University of Nevada, Reno

# Optimal Rebalancing Strategy for a Two-Asset Stock and Bond Portfolio

A thesis submitted in partial fulfillment of the
requirements for the degree of Master of Science in
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By

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# Abstract

It is commonly believed that a continuously rebalanced investment portfolio achieves the optimal investment returns by maintaining a diversified portfolio that satisfies each individual investor's risk aversion or tolerance level. However, frequent rebalancing is costly. This paper sought to determine the optimal rebalancing method of a 60/40 stock and bond portfolio, taking into consideration transaction costs and capital gains taxes. This rebalancing method looked into two strategies of approach: time interval and percentage movement rebalancing. The first strategy rebalances a portfolio by a set frequency, be it monthly, quarterly, or yearly, etc. The second strategy rebalances a portfolio by a certain percentage movement away from the portfolio's original 60/40 mix. The findings from this exercise indicated that less frequent rebalancing is ideal for achievable maximum investment returns, including or excluding transaction costs and taxes. This finding is also consistent with the fact that rebalancing by higher percentage movement also achieves the same results of higher portfolio returns. However, with either strategy mentioned above, the longer the interval or the bigger the percentage movements away from a portfolio's original mix, the higher the standard deviation because in either case, the stocks eventually dominate the portfolio and causing it to deviate significantly from its original mix of 60/40.

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# I. Introduction

Many investors, once their investment portfolio mix is determined based on their personal comfort level in risk taking, desire to keep that proportional mix by periodically rebalancing their portfolios. The belief is that rebalancing at a certain frequency or percentage change or movement from its original portfolio mix will ensure optimal portfolio returns and the most efficient diversification. However, rebalancing costs money, frequent rebalancing lessens potential profitable gains. Furthermore, capital gains taxes further reduce the investors' net gain. On the other hand, the lack of rebalancing will cause the portfolio to deviate from its original mix, thereby losing the objective of the investors' desired portfolio mix and the effects of diversification.

There are numerous rebalancing strategies available for the investment portfolio management. In general, these strategies can be broadly categorized into two distinct groups: frequency and percentage movement rebalancing. For the frequency rebalancing strategy, an investment portfolio is rebalanced based on timelines: monthly, quarterly, semi-annually, annually, or every few years. The percentage movement rebalancing strategy rebalances a portfolio when a predetermined percentage, set by an investor or his financial advisor, moves beyond the boundaries of the desired portfolio mix. For example, a five percent movement portfolio rebalancing strategy for a 60/40 Stock and bond portfolio would take place when either the stock or bond portion of the portfolio is five percent more or less than its respective, original designated percentage.

The intention of this study is to determine how an individual, non-institutional investor can benefit from a rebalanced portfolio for his/her individual investment strategy. To this end, daily exchange traded funds (ETF) data for the Lehman aggregate bond Index (AGG) and Standard and Poor's 500 stocks (SPY) were used to calculate the optimal rebalancing strategies, since these are available to individual investors. However, ETF data were only available since 2004, so data used were from 2004 to 2010 (Wharton Research Data Services). This is an insufficient length of time to study the true effects of each rebalancing strategy. Consequently, indexed data from 1976 to 2009 for both AGG and SPY were used in the calculation for long term rebalancing determination, after the initial calculation of the strategies using the ETF data. Although indexed data of the stocks and bonds are not available to an individual investor, the results should show a trend similar to the ETF data.

Testing the two portfolio rebalancing strategies using indexed data posed some other minor challenges besides the fact that they are not available for individual investors. Other minor issues encountered include indexed data was available to me only on a monthly basis, and this data is in the form of average bid/ask prices for each index, instead of individual bid and ask prices. However, based on the preliminary calculation using the ETF data, the difference between using the average bid/ask prices versus that of the actual bid or ask prices only accounted for a very small percentage difference in the rebalanced portfolio value (±0.1%) between the two methods. Another minor issue is the use of monthly indexed data versus the use of daily ETF data for percentage movement rebalancing strategy, since the former uses monthly data and the latter uses daily data, the

calculation may not compare equally. Although the loss of accuracy further introduced a slight difference in the calculation outcome for the percentage movement rebalancing strategy, it is still not significant enough to alter the pattern/trend of the underlying data. On the other hand, the issue of the monthly versus daily data difference is not a concern for frequency rebalancing since both sets of data had to start from the monthly numbers and further converted to quarterly and yearly figures.

## II. Rebalancing Analysis Using ETF Data (2004-2010)

For the ETF data, the frequency rebalancing strategy was examined by using the monthly, quarterly, and yearly data. For the percentage movement rebalancing strategy, the percentages of 3, 5, 10 and 15% were considered. Both categories of strategies are based on a two-asset portfolio made up of 60% stocks and 40% bonds, with an initial capital investment of $10 million U.S. Dollars (USD).

To determine the net rebalanced portfolio value, transaction costs and capital gains taxes were also included in this analysis. A transaction cost of $10 was assumed for each transaction of stocks and bonds sold or bought to rebalance the portfolio. Capital gains tax rates used in this study ranged from 25 to 35 percent, by 5 percent increment for sensitivity analysis. Risk free rates (USFR) over the same period of time were used to calculate Sharpe and Treynor Ratios and Jensen's Alphas. These criteria were then used to calculate and determine the best performer among both rebalancing strategies, taking into account the consequences of transaction costs and capital gains

taxes. The sensitivity analysis revealed that the portfolio returns were consistent across the different tax rates. Therefore, for the sake of brevity, only the results for the 30% tax rate is reported and discussed in this paper. All calculations were performed using Microsoft EXCEL 2010 software, since it is a powerful tool well suited for the task.

## A. Strategy 1 – ETF Portfolio Frequency Rebalancing

The first strategy tested is that of the rebalancing by a specific period of time. For this strategy, monthly, quarterly, and yearly frequencies were chosen. Daily ETF data from 2004 to 2010 were used first to determine the trends of this strategy. After that, indexed data from 1976 to 2009 were used to test long range trends and confirm results observed from the analysis using the ETF data. The results of the Indexed data are presented in Section III. Monthly numbers were calculated from daily returns and bid and ask prices. Likewise, quarterly and yearly numbers were calculated in the same manner.

### 1. Frequency Rebalancing Strategy Results for ETF data (2004 -2010)

The main results of the ETF portfolio frequency rebalancing strategy are listed in Table 1 and Figure 1. Results from this strategy indicated that monthly rebalancing incurred the most transaction costs, whereas quarterly rebalancing resulted in the highest amount of taxes paid. There is no change in the average portfolio returns and the standard deviations before or after the transaction costs and taxes were deducted from the rebalanced portfolio because the transaction costs and capital gains taxes were small compared to the overall portfolio value. The average portfolio returns and standard deviations for the monthly and quarterly rebalancing were converted to an annualized

basis so that they can be compared with the yearly rebalancing period. Based on the annualized figures, yearly rebalancing provided the highest average portfolio excess return and also the greatest standard deviation. Excess return is the difference between the actual rate of return on a stock and the actual risk-free rate, where risk-free rate is the rate that an investor can earn by leaving money in risk-free assets such as T-bills, money market funds, or the bank.[1]

**Table 1 – ETF Portfolio Rebalancing by Frequency (2004 – 2010) Using a 30% Tax Bracket**

| Rebalancing Frequency | PF before Trans Costs & Taxes ($) | Trans Cost ($) | Taxes ($) | PF after Trans Cost & Taxes ($) | Avg PF Excess Rtn* | PF Std Dev* |
|---|---|---|---|---|---|---|
| **Monthly** | 8,066,629 | 1,680 | 53,339 | 8,011,610 | -4.66% | 10.74% |
| **Quarterly** | 9,075,192 | 560 | 55,712 | 9,018,920 | -2.92% | 10.91% |
| **Yearly** | 10,147,316 | 140 | 41,261 | 10,105,916 | -1.00% | 12.58% |

*Annualized

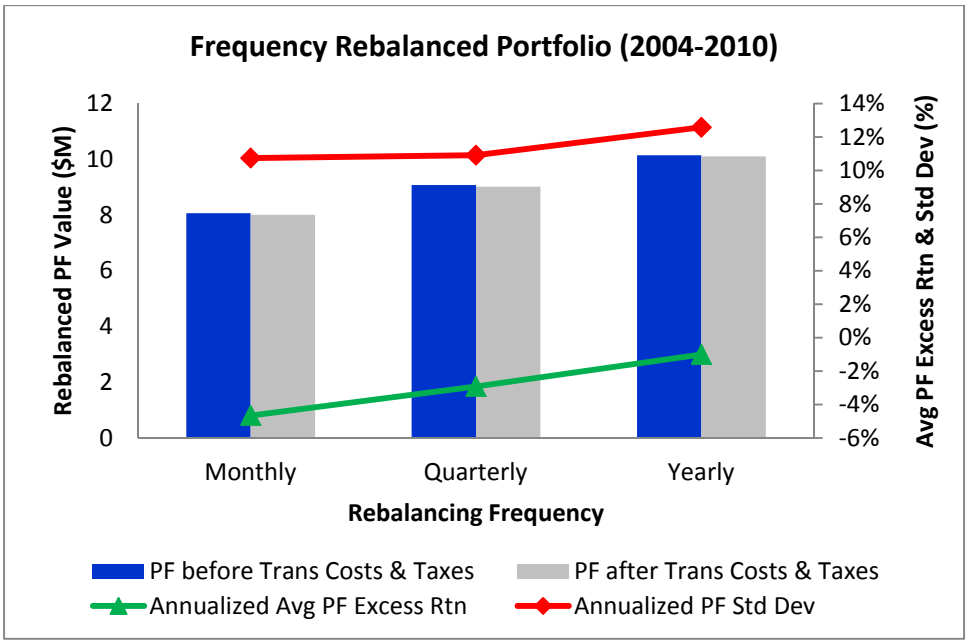


**Figure 1 – Frequency Rebalanced ETF Portfolio (2004-2010)**

[1] Bodie Kane Marcus, "Investments", p. 129, 9th Edition, McGraw-Hill/Irwin, 2011.

It makes sense that the annual rebalancing resulted in the highest average portfolio excess return as well as standard deviation since the portfolio is made up of 60% of stocks, which have a higher beta and standard deviation hence higher average return than the bonds. The yearly rebalancing allowed the stocks to increase in value faster than did the bonds. As the stock value increases, its percentage grows proportionally larger and deviates further away from its original mix of 60%. Another reason that the yearly rebalanced portfolio outperformed the monthly or quarterly rebalanced portfolios may be due to the fact that rebalancing at a higher frequency could inevitably cause the portfolio to miss the best timing when selling off excess stocks or bonds or buying additional stocks or bonds thus resulting in lower gains than could otherwise be achieved through yearly rebalancing, which rode out the highs and lows of the market.

Further explanation can be found in Table 2, which lists the average, maximum and minimum bid/ask average prices for ETF stocks and bonds from 2004 to 2010, and Table 3, which shows the annualized and non-annualized average returns for the individual stocks and bonds and the frequencies of which the stock and bond returns were above or below their respective average return for the monthly, quarterly and yearly period. From Table 2, it is noted that stock prices were 43.6% higher than bond prices at the maximum, 23% lower at the minimum, and 18.6% at an average. From Table 3, it is easily seen that bonds had more below average returns on a monthly and yearly basis while stocks exhibited above average returns for all three time periods.

**Table 2 – ETF Prices for Stocks and Bonds (2004-2010)**

| ETF Prices | Bonds ($) | %Δ from Avg | Stocks ($) | %Δ from Avg |
|---|---|---|---|---|
| **Average Bid/Ask** | 101.72 | - | 120.63 | - |
| **Maximum Bid/Ask** | 108.94 | 7.10% | 156.48 | 29.72% |
| **Minimum Bid/Ask** | 88.40 | -13.09% | 68.11 | -43.54% |

**Table 3 – Average Returns for ETF Stocks and Bonds (2004-2010)**

| | Bonds | | | Stocks | | |
|---|---|---|---|---|---|---|
| **Rebalancing Frequency** | **Monthly** | **Quarterly** | **Yearly** | **Monthly** | **Quarterly** | **Yearly** |
| **Avg Return** | 0.40% | 1.21% | 4.83% | 0.51% | 1.53% | 6.10% |
| **Avg Return*** | 4.83% | 4.83% | 4.83% | 6.10% | 6.10% | 6.10% |
| **Freq < Avg Rtn** | 38 | 14 | 4 | 35 | 11 | 2 |
| **Freq > Avg Rtn** | 46 | 14 | 3 | 49 | 17 | 5 |

*Annualized

All the above pointed to the conclusion that yearly rebalanced portfolio results in higher returns for the investor than monthly or quarterly returns. However, with higher return comes a higher standard deviation, which means that there is a higher chance that the stocks would devalue just as easily as gaining value. But based on the 7-year ETF data analysis, that did not happen. Despite this fact, one should always remember that although historic data is used to determine future trend, the future is unpredictable and could easily upset the trend as predicted using historic data.

Table 4 and Figure 2 represent the ranges of stock and bond percentages in the rebalanced ETF portfolio for each rebalancing frequency and the portfolio betas before and after rebalancing. The portfolio betas after rebalancing are higher than the original portfolio mix before rebalancing for all three rebalancing frequencies. However, it is

interesting to note that among the three rebalancing scenarios, betas before and after rebalancing were consistently the lowest for the annual rebalancing scenario. This is possibly because given a longer period of time between rebalancing activities, any losses incurred during that time period were balanced out and surpassed by the gains within that same period. They further support the findings discussed in the last paragraph. From Figure 2, it is proof that the earlier hypothesis that higher stock mix with higher return and standard deviation do increase overall portfolio value.

