University of Nevada, Reno

Stock Option Valuation for Thinly Traded Enterprises: Comparing the Historically Based Intrinsic Value Model to the Black-Scholes-Merton Model

A thesis submitted in partial fulfillment
of the requirements for the degree of

Bachelor of Science in Business Administration with a Major in Accounting and the Honors Program

by

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December, 2015

UNIVERSITY OF NEVADA RENO

THE HONORS PROGRAM

We recommend that the thesis
prepared under our supervision by

LUKE TANAKA

entitled

Stock Option Valuation for Thinly Traded Enterprises: Comparing the Historically Based Intrinsic Value Model to the Black-Scholes-Merton Model

be accepted in partial fulfillment of the
requirements for the degree of

BACHELOR OF SCIENCE IN BUSINESS ADMINISTRATION WITH A MAJOR IN ACCOUNTING

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Charles Carslaw, M.A., Thesis Advisor

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December, 2015

Abstract

A publicly traded company's reporting is often affected by the valuation of stock options. For large, regularly traded companies, the valuation of stock options isn't an issue because these companies have valuation data for publicly traded options. For thinly traded, highly volatile companies, the issue of establishing a fair value can seriously impact thinly traded, highly volatile companies' bottom line. Generally Accepted Accounting Principles or GAAP as promulgated by accounting standard setters such as the Financial Accounting Standards Board (FASB) or the International Accounting Standards Board (IASB) require that stock options issued by companies must be valued at their fair value. In order to value these options, most accountants use the Black-Scholes-Merton (BSM) option pricing model because of its simplicity. While evidence suggests that the model is effective for larger entities with regularly traded stocks, the BSM model becomes less effective when a stock's price is highly volatile or trading is less regular. The Historically Based Intrinsic Value (HBIV) model is a proposed alternative model that makes similar assumptions to the BSM model. In this thesis, the author will test the two models on theoretical call options for 59 highly volatile, thinly traded stocks to establish whether or not the HBIV model is a valid alternative to the BSM model, which could improve the accuracy of financial reporting for thinly traded, highly volatile companies.

Acknowledgements

First and foremost, I would like to extend a tremendous thank you to Professor Charles Carslaw, my mentor for this undertaking. He has worked with me from Day One, helping me select my thesis topic and guiding me through until its conclusion. Thank you for taking the time to thoroughly edit my drafts, hastily drive to the University for last minute meetings, and conversations about traveling the world.

Second, thank you to Dr. Rahul Bhargava for everything. When I started this thesis, I knew that I would have to consult him regarding his model and his suggestions for implementation; however, he has felt like a second mentor to me during this past semester. His advice went beyond the thesis and often drifted on to the broader topics of personal fulfillment and following your passions.

Thank you to the Honors Program of the University of Nevada, Reno and its Director, Dr. Tamara Valentine for the structure and assistance with the formalities and intricacies of thesis writing and source citing.

Lastly, thank you to Miwako Schlageter, my thesis buddy. Thanks to you I have learned the difference between good and bad coffee and know more about CSR than I ever wanted to. I'm thrilled to have pushed through the thesis writing experience with you and to complete our degrees simultaneously. Thank you to my roommates and friends for listening to my concerns and always being supportive of me.

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Introduction & Literature Review

Under Generally Accepted Accounting Principles (“GAAP”), accountants are required to value options and warrants in a number of situations. An option is a financial instrument that provides the owner the right, but not the obligation, to purchase stock at a set price at a point in the future. The seller conversely must, if the option is exercised, sell stock to the option owner at the agreed-upon price. Options of large companies are commonplace now and can be bought and sold in many capital markets. Options of smaller companies do not enjoy a similar degree of marketability; therefore, the market value of these options may be difficult to determine thus creating the need for a pricing model.

While there are several models available to estimate the fair value of options, the Black-Scholes-Merton model is by far the most common, as can be seen from observing the practices of firms (Rudkin & Bosco, 2014). The majority of models base the estimates on data from the underlying security (FASB, 2004). The Black-Scholes-Merton model (“BSM”) works well for stocks that are frequently traded, as the levels of volatility for these stocks tend to be lower (Long & Officer, 1997). Stocks that are thinly traded tend to have higher levels of volatility, which in turn make models like the Black-Scholes-Merton model inappropriate at predicting the prices of options, particularly where volatility exceeds 100% (1997).

The Financial Accounting Standards Board (“FASB”) has expressed a desire for accountants to seek a more technically advanced model than the BSM model to attain more accurate estimates of fair value (FASB, 2004). The Historically Based Intrinsic

Value ("HBIV") model is an alternative to the BSM model and was developed by Dr. Rahul Bhargava, Associate Professor of Finance at the University of Nevada, Reno. Preliminary work with the HBIV model showed promise and this thesis builds off of the initial study. This alternative model, like the BSM model, utilizes the past prices of underlying stocks to predict the fair value of stock options. Accountants are required to segregate the value of the option from the host contract in the case of an embedded option or to allocate cost to the warrant where it is a separate security. This HBIV model may allow those in the accounting profession to improve accuracy in the valuation process.

