu(z) + \mu_1(z) - \mu_2(z). \quad \text{(A11)}$$

Since formal Taylor Expansions of $\mu_1,\ \mu_2$ and ν are written in terms of the exponential function in terms of e^{-cz} as z approaches infinity, we can obtain the leading terms in the expansion form our equations(A11). The translated system of equation can write with some constants A and B:

$$\mu_1(z) = A \cdot e^{-cz}, \quad \mu_2(z) = B \cdot e^{-cz}, \quad \nu(z) = -c + \frac{2 \cdot A}{c} e^{-cz}. \quad \text{(A12)}$$

Applying the von Karman's method to obtain an approximate solution we integrate (A11). We then need to analyze the rate of decay in the system. So we match the expressions for some finite value of $z : z_0$. As a result new boundary conditions are:

$$\mu_1(z_0) = 0,\ \ \mu'_1(z_0) = 0,\ \ \mu_2(z_0) = 0,\ \ \mu_2(z_0) = 0,\ \ \mu_1(z) = 0,\ \ \mu_2(z) = 1.$$

(A13)

Using (A13), we construct the lowest degree polynomial that satisfies all of the above boundary conditions. Specifically, $\mu_1(z)$ becomes a quadratic polynomial equation and $\mu_2(z)$ becomes a cubic polynomial equation. Using the conditions on higher order derivatives of $\mu_1(z),\ \mu_2(z)$ we obtain and derive the following equations:

$$\mu_1(z) = (1 - \frac{z}{z_0})^2(a \cdot z + (\frac{2 \cdot a}{z_0} - \frac{1}{2})z^2),$$
$$\mu_2(z) = (1 - \frac{z}{z_0})^2(1 + \frac{1}{2 \cdot z_0}z). \quad \text{(A14)}$$

Based on (A14), we have the derivatives of each function. Substituting the obtained expressions into (A11), and using the mathematical software Maple, we obtain:

$$a = 0.51433880,$$

$$z_0 = 2.79407820.$$

These calculations and estimations gave us the values of a and of particular distance z. Cochran [2] used these variables to conduct various numerical simulations to obtain meaningful and reasonable values of parameters. The method he used was the multidimensional Newton's Method. He used (A4) and numerical integration of the equations (A9). We did not replicate all the details of Cochran's derivation since it requires in cumbersome and tedious manipulation of the expressions. Thus, we used the solution given in [2] and other sources, too. The solution to our system of ordinary differential equations finally turns into the form:

$$\mu_1(z) \simeq 0.5143z - 0.5143z^2,$$
$$\mu_2(z) \simeq 1 - 0.616z,$$
$$\nu(z) \simeq -0.5143z^2 + 0.333z^3,$$
$$p(z) = 0.393 - 1.02z, \quad \text{(A15)}$$

as z approaches zero, and

$$\mu_1(z) \simeq 0.934e^{-0.886z},$$
$$\mu_1(z) \simeq 1.208e^{-0.886z},$$
$$\nu(z) \simeq -0.886,$$
$$p(z) = 0.393, \quad \text{(A16)}$$

as z approaches infinity. Using the expressions in (A16), we can estimate the perturbations caused by the rotating disk in the fluid away from the surface of the disk.

These equations reproduce boundary conditions (A2). However, the remote dimensionless axial velocity is not zero. This is the rate at which the disk draws the ambient fluid. The radial and angular velocities are perturbed only near the disk surface, in the so-called dynamic boundary layer. The thickness of this disk is independent of the rotational coordinate and is approximately equal to σ, which represents the thickness of the hydrodynamic boundary layer:

$$\sigma = 3\frac{\sqrt{\nu}}{\omega}.$$

Appendix B:
Mass Transfer Equation

Mass Transfer Equation

Under the assumption that the medium's density and viscosity are independent of the concentration and temperature distributions, we can conclude that they do not affect the flow field. Then, we can decouple the hydrodynamic problem of the fluid motion and the diffusion-heat problem of finding the concentration and temperature fields. In the Cartesian coordinate system $X,\ Y$, and Z, solute transfers in the absence of homogeneous transformations is described by the equation [5]:

$$\frac{\partial C}{\partial t} + V_X \frac{\partial C}{\partial X} + V_Y \frac{\partial C}{\partial Y} + V_Z \frac{\partial C}{\partial Z} = D\left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} + \frac{\partial^2 C}{\partial Z^2}\right), \qquad \text{(B1)}$$

where C represents the concentration, and D represents the diffusion coefficient mass diffusivity with $V_X,\ V_Y,\ V_Z$, being the fluid velocity components.