**Table 4 – Stock and Bond Percentages in Frequency Rebalanced ETF Portfolio (2004-2010)**

| | **Bond % in Rebalanced PF** | | | **Stock % in Rebalanced PF** | | |
|---|---|---|---|---|---|---|
| **Rebalancing Frequency** | **Monthly** | **Quarterly** | **Yearly** | **Monthly** | **Quarterly** | **Yearly** |
| **Avg PF %** | 40.43% | 40.71% | 40.63% | 59.57% | 59.29% | 59.37% |
| **Max PF %** | 47.48% | 50.06% | 51.86% | 62.00% | 62.57% | 63.66% |
| **Min PF %** | 38.00% | 37.43% | 36.34% | 52.52% | 49.94% | 48.14% |
| | **Beta for Original 60/40 PF** | | | **Beta after PF Rebalancing** | | |
| **PF Beta** | 0.61 | 0.57 | 0.57 | 0.69 | 0.67 | 0.62 |
| **P-Value (95% C.L.)** | N/A | N/A | N/A | 6.53E-11 | 2.00E-17 | 9.81E-06 |

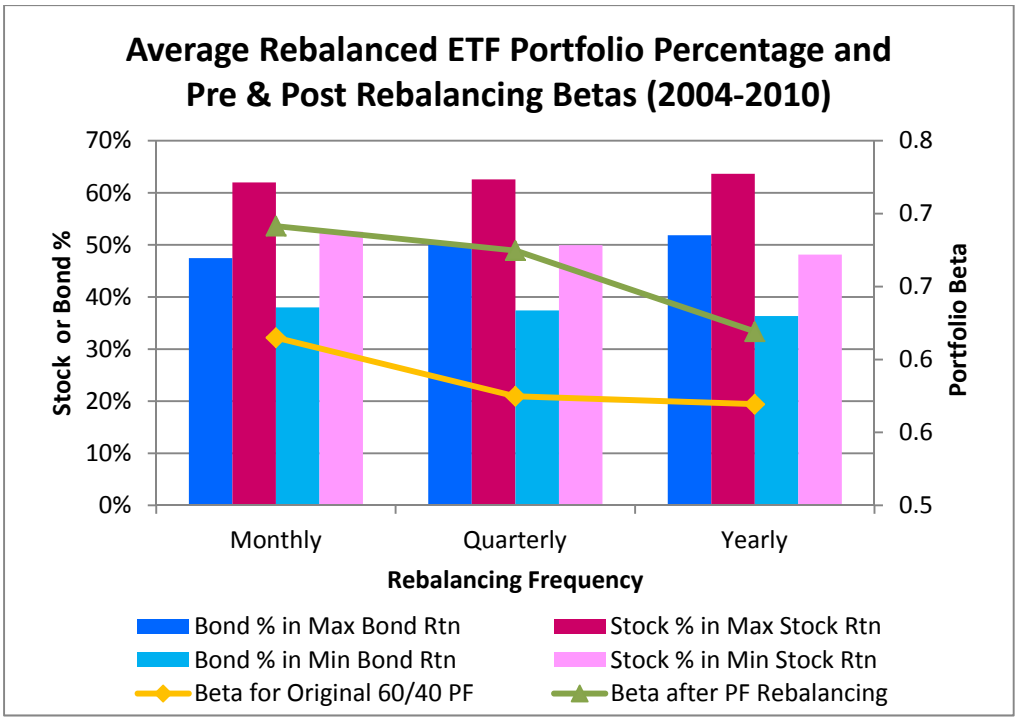


**Figure 2 – Average Rebalanced ETF Portfolio Percentages and Pre & Post Rebalancing Betas**

The original portfolio betas were calculated by first finding the bond beta using linear regression, with stock returns as the dependent variable and bond returns as the independent variable. The portfolio beta was then calculated by the sum of the bond beta fraction (multiplying 0.4 weight percent of bonds in the portfolio by the bond beta from the linear regression result) and the stock beta fraction (multiplying 0.6 weight percent of stocks in the portfolio by stock beta, which is assumed to be the market beta of one). Because it is a calculated beta instead of a linear regression generated beta, the p-value for the portfolio beta cannot be determined.

## 2. Sharpe & Treynor Ratios and Jensen's Alphas for the Frequency Rebalanced ETF Portfolio (2004-2010)

To further verify that one frequency outperforms another, Sharpe and Treynor Ratios, and Jensen's Alphas were calculated for all three rebalancing frequencies (Table 5). Sharpe Ratio determines a portfolio's reward-to-total volatility trade-off by dividing average portfolio excess return over the sample period by the standard deviation of the excess return over that period.[2] Similar to Sharpe Ratio, Treynor Ratio measures excess return per unit of systematic risk.[3] Systematic risk is the risk that remains after diversification of an investor's portfolio, better known as market risk or beta. Jensen's Alpha measures a portfolio's alpha value, or the excess return as predicted by the Capital Asset Pricing Model (CAPM), given the portfolio's beta and the average market return.[4] The CAPM model gives us a precise prediction of the relationship that we should observe between the risk of an asset and its expected return. In other words, it provides a benchmark rate of return for evaluating possible investments, and it helps investors to make an educated guess as to the expected return on assets that have not yet been traded in the marketplace.[5] In general, positive Jensen's Alpha for a portfolio is an indication that the portfolio truly has value and the portfolio manager possesses real stock-picking skills that "beat the market" instead of being merely lucky at the right time.

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[2] Bodie Kane Marcus, "Investments", p.822, 9th Edition, McGraw-Hill/Irwin, 2011.
[3] Ibid, p. 822.
[4] Ibid, p. 822.
[5] Bodie Kane Marcus, "Investments", p.280, 9th Edition, McGraw-Hill/Irwin, 2011.

**Table 5 – Sharpe & Treynor Ratios and Jensen's Alphas from the ETF Portfolio Frequency Rebalancing (2004-2010)**

| Refrequency Rebalancing | Before Trans Costs & Taxes | | | After Trans Costs & Taxes | | |
|---|---|---|---|---|---|---|
| | Monthly | Quarterly | Yearly | Monthly | Quarterly | Yearly |
| Sharpe Ratio | -0.22905 | -0.07012 | 0.08091 | -0.43370 | -0.26767 | -0.07976 |
| Treynor Ratio | -0.03566 | -0.01145 | 0.01696 | -0.06735 | -0.04330 | -0.01616 |
| Jensen's Alpha | -0.00614 | -0.01397 | -0.03510 | -0.00614 | -0.01399 | -0.03513 |
| P-Value (95% C.L.) | 6.50E-11 | 1.24E-05 | 3.23E-03 | 6.53E-11 | 1.19E-05 | 3.13E-03 |

Based on the numbers in Table 5, it can be deduced that yearly rebalancing is indeed superior to monthly or quarterly rebalancing, since the Sharpe and Treynor ratios are positive only for that frequency. However, based on the definition given for Jensen's alpha in the last paragraph, it does not look as though the yearly rebalancing occurred by skill, rather it was just good timing, since all numbers are negative. However, after the effect of capital gains taxes, the Sharpe and Treynor Ratios are also negative, although they are still higher than the monthly or quarterly numbers. Comparisons between Sharpe and Treynor Ratios and Jensen's Alpha are provided in Figure 3. Since there are insignificant differences between the rebalanced portfolios before and after transaction costs and taxes were accounted for, only the post transaction costs and taxes figures are included below for simplicity sake.

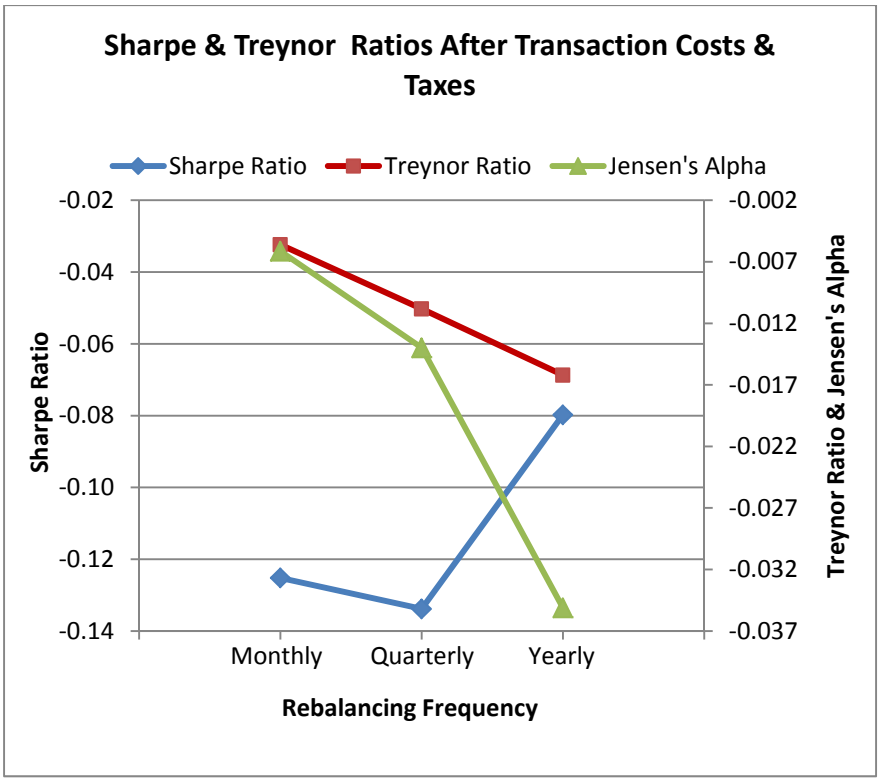


**Figure 3 – Sharpe & Treynor Ratios and Jensen's Alpha after Transaction Costs and Taxes**

## B. Strategy 2 – ETF Portfolio Percentage Movement Rebalancing

The second rebalancing strategy tested portfolio returns using percentage movement rebalancing. This strategy rebalances a portfolio when the percentages of stocks and bonds move beyond the initial, preset, portfolio mix. As an example, a five percent movement portfolio rebalancing strategy for a 60/40 Stock and bond portfolio would take place when either the stock or bond portion of the portfolio is five percent more or less than its respective, original, predetermined percentage. For this analysis, 3, 5, 10 and 15 percentage movements were used for this strategy. As before, tax brackets

of 25, 30 and 35% were used for the sensitivity analysis with similar findings indicating that the tax brackets used did not affect the general outcome of the results. Subsequent calculations and results were based on the 30% tax bracket to make calculations and explanations simpler.

### 1. Percentage Movement Rebalancing Strategy Results for ETF data (2004 -2010)

A summarized result of the ETF portfolio percentage movement rebalancing strategy for the 3, 5, 10 and 15% percentage movement scenarios analyzed is provided in Table 6. The average portfolio excess returns for the different rebalancing percentage movements are monotonic, or strictly increasing in one direction. This means that when rebalanced at a higher percentage movement, at 10 or more percent but below 15 percent, the rebalanced portfolio value is higher. Note that no rebalancing occurred at the 15 percent movement. It makes sense that the portfolio value in terms of average portfolio excess return is higher since the portfolio is rebalanced less often, thereby reducing transaction costs and capital gains taxes. However, although the average portfolio excess returns are increasing in one direction, their corresponding standard deviations are decreasing in the same direction. However, both the portfolio excess returns and the standard deviations are small as compared to those from the frequency rebalancing strategy. Figure 4 provides a graphical representation of the numeric results listed in Table 6.

**Table 6 – ETF Portfolio Rebalancing by Percentage Movement (2004-2010) Using a 30% Tax Bracket**

| Percentage Movement Rebalancing | PF before Trans Costs & Taxes ($) | Trans Cost ($) | Taxes ($) | PF after Trans Costs & Taxes ($) | Avg PF Excess Rtn | PF Std Dev |
|---|---|---|---|---|---|---|
| **3%** | 8,711,715 | 5,240 | 19,548 | 8,686,927 | -0.010% | 0.850% |
| **5%** | 9,345,725 | 3,160 | 25,864 | 9,316,701 | -0.006% | 0.842% |
| **10%** | 10,740,926 | 100 | -867 | 10,741,693 | 0.001% | 0.837% |
| **15%** | 10,913,345 | 0 | 0 | 10,913,345 | 0.002% | 0.832% |

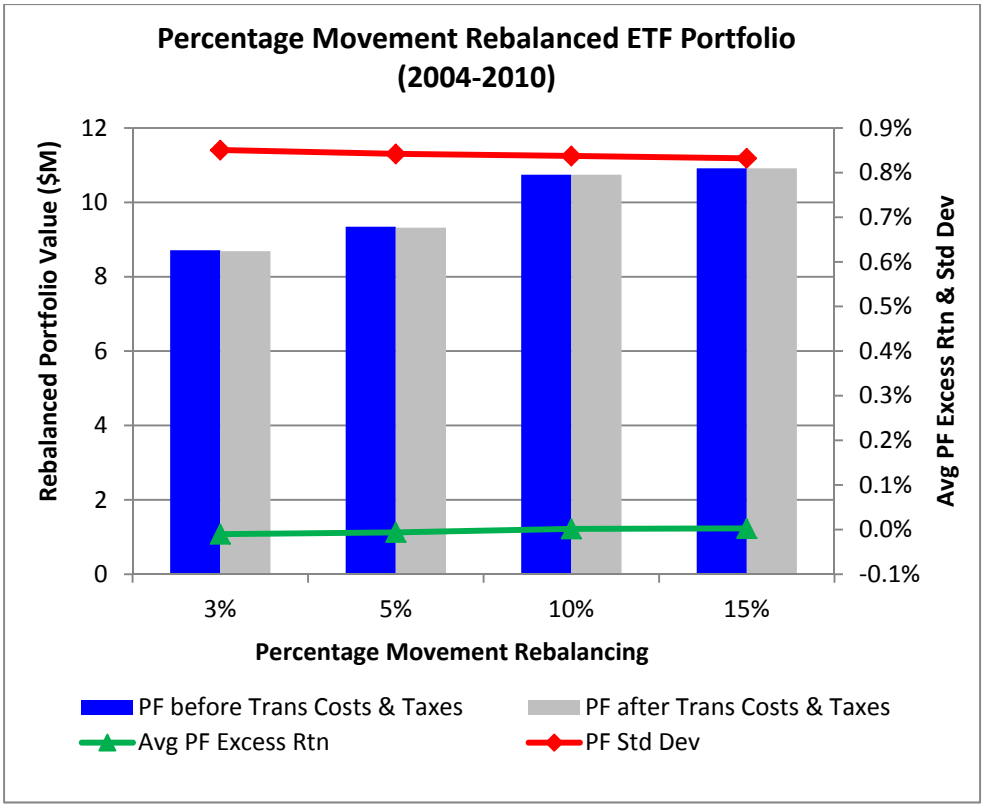


**Figure 4 – ETF Portfolio Rebalancing by Percentage Movement (2004-2010)**

Tables 7 and 8 present the average, maximum and minimum ranges of stock and bond percentages observed in the rebalanced portfolio for each percentage movement scenario. The portfolio bond percentages had higher occurrences of below average

portfolio bond average for the 3 and 10% rebalancing movements, with alternating above average percentages at 5 and 15% rebalancing movements. On the other hand, the portfolio stock percentages had more above average averages than the portfolio stock average for three out of the four percentage rebalancing movements, at 3, 5, and 15**%.**

**Table 7 – Stock and Bond Percentages in Percentage Movement Rebalanced ETF Portfolio (2004-2010)**

| Percentage Movement Rebalancing | Bond % in Rebalanced PF | | | | Stock % in Rebalanced PF | | | |
|---|---|---|---|---|---|---|---|---|
| | 3% | 5% | 10% | 15% | 3% | 5% | 10% | 15% |
| Avg PF % | 38.84% | 37.90% | 38.02% | 38.30% | 61.16% | 62.10% | 61.98% | 61.70% |
| Max PF % | 46.44% | 48.44% | 51.06% | 51.63% | 63.75% | 65.44% | 68.60% | 68.60% |
| Min PF % | 36.25% | 34.56% | 31.40% | 31.40% | 53.56% | 51.56% | 48.94% | 48.37% |
| Freq < Avg PF % | 1072 | 937 | 855 | 882 | 691 | 826 | 908 | 881 |
| Freq > Avg PF % | 691 | 826 | 908 | 881 | 1072 | 937 | 855 | 882 |

**Table 8 – Stock and Bond Rate of Change in Percentage Movement Rebalanced ETF Portfolio (2004-2010)**

| Percentage Movement Rebalancing | Bond % Δ in Rebalanced PF | | | | Stock % Δ in Rebalanced PF | | | |
|---|---|---|---|---|---|---|---|---|
| | 3% | 5% | 10% | 15% | 3% | 5% | 10% | 15% |
| Avg PF % Δ | -1.16% | -2.10% | -1.98% | -1.70% | 1.16% | 2.10% | 1.98% | 1.70% |
| Max PF % Δ | 6.44% | 8.44% | 11.06% | 11.63% | 3.75% | 5.44% | 8.60% | 8.60% |
| Min PF % Δ | -3.75% | -5.44% | -8.60% | -8.60% | -6.44% | -8.44% | -11.06% | -11.63% |

Figures 5 through 7 illustrate the bond mix percentages from Tables 7 and 8 in graphical format. On average, the bond mix percentage s of the ETF portfolio declined at a faster rate from 3 to 5% rebalancing movements then at a slower rate from 5 to 15% rebalancing movements. Similarly, the minimum bond mix percentages decreased rapidly then flattened out at -8.6% between 10 and 15% rebalancing movements, while

the maximum bond mix percentages increased at a faster rate from 3 to 5% then at a slower rate from 10 to 15% rebalancing movements.