In this thesis, the author will compare the Historically Based Intrinsic Value model to the Black-Scholes-Merton model for accuracy in predicting the fair value of stock options of thinly traded, highly volatile, publicly traded companies. The author will also provide a thorough introduction to options, the accounting standards impacting options, and prominent valuation models that currently exist before discussing the methodology, results, and concluding observations.

Overview of Options

When an option contract is created, the buyer has the right, but not the obligation, to purchase a security for an agreed upon price, called the strike price. There are two types of options: European and American. European type options assume that an option can only be exercised on their expiration date whereas American type options assume that they can be exercised at any point in time up until the expiration date (Black & Scholes, 1973). Because an option derives its value from the price of the underlying security, whether or not an option holder chooses to exercise the option will depend on the price.

If an option can be exercised at a price that is below the market value of the underlying stock, the option is considered to be “in the money” (ITM) because the holder is able to make an immediate profit on the sale of the stock. Conversely, if the stock option exercise price exceeds the available market price, the option is “out of the money” (OTM) as there is no financial gain to be realized by the holder of the option. When first issued, options have a “time value” component that represents the expected rise in the stock price until the exercise date. In the case of “at-the-money” (ATM) or OTM stocks, the cost of the option represents the time value component. For ITM options, the difference between the price of the option and the profit to be made is the time value. There are several types of options in the market, including warrants, embedded options, and employee stock options.

Warrants are call options that are created by corporations that relate to their common stock and are commonly associated with debt offerings (Handley, 2002). When a debt offering occurs, warrants may often be included with the initial offering to make the offer more appealing. If a warrant is detachable, the warrant can be traded on a secondary market. Embedded options are similar to warrants in that the embedded option is associated with debt offerings; however, the embedded option is inseparable from the security itself.

Employee stock options share similar characteristics to regular stock options and warrants, with some exceptions. Employee stock options are used as a form of compensation to a company’s employees and are designed to align the employees’ interests with those of the company. This alignment of interests serves to encourage employees to make smart business decisions and actions that will raise the company's stock price (U.S. Securities and Exchange Commission, 2014). Similar to regular stock options, employee stock options guarantee that employees can buy stock, or exercise their option, at a certain price called the exercise price or strike price; however, employee stock options are subject to some restrictions that regular stock options are not. When an employee stock option is awarded to an employee, the particular day is referred to as the grant date and the exercise price is set. Other conditions accompanying employee stock options are restrictions in order to ensure the goals of the employee are in line with those of the company. The time required for an option to vest, or become exercisable, is called the service period. During this time, the employee must continue to work for the company or the options are forfeited. Furthermore, an employee's action with the employee stock option may be further restricted by preventing the options from being traded or hedging the value of the option against price shifts. These characteristics are not found in regular stock options.

The proliferation of the other aforementioned options has played a large role in the development of accounting rules relating to option valuation and therefore the need for models to estimate fair value.

The Emergence of Option Valuation Models

The use of employee stock options as compensation became popular during the technology boom at the turn of the twentieth century, and regulations regarding the disclosure of options use have changed dramatically in the last few decades (U.S.

Securities and Exchange Commission, 2007). Under the now superseded Opinion 25 of the FASB, these employee stock options were valued at their intrinsic value (APB, 1972). The issue arises from the nature of measuring these employee stock options at their intrinsic value, which is the difference between the underlying stock price and the exercise price. The option has no value if the exercise price is less than or equal to the current stock price. Qualified stock options commonly had no intrinsic value, which had special tax benefits for recipients. A company would not need to recognize an expense if there was no intrinsic value to the award granted to the employee.

In order to provide the users of financial statements with accurate and reliable information, the FASB issued Financial Accounting Standard 123 (“FAS 123”) in 1995, which pushed for companies to recognize compensation expense for employee stock options at the fair value of the option rather than intrinsic value. Fundamental accounting principles require that expenses be recorded in the same period as the related revenues are earned. In the case of employee stock options, the employee who receives the options provides a service to the company through his or her employment, and the value of the compensation expense determined from the employee stock options must be recognized much like a salary would be. Fair value, as defined by GAAP, is the price at which the asset could be acquired or sold in the current market (FASB, 2004). The fair value of an option comprises of both its intrinsic value, which is the difference between the exercise price and the current market price, and the option’s “time value”. The time value gives option holder the ability to take advantage of an extended time period to pick the optimal moment to exercise the option at a favorable price.

FAS 123 suggested the use of the BSM model to compute the employee stock option's fair value. In the BSM model, the value is computed in the case of a transaction involving an option that takes into account not only the intrinsic value, but also other factors, including volatility, term, expected dividends, and the risk-free interest rates (Black & Scholes, 1973). When FAS 123 was issued, it did not require the use of the fair value valuation and still allowed for companies to report under Opinion 25, but did require footnote disclosure. Starting in 2006, FAS 123(R), which is now Accounting Standards Codification Section 718, changed the accounting rules pertaining to fair value by requiring companies to recognize compensation expense for employee stock options over the service period based exclusively on fair value (FASB, 2014). FAS 123(R) was also broader in scope than its predecessor since it related to share-based payments in general and not just to employee stock options. This change expanded the need to value a broader range of share transactions using a valuation model.