Dimensionless Equation and Boundary Conditions

If a surface (heterogeneous) chemical reaction with a finite rate occurs on the interface, then the boundary condition that represents an infinitely fast heterogeneous chemical reaction must be replaced by the more complicated boundary conditions. When we defined terms in the equation, we used a characteristic length a(ex: the characteristic length) and a characteristic velocity U(ex: non-perturbed flow velocity away from a particle on the axis of the rotating disk). Then, expression (B1) is converted with the following with dimensionless variables:

$$\tau = \frac{Dt}{a^2}, \quad x = \frac{X}{a}, \quad y = \frac{Y}{a}, \quad z = \frac{Z}{a}, \quad \xi = \frac{\xi_d}{a},$$
$$v_x = \frac{V_X}{U}, \quad v_y = \frac{V_Y}{U}, \quad v_z = \frac{V_Z}{U}, \quad c = \frac{C_i - C}{C_i - C_S}. \tag{B2}$$

where ξ_d be the distance measured along the normal to the surface [5].

Using variables defined by (B2), we can rewrite (B1) as:

$$\frac{\partial c}{\partial \tau} + Pe(V_X \frac{\partial c}{\partial x} + V_Y \frac{\partial c}{\partial y} + V_Z \frac{\partial c}{\partial z}) = \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2}, \tag{B3}$$

where Pe represents a dimensionless parameter (Peclet number) characterizing the ratio of convective transfer to diffusion transfer:

$$Pe = \frac{U \cdot a}{D}.$$

(B3) expresses the concentration term without chemical reaction, since we are considering overall reaction in the system, we will add the reaction term later in this procedure.

Using the obtained equation (B3), we need to rewrite the boundary condition given on the surface of the disk and away from it. Then the boundary conditions translate into:

$$c = 0, \text{ as } \xi \to \infty,$$

$$c = 1, \text{ as } \xi \to 0.$$

If we have heterogeneous chemical reaction with finite velocity of fluid, then instead of the condition, $\xi_d = 0,\ c \to 1$,we used:

$$\xi_d = 0,\ D \frac{\partial C}{\partial \xi_d} = K_S \cdot F_S(C),$$

where K_S represents the rate of constant of the surface chemical reaction, and $K_S F_S(C)$ represents the rate of the surface reaction. For this different setting, we take $c = C_i - C/C_i$, and rewrite the convective diffusion equation, which was obtained in (B3) as:

$$\frac{\partial c}{\partial \xi} + Pe(v\nabla)c = \Delta c, \tag{B4}$$

where ∇ represents the Hamilton operator and Δ represents the Laplace operator. Rewriting expression (B4) in the cylindrical coordinates, the differential operators translate into:

$$(v\nabla)c = v_R \frac{\partial c}{\partial R} + v_Z \frac{\partial c}{\partial Z} + \frac{\mu_\phi}{R}\frac{\partial c}{\partial \phi},$$
$$\Delta c = \frac{1}{R}\frac{\partial}{\partial R}(R\frac{\partial c}{\partial R}) + \frac{\partial^2 c}{\partial Z^2} + \frac{1}{R^2}\frac{\partial^2 c}{\partial \phi^2},$$
$$R = \sqrt{x^2 + y^2}. \tag{B5}$$

Rewriting the expression (B4) with (B5) and $Pe = aU/D = Sc,$ we obtain:

$$\frac{\partial c}{\partial \xi}\frac{1}{Sc} + (v_R \frac{\partial c}{\partial R} + v_Z \frac{\partial c}{\partial Z} + \frac{\mu_\phi}{R}\frac{\partial c}{\partial \phi}) = \frac{1}{Sc}[\frac{1}{R}\frac{\partial}{\partial R}(R\frac{\partial c}{\partial R}) + \frac{\partial^2 c}{\partial Z^2} + \frac{1}{R^2}\frac{\partial^2 c}{\partial \phi^2}]. \tag{B6}$$

Overall, the equation describes the physical and chemical hydrodynamic problem without boundary conditions in terms of dimensionless variables. *Sc* is called the large Schmidt number [13], which is a dimensionless number. It is defined as the ratio of momentum diffusivity (viscosity) to mass diffusivity, and is used to characterize fluid flow in which there are the simultaneous momentum and the mass diffusion convection processes. In other words, it is the ratio of the shear component for diffusivity viscosity/density to the diffusivity for mass transfer D.Specifically, it is expressed as:

$$Sc = \frac{v}{D} = \frac{\mu}{\rho \cdot D} = \frac{viscousdiffusionrate}{molecular(mass)diffusionrate}.$$

Diffusion to a Rotating Disk: (Infinite Plane Disk)

Consider steady-state mass transfer to the surface of a disk rotating around its axis at a constant angular velocity ω. Our assumption was that remote from the disk, the concentration is equal to the constant C_i. The z-axis is normal to the surface of the disk. The motion of a fluid was obtained in Appendix A [5] for this nonlinear boundary value problem for the Navier-Stokes equations.