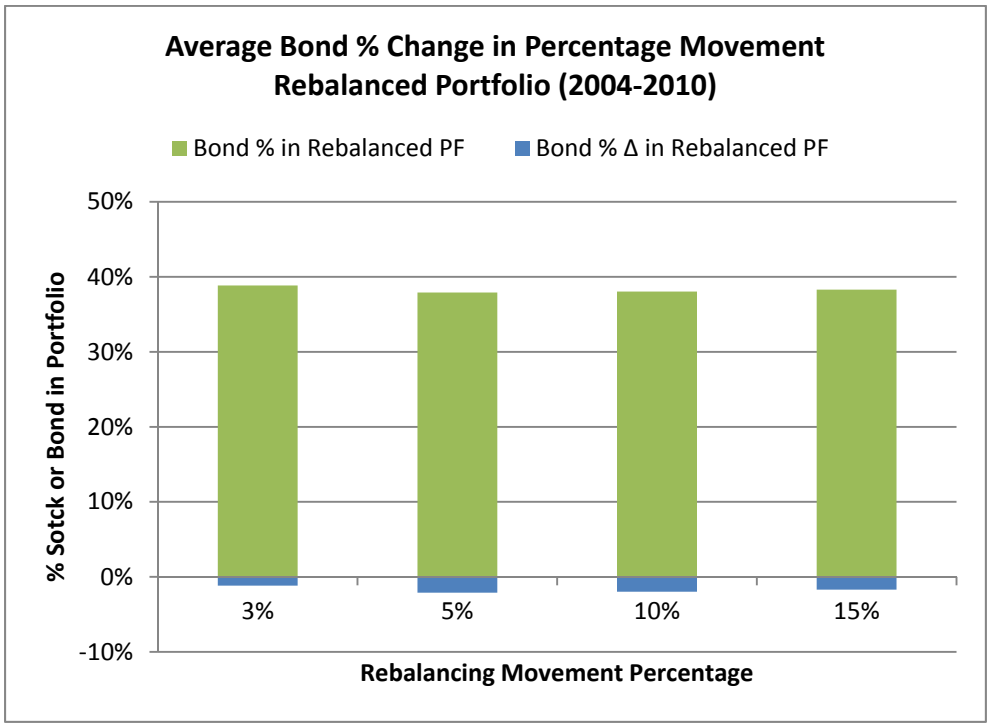


**Figure 5 – Average Bond % Change in Percent Movement Rebalanced ETF Portfolio (2004-2010)**

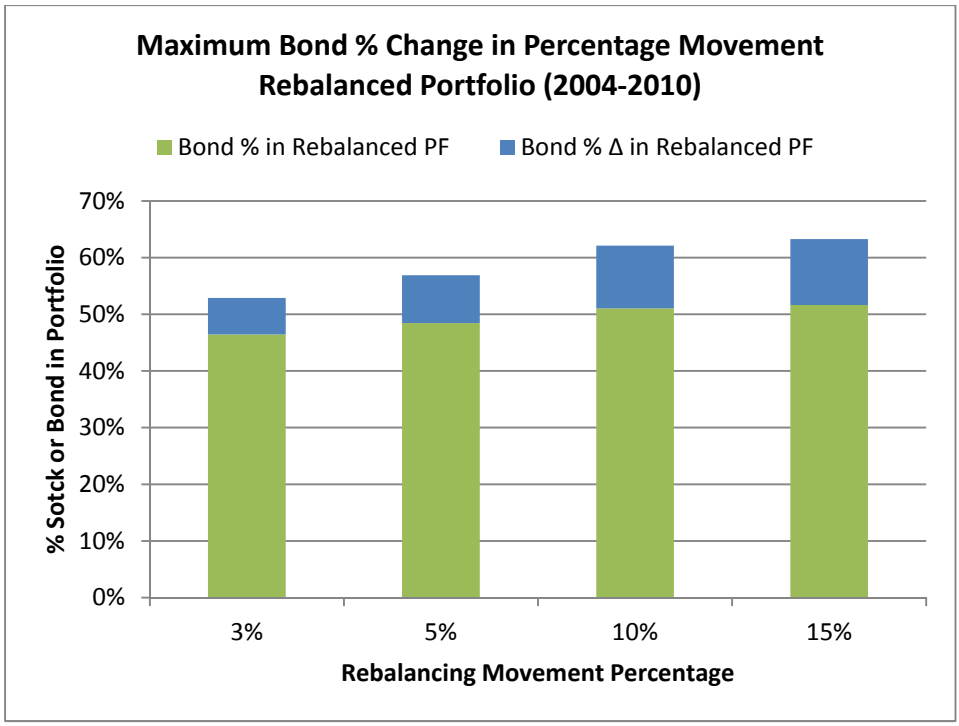


Figure 6 – Maximum Bond % Change in Percentage Movement Rebalanced Portfolio (2004-2010)

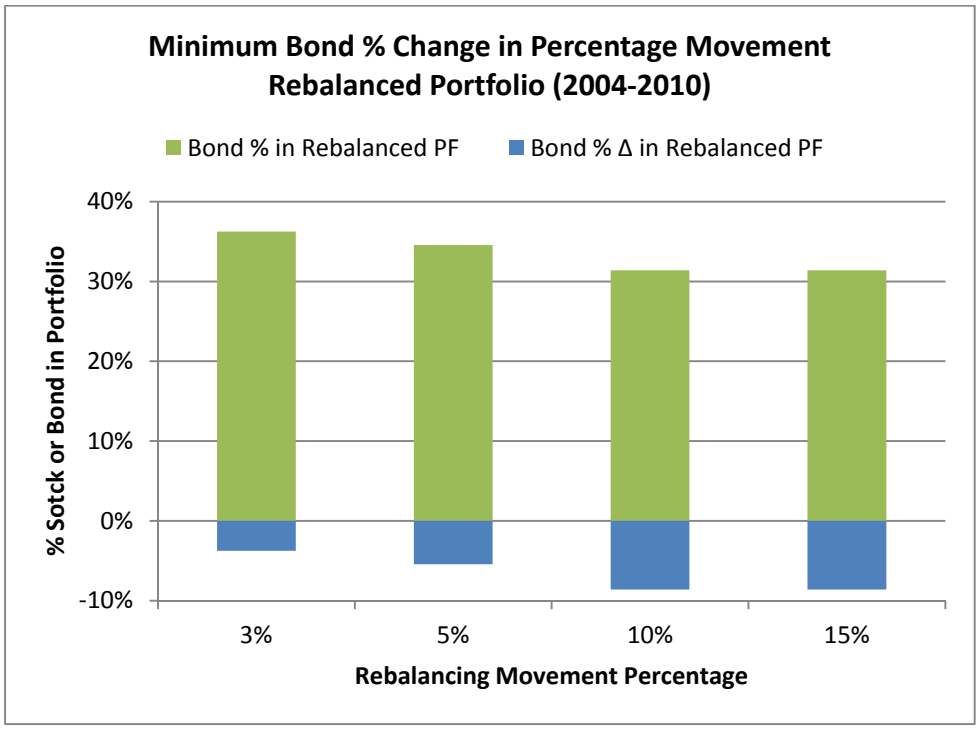


Figure 7 – Minimum Bond % Change in Percentage Movement Rebalanced Portfolio (2004-2010)

Figures 8 through 10 illustrate the stock mix percentages from Tables 7 and 8. In a similar manner, stock mix percentages increased at a faster rate from 3 to 5% rebalancing movements then decreased at a slower rate than the increasing rate from 5 to 15% rebalancing movements. The maximum stock mix percentages mirrored that of the minimum bond mix percentages in the opposite direction as it increased at a higher rate from 3 to 5% rebalancing movements then flattened out at a maximum of 68% from 10 to 15% rebalancing movements. The minimum stock mix percentages likewise mirrored the maximum bond mix percentages in the opposite direction by decreasing at a faster rate from 3 to 5% rebalancing movements then tapered off from 10 to 15% rebalancing movements.

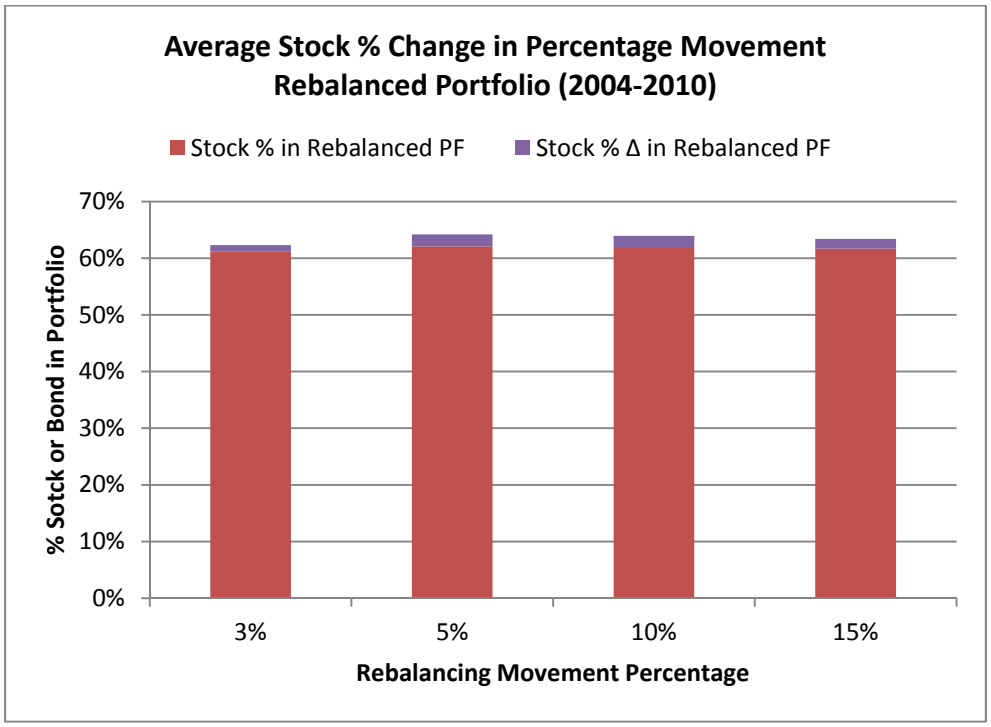


**Figure 8 – Average Stock % Change in Percentage Movement Rebalanced Portfolio (2004-2010)**

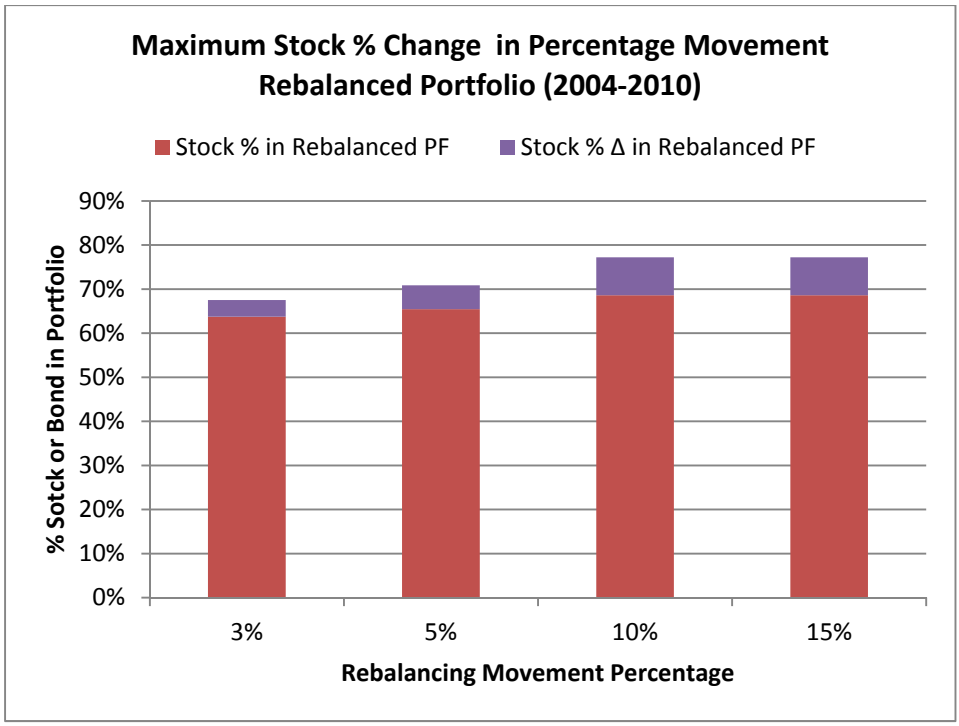


Figure 9 – Maximum Stock % Change in Percentage Movement Rebalanced Portfolio (2004-2010)

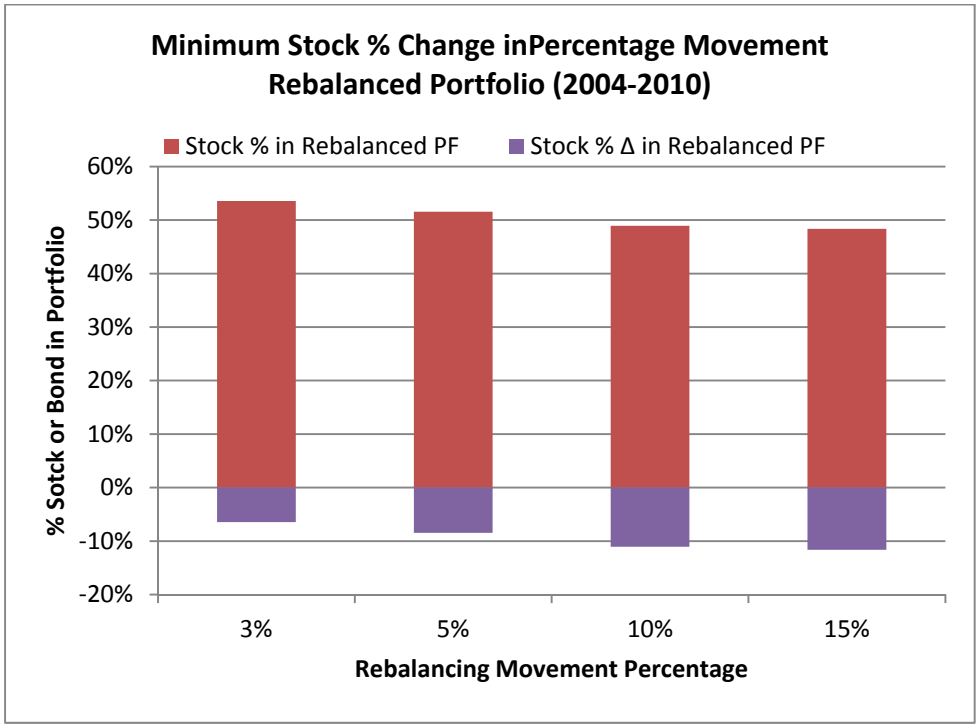


Figure 10 – Minimum Bond % Change in Percentage Movement Rebalanced Portfolio (2004-2010)

Table 9 is a summary of the ETF portfolio betas before and after rebalancing by percentage change in movements. The original betas were constant because of the percentage movement rebalancing strategy, which had the same set of data points for the various percentage movements tested. The rebalanced portfolio started with a higher beta for the 3% rebalancing movements that gradually declined over the subsequent, higher percentage rebalancing movements. At the 3 to 5% rebalancing movements, the rebalanced betas were higher than the original portfolio betas, but at the 10 to 15% rebalancing movements, these betas were lower than the original ones. Figure 11 displays the results of these betas together with the maximum to minimum ranges of stock and bond percentages within the portfolio for each rebalancing movements.