Throughout this time period, accountants were implementing FAS 133: *Accounting for Derivative Instruments and Hedging Activities* (FASB, 1998), which was originally issued in 1998 and became effective for companies for all financial quarters beginning after June 15, 1999. FAS 133 required companies to value a wide range of derivative financial instruments at fair value. These financial instruments often explicitly included option features, such as stock warrants, or implicitly included an option feature, such as the embedded option in a convertible debt contract. These option elements also had to be valued by accountants using an option pricing model to determine their fair value.

The BSM model became the most commonly-used model to value options and similar securities (Rudkin & Bosco, 2014).

Black-Scholes-Merton Model and Assumptions

The BSM model was originally developed in the late 1960s and early 1970s to value certain types of options. Fischer Black, Myron Scholes, and Robert Merton's Nobel Prize winning model uses stochastic calculus to calculate for the random, but statistically probable elements found in pricing options (Black & Scholes, 1973). The BSM model makes several assumptions regarding stock prices and returns; the risk free rate; volatility; taxes and transaction costs; payment of dividends; and the option type (Chance & Brooks, 2007; Madhani, 2007). The BSM model formula for a call option can be found in Figure 1.

Figure 1: BSM Model Formula

$$Value\ of\ Call\ Option = Se^{-qt}N(d_1) - N(d_2)Ke^{-rt}$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Where:

S = Current Stock Price

K = Strike Price

ln = Natural Log

e = Exponential Term

t = Time until Exercise

r = Risk Free Interest Rate

σ = Standard Deviation of Stock Price

q = Dividend Yield

N = Cumulative Standard Normal Distribution

Stock prices are difficult to predict because the returns on stocks tend to be random and thus unpredictable (Chance & Brooks, 2007). When plotted, these returns do not appear to come close to adhering to the normal distribution; however, it is possible to look at the stock return in another form, continuous compounding. Using continuous compounding, the model is able to produce another return called the continuous compounded return or log return, which can be defined as $r = \ln(FV/PV)$ where r is the return, FV is the price at the end of the period, PV is the price at the beginning of the period, and ln is the natural log. The log return for stocks ends up being more normally distributed than a regular stock return. Furthermore, by continuously compounding the returns, the lognormal distribution will always be positive and therefore will not allow for any stock price to dip below zero even though some returns will be negative. These factors make the log return more applicable for use in the BSM model.

The BSM model further assumes that the risk-free rate, and therefore interest rates, will remain constant throughout the term of the option. While interest rates will undoubtedly change, the impact made by interest rates on the estimation of fair value is small so assuming interest rates are constant simplifies the model (Gardner et al., 2011). The effect is further minimized in times when risk-free interest rates are low.

Volatility, or the standard deviation of the stock price, is not directly observable when pricing options, unlike many of the other parameters in the BSM model, and therefore, must be estimated. The volatility has a larger effect on the pricing of options than other parameters, and the assumption that volatility is constant is not true in the real world. Again, in order to keep the model simplified, the assumption is that volatility is constant throughout the term of the option even though the volatility level is sure to change (Fortune, 1996).

The taxes and transaction costs that accompany the trading of options, stocks, and other securities are ignored in order to focus on the price of the option in as pure a manner as possible.

The original model also assumes that there are no dividend payments (Black & Scholes, 1973). Where the dividend payment assumption is not realistic, the model has been adjusted by subtracting the discounted value of a dividend from the stock's price (Madhani, 2007).

Lastly, the BSM model assumes that the options are European type options, as the BSM model is unable to effectively incorporate the possibility of exercising the option prior to the lapse date. This is clearly unrealistic for many types of option. Although there are these numerous criticisms to the BSM model due to the reliance on the aforementioned assumptions, the BSM model is still widely used to value stock options.

Criticisms of Black-Scholes-Merton Model

The BSM model was determined to work with options that were highly liquid and had few restrictions on them, such as the vesting period with employee stock options (Madhani, 2007). While the BSM model is effective at predicting American options of large, highly-traded companies, the BSM model begins to fall apart when stocks become thinly traded as the fair value of the stock is difficult to ascertain (Long & Officer, 1997). The price may jump relatively large amounts when trading does occur and would therefore not result in a normal distribution in regard to the log return, which makes hedging impractical and violates an assumption of the BSM model (Chance & Brooks, 2007). Furthermore, the model encounters issues when the volatility is over one hundred percent. A stock's price can go to zero, but can never be negative as the owner of that stock would have lost all of his or her investment but would not owe anyone anything further. The BSM model does not reflect this minimum price as it allows stock prices to drop below zero due to volatilities that can be well over one hundred percent in some cases. Furthermore, as discussed below, the dependence on somewhat arbitrarily determined inputs makes the valuation process subjective (2007).

There are five inputs required for the BSM: current stock price, strike price, time to maturity, annualized risk-free interest rate, and annualized volatility. The first two inputs can be determined relatively easily because the current stock price is known for a stock that has a stock exchange quotation and the strike price is stated in the option contract. There are some issues that exist for the observable stock price of a thinly traded security. The quoted stock price may be for a small parcel of shares and may not be applicable to large blocks of shares. Valuation issues relating to "blockage discounts" are beyond the scope of this thesis. The time to maturity for American type options and employee stock options can be more difficult to determine as the option may be exercised prematurely. The BSM model assumes exercise at the expiry date of the option, though the exercise at expiry may not be the reality for employee stock options in particular. However, this early exercise date can be compensated for and the ASC 718 requires the use of the expected term, rather than the full term, of the option to be used in valuing employee stock options (FASB, 2014). Expected term is typically determined from past history of exercise by the option holder group. In the case of smaller entities with fewer option holders, there may be limited reliable history about past exercise practices.