Under the assumption of a steady-state process we can modify (B6) as:

$$v_R \frac{\partial c}{\partial R} + v_Z \frac{\partial c}{\partial Z} + \frac{\mu_\phi}{R} \frac{\partial c}{\partial \phi} = \frac{1}{Sc} [\frac{1}{R} \frac{\partial}{\partial R} (R \frac{\partial c}{\partial R}) + \frac{\partial^2 c}{\partial Z^2} + \frac{1}{R^2} \frac{\partial^2 c}{\partial \phi^2}], \quad \text{(B7)}$$

with the following boundary conditions:

$z = 0, c = 1,$

$z \to \infty, c \to 0.$

We assume that solution c depends on Z only, i.e. $c = c(Z)$, then (B7) will be:

$$v_Z \frac{\partial}{\partial Z} [c(Z)] = \frac{1}{Sc} \frac{\partial^2}{\partial z^2} [c(Z)] \text{ and } W = \frac{\partial c}{\partial Z},$$

So for the new variable W we have:

$$Sc \cdot v_Z \cdot W = \frac{dW}{dZ}. \quad \text{(B8)}$$

Solving the differential equation (B8), we obtaine:

$$W(z) = W(0) \exp [Sc \int_0^z v_Z(Z) dz] \quad \text{(B9)}$$

We analyze the differential equation as $z \to 0$:

$$W(0) = \frac{-1}{\int_0^\infty \exp [Sc \int_0^z v_Z(Z) dZ] dz}$$

The final obtained expression is as follows:

$$c(z) = \frac{\int_0^\infty \exp [Sc \int_z^z v_Z(Z) dZ] dz}{\int_0^\infty exp [Sc \int_0^z v_Z(Z) dZ] dz} \quad \text{(B10)}$$

Substituting the variables from (A15) and (A16) into the above the expression (B10) we get:

$$c(z) = \frac{\int_0^{\infty} \exp\,[Sc\int_z^{z} (\frac{-0.51Z^3}{3} + \frac{0.333Z^4}{4})dZ]dz}{\int_0^{\infty} \exp\,[Sc\int_0^{z} (\frac{-0.51Z^3}{3} \frac{0.333Z^4}{4})dZ]dz} \qquad \text{(B11)}$$

Appendix C:
Construction of the Adomian Polynomials

The Adomian Decomposition (Construction of the Adomian Polynomials)

The Adomian decomposition method is the recurrent procedure to obtain an approximate solution in the form of a series. Applying this method, we can express the nonlinear reaction term by Adomian polynomials. That is to say, the Adomian method is used to construct the solutions in the form of a series in general:

$$u(z,t) = \sum_{n=0}^{\infty} u_n(z,t),$$

where terms are found consequently. To apply the Adomian method, we represented a differential equation as:

$$L_u + R_u + N_u = S,$$

where L_u is the linear term that is easily invertible, R_u is the remainder of the linear operator, N_u is the nonlinear term and S is the right-hand side expression. The nonlinear term is then decomposed into a special series where each term is represented by the Adomian polynomial:

$$Nu_n = \sum_{n=0}^{\infty} A_n \,. \tag{C1}$$

This approach was first introduced in analysis of the stochastic differential equations. Specifically, we apply the method introduced by Zhu [8]. We use a third degree polynomial function to obtain the approximate concentration. Applying Zhu's method, (C1) is rewritten as:

$$\sum_{k=0}^{\infty} k_v \lambda^k A_k = k_v (c_0 + \lambda c_1 + \lambda^2 c_2 + \lambda^3 c_3 \ldots)^3 \quad \text{(C2)}$$

The initial concentration has to satisfy our initial conditions, so we decided:

$$c_0 = 1 + c'(0)z. \quad \text{(C3)}$$

We substitute $\lambda = 0$, it eliminates all the terms except for the very first term. So the first the Adomian polynomial is expressed as:

$$A_0 = k_v (c_0)^3. \quad \text{(C4)}$$

Similarly, taking the first derivative (C2) with respect to λ and setting $\lambda = 0$, we have:

$$\frac{\partial A_0 + A_1 \lambda}{\partial \lambda} = \frac{\partial [k_v (c_0 + c_1 \lambda)^3]}{\partial \lambda},$$

$$A_1 = 3k_v \cdot c_0^2 \cdot c_1,$$

Similarly, taking the second order derivative (C2) with respect to λ and setting $\lambda = 0$, we have:

$$\frac{\partial^2 (A_0 + A_1 \lambda + A_2 \lambda^2)}{\partial \lambda^2} = \frac{\partial^2 [k_v (c_0 + c_1 \lambda + c_2 \lambda^2)^3]}{\partial \lambda^2},$$

$$A_2 = 3k_v (c_0 c_1^2 + c_0^2 c_1).$$

Using similar derivations we can obtain the Adomian polynomials of a higher degree, for example.