**Table 9 – ETF Portfolio Betas before and after Percentage Movement Rebalancing (2004-2010)**

| Percentage Movement Rebalancing | Betas for Original 60/40 PF | | | | Betas after PF Rebalancing | | | |
|---|---|---|---|---|---|---|---|---|
| | 3% | 5% | 10% | 15% | 3% | 5% | 10% | 15% |
| PF Beta | 0.594 | 0.594 | 0.594 | 0.594 | 0.601 | 0.596 | 0.590 | 0.585 |
| P-Value (95% C.L.) | N/A | N/A | N/A | N/A | 0 | 0 | 0 | 0 |

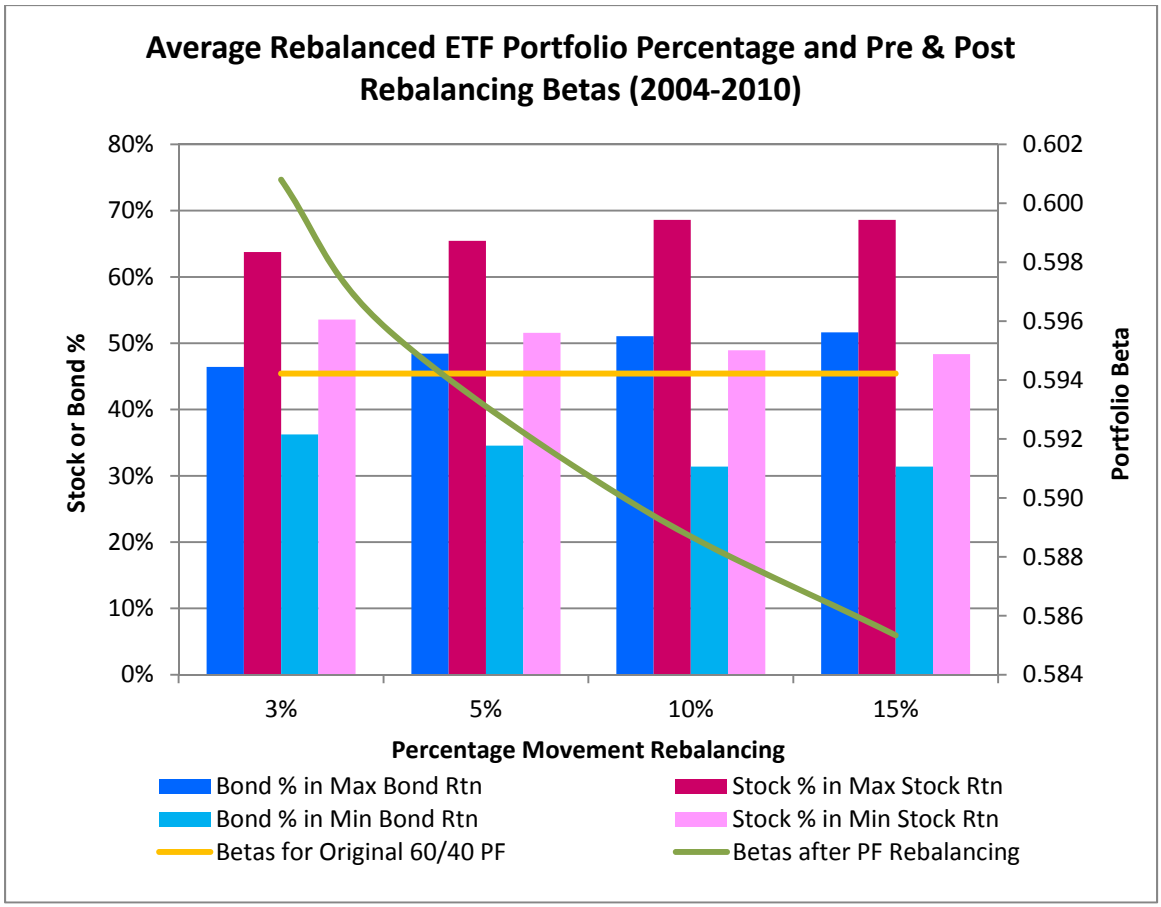


**Figure 11 – Average Rebalanced ETF Portfolio Percentages and Pre & Post Rebalancing Betas (2004-2010)**

## 2. Sharpe & Treynor Ratios and Jensen's Alphas for the Percentage Movement Rebalanced ETF Portfolio (2004-2010)

There was no change in values for the Sharpe and Treynor Ratios as well as Jensen's Alphas for the rebalanced ETF portfolio before or after accounting for transaction costs and capital gains taxes. Therefore, only one set of data is provided here, as outlined in Table 10. Graphical representations are displayed in Figures 12 through 14.

**Table 10 – Sharpe & Treynor Ratios and Jensen's Alphas from the ETF Portfolio Percentage Movement Rebalancing (2004-2010)**

| Rebalancing Percent | 3% | 5% | 10% | 15% |
|---|---|---|---|---|
| **Sharpe Ratio** | -0.012065 | -0.007546 | 0.001777 | 0.002821 |
| **Treynor Ratio** | -0.000171 | -0.000107 | 0.000025 | 0.000040 |
| **Jensen's Alpha** | -0.000212 | -0.000172 | -0.000092 | -0.000083 |
| **P-Value (95% C.L.)** | 1.82E-05 | 2.87E-04 | 6.98E-02 | 1.06E-01 |

Sharpe and Treynor Ratios increased with increasing percentage movement rebalancing; however they plateaued around the 12% rebalancing movement. Rebalancing beyond the 12 percentage shift from the portfolio's original mix will not enhance effective portfolio rebalancing, rather, it would cause the portfolio to be off balance thus losing the diversification objective. Likewise, Jensen's Alphas increased in the same trend; however, since Jensen's Alphas for all rebalancing percentage movements are negative, results of the rebalanced portfolios are considered to be purely from good timing.

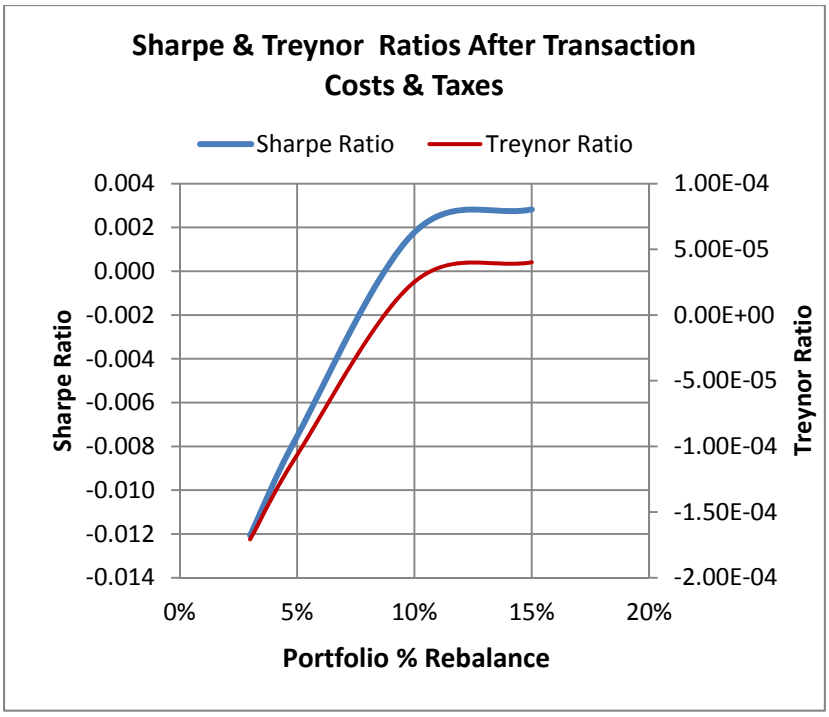


Figure 12 – Sharpe and Treynor Ratios after Transaction Costs & Taxes

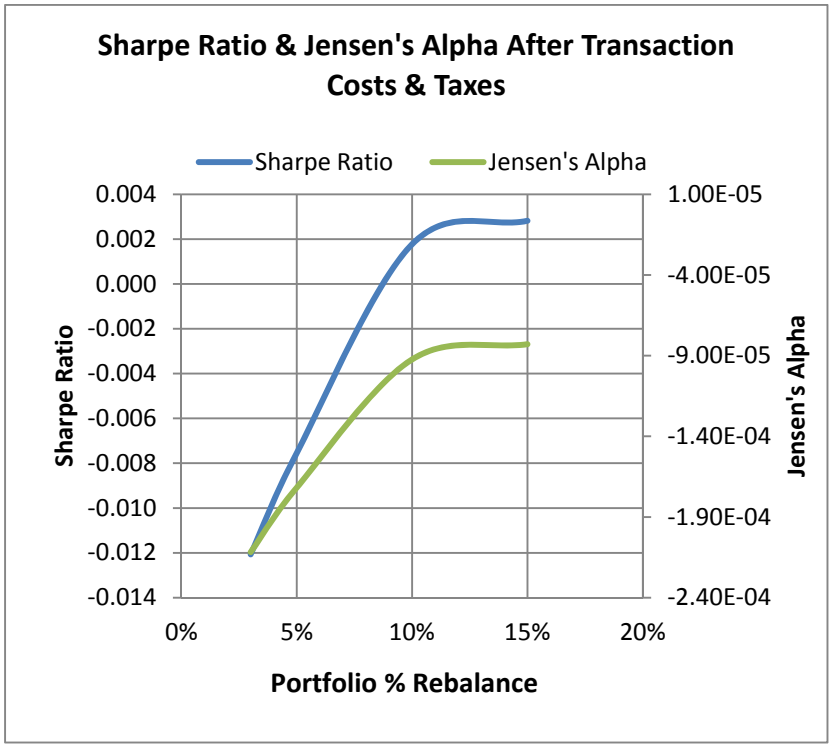


Figure 13 – Sharpe Ratio & Jensen's Alpha after Transaction Costs & Taxes

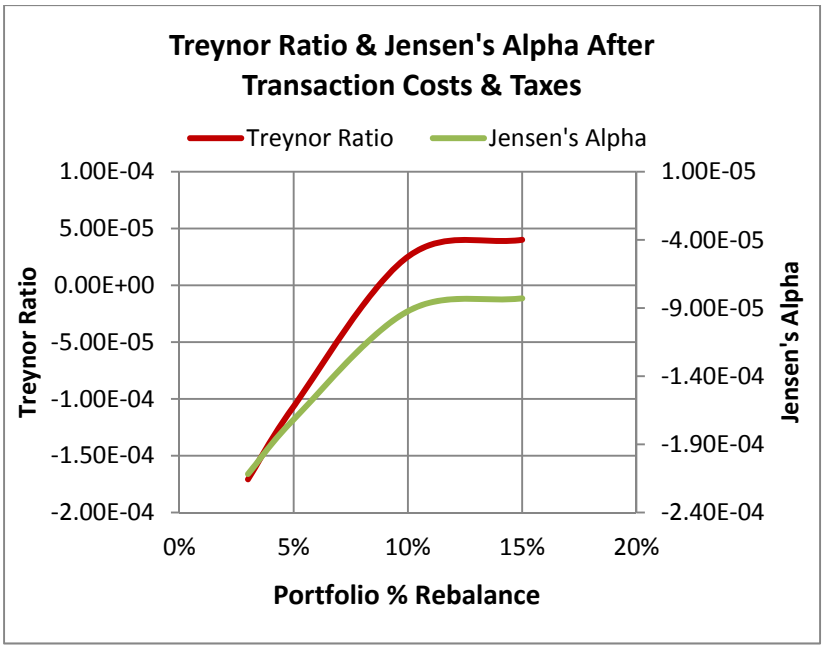


**Figure 14 –Treynor Ratio and Jensen's Alpha after Transaction Costs & Taxes**

## C. Strategy Comparison - Frequency vs. Percentage Movement Rebalancing for the ETF Portfolio (2004-2010)

When the ETF portfolio is rebalanced at 10 percent or more but less than 15 percent movement from its original mix, the rebalanced portfolio value, after transaction costs and taxes, is higher than the same portfolio using an annually rebalanced portfolio. Moreover, standard deviation for each of the percentage movement scenario is much lower than each of the frequency rebalancing scenario. However, the average excess return on the rebalanced portfolio for the percentage movement strategy is a lot lower when compared to the frequency rebalancing strategy's average excess return. This is because the average portfolio excess returns for the percentage movement rebalancing strategy are much lower as compared to the excess returns for the frequency rebalancing

strategy. Betas are also higher for the frequency rebalancing strategy than for the percentage movement rebalancing strategy. Table 11 is a summary ETF portfolio comparison between the best performers of each rebalancing strategy.

**Table 11 –Summary Comparison between the Best Performing Frequency and Percentage Movement Rebalancing Strategies for the ETF Portfolio (2004-2010)**

| Rebalancing Strategy Comparison | Yearly Rebalancing | 10% Movement Rebalancing |
|---|---|---|
| **PF Value before Trans Costs &Taxes ($)** | 10,147,316 | 10,740,926 |
| **Transaction Costs ($)** | 140 | 100 |
| **Capital gains taxes ($)** | 41,260.57 | -866.58 |
| **PF Value after Trans Costs & Taxes ($)** | 10,105,916 | 10,741,693 |
| **Avg PF Excess Rtn** | -1.00% | 0.00% |
| **Std Dev** | 12.58% | 0.84% |
| **PF Beta** | 0.62 | 0.59 |
| **Sharpe Ratio** | -0.0798 | 0.0018 |
| **Treynor Ratio** | -0.0162 | 0.0000 |
| **Jensen's Alpha** | -0.0351 | -0.0001 |

## III. Rebalancing Analysis Using Indexed Data (1976-2009)

Following the same established procedure used for the ETF data analysis, indexed data from 1976 to 2009 were used to perform monthly, quarterly and yearly calculation for the Index portfolio frequency rebalancing strategy. For the percentage movement rebalancing strategy, extra percentage movements were added based on the results of the ETF data analysis. This is done because prices for the indexed stocks and bonds increased at much greater percentages than the ETF prices over the 34-year study period. The indexed data was only available in a monthly format, with the bid/ask average prices

given. Besides the percentage movements used for the ETF data analysis, additional percentages of 8, 12, 20, 25 and 28% were also included in the analysis. Twenty-eight percent was the maximum limit for the percentage movement because the returns did not change beyond that amount over the 34-year period, from 1976 to 2009. In other words, no rebalancing occurred at more than 28 percent rebalancing movement.

## A. Strategy 1 – Index Portfolio Frequency Rebalancing

This is the repeat performance of frequency rebalancing as outlined under the ETF frequency rebalancing analysis section, except that indexed data of stocks and bonds from 1976 to 2009 were used because the ETF data was not available before 2004.