The last two inputs, the interest rate and volatility, are more difficult to predict as they can change in unpredictable directions and will not remain constant throughout the term of the option. Therefore, these inputs are defined somewhat arbitrarily. Currently, with interest rates at historic lows and remaining relatively steady, the interest rate parameter is not of major consequence (Gardner et al., 2011). The final parameter is the volatility of the entity's stock price, which, in the case of highly volatile or thinly traded options, can heavily impact the BSM model. Because the future volatility of an option is not known at the inception of the option, assumptions have to be made about what the volatility will be. Typically, if a security has a price history of a sufficient length, the expected volatility will be derived from the historical price pattern of the security. The SEC's recommendation is that a time period preceding the option grant date equivalent to the term of the option should be used to determine the volatility of the stock (U.S. Securities and Exchange Commission, 2011). If there is insufficient data on stock prices, companies may use the price history and volatility of similar companies as surrogates to determine their volatility. Unfortunately, evidence suggests that "implied volatility is a poor forecast of actual future volatility" and has negative repercussions for the BSM model (Fortune, 1996, p. 27).

The research conducted on the BSM model suggests that while the BSM model can be a good fit for heavily traded securities, securities that do not have similar market power can be severely mispriced due to a number of reasons, the most significant being the volatility parameter. If options are overpriced, it will cause an unwarranted increase in expenses for the company issuing the options and misstate the financial statements accordingly. As a result, since the development of the BSM model in the early 1970s, there have been a number of models created which attempt to improve upon the flaws of the BSM model. These flaws include the BSM model's problem with high volatility in an underlying security and its performance with thinly trade stocks.

Other Valuation Models

There are several other models that exist for companies to utilize; however, not many are used in practice due to their more intricate and complex nature. Two of the major model variants are lattice type models and Monte Carlo simulations.

The FASB allows the use of lattice models to calculate the fair value of options since the lattice model also takes into account "financial economic theory and reflects all substantive characteristics of the options" (Baril et al., 2005, p. 57). A lattice models seeks to model possible paths of the options' price in discrete time periods (2005). In many respects, the lattice model is able to outperform the BSM model when valuing employee stock options because the lattice model allows for greater flexibility with exercise dates, volatility, and interest rates (2005). Unfortunately, the lattice models are much more complicated than the BSM model, and lattice model implementation requires more technically experienced staff and the use of in-house programming or outside software, and possibly more funds/expenses (2005). When these lattice models are implemented, they tend to decrease the valuation of the options and the expense related to options for firms because managers can take advantage of the flexibility inherent with this model (Bratten et al., 2015). Bratten et al.'s research further suggests that the lattice models do not make up for the BSM model's shortcomings and firms that make the switch to the lattice model from the BSM model do so primarily to reduce the compensation expense impact on the income statement (2015).

Another type of acceptable model is Monte Carlo simulation. This simulation works by calculating future stock price movements in order to find the possible stock prices at the option expiration date (Boyle, 1977). The benefit of a Monte Carlo simulation is that it is "independent of the number of state variables" (Averbukh, 1997, p. 19). Therefore this simulation can still provide a statistical likelihood of an outcome regardless of the number of inputs that may vary (1997). The downside is that the simulation requires a large number of complex calculations, which increase in quantity when more unknown variables are introduced thus making the simulation very expensive. While there are ways to reduce the number of calculations required (1997), implementing a Monte Carlo simulation is still has a high economic cost that small companies may be unwilling to incur.

Unfortunately, while there have been many models developed and researched, the majority of these models are unable to improve upon the BSM model significantly or provide explanations for the shortcomings of the BSM model at an implementable level (Guidolin & Timmermann, 2003). For the thinly traded, highly volatile companies examined in this thesis, many of these models are impractical due to the high levels of resources and technical expertise required. The BSM model is still the simplest model. It is widely used and allowed under GAAP even though it may not be accurate. The lack of significant progress in finding an alternative to the BSM model that is used in practice leaves the field wide open for further research into this topic, and this thesis aims to examine a simple alternative.

Historically Based Intrinsic Value (HBIV) Model

An alternative to the BSM model, developed by Dr. Rahul Bhargava, uses the historical intrinsic value to determine the fair value of the option at grant date. The HBIV model is relatively simple compared to many of the other BSM model alternatives, such as Monte Carlo simulations and lattice models. The HBIV model is able to calculate the fair value using readily available historical stock prices and comparable assumptions to the BSM model.

The assumption for HBIV is that the stock will be as equally volatile for the upcoming year as it has been for the past year. This is a similar assumption to the BSM model but it is used to calculate a stock option’s fair value under the HBIV model. The value of the stock option is calculated for each day until maturity. The values each day will either be the difference between the closing stock price for each day and the strike price for the option if the option is in-the-money or zero if the option is out-of-the-money. These values are then averaged to create the average payoff of the option over the historical time period equal to the time to maturity when the stock option was first written. This average payoff of the option is the fair value of the stock option per the HBIV model. The formula for the HBIV model is shown in Figure 2.