$$A_3 = 3k_v (c_0)^2 c_3 + 6k_v (c_0 \cdot c_1 \cdot c_2) + k_v (c_1)^3.$$

Calculation of the total concentration using Adomian polynomial

Let's analyze the differential equation:

$$L_n = R_n + N_n. \quad \text{(C5)}$$

We already know the expression of the term R_n:

$$R_n = -Pe \cdot z^2 \frac{dc_0}{dz},$$

Moreover, we accomplished the non-linear term of the differential equation will be represented as:

$$N_n = \sum_{n=0}^{z} A_n,$$

where A_n are the Adomian polynomials. Since our L_n is the second order derivative, we need to integrate twice to determine the concentration. Let's write the transformation step of the Adomian method in the general form:

$$c_n = L^{-1}R_n + L^{-1}A_n. \tag{C6}$$

Here are the first terms when we start the procedures. For the A_0 term:

$$L^{-1}A_0 = \int_{0_1}^{y} \int_{0}^{y} k_v(1 + c'(0)z)^3 dz dy_1,$$

$$= \frac{1}{20} k_v \cdot c^3 \cdot y^5 + \frac{1}{4} k_v \cdot c^2 \cdot y^4 + \frac{1}{2} k_v \cdot c \cdot y^3 + \frac{1}{2} k_v y^2. \tag{C7}$$

For the R_0 term:

$$L^{-1}R_0 = \int_{0_1}^{y} \int_{0}^{y} - Pe \cdot z^2 \frac{dc_0}{dz},$$

$$= -\frac{1}{12} Pe \cdot c \cdot y^4. \tag{C8}$$

The first term c_1 is obtained by the addition of (C7) and (C8):

$$c_1 = \frac{1}{20} k_v \cdot c^3 \cdot y^5 + \frac{1}{4} k_v \cdot c^2 y^4 + \frac{1}{2} k_v \cdot c y^3 + \frac{1}{2} k_v y^2 - \frac{1}{12} Pe \cdot c y^4. \tag{C9}$$

Similarly, the second term c_2 is determined by two terms. The inverse of the Adomian polynomial is determined by:

$$L^{-1}A_1 = \int_0^{y1} \int_0^y [3k_v \cdot c_0^2 \cdot c_1]dzdy_1. \quad \text{(C10)}$$

Substituting the known expressions for of the concentration terms, we obtained the following result:

$$L^{-1}A_1 = \int_0^{y1}\int_0^{y} [3k_v(1 + c'(0)z^2)(\tfrac{1}{20}k_v c^3 y^5 + \tfrac{1}{4}k_v c^2 y^4 + \tfrac{1}{2}k_v \cdot cy^3 + \tfrac{1}{2}k_v - \tfrac{1}{12}Pe \cdot c \cdot y^4)]dzdy_1$$

$$\begin{aligned} &= -\tfrac{1}{480}k_v^2 c^5 y^9 + \tfrac{1}{8}(\tfrac{3}{70}k_v^2 c^4 + \tfrac{3}{7}k_v c^2(-\tfrac{1}{12}Pe \cdot c + \tfrac{1}{4}k_v c^2))y^8 \\ &+ \tfrac{1}{7}(\tfrac{11}{40}k_v^2 \cdot c^3 + k_v \cdot c(-\tfrac{1}{12}Pe \cdot c + \tfrac{1}{4}k_v c^2))y^7 + \tfrac{1}{8k_v^2 y^4} \\ &+ \tfrac{1}{6}(\tfrac{3}{5}k_v(-\tfrac{1}{12}Pe \cdot c + \tfrac{1}{4}k_v^2 c^2) + \tfrac{9}{10}k_v^2 \cdot c^2)y^6 + \tfrac{9}{40}k_v^2 \cdot cy^5. \end{aligned} \quad \text{(C11)}$$

R_1 term:

$$L^{-1}R_1 = \int_0^{y1}\int_0^{y} -Pez^2 \frac{dc_1}{dz},$$

$$= -\tfrac{1}{224}Pek_v c^3 \cdot y^8 - \tfrac{1}{42}Pe\tfrac{-1}{3}Pe \cdot c + k_v c^2\, y^7 - \tfrac{1}{20}Pek_v \cdot cy^6 - \tfrac{1}{20}Pek_v y^5. \quad \text{(C12)}$$