### 1. Frequency Rebalancing Strategy Results for Indexed Data (1976-2009)

The main results of the Index portfolio frequency rebalancing strategy are listed in Table 12 and Figure 15. Once again, monthly rebalancing incurred the most transaction costs. Yearly rebalancing is still the most profitable as compared to the other two rebalancing time frames, even though it also paid out the most capital gains taxes. This is in line with the highest average excess portfolio return of 3.7% as compared to the annualized -0.3% and 1.8% for the monthly and quarterly rebalancing, respectively. Higher return is also associated with higher standard deviation, as is evident from Table 12 and Figure 15.

Table 12 – Index Portfolio Rebalancing by Frequency (1976-2009) Using a 30% Tax Bracket

| Rebalancing Frequency | PF before Trans Costs & Taxes ($) | Trans Costs ($) | Taxes ($) | PF after Trans Costs & Taxes ($) | Avg PF Excess Rtn* | PF Std Dev* |
|---|---|---|---|---|---|---|
| **Monthly** | 51,403,629 | 8,160 | 734,354 | 50,661,114 | -0.28% | 10.54% |
| **Quarterly** | 100,189,357 | 2,720 | 29,247,053 | 70,939,584 | 1.76% | 10.57% |
| **Yearly** | 179,194,013 | 680 | 22,925,366 | 156,267,967 | 3.74% | 11.07% |

*Annualized

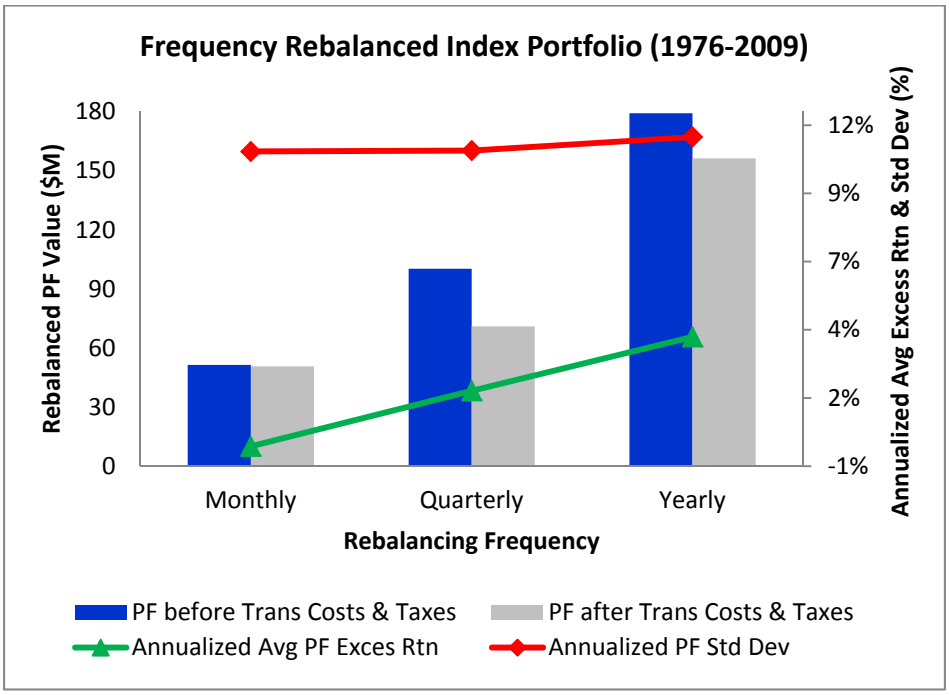


Figure 15 – Frequency Rebalanced Index Portfolio (1976-2009)

Table 13 displays the annualized and non-annualized average returns for the stocks and bonds for each frequency rebalancing portfolio and the frequencies of which the stock and bond returns were above or below their respective average return for the monthly, quarterly and yearly period. The average annualized returns for the respective stocks and bonds are the same because these are individual returns for the stocks and bonds without looking at them from a portfolio perspective. In this case, bonds had more

below average returns on a quarterly and yearly basis but a higher above average return on a monthly basis. Stocks, as demonstrated with the ETF data before, showed more above average returns for all three time periods.

**Table 13 – Indexed Stock and Bond Returns and Standard Deviations (1976-2009)**

| Frequency Rebalancing | Bonds | | | Stocks | | |
|---|---|---|---|---|---|---|
| | Monthly | Quarterly | Yearly | Monthly | Quarterly | Yearly |
| Avg Return | 0.69% | 2.06% | 8.24% | 0.98% | 2.93% | 11.72% |
| Avg Return* | 8.24% | 8.24% | 8.24% | 11.72% | 11.72% | 11.72% |
| Freq < Avg Rtn | 201 | 73 | 20 | 192 | 64 | 16 |
| Freq > Avg Rtn | 207 | 63 | 14 | 216 | 72 | 18 |

*Annualized

It is not surprising to learn that yearly rebalancing produced the highest net portfolio value. However, what is surprising is the huge disparity in the amount of each portfolio value at the end of the rebalancing period. Yearly rebalanced portfolio outperformed monthly and quarterly rebalanced portfolio by 250% and 80%, respectively (please refer back to Table 12). One plausible explanation is that over the 34-year period, prices for the indexed stocks have outpaced that of the bonds. Figure 16 illustrates the divergence of stock and bond prices over the 34-year period.

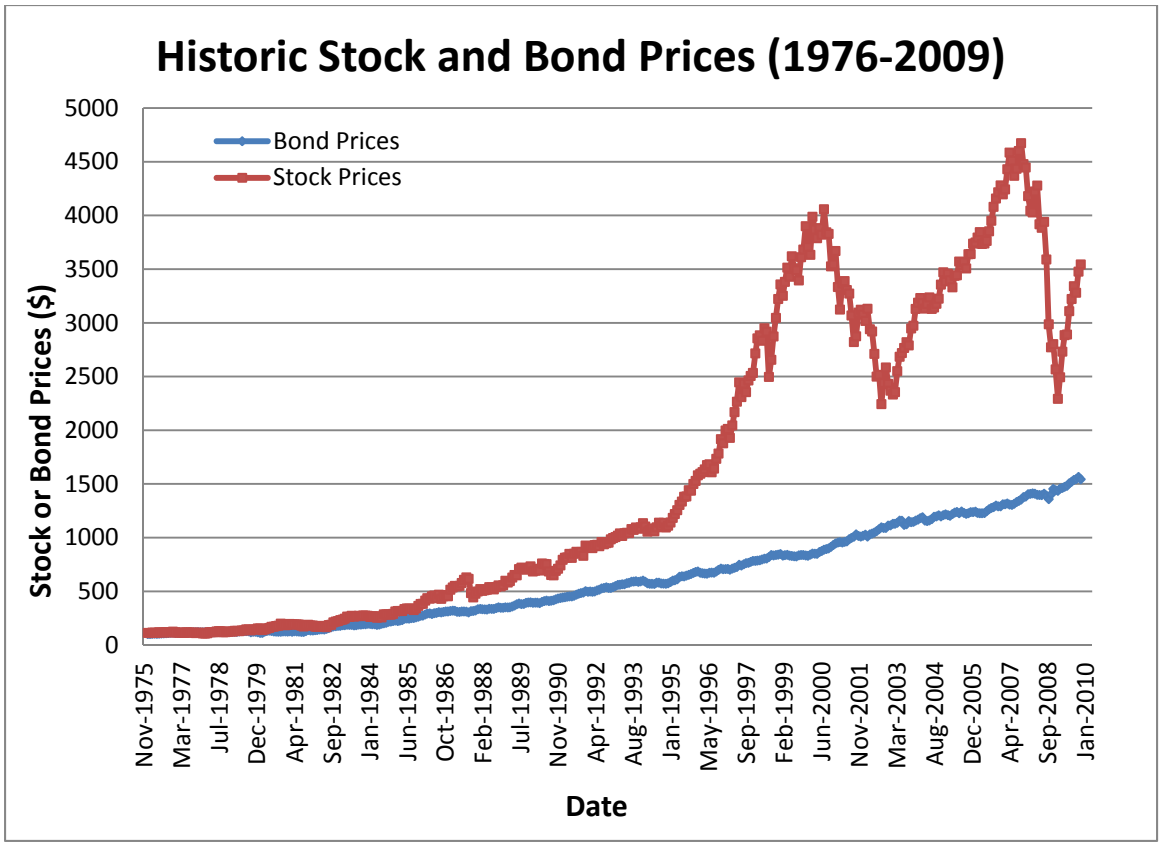


Figure 16 – Historic Stock and Bond Prices (1976-2009)

A closer look at the indexed prices revealed that over the 34-year period, although stock and bond indices initially priced around $100 each, 34 years later, they had diverged greatly from each other. Table 14 shows that stock prices, on average, were more than twice that of the bond prices, and at the maximum, more than 3 times that of the bond prices.

Table 14 – Indexed Prices for Stocks and Bonds (1976-2009)

| Indexed Prices | Bonds ($) | %Δ from Avg | Stocks ($) | %Δ from Avg |
|---|---|---|---|---|
| **Average Bid/Ask** | 618.04 | 236.69% | 1,602.37 | 285.04% |
| **Maximum Bid/Ask** | 1,564.80 | 1434.87% | 4,673.81 | 4291.65% |
| **Minimum Bid/Ask** | 101.95 | 93.48% | 106.42 | 97.72% |

Further analysis also discovered that more above average than below average returns for the indices occurred over the 34-year period, as shown in the bottom two rows of Table 13. However, bonds had slightly higher occurrences of below average rather than above average returns for the quarterly and yearly rebalancing time frames, whereas stocks had higher occurrences of above average than below average returns for all three rebalancing periods. This means that in addition to the much higher prices, the stocks also enjoyed more above average gains than did the bonds. When the portfolio was rebalanced once per year, the portfolio was able to capitalize on the gains of increasing stock prices before another rebalancing occurred. All these factors compounded into huge portfolio value gain by rebalancing yearly, as compared to the less than optimal gains obtained from the monthly or quarterly rebalancing.

Table 15 and Figure 17 display the ranges of stock and bond percentages in the rebalanced index portfolio for each rebalancing frequency. The portfolio beta was higher after rebalancing for the monthly return, whereas the portfolio betas for the quarterly and yearly rebalancing period were higher for the original portfolio rather than after the rebalancing.

Table 15 – Stock and Bond Percentages in Frequency Rebalanced Index Portfolio (1976-2009)

| Rebalancing Frequency | Bond % in Rebalanced PF | | | Stock % in Rebalanced PF | | |
|---|---|---|---|---|---|---|
| | Monthly | Quarterly | Yearly | Monthly | Quarterly | Yearly |
| Avg PF % | 40.16% | 40.08% | 38.72% | 59.84% | 59.92% | 61.28% |
| Max PF % | 47.46% | 51.65% | 54.19% | 63.24% | 65.26% | 70.40% |
| Min PF % | 36.76% | 34.74% | 29.60% | 52.54% | 48.35% | 45.81% |
| | Beta for Original 60/40 PF | | | Beta after PF Rebalancing | | |
| PF Beta | 0.63835 | 0.63034 | 0.63920 | 0.67118 | 0.63435 | 0.62470 |
| P-Value (95% C.L.) | 5.13E-264 | 8.51E-70 | 1.09E-14 | 2.12E-262 | 1.78E-68 | 1.22E-13 |

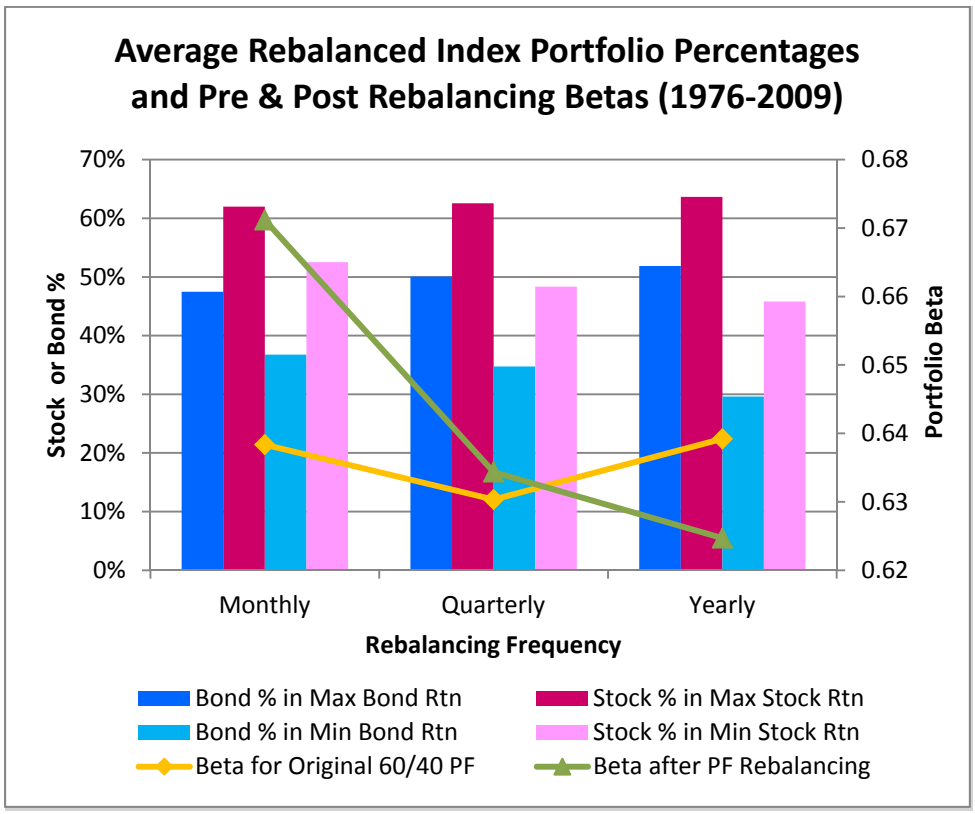


Figure 17 – Average Rebalanced Index Portfolio Percentages and Pre & Post Rebalancing Betas

## 2. Sharpe & Treynor Ratios and Jensen's Alphas for the Frequency Rebalanced Index Portfolio (1976-2009)

Further verification of each portfolio's rebalanced value was determined by performing calculations for Sharpe and Treynor Ratios as well as Jensen's Alphas. The results are shown in Table 16 and Figure 18. Differences between the pre and post transaction costs and capital gains taxes for all three indicators were minimal, so only a graph is shown for simplicity sake.