Figure 2: HBIV Model Formula

$$Value\ of\ Option\ Call = \frac{\sum_{i=0}^{n} Maximum(Current\ Stock\ Price - Strike\ Price, 0)}{n}$$

The HBIV model is inexpensive to implement as it uses readily available historic data and limited computational power to determine an option’s fair value. Furthermore, because the HBIV model takes into account the average payoff of the option historically, the HBIV model is suited for use on American style options.

Given the ever-increasing amounts of regulation imposed on the financial industry by the FASB, SEC, and other governing bodies, there will continue to be research on valuation models for derivatives. Currently, there is an opportunity for a low-cost, simple, and more accurate valuation model to impact the reporting process for firms whose stocks experience high volatility. By discerning whether the BSM or the HBIV model is more effective at establishing the fair value of thinly traded, highly volatile stock options, the author hopes to facilitate future research on stock options and similar financial instrument valuation that will result in more fairly stated financial statements and efficient capital markets. The methods used to conduct the research with the HBIV and BSM models are discussed in detail in the following section.

Methodology

Explanation of Methods

To test whether or not the HBIV model is more effective than the BSM model at estimating the fair value of American style call options of highly volatile companies, a number of criteria were established to select companies that would qualify as highly volatile and thinly traded.

The sample selection was based on the following: small market capitalization, low average daily trade volume, and low 50-day average prices. A small market capitalization, the number of outstanding shares multiplied by the stock price, ensured that the company had neither a large number of shares nor an extremely high stock price, which are indicative of larger, stable companies. A micro market capitalization between $10 and $150 million was selected to ensure that all stocks selected still had a value and outstanding shares. Furthermore, this market capitalization range decreased the likelihood of a publicly traded option. There is no widely accepted definition of “thinly traded” and for the purposes of this study thinly traded was determined to be a low average daily trade volume between 5 and 200 trades. This low volume suggests that the company’s stock did not regularly trade, and therefore, did not have a stable price that a regularly traded stock would have. When trades do occur with a low average daily trade volume stock, the price tends to change by a large percentage, thus creating a high volatility. Lastly, the low 50-day average price of the stock that ranged between $0.50 and $5.00 ensured that small shifts in price would lead to major differences in the daily return, thus increasing the volatility of the stock. Additionally, the lowest price of $0.50 ensured that a $0.01 change would not lead to a massive daily return difference. The cap at $5.00 served to ensure that none of the stocks belonged to a closely held company that did not trade.

The companies selected had to have at least 5 consecutive years of historic stock data to run the models on assuming a one-year time to maturity for the hypothetical options. The models require one year of prior year data, where one year is the equivalent to 250 trading days, to estimate the fair value of a one-year option. This estimate was then compared to the actual payoff of the option at the maturity date, one year later, and to the average payoff of the option during the one year period. The BSM and HBIV models were both tested on hypothetical options assuming that they were issued at-the-money, in-the-money, and out-of-the-money each year for four years.

These criteria were inputted into the Google Finance Stock Screener, which filtered out all stocks that did not meet all three of these criteria. The Stock Screener provided 131 companies whose stocks met the initial criteria. The remaining stocks' historical prices and trade volume data were downloaded and further vetted to ensure that there were at least 5 years' worth of trading data. A "year" in this sense constitutes 250 trading days, which is approximately the number of days each year the major exchanges are open. Any company that did not have at least 5 years of data was removed from the sample leaving 59 companies, which are listed in Appendix A. The remaining companies' stocks represented the sample of highly volatile, thinly traded stocks. Once the sample of stocks was gathered, the BSM and HBIV models were implemented for each stock to produce an option call price for years 2, 3, 4, and 5.

To implement the BSM model as shown in Figure 1, several other inputs were required. Those inputs that could be derived from the historic stock data included: annual standard deviation, the stock's current price, and the option's strike price. The remaining inputs, including the time to maturity, risk free rate, and dividend yield were arbitrarily defined to reflect the nature of the current market and the theoretical options being valued.

The annual standard deviation was calculated by first taking the daily stock returns and finding the daily standard deviation of each year's daily returns. The daily standard deviation was then annualized by multiplying it by the square root of the number of days per year. In this case, 250 days to represent each trading year.

The stock's current price was taken at each year interval from the historic data. The stock option strike price was derived from the current price. If the option was simulated to be at-the-money, the strike price was equal to the current price at the year interval. If the option was simulated to be in-the-money, the strike price was 10% less than the current price at the year interval. Alternatively for out-of-the-money options, the strike price was calculated to be 10% more than the current stock price at the year interval.

The arbitrary input, time to maturity, was 1 year or 250 trading days, as stated above. The risk free rate was set to 0.25%, a comparable amount to the current market's risk free rate determined by one-year Treasury securities. The dividend yield was assumed to be zero as these high volatility companies very rarely pay dividends.

The value of the call option was calculated using the prior year stock data and simply took the average of the sum of the larger of the difference between the daily stock price and the strike price or zero. The average payoff of the prior year's option was considered to be the estimated fair value of the stock option. This process was repeated every year until the end of year four.