The second term c_2 is determined by the addition of (C9) and (C10):

$$\begin{aligned} c_2 = &\tfrac{1}{480}k_v^2 c^5 \cdot y^9 + \tfrac{1}{8}(\tfrac{3}{70}k_v^2 \cdot c^4 + \tfrac{3}{7}k_v \cdot c^2(-\tfrac{1}{12}Pe \cdot c + \tfrac{1}{4}k_v \cdot c^2))y^8 \\ &+ \tfrac{1}{7}(\tfrac{11}{40}k_v^2 \cdot c^3 + k_v \cdot c(-\tfrac{1}{12}Pe \cdot c + \tfrac{1}{4}k_v \cdot c^2))y^7 \\ &+ \tfrac{1}{6}(\tfrac{3}{5}k_v(-\tfrac{1}{12}Pe \cdot c + \tfrac{1}{4}K_v \cdot c^2) + \tfrac{9}{10}k_v^2 \cdot c^2)y^6 \\ &+ \tfrac{9}{40}k_v^2 \cdot cy^5 + \tfrac{1}{8}k_v^2 \cdot y^4 - \tfrac{1}{224}Pek_v \cdot c^3 \cdot y^8 \\ &- \tfrac{1}{42}Pe(-\tfrac{1}{3}Pe \cdot c + k_v \cdot c^2)y^7 - \tfrac{1}{20}Pe \cdot k_v \cdot cy^6 - \tfrac{1}{20}Pe \cdot k_v \cdot y^5. \end{aligned} \quad \text{(C13)}$$

The third term c_2 for the concentration is defined by the following expression:

$$c_3 := \frac{1}{20800}k_v^3 c^7 y^{13} + \frac{1}{1600}k_v^3 c^6 y^{12} + \frac{3}{800}k_v^3 c^5 y^{11} + \frac{11}{800}k_v^3 c^4 y^{10} + \frac{1}{9}\Big(\frac{9}{80}k_v^3 c^3 + \frac{3}{8}k_v c\Big(-\frac{1}{19200000}Pe\, c^4 k_v + \frac{1}{2}k_v^2 c^2\Big)\Big)y^9 + \frac{1}{8}\Big(\frac{3}{7}k_v\Big(-\frac{1}{19200000}Pe\, c^4 k_v + \frac{1}{2}k_v^2 c^2\Big) + \frac{3}{7}k_v c\Big($$

$$- \frac{1}{3840000} Pe\, k_v c^3 + \frac{1}{2} k_v^2 c)) y^8 + \frac{1}{7} (\frac{1}{2} k_v (- \frac{1}{3840000} Pe\, k_v c^3 + \frac{1}{2} k_v^2 c) + \frac{1}{2} k_v c ($$
$$- \frac{1}{1920000} Pe\, c^2 k_v + \frac{1}{4} k_v^2)) y^7 + \frac{1}{6} (\frac{3}{5} k_v (- \frac{1}{1920000} Pe\, c^2 k_v + \frac{1}{4} k_v^2) - \frac{1}{3200000} k_v^2 c^2 Pe) y^6$$
$$+ \frac{1}{5} (- \frac{1}{2560000} k_v^2 Pe\, c + \frac{1}{4915200000000} k_v c^3 Pe^2) y^5 + \frac{1}{4} (\frac{1}{3686400000000} k_v Pe^2 c^2$$
$$+ \frac{1}{3} c^2 (\frac{1}{480} k_v^2 c^5 y^9 + \frac{1}{8} (\frac{3}{70} k_v^2 c^4 + \frac{3}{7} k_v c^2 (- \frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2)) y^8 + \frac{1}{7} (\frac{11}{40} k_v^2 c^3 + k_v c ($$
$$- \frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2)) y^7 + \frac{1}{6} (\frac{3}{5} k_v (- \frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2) + \frac{9}{10} k_v^2 c^2) y^6 + \frac{9}{40} k_v^2 c y^5 + \frac{1}{8} k_v^2 y^4$$
$$- \frac{1}{224} Pe\, k_v c^3 y^8 - \frac{1}{42} Pe (- \frac{1}{3} Pe\, c + k_v c^2) y^7 - \frac{1}{20} Pe\, k_v c y^6 - \frac{1}{20} Pe\, k_v y^5)) y^4 + \frac{1}{3} c (\frac{1}{480}$$
$$k_v^2 c^5 y^9 + \frac{1}{8} (\frac{3}{70} k_v^2 c^4 + \frac{3}{7} k_v c^2 (- \frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2)) y^8 + \frac{1}{7} (\frac{11}{40} k_v^2 c^3 + k_v c (- \frac{1}{12} Pe\, c$$
$$+ \frac{1}{4} k_v c^2)) y^7 + \frac{1}{6} (\frac{3}{5} k_v (- \frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2) + \frac{9}{10} k_v^2 c^2) y^6 + \frac{9}{40} k_v^2 c y^5 + \frac{1}{8} k_v^2 y^4$$
$$- \frac{1}{224} Pe\, k_v c^3 y^8 - \frac{1}{42} Pe (- \frac{1}{3} Pe\, c + k_v c^2) y^7 - \frac{1}{20} Pe\, k_v c y^6 - \frac{1}{20} Pe\, k_v y^5) y^3 + \frac{1}{2} (\frac{1}{480} k_v^2 c^5 y^9$$
$$+ \frac{1}{8} (\frac{3}{70} k_v^2 c^4 + \frac{3}{7} k_v c^2 (- \frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2)) y^8 + \frac{1}{7} (\frac{11}{40} k_v^2 c^3 + k_v c (- \frac{1}{12} Pe\, c$$
$$- \frac{1}{224} Pe\, k_v c^3 y^8 - \frac{1}{42} Pe (- \frac{1}{3} Pe\, c + k_v c^2) y^7 - \frac{1}{20} Pe\, k_v c y^6 - \frac{1}{20} Pe\, k_v y^5) y^2 + \frac{1}{2} (\frac{3}{160} k_v^2 c^5 y^8$$
$$+ \frac{1}{4} k_v c^2)) y^7 + \frac{1}{6} (\frac{3}{5} k_v (- \frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2) + \frac{9}{10} k_v^2 c^2) y^6 + \frac{9}{40} k_v^2 c y^5 + \frac{1}{8} k_v^2 y^4$$
$$+ (\frac{3}{70} k_v^2 c^4 + \frac{3}{7} k_v c^2 (- \frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2)) y^7 + (\frac{11}{40} k_v^2 c^3 + k_v c (- \frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2)) y^6$$
$$+ (\frac{3}{5} k_v (- \frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2) + \frac{9}{10} k_v^2 c^2) y^5 + \frac{9}{8} k_v^2 c y^4 + \frac{1}{2} k_v^2 y^3 - \frac{1}{28} Pe\, k_v c^3 y^7 - \frac{1}{6} Pe ($$
$$- \frac{1}{3} Pe\, c + k_v c^2) y^6 - \frac{3}{10} Pe\, k_v c y^5 - \frac{1}{4} Pe\, k_v y^4) y^2 :$$