**Table 16 – Sharpe & Treynor Ratios and Jensen's Alphas from the Index Portfolio Frequency Rebalancing (1976-2009)**

| PF Performance Indicator | Before Trans Costs & Taxes | | | After Trans Costs & Taxes | | |
|---|---|---|---|---|---|---|
| | Monthly | Quarterly | Yearly | Monthly | Quarterly | Yearly |
| Sharpe Ratio | -0.0065 | 0.0843 | 0.3425 | -0.0076 | 0.0832 | 0.3375 |
| Treynor Ratio | -0.0003 | 0.0066 | 0.0598 | -0.0003 | 0.0069 | 0.0598 |
| Jensen's Alpha | -0.0036 | -0.0052 | -0.0003 | -0.0036 | -0.0053 | -0.0003 |
| P-Value (95% C.L.) | 5.64E-23 | 5.64E-23 | 9.69E-01 | 5.78E-23 | 4.69E-04 | 9.71E-01 |

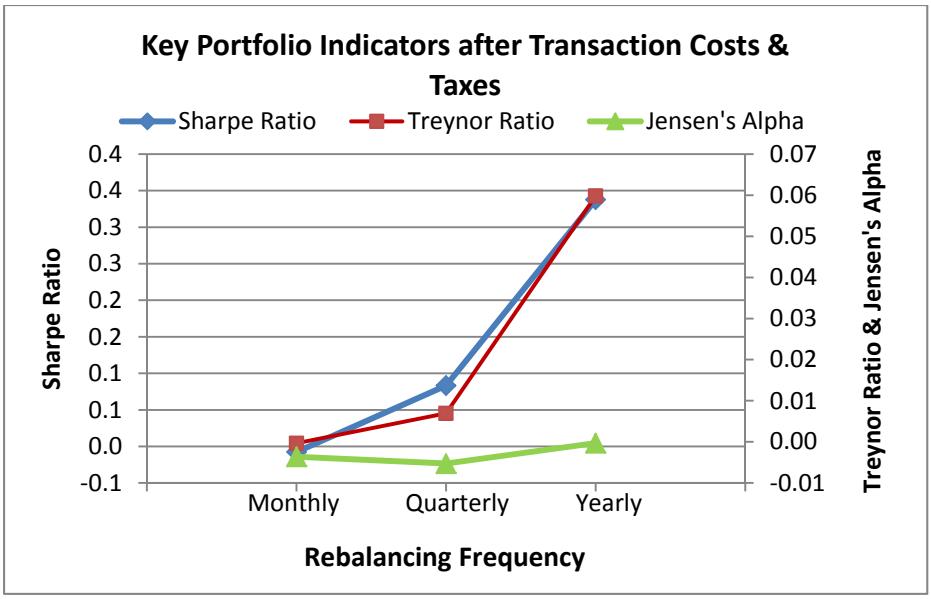


**Figure 18 – Sharpe & Treynor Ratios and Jensen's Alphas after Transaction Costs & Taxes**

As illustrated in Table 16, all three performance indicators were negative in value for the monthly portfolio rebalancing. Each indicator also showed an increasing trend from monthly to quarterly to yearly portfolio rebalancing, proving again that yearly rebalancing is the best interval for long term portfolio value. Although Jensen's Alphas has the same trend as the Sharpe and Treynor Ratios, its numbers are negative for all three rebalancing periods, indicating that these value were not due to skilled portfolio management but rather being lucky. However, the p-value of Jensen's Alpha for the yearly rebalanced portfolio is statistically insignificant at the 95% confidence level, which means that no conclusive outcome can be drawn by those negative numbers for the yearly rebalancing scenario. Additionally, all three indicators fared slightly worse after, as compared to before, the transaction costs and taxes were accounted for, which makes sense since the overall value of each frequency rebalanced portfolio was reduced by the amount of transaction costs and capital gains taxes.

## B. Strategy 2 – Index Portfolio Percentage Movement Rebalancing

As mentioned earlier in the ETF portion, percentage movement rebalancing involves rebalancing a portfolio when the portfolio's stock and bond mix moves beyond the preset percentage limit. For the indexed data, movement percentages of 3, 5, 8, 10, 12, 15, 20, 25, and 28 were chosen for this strategy analysis.

### 1. Percentage Movement Rebalancing Strategy Results for Indexed Data (1976-2010)

Table 17 and Figure 19 are the summarized Index portfolio values, average returns and standard deviations for the different rebalancing percentage movements. Similar to the ETF portfolio, when the Index portfolio was rebalanced at a higher percentage, the portfolio value was greater. Likewise, transaction costs and capital gains taxes are higher at the smaller rebalancing percentages as compared to the higher rebalancing percentages. Over the percentage movements, average excess returns and corresponding standard deviations both increased in monotonic directions, from 0.2% to 0.4% in average portfolio excess returns and 3% to 3.4% in portfolio standard deviations. These findings parallel those of the ETF portfolios as well, showing the robustness of the result. Likewise, as a whole, average excess returns and corresponding standard deviations are much smaller for the percentage movement rebalancing strategy as compared to the frequency rebalancing strategy.

Table 17 – Index Portfolio Rebalancing by Percentage Movement (1976-2009) Using a 30% Tax Bracket

| Percentage Movement Rebalancing | PF before Trans Costs & Taxes ($) | Trans Cost ($) | Taxes ($) | PF after Trans Costs & Taxes ($) | Avg PF Excess Rtn | PF Std Dev |
|---|---|---|---|---|---|---|
| **3%** | 134,463,052 | 2,840 | 877,272 | 133,582,940 | 0.21% | 2.96% |
| **5%** | 167,444,307 | 2,000 | 789,217 | 166,653,091 | 0.27% | 2.95% |
| **8%** | 196,368,157 | 1,620 | 846,603 | 195,519,934 | 0.31% | 2.98% |
| **10%** | 214,179,219 | 1,400 | 614,241 | 213,563,578 | 0.33% | 3.01% |
| **12%** | 226,209,718 | 1,280 | 617,518 | 225,590,920 | 0.35% | 3.06% |
| **15%** | 233,964,669 | 1,120 | 583,918 | 233,379,630 | 0.36% | 3.15% |
| **20%** | 248,121,391 | 700 | 472,266 | 247,648,425 | 0.38% | 3.26% |
| **25%** | 265,552,302 | 320 | 249,631 | 265,302,351 | 0.40% | 3.39% |
| **28%** | 274,324,880 | 0 | 0 | 274,324,880 | 0.41% | 3.45% |

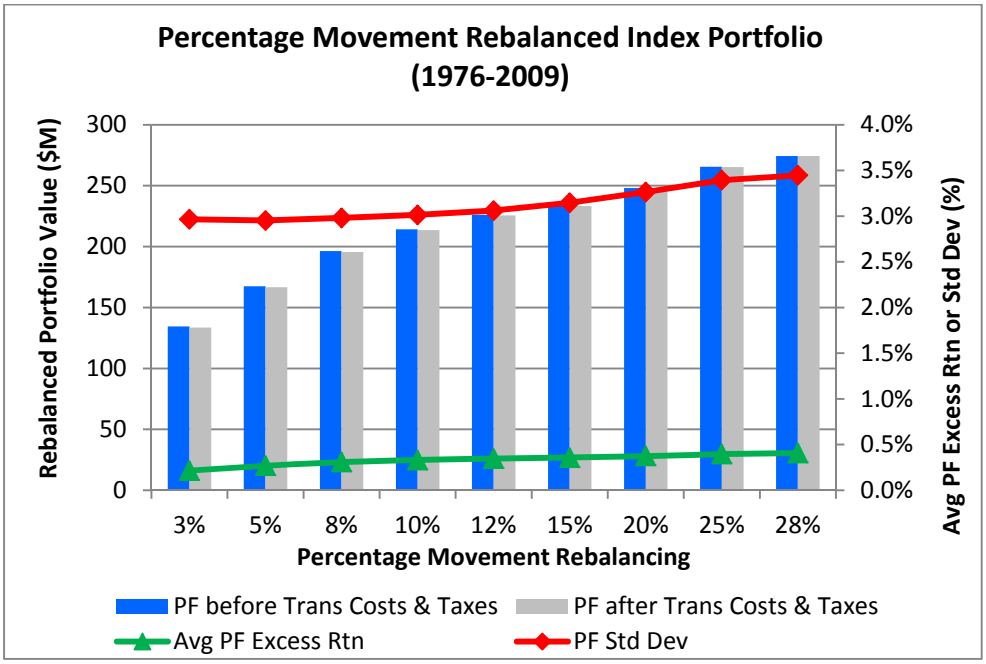


Figure 19 – Percentage Movement Rebalanced Index Portfolio (1976-2009)

Tables 18 and 19 summarized the maximum to minimum bond percentages and the rate of change of the average, maximum and minimum bond percentages observed for the Index portfolio rebalancing strategy by percentage movements. In addition, it also

delineates the occurrences of above versus below average portfolio bond mix percentages. As we can see, the bond mix percentages were consistently below average from 3 to 25% rebalancing movements. At 28% rebalancing movement, which is when the portfolio was not rebalanced at all, it showed that the bond mix percentage was finally above the average percentage.

**Table 18 – Bond Percentages for Percentage Movement Rebalanced Index Portfolio (1976-2009)**

| Rebalancing Percentage | 3% | 5% | 8% | 10% | 12% | 15% | 20% | 25% | 28% |
|---|---|---|---|---|---|---|---|---|---|
| Avg PF % | 38.93% | 38.02% | 36.53% | 35.24% | 33.77% | 31.69% | 29.28% | 27.06% | 26.18% |
| Max PF % | 50.02% | 50.86% | 53.40% | 54.31% | 52.48% | 48.62% | 42.77% | 42.77% | 42.77% |
| Min PF % | 34.00% | 32.13% | 29.52% | 27.95% | 26.04% | 23.18% | 18.48% | 13.88% | 12.47% |
| Freq < Avg PF % | 250 | 237 | 230 | 232 | 231 | 206 | 217 | 215 | 189 |
| Freq > Avg PF % | 158 | 171 | 178 | 176 | 177 | 202 | 191 | 193 | 219 |

**Table 19 – Bond Rate of Change in Movement for Percentage Movement Rebalanced Index Portfolio (1976-2009)**

| Rebalancing Percentage | 3% | 5% | 8% | 10% | 12% | 15% | 20% | 25% | 28% |
|---|---|---|---|---|---|---|---|---|---|
| Avg PF % Δ | -1.07% | -1.98% | -3.47% | -4.76% | -6.23% | -8.31% | -10.72% | -12.94% | -13.82% |
| Max PF % Δ | 10.02% | 10.86% | 13.40% | 14.31% | 12.48% | 8.62% | 2.77% | 2.77% | 2.77% |
| Min PF % Δ | -6.00% | -7.87% | -10.48% | -12.05% | -13.96% | -16.82% | -21.52% | -26.12% | -27.53% |

From the corresponding graphs in Figures 20 through 22, we can clearly see that on average, the bond mix percentages of the Index portfolio declined at an increasing rate as the rebalancing percentage movements became larger. In a similar trend, the minimum bond mix percentages also decreased with an increasing rate at higher rebalancing percentages, but at twice the rate as that of the average bond percentages.

However, it is quite another story for the maximum bond mix percentages: they increased at an increasing rate from 50% when the portfolio was rebalanced by 3% movement, plateaued around 54% when rebalancing occurred by 10% change in movement, then decreased at a slightly slower rate than the increasing rate to around 43% when rebalancing occurred at 28% movement. No rebalancing occurred at or above the 28% movement because the highest percentage of either the stocks or the bonds moved just below that percentage.

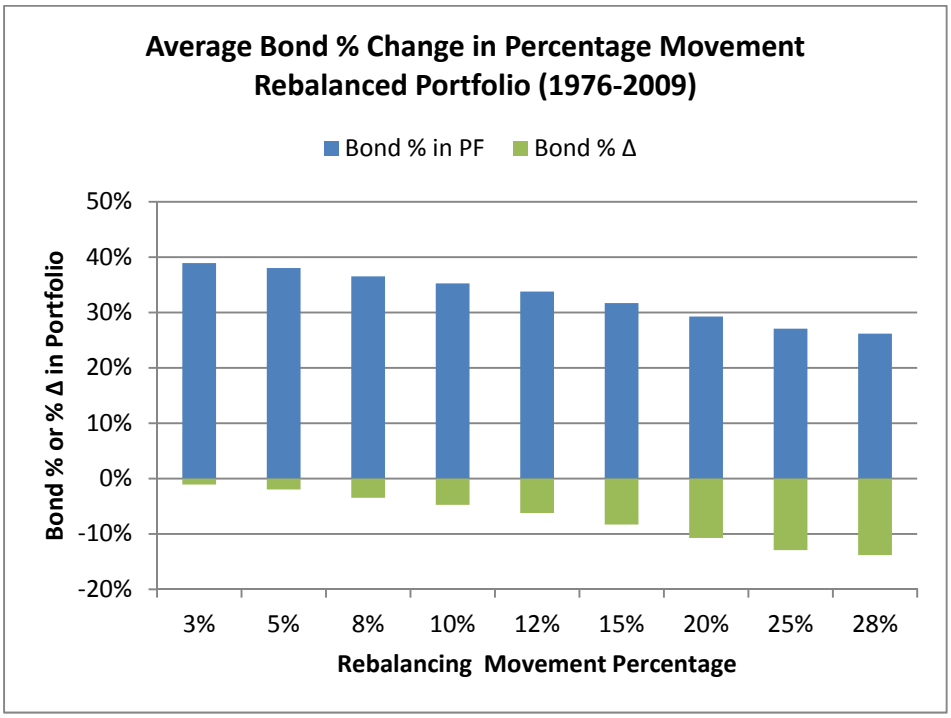


**Figure 20 – Average Bond % Change in Percentage Movement Rebalanced Index Portfolio (1976-2009)**

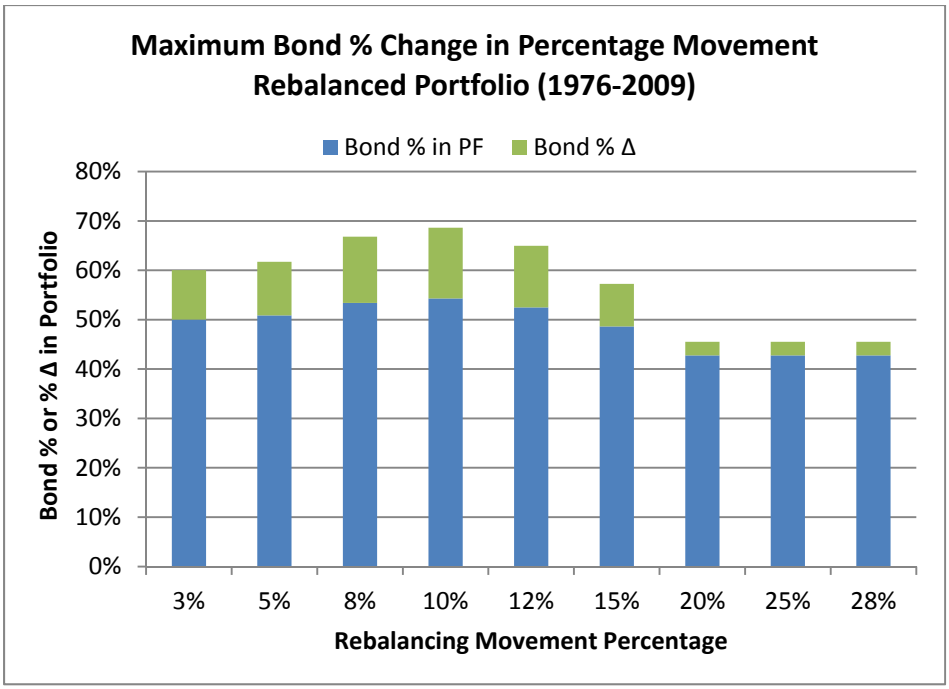


Figure 21 – Maximum Bond % Change in Percentage Movement Rebalanced Index Portfolio (1976-2009)

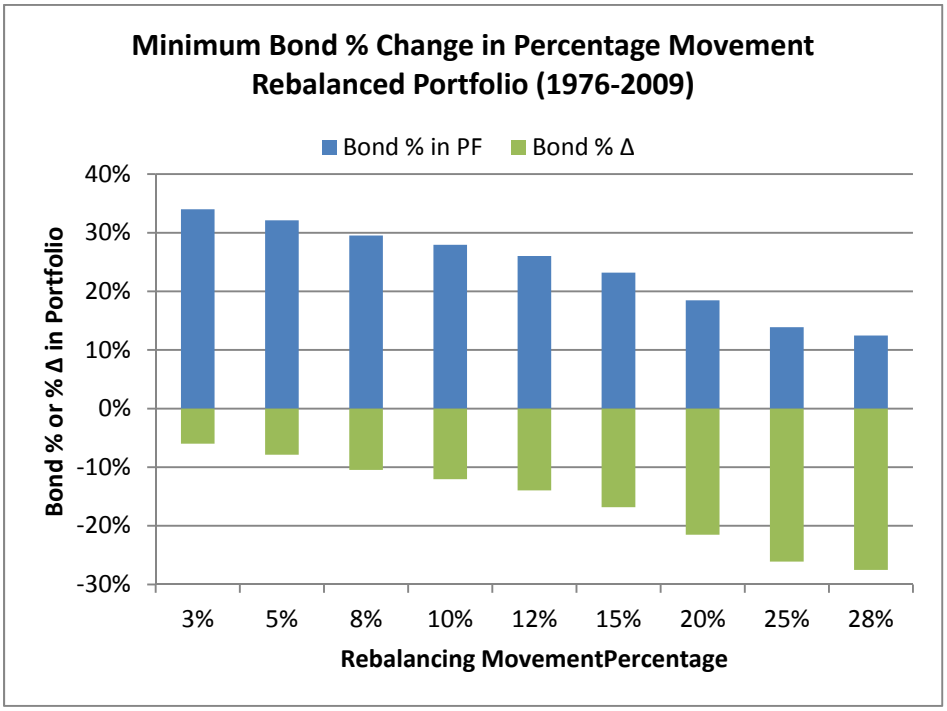


Figure 22 – Minimum Bond % Change in Percentage Movement Rebalanced Index Portfolio (1976-2009)

Tables 20 and 21 summarized the maximum to minimum stock percentages observed for the Index portfolio percentage movement rebalancing strategy. Opposite to the portfolio bond mix percentages, the portfolio stock mix percentages were consistently above average for the 3 to 25% rebalancing movements. Likewise, at 28% rebalancing movement, since the portfolio was not rebalanced at all, it finally had a below average stock mix percentage.