The actual payoff of the option is the value of the stock option at maturity, which will either be the difference between the stock's current price and the strike price or zero if the strike price exceeds the current price. The average payoff of the option follows the same formula as the HBIV model, which is the average of the daily value of the option during the time from when the option is written to the maturity date.

To ultimately test the effectiveness of both models, the difference between each model's estimate of fair value and the option's average and actual payoff were tabulated for each stock. This difference represents the prediction error of each model. To account for varying stock prices and option values, the differences were normalized by dividing the difference by the strike price for each option. The differences for each stock were then compiled into groups based on whether the option strike prices were initially at-the-money (ATM), in-the-money (ITM), or out-of-the-money (OTM) and whether the difference was between the average payoff or actual payoff to two sample T tests.

Development of Statistical Hypotheses

The statistical hypothesis for each T test was created from the overall thesis hypothesis that the HBIV model would outperform the BSM model in predicting the fair value of American style call options for highly volatile, thinly traded stocks. The two sample T test as able to determine whether or not "the means of two independent groups differ" (Minitab).

Therefore, the following hypotheses are created for each of the scenarios based on whether the differences were from the average payoff (AVG PO) or the actual payoff (ACT PO) and whether the options were initially at-the-money (ATM), in-the-money (ITM), or out-of-the-money (OTM). Each is to be evaluated at a significance level of $\alpha = .05$ and the null hypothesis will be rejected if the p-value < .05.

Two Sample T Test for ATM and AVG PO

$$H_0: \mu_{BSM\ ATM-AVG\ PO\ ATM} - \mu_{HBIV\ ATM-AVG\ PO\ ATM} = 0$$

$$H_a: \mu_{BSM\ ATM-AVG\ PO\ ATM} - \mu_{HBIV\ ATM-AVG\ PO\ ATM} > 0$$

Two Sample T Test for ITM and AVG PO

$$H_0: \mu_{BSM\ ITM-AVG\ PO\ ITM} - \mu_{HBIV\ ITM-AVG\ PO\ ITM} = 0$$

$$H_a: \mu_{BSM\ ITM-AVG\ PO\ ITM} - \mu_{HBIV\ ATM-AVG\ PO\ ITM} > 0$$

Two Sample T Test for OTM and AVG PO

$$H_0: \mu_{BSM\ OTM-AVG\ PO\ OTM} - \mu_{HBIV\ OTM-AVG\ PO\ OTM} = 0$$

$$H_a: \mu_{BSM\ OTM-AVG\ PO\ OTM} - \mu_{HBIV\ OTM-AVG\ PO\ OTM} > 0$$

Two Sample T Test for ATM and ACT PO

$$H_0: \mu_{BSM\ ATM-ACT\ PO\ ATM} - \mu_{HBIV\ ATM-ACT\ PO\ ATM} = 0$$

$$H_a: \mu_{BSM\ ATM-ACT\ PO\ ATM} - \mu_{HBIV\ ATM-ACT\ PO\ ATM} > 0$$

Two Sample T Test for ITM and ACT PO

$$H_0: \mu_{BSM\ ITM-ACT\ PO\ ITM} - \mu_{HBIV\ ITM-ACT\ PO\ ITM} = 0$$

$$H_a: \mu_{BSM\ ITM-ACT\ PO\ ITM} - \mu_{HBIV\ ITM-ACT\ PO\ ITM} > 0$$

Two Sample T Test for OTM and ACT PO

$$H_0: \mu_{BSM\ OTM-AC\ PO\ OTM} - \mu_{HBIV\ OTM-ACT\ PO\ OTM} = 0$$

$$H_a: \mu_{BSM\ OTM-ACT\ PO\ OTM} - \mu_{HBIV\ OTM-ACT\ PO\ OTM} > 0$$

where μ is representative of the prediction error under each model divided by the corresponding strike price.

Results

After running the two different T tests on the data collected in the methods surmised above, the T statistic results are displayed in Table 1. The p-value required to reject the null hypothesis for all of the aforementioned hypotheses is one that is less than .05. Given the 239 degrees of freedom, the t statistic would need to have an absolute value of 1.65 or greater. As seen in Table 1, the HBIV model does not outperform the BSM model in any scenario given a significance level of alpha = .05. Therefore, the alternative hypothesis for each scenario must be rejected and it can be concluded that overall the HBIV model is not superior to the HBIV model in any of the tested scenarios.

Table 1: P Values of Single and Double Sample T Tests

Scenario	Two Sample T Statistic
AVG PO ATM	-1.349
ACT PO ATM	-1.359
AVG PO ITM	-1.393
ACT PO ITM	-1.403
AVG PO OTM	-1.311
ACT PO OTM	-1.322

The data contained in Table 2 – Table 4 further reveal that the BSM model has a tendency to consistently overprice the options in all scenarios. Regardless of whether the option was issued ATM, ITM, or OTM, the BSM overpriced more than 80% of the call options. The HBIV model overpriced and underpriced options at similar rates with the majority of scenarios having less than 55% of the options overpriced. Only the HBIV ACT PO ITM and HBIV ACT PO OTM had options overpriced 60% and 70% of the time, respectively. While this would indicate the BSM has a pricing bias, the percentage by which the BSM overprices options was approximately 70% - 86% on average with a standard deviation that ranged from 24% to 34% whereas the HBIV model overprices options by 981% - 1784% with a massive standard deviation ranging from 8383% to 12448%. The massive standard deviation found with the overpriced HBIV options suggests that the model is not reliable and lacks consistency.