(C14)

(We decide to stop here, since further terms c_i, $i \geq 4$ will be even lengthier. For example c_4 is longer than a page. In our work we use the first four terms c_0, c_1, c_2, and c_3) Then the approximation for the total concentration is expressed in form of

$$C = c_0 + c_1 + c_2 + c_3 + \ldots,$$

Or using summation notation we can write:

$$C = \sum_{i=0}^{n} c_i \,. \qquad \text{(C15)}$$

With $n = 3$, the total concentration is expressed in the following form:

$$C := 1 + \frac{9}{40} k_v^2 c y^5 + \frac{1}{480} k_v^2 c^5 y^9 + \frac{1}{8} k_v^2 y^4 + \frac{1}{3} c (\frac{1}{480} k_v^2 c^5 y^9 + \frac{1}{8} (\frac{3}{70} k_v^2 c^4 + \frac{3}{7} k_v c^2 (- \frac{1}{12} Pe\, c$$
$$+ \frac{1}{4} k_v c^2)) y^8 + \frac{1}{7} (\frac{11}{40} k_v^2 c^3 + k_v c (- \frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2)) y^7 + \frac{1}{6} (\frac{3}{5} k_v (- \frac{1}{12} Pe\, c + \frac{1}{4} k_v c^2)$$
$$+ \frac{9}{10} k_v^2 c^2) y^6 + \frac{9}{40} k_v^2 c y^5 + \frac{1}{8} k_v^2 y^4 - \frac{1}{224} Pe\, k_v c^3 y^8 - \frac{1}{42} Pe (- \frac{1}{3} Pe\, c + k_v c^2) y^7$$