**Table 20 – Stock Percentages for Percentage Movement Rebalanced Index Portfolio (1976-2009)**

| Rebalancing Percentage | 3% | 5% | 8% | 10% | 12% | 15% | 20% | 25% | 28% |
|---|---|---|---|---|---|---|---|---|---|
| **Avg PF %** | 61.07% | 61.98% | 63.47% | 64.76% | 66.23% | 68.31% | 70.72% | 72.94% | 73.82% |
| **Max PF %** | 66.00% | 67.87% | 70.48% | 72.05% | 73.96% | 76.82% | 81.52% | 86.12% | 87.53% |
| **Min PF %** | 49.98% | 49.14% | 46.60% | 45.69% | 47.52% | 51.38% | 57.23% | 57.23% | 57.23% |
| **Freq < Avg PF %** | 158 | 171 | 178 | 176 | 177 | 202 | 191 | 193 | 219 |
| **Freq > Avg PF %** | 250 | 237 | 230 | 232 | 231 | 206 | 217 | 215 | 189 |

**Table 21 – Stock Rate of Change in Movement for Percentage Movement Rebalanced Index Portfolio (1976-2009)**

| Rebalancing Percentage | 3% | 5% | 8% | 10% | 12% | 15% | 20% | 25% | 28% |
|---|---|---|---|---|---|---|---|---|---|
| **Avg PF % Δ** | 1.07% | 1.98% | 3.47% | 4.76% | 6.23% | 8.31% | 10.72% | 12.94% | 13.82% |
| **Max PF % Δ** | 6.00% | 7.87% | 10.48% | 12.05% | 13.96% | 16.82% | 21.52% | 26.12% | 27.53% |
| **Min PF % Δ** | -10.02% | -10.86% | -13.40% | -14.31% | -12.48% | -8.62% | -2.77% | -2.77% | -2.77% |

Figures 23 through 25 are the corresponding charts for the stock mix percentages for the percentage rebalancing movements. On average, the stock mix percentages of the Index portfolio increased from 61 to 74% at an increasing rate, from one to 14%, over the range of the rebalancing movements. Likewise, maximum stock mix percentages in the

portfolio rose from 66 to 87% over the same rebalancing movements at an increasing rate, from 6 to 28%. Conversely, the minimum stock mix percentages decreased from 50% when the portfolio was rebalanced by 3% movement, at an increasing rate, hit the bottom around 46% when rebalancing occurred by 10% in movement, then increased at a slower increasing rate than did the decreasing rate, to around 57% when rebalancing occurred at 28% movement. Note that this is exactly the opposite of the bond percentages at its maximum portfolio mix percentage because of the 60/40 stock and bond mix for the portfolio. When one asset increases in percentage, the other must decrease so that the overall portfolio is still 100%.

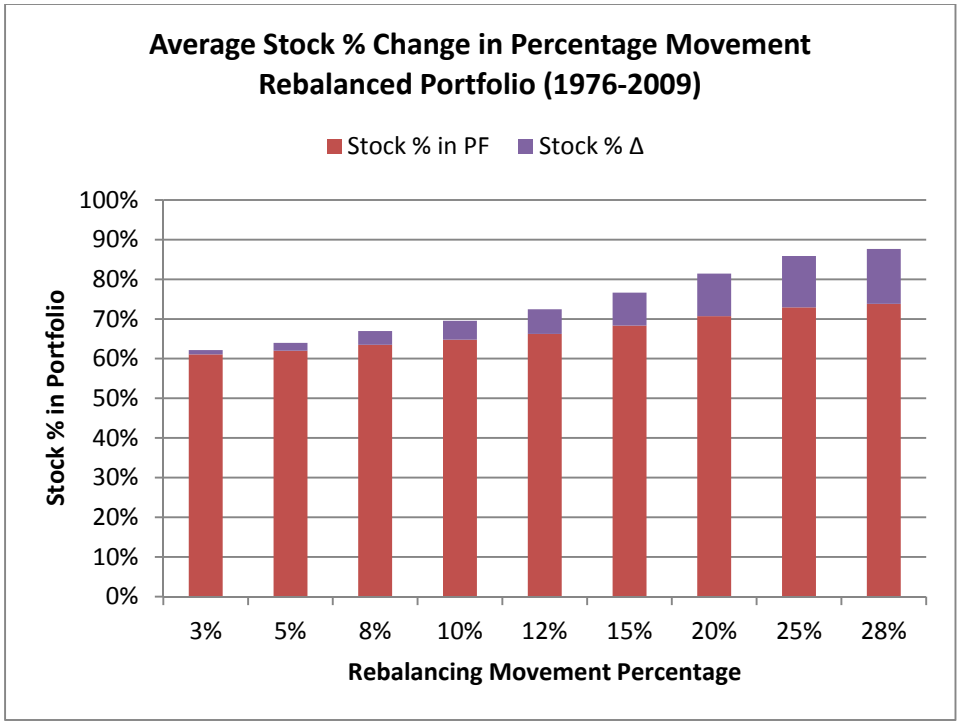


**Figure 23 – Average Stock % Change in Percentage Movement Rebalanced Index Portfolio (1976-2009)**

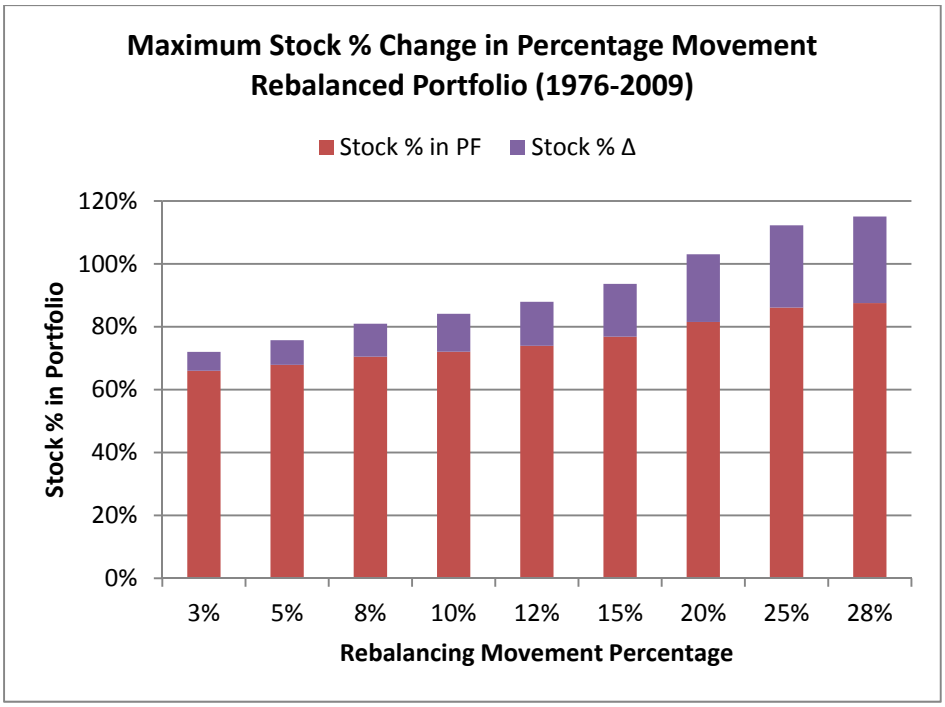


Figure 24 – Maximum Stock % Change in Percentage Movement Rebalanced Index Portfolio (1976-2009)

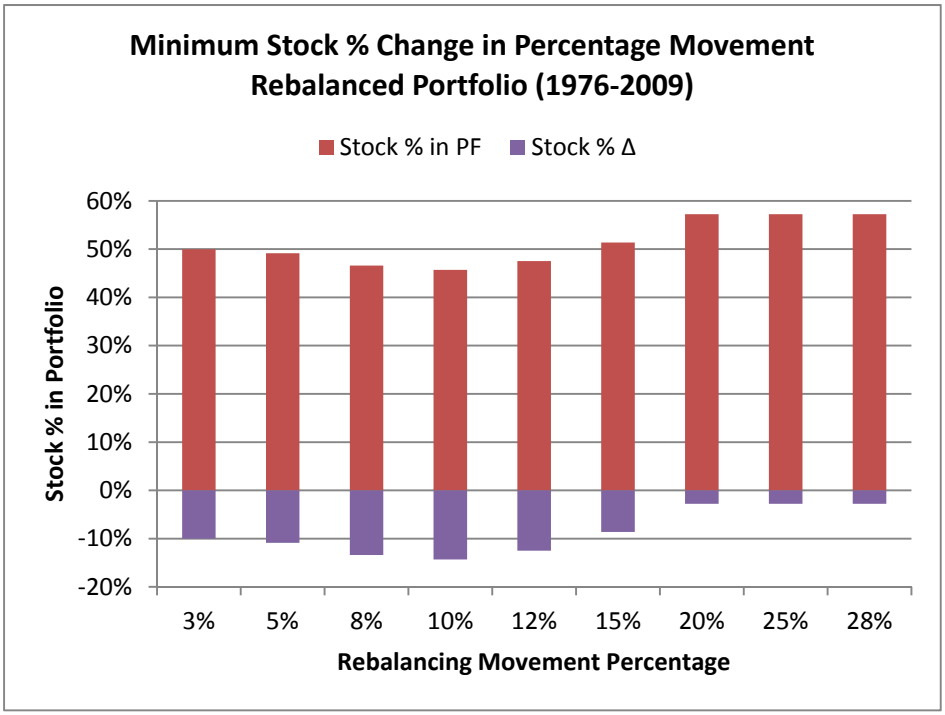


Figure 25 – Minimum Stock % Change in Percentage Movement Rebalanced Index Portfolio (1976-2009)

Table 22 represents the Index portfolio betas before and after rebalancing by percentage change in movements. The original betas were constant because this rebalancing strategy is by percentage movement instead of by frequency change thus having the same set of data points. The rebalanced portfolio betas increased more than 18%, from 0.65 to 0.77, over the rebalancing percentage range. This is consistent with the findings that for the 34-year period, stocks and bonds had higher occurrences of above average than below average return, as noted on the last two rows of numbers in Table 20. Figure 26 showcases the Index portfolio's betas, before and after rebalancing, along with the maximum and minimum stock and bond percentages at the rebalancing percentage movements.

**Table 22 – Index Portfolio Betas before and after Percentage Movement Rebalancing (1976-2009)**

| Rebalancing Percentage | 3% | 5% | 8% | 10% | 12% | 15% | 20% | 25% | 28% |
|---|---|---|---|---|---|---|---|---|---|
| Original PF Beta | 0.6024 | 0.6024 | 0.6024 | 0.6024 | 0.6024 | 0.6024 | 0.6024 | 0.6024 | 0.6024 |
| P-Value (95% C.L.) | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| Rebalanced PF Beta | 0.6529 | 0.6516 | 0.6591 | 0.6668 | 0.6789 | 0.7003 | 0.7296 | 0.7602 | 0.7724 |
| P-Value (95% C.L.) | 3.42E-256 | 4.82E-263 | 4.19E-270 | 8.00E-272 | 7.77E-281 | 5.09E-296 | 0 | 0 | 0 |

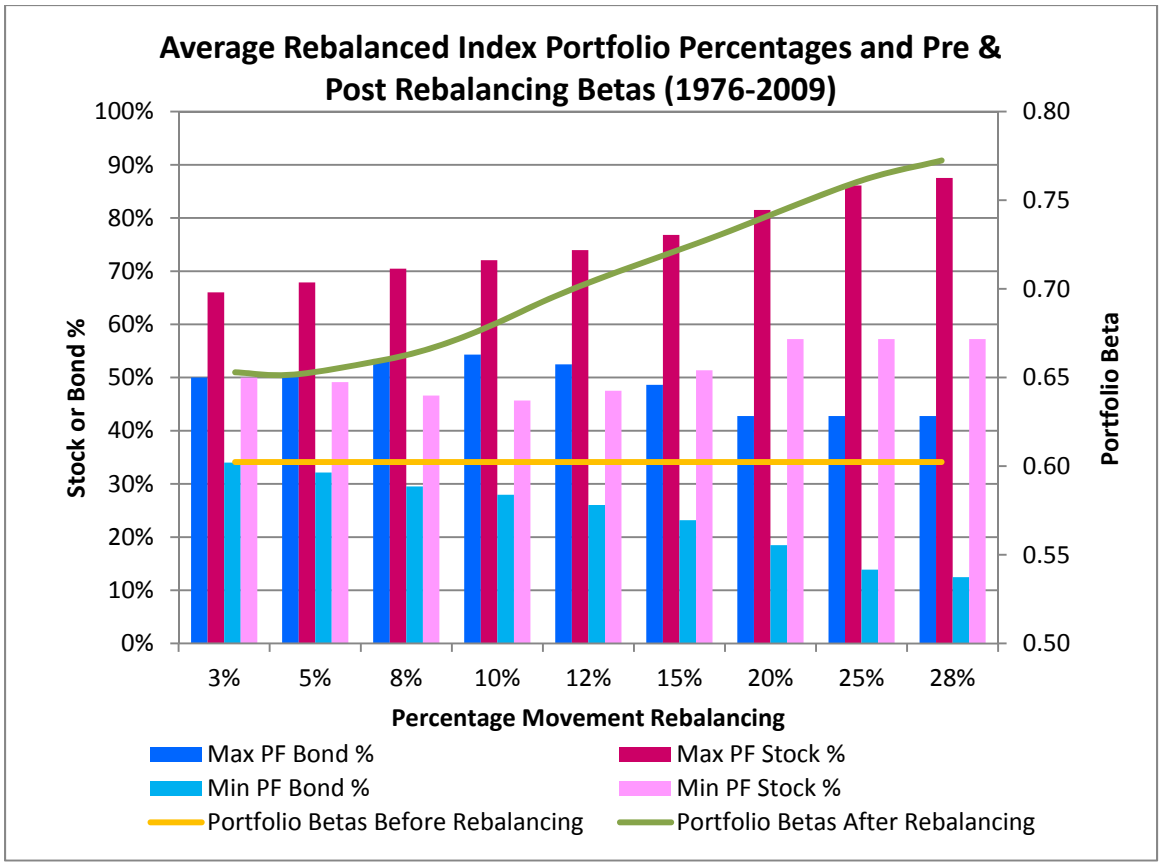


**Figure 26 – Average Rebalanced Index Portfolio Percentages and Pre and Post Rebalancing Betas**

## 2. Sharpe & Treynor Ratios and Jensen's Alphas for the Percentage Movement Rebalanced Index Portfolio (1976-2009)

Sharpe and Treynor Ratios, together with Jensen's Alphas, were once again used as indicators to evaluate the effectiveness of the percentage movement rebalancing strategy. Sharpe and Treynor Ratios did not show any significant differences before or after the transaction costs and capital gains taxes were accounted for. However, Jensen's Alphas displayed a noticeable difference between the pre and post transaction costs and tax accounting. Tables 23 and 24 are summaries of these indicators accounting for the

effectiveness or the lack thereof for the rebalanced Index portfolio before and after transaction costs and taxes.