In terms of underpricing options, the BSM model underpriced on average between 877% and 1021% whereas the HBIV model underpriced on average by 302% to 505%. For underpriced options, the HBIV model had a slightly smaller standard deviation that ranged from 2145% to 2567% whereas the BSM model ranged from 3170% to 4527%. These metrics suggest that neither the BSM model nor the HBIV model are accurate with underpricing, although the HBIV model's performance for underpriced options was better than that of the BSM model.

Table 2: Overpricing and Underpricing Statistics for Normalized Differences for AVG/ACT PO ATM Options

Overpricing and Underpricing Statistics for Normalized Differences for Average & Actual Payoff and Options Written At-The-Money				
	Average Payoff		**Actual Payoff**	
Statistic	**BSM**	**HBIV**	**BSM**	**HBIV**
Percent Overpriced	85.59%	52.12%	81.78%	54.24%
Percent Underpriced	14.41%	47.88%	18.22%	45.76%
Average Percent by which the Option was Overpriced	75.64%	1533.3%	75.2%	1474.4%
Standard Deviation for Overpricing	28.0%	11011.9%	30.8%	10788.0%
Average Percent by which the	-1050.4%	-325.5%	-904.0%	378.0%

Option was Underpriced				
Standard Deviation for Under Pricing	4245.7%	2350.8%	3373.2%	2155.0%

Table 3: Overpricing and Underpricing Statistics for Normalized Differences for AVG/ACT PO ITM Options

Overpricing and Underpricing Statistics for Normalized Differences for Average & Actual Payoff and Options Written In-The-Money				
	Average PO		**Actual PO**	
Statistic	**BSM**	**HBIV**	**BSM**	**HBIV**
Percent Overpriced	84.75%	51.27%	82.63%	59.75%
Percent Underpriced	15.25%	48.73%	17.37%	40.25%
Average Percent by which the Option was Overpriced	79.3%	1812.5%	83.2%	1569.8%
Standard Deviation for Overpricing	31.5%	12649.8%	34.5%	11719.7%
Average Percent by which the Option was Underpriced	-1109.7%	-357.0%	-1050.2%	-469.1%
Standard Deviation for Under Pricing	4587.5%	2588.8%	3831.4%	2548.1%

Table 4: Overpricing and Underpricing Statistics for Normalized Differences for AVG/ACT PO OTM Options

Overpricing and Underpricing Statistics for Normalized Differences for Average Payoff and Options Written Out-Of-The-Money				
	Average PO		**Actual PO**	
Statistic	**BSM**	**HBIV**	**BSM**	**HBIV**
Percent Overpriced	86.44%	54.24%	83.47%	69.92%
Percent Underpriced	13.56%	45.76%	16.53%	30.08%
Average Percent by which the Option was Overpriced	70.6%	1284.7%	73.5%	998.7%
Standard Deviation for Overpricing	25.3%	9599.1%	24.9%	8458.2%
Average Percent by which the Option was Underpriced	-1008.4%	-307.3%	-887.7%	-504.4%
Standard Deviation for Under Pricing	3974.1%	2185.3%	3221.0%	2403.4%

The data were heavily skewed by several observations contained in the data set that drove the averages and standard deviations to extreme levels. The stock data for several stocks were volatile and showed indicators of unusual price shifts that are not accompanied by trades to drive the price shifts, which is highly irregular. Additionally, some of the companies selected for this study, such as Jolen Inc., appear to have been bankrupt for a period of time within the sample data and had a stock price that was essentially zero. These companies were then later rejuvenated and the stock price jumped increased an enormous amount.

Limitations

This study was limited in several respects in regard to the sample. There are not a tremendous number of companies whose stocks fit the filtering criteria set stated in the Methodology section. The sample was further cut by the number of companies that had at least five years of data, and, while the selection of 59 companies is an appropriate sample, the lack of further historic data prevented the testing of longer term options. The companies require more historic data in order to test longer option and the sample would have decreased a considerable amount. Furthermore, the outliers within the data also clouded the results. The data could be cleansed and further reviewed; however, given the time frame for the study and the sheer amount of data collected and inputted manually, the cleaning of data proved to be impracticable for this thesis. Many of the finance tools available to more easily manipulate data or select a different time period for historical stock prices, lacked other tools necessary to screen out companies whose stocks did not match criteria.

Conclusion

The comparison of the BSM and the HBIV models revealed three key points: 1) neither model worked particularly well, 2) no evidence that the HBIV model is superior to the BSM model, and 3) some evidence that the HBIV model reduces overpricing relative to the BSM model.

Ultimately, neither the BSM nor the HBIV were effective at predicting the fair value of the call options for this group of stocks listed in Appendix A. Both models under all circumstances had large normalized prediction differences and massive standard deviations indicating the models were not very reliable. There is still a need for a more reliable model for this niche group of companies' stocks.