$$
\begin{aligned}
&- \frac{1}{20} Pe\, k_v\, c\, y^6 - \frac{1}{20} Pe\, k_v y^5 \Big) y^3 - \frac{1}{20} Pe\, k_v y^5 + \frac{1}{1600} k_v^3\, c^6\, y^{12} + \frac{11}{800} k_v^3\, c^4\, y^{10} + \frac{1}{9} \Big(\frac{9}{80} k_v^3\, c^3 \\
&+ \frac{3}{8} k_v\, c \Big(- \frac{1}{19200000} Pe\, c^4\, k_v + \frac{1}{2} k_v^2\, c^2 \Big) \Big) y^9 + \frac{1}{8} \Big(\frac{3}{7} k_v \Big(- \frac{1}{19200000} Pe\, c^4\, k_v + \frac{1}{2} k_v^2\, c^2 \Big) \\
&+ \frac{3}{7} k_v\, c \Big(- \frac{1}{3840000} Pe\, k_v\, c^3 + \frac{1}{2} k_v^2\, c \Big) \Big) y^8 + \frac{1}{7} \Big(\frac{1}{2} k_v \Big(- \frac{1}{3840000} Pe\, k_v\, c^3 + \frac{1}{2} k_v^2\, c \Big) + \frac{1}{2} k_v\, c \Big(\\
&- \frac{1}{1920000} Pe\, c^2\, k_v + \frac{1}{4} k_v^2 \Big) \Big) y^7 + \frac{1}{6} \Big(\frac{3}{5} k_v \Big(- \frac{1}{1920000} Pe\, c^2\, k_v + \frac{1}{4} k_v^2 \Big) - \frac{1}{3200000} k_v^2\, c^2\, Pe \Big) y^6 \\
&+ \frac{1}{5} \Big(- \frac{1}{2560000} k_v^2\, Pe\, c + \frac{1}{4915200000000} k_v\, c^3\, Pe^2 \Big) y^5 - \frac{1}{224} Pe\, k_v\, c^3\, y^8 - \frac{1}{12} Pe\, c\, y^4 + \frac{1}{2} k_v\, c\, y^3 \\
&- \frac{1}{20} Pe\, k_v\, c\, y^6 + \frac{1}{2} k_v y^2 + \frac{1}{20} k_v\, c^3\, y^5 + \frac{1}{20800} k_v^3\, c^7\, y^{13} + \frac{1}{8} \Big(\frac{3}{70} k_v^2\, c^4 + \frac{3}{7} k_v\, c^2 \Big(- \frac{1}{12} Pe\, c \\
&+ \frac{1}{4} k_v\, c^2 \Big) \Big) y^8 + \frac{1}{7} \Big(\frac{11}{40} k_v^2\, c^3 + k_v\, c \Big(- \frac{1}{12} Pe\, c + \frac{1}{4} k_v\, c^2 \Big) \Big) y^7 + \frac{1}{6} \Big(\frac{3}{5} k_v \Big(- \frac{1}{12} Pe\, c + \frac{1}{4} k_v\, c^2 \Big) \\
&+ \frac{9}{10} k_v^2\, c^2 \Big) y^6 + \frac{1}{4} \Big(\frac{1}{3686400000000} k_v\, Pe^2\, c^2 + \frac{1}{3} c^2 \Big(\frac{1}{480} k_v^2\, c^5\, y^9 + \frac{1}{8} \Big(\frac{3}{70} k_v^2\, c^4 + \frac{3}{7} k_v\, c^2 \Big(\\
&- \frac{1}{12} Pe\, c + \frac{1}{4} k_v\, c^2 \Big) \Big) y^8 + \frac{1}{7} \Big(\frac{11}{40} k_v^2\, c^3 + k_v\, c \Big(- \frac{1}{12} Pe\, c + \frac{1}{4} k_v\, c^2 \Big) \Big) y^7 + \frac{1}{6} \Big(\frac{3}{5} k_v \Big(- \frac{1}{12} Pe\, c \\
&+ \frac{1}{4} k_v\, c^2 \Big) + \frac{9}{10} k_v^2\, c^2 \Big) y^6 + \frac{9}{40} k_v^2\, c\, y^5 + \frac{1}{8} k_v^2 y^4 - \frac{1}{224} Pe\, k_v\, c^3\, y^8 - \frac{1}{42} Pe \Big(- \frac{1}{3} Pe\, c + k_v\, c^2 \Big) y^7 \\
&- \frac{1}{20} Pe\, k_v\, c\, y^6 - \frac{1}{20} Pe\, k_v y^5 \Big) \Big) y^4 + \frac{1}{2} \Big(\frac{1}{480} k_v^2\, c^5\, y^9 + \frac{1}{8} \Big(\frac{3}{70} k_v^2\, c^4 + \frac{3}{7} k_v\, c^2 \Big(- \frac{1}{12} Pe\, c \\
&+ \frac{1}{4} k_v\, c^2 \Big) \Big) y^8 + \frac{1}{7} \Big(\frac{11}{40} k_v^2\, c^3 + k_v\, c \Big(- \frac{1}{12} Pe\, c + \frac{1}{4} k_v\, c^2 \Big) \Big) y^7 + \frac{1}{6} \Big(\frac{3}{5} k_v \Big(- \frac{1}{12} Pe\, c + \frac{1}{4} k_v\, c^2 \Big) \\
&+ \frac{9}{10} k_v^2\, c^2 \Big) y^6 + \frac{9}{40} k_v^2\, c\, y^5 + \frac{1}{8} k_v^2 y^4 - \frac{1}{224} Pe\, k_v\, c^3\, y^8 - \frac{1}{42} Pe \Big(- \frac{1}{3} Pe\, c + k_v\, c^2 \Big) y^7 \\
&- \frac{1}{20} Pe\, k_v\, c\, y^6 - \frac{1}{20} Pe\, k_v y^5 \Big) y^2 + \frac{1}{2} \Big(\frac{3}{160} k_v^2\, c^5\, y^8 + \Big(\frac{3}{70} k_v^2\, c^4 + \frac{3}{7} k_v\, c^2 \Big(- \frac{1}{12} Pe\, c \\
&+ \frac{1}{4} k_v\, c^2 \Big) \Big) y^7 + \Big(\frac{11}{40} k_v^2\, c^3 + k_v\, c \Big(- \frac{1}{12} Pe\, c + \frac{1}{4} k_v\, c^2 \Big) \Big) y^6 + \Big(\frac{3}{5} k_v \Big(- \frac{1}{12} Pe\, c + \frac{1}{4} k_v\, c^2 \Big) + \frac{9}{10} \\
&k_v^2\, c^2 \Big) y^5 + \frac{9}{8} k_v^2\, c\, y^4 + \frac{1}{2} k_v^2 y^3 - \frac{1}{28} Pe\, k_v\, c^3\, y^7 - \frac{1}{6} Pe \Big(- \frac{1}{3} Pe\, c + k_v\, c^2 \Big) y^6 - \frac{3}{10} Pe\, k_v\, c\, y^5 \\
&- \frac{1}{4} Pe\, k_v y^4 \Big) y^2 + \frac{3}{800} k_v^3\, c^5\, y^{11} - \frac{1}{42} Pe \Big(- \frac{1}{3} Pe\, c + k_v\, c^2 \Big) y^7 + \frac{1}{4} k_v\, c^2\, y^4 :
\end{aligned}
$$