**Table 23 – Sharpe and Treynor Ratios and Jensen's Alphas – before Index Portfolio Rebalancing (1976-2009)**

| Rebalancing Percentage | 3% | 5% | 8% | 10% | 12% | 15% | 20% | 25% | 28% |
|---|---|---|---|---|---|---|---|---|---|
| Sharpe Ratio | 0.0724 | 0.0909 | 0.1035 | 0.1097 | 0.1129 | 0.1134 | 0.1149 | 0.1168 | 0.1178 |
| Treynor Ratio | 0.0033 | 0.0041 | 0.0047 | 0.0050 | 0.0051 | 0.0051 | 0.0052 | 0.0052 | 0.0053 |
| Jensen's Alpha | 0.0035 | 0.0041 | 0.0044 | 0.0046 | 0.0047 | 0.0047 | 0.0047 | 0.0048 | 0.0048 |
| P-Value (95% C.L.) | 2.22E-18 | 2.22E-25 | 2.73E-31 | 5.69E-34 | 4.04E-37 | 3.70E-40 | 7.49E-46 | 4.40E-49 | 1.23E-49 |

**Table 24 – Sharpe and Treynor Ratios and Jensen's Alphas – after Index Portfolio Rebalancing (1976-2009)**

| Rebalancing Percentage | 3% | 5% | 8% | 10% | 12% | 15% | 20% | 25% | 28% |
|---|---|---|---|---|---|---|---|---|---|
| Sharpe Ratio | 0.0725 | 0.0909 | 0.1035 | 0.1098 | 0.1129 | 0.1134 | 0.1168 | 0.1178 | 0.1178 |
| Treynor Ratio | 0.0033 | 0.0041 | 0.0047 | 0.0050 | 0.0051 | 0.0051 | 0.0052 | 0.0053 | 0.0053 |
| Jensen's Alpha | -1.2E-03 | -6.3E-04 | -2.6E-04 | -8.0E-05 | 8.0E-06 | 9.8E-06 | 4.5E-05 | 1.0E-04 | 1.4E-04 |
| P-Value (95% C.L.) | 0.0009 | 0.0626 | 0.4168 | 0.8069 | 0.9797 | 0.9737 | 0.8686 | 0.6970 | 0.5949 |

As noted in Tables 23 and 24, from 3 to 28% rebalancing movements before transaction costs and capital gains taxes were accounted for, Jensen's Alphas were all positive and increasing at a small increment. However, Jensen's Alphas were only positive from 12 to 28% rebalancing movements after the transaction costs and taxes were taken out. In addition, p-values for Jensen's Alphas were all statistically significant for the pre transaction costs and taxes scenario but were only significant for the 3% rebalancing movement for the post transaction costs and taxes. This is an indication that

each of the Index portfolios at the various percentage movements rebalancing before transaction costs and taxes was more valuable than its counterpart after the transaction costs and taxes. This is especially true at the higher percentage rebalancing movements. At the 3% rebalancing movement after transaction costs and taxes, the negative Jensen's Alpha and the significant p-value confirms that rebalancing at that percentage movement was of no value to the portfolio. The positive Jensen's Alphas from 12 to 28% rebalancing movements after the transaction costs and taxes and their respective insignificant p-value indicated that it is indeterminate if the portfolio at those percentage movements was of value.

By presenting these indicators in graphical format, as shown in Figures 27 through 29, it is easier to see that there is an interesting trend emerging for these indicators. Since there was no much difference accounting for the transaction costs and capital gains taxes, only one graph is displayed for the comparison between Sharpe and Treynor Ratios. Figure 30 highlights the different outcome of Jensen's Alphas between the pre and post transaction costs and capital gains tax scenarios.

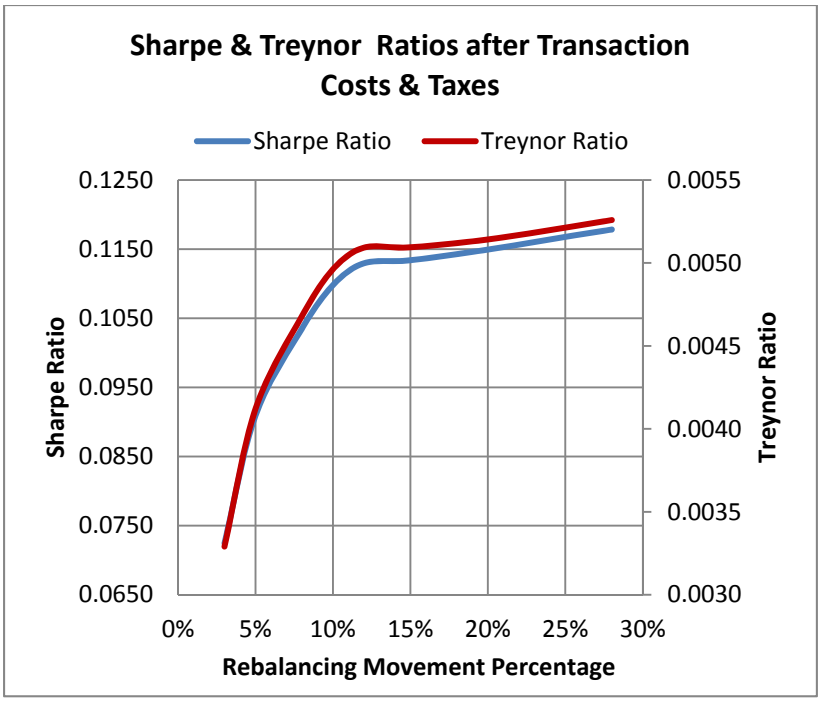


Figure 27 – Sharpe & Treynor Ratios after Transaction Costs and Taxes

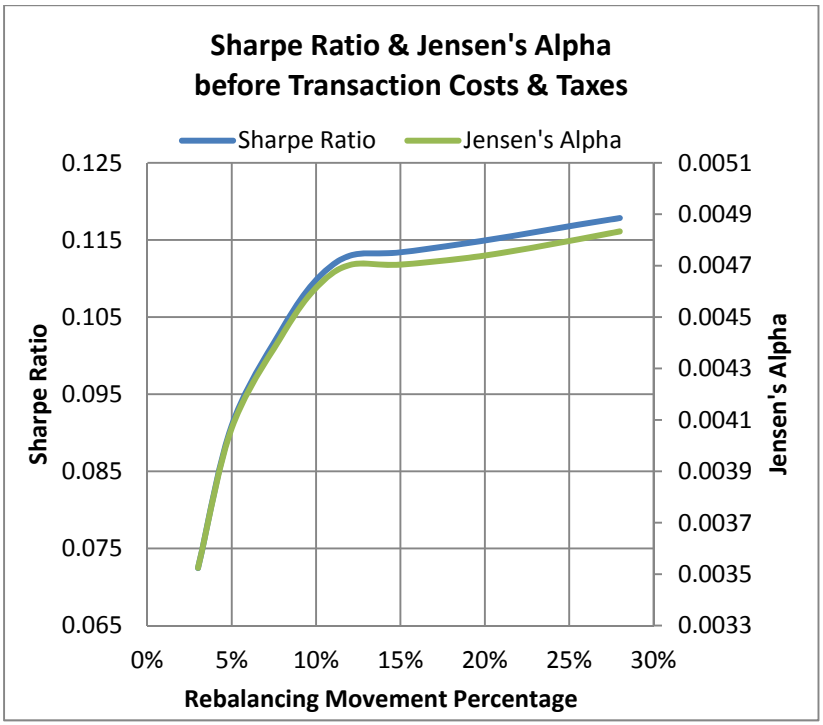


Figure 28 – Sharpe Ratio and Jensen's Alphas before Transaction Costs and Taxes

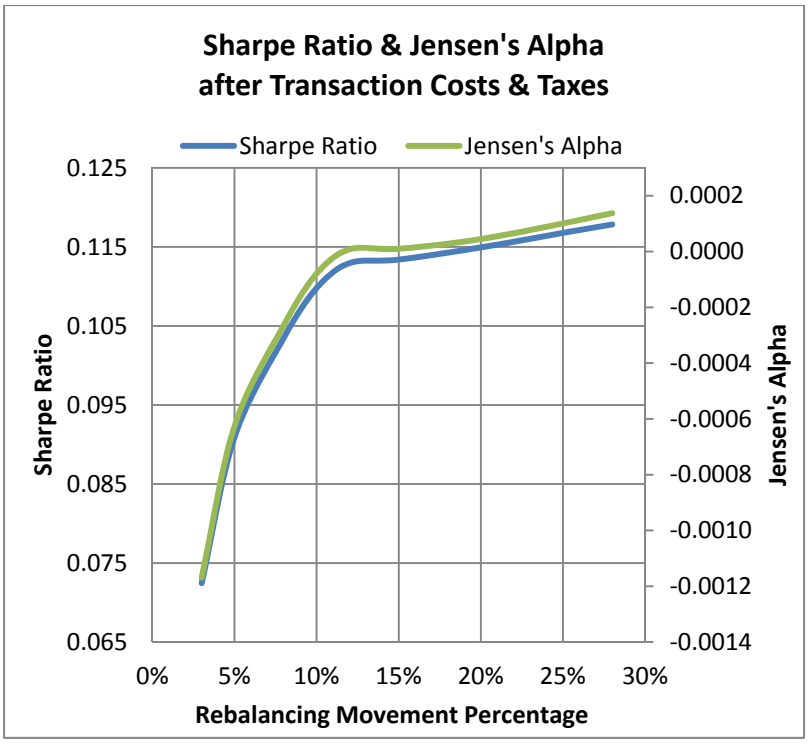


Figure 29 – Sharpe Ratio and Jensen's Alphas after Transaction Costs and Taxes


Figure 30 – Jensen's Alphas before and after Transaction Costs and Taxes

From these graphs, it is obvious that Sharpe and Treynor Ratios and Jensen's Alphas increased rapidly from 3 to 12% rebalancing movements; they stayed flat until the 15% rebalancing movement then increased at a much slower rate from 15 to 28% rebalancing movements. This trend is an indication that if choosing a percentage movement rebalancing strategy for the 60/40 stock and bond portfolio, the most ideal rebalancing percentages would be between 12 and 15%, because rebalancing higher than 15% movements would lose the objective of portfolio diversification and would lead to an inevitable loss of portfolio value if the stock market experienced a serious downturn.

## C. Strategy Comparison – Frequency vs. Percentage Movement Rebalancing for the Index Portfolio (1976-2009)

Indexed stocks and bonds exhibited a similar trend as the ETF stocks and bonds in portfolio performance for both the frequency as well as the percentage movement rebalancing strategy. From the post transaction costs and capital gains taxes' perspective, when rebalanced above 8 percent from its original mix, the percentage movement rebalancing strategy netted a higher portfolio value than the frequency rebalanced portfolio. Standard deviations are much lower for the percentage rebalancing movements than the frequency rebalancing strategy. Just as it was the case for the ETF portfolio, the average returns for the rebalanced Index portfolio using the percentage movement rebalancing strategy is significantly lower than those from using the frequency rebalancing strategy for the Index portfolio. The corresponding standard deviations are

also significantly lower. Because of the higher average returns enjoyed by the frequency rebalancing strategy, its portfolio betas are also higher than those for the percentage movement rebalancing strategy. Table 25 compares the best performer between the two index portfolio rebalancing strategies.

**Table 25 – Summary Comparison between the Best Performing Frequency and Percentage Movement Rebalancing Strategies for the index Portfolio (1976-2009)**

| Rebalancing Strategy Comparison | Yearly Rebalancing | 12% Movement Rebalancing |
|---|---|---|
| **PF Value before C&T ($)** | 179,194,013 | 226,209,718 |
| **Transaction Costs ($)** | 680 | 1,280 |
| **Capital gains taxes ($)** | 22,925,366 | 617,518 |
| **PF Value after C&T ($)** | 156,267,967 | 225,590,920 |
| **Avg PF Excess Rtn** | 3.74% | 0.35% |
| **Std Dev** | 11.07% | 3.06% |
| **PF Beta** | 0.625 | 0.679 |
| **Sharpe Ratio** | 0.33755 | 0.11295 |
| **Treynor Ratio** | 0.05980 | 0.00509 |
| **Jensen's Alpha** | -0.00031 | 0.00001 |

# IV. Conclusion

For an investor whose goal is to maximize portfolio returns and at the same time, diversify his/her portfolio, it is vitally important to rebalance his/her portfolio to achieve this objective. An important consideration before undertaking such a task is to determine the type of rebalancing strategy that best suits an investor's aversion level and the amount of extra cost he/she wishes to pay his portfolio manager, not to mention the amount of capital gains taxes he/she would have to pay.

Two rebalancing strategies were tested in this study: frequency and percentage movement rebalancing. The results indicated that depending on an investor's investment goal and how much money he/she would like to pay a portfolio manager to manage the chosen rebalancing strategy, either strategy would produce a desired result.

For the frequency rebalancing strategy, annual rebalancing achieved the objective of netting the highest return when compared with the monthly and quarterly rebalancing periods tested. For the percentage movement rebalancing, an ideal percentage movement is between 10 to 15 percent, based on the results for both the ETF data and the Index data. Rebalancing beyond this percentage range would deviate from the original portfolio mix by a huge margin thus losing the objective for diversification.

Finally, when the percentage rebalancing movements are higher than 8 percent (for the Index portfolio) or 10 percent (for the ETF portfolio), the percentage movement

rebalancing strategy outperforms the yearly rebalancing strategy. Therefore, for this chosen portfolio, the best strategy is the percentage movement rebalancing strategy, to be rebalanced when its portfolio mix moves between12 to 15 percent beyond its original portfolio mix, since those are the optimal rebalancing percentage movements as predicted by the Sharpe and Treynor Ratios. However, since this strategy requires a closer monitor due to percentage movement change, I suspect the cost incurred from the portfolio manager would be higher than if the portfolio is rebalanced once a year.

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