The comparison of the two models reveals that the HBIV model is not superior to the BSM model at predicting the fair value of call options of highly volatile, thinly traded stocks. The HBIV model, as it does not exceed the effectiveness of the BSM model, should not be used for valuation purposes unless it is modified and thoroughly vetted in future research.

If the prediction errors of the models are split into the instances where the model overestimated and underestimated the actual/average price, the data suggest the HBIV model results in fewer instances of overpricing for every scenario. Furthermore, the average amount by which the HBIV model underpriced the call options was less than the BSM model. The HBIV model results in a smaller standard deviation for the underpriced options as well. The fewer instances of overpricing could lead to more accurate financial statements and should be investigated further.

Future research on models predicting the fair value of stock options of highly volatile stocks could involve the comparison of the BSM model and Monte Carlo simulation or lattice models. If further work were to be done on the HBIV model, a more refined data set that excluded non-normal market conditions could yield different results. Accounting Standards Codification paragraph 718-10-55-37 justifies “disregarding an identifiable period of time in which [the] share price was extraordinarily volatile” (FASB, n.d.), which could provide reason for cleansing the data. Lastly, additional research could be done to determine whether or not these thinly traded, highly volatile stock options should be valued at anything other than the intrinsic value. If the intrinsic value is superior to existing models, it would further prove the ineffectiveness of the BSM model for this subset of options.

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Appendix A: Companies Tested in Study

Company name	Symbol	Start of Hist. Data
Aamaxan Transport Group, Inc.	AAXT	8/7/98
Aly Energy Services Inc	ALYE	12/21/98
American Church Mortgage Company	ACMC	1/10/07
Arista Investors Corp.	ARINA	1/18/89
Asia Travel Corp	ATSR	12/1/09
Avesis Incorporated	AVSS	1/10/07
Avix Technologies, Inc.	AVIX	3/26/99
Aytu Bioscience Inc	AYTU	9/25/08
Bonds.com Group Inc	BDCG	1/10/05
Clarkston Financial Corporation	CKFC	12/1/98
CoConnect Inc.	CCON	5/18/00
Comdisco Holdings Company, Inc.	CDCO	4/3/03
Comtrex Systems Corporation	COMX	1/29/08
Cornerstone Bancorp (SC)	CTOT	7/10/06
Cuisine Solutions, Inc.	CUSI	10/2/09
Dewey Electronics Corp	DEWY	3/8/99
ES Bancshares Inc	ESBS	9/4/07
Elamex, S.A. de C.V.	ELAMF	12/18/09
Emtec, Inc.	ETEC	3/31/97
Head N.V. (ADR)	HEDYY	10/3/00
Huayue Electronics Inc	HUAY	10/15/10
I-Cable Communications Ltd (ADR)	ICABY	11/24/99
ICTS International N.V.	ICTSF	6/26/96
Imperalis Holding Corp	IMHC	2/15/07
Imperial Ginseng Products Ltd. (USA)	IGPFF	10/27/95
Iris BioTechnologies Inc	IRSB	9/23/08
Jaclyn, Inc.	JCLY	8/2/91
Janel Corp	JANL	7/15/03
Jolen Inc	JOLE	2/1/95
Katy Industries, Inc.	KATY	11/5/87
Kyto Biopharma Inc	KBPH	5/1/07
Marketing Alliance Inc(NDA)	MAAL	7/21/08
MobileSmith Inc	MOST	4/15/05

Mutual Federal Bancorp Inc (Illinois)	MFDB	4/7/06
NCAL Bancorp	NCAL	5/8/97
Northern California Bancorp Inc	NRLB	5/16/07
Oakridge Energy Inc	OAKR	1/2/97
Omtool, Ltd.	OMTL	12/27/07
PASSUR Aerospace, Inc.	PSSR	2/1/95
Paragon R.E. Equity & Investment Trust	PRLE	4/24/96
Pharmaxis Ltd. (ADR)	PXSLY	1/3/06
Princeton Capital Corp	PIAC	1/8/97
Roine International Holding Corp	TSHN	5/3/10
Safco Investment Holding Corp	SIHC	4/3/03
Shale Oil International Inc	SHLE	10/8/10
Smart Energy Solutions, Inc.	SMGY	4/13/00
Smart Technologies Holding Corp	SMTE	3/26/10
Spindletop Oil & Gas Co	SPND	11/9/99
Stack-It Storage Inc	STAK	11/30/05
Strategic Acquisitions Inc	STQN	10/25/00
TexStar Oil Corp	TEXS	8/8/07
Trans-Lux Corporation	TNLX	6/17/92
Trycera Financial Inc	TRYF	8/23/06
USA Zhimingde International Group Corp	ZMDC	3/26/07
Vibe I Inc	VIBE	11/11/02
Waxman Industries, Inc.	WXMN	10/27/99
Williams Industries, Incorporated	WMSI	3/26/90
Zap.com Corp	ZPCM	11/30/99
eDoorways International Corp	ESCU	6/23/03

Appendix B: Stock Data and Test Spreadsheet

The Excel Spreadsheet utilized in this thesis can be access and reviewed at the following link:

https://goo.gl/YqqWVK