(C16)

Appendix D:
Calculation of values for Pe and k_v for our system

Calculation of values for Pe and k_v for our system

As we mentioned earlier in Abstract, we use Ethanol as a solvent and Sodium Ethoxide (NaC2H5O) and Methyl Iodide (CH3I) as a solute. Let introduce again the equations for Pe and k_v for our system:

$$Pe = 0.51\frac{v}{D},$$

$$k_v = \frac{a^2 K_v F_v(C_s)}{DC_s}, \tag{D1}$$

where v describes the kinetic viscosity, a is the characteristic length (It was derived in Appendix A), K_v is the rate constant for the volume reaction, C_s describes the concentration at the surface of the rotating disk, $F_v \cdot C_s$ is the kinetic reaction term for the volume reaction at the surface of the rotating disk, and D represents the expression of the diffusion constant. From Appendix A:

$$\alpha = 0.51433880[m/radian]. \tag{D2}$$

From [10] and the equation $v = \mu/\rho$, and μ represents the dynamic viscosity and ρ represents the density of the liquid, the diffusion coefficient, we can solve the expression of the particular chemical reaction. The diffusion coefficients for the most ions are similar and at the room temperature they will be in the range of 8.0×10^{-9} to $9.5 \times 10^{-9}[m^2/s]$ [11].

We then calculated the value of v:

$$\mu = 0.001095[Ns/m^2], [Pa \cdot s],$$

$$\rho = 0.789[g \cdot cm^{-3}],$$

$$v = 1.878327 \cdot 10^{-6}[m^2/s]. \quad \text{(D3)}$$

Then we obtain the dimensionless Peclet number for the Ethanol reaction, using the lower range of the diffusion constant:

$$Pe_1 = 0.51 \frac{1.878327 \times 10^{-6}}{8 \times 10^{-9}},$$

$$= 119.7433463.$$

Similarly, using the upper range of the diffusion constant:

$$Pe_2 = 0.51 \frac{1.878327 \times 10^{-6}}{9.5 \times 10^{-9}},$$

$$= 100.8365021. \quad \text{(D4)}$$

We assume that the kinetic reaction term for $F_v(C_s)$ as that the volume reaction at the surface of the rotating disk has the similar structure as the general kinetic reaction. So we can obtain the rate of reaction constant using the Arrhenius parameters given in [12]:

$$\ln F_v(C_s) = \ln A - \frac{E_a}{RT}, \quad \text{(D5)}$$

where A is the Arrhenius parameter for our chemical reaction. The $-E_a/R$ is the activation energy, and T is temperature. Using data from [12]:

$$A = 2.42 \times 10^{11}[dm^3 mol^{-1} s^{-1}]:$$

$$E_a = 81.6[kJ \cdot mol^{-1}]:$$

$$R = 8.314[J \cdot K^{-1} mol^{-1}]:$$

$$T = 285.15[K]: \quad \text{(D6)}$$

Then, we obtained the rate of reaction constant:

$$F_v(C_s) = exp(-\frac{E_a}{RT}) \times A,$$

$$F_v(C_s) = 2.726092 \times 10^{-4}[dm^3 mol^{-1} s^{-1}]. \quad \text{(D7)}$$

We used the values from (D4) and (D7) and substitute them into (3.1), and we call this newly constructed equation as a total concentration equation.