Air Flow Distribution Optimization in Mine Network

A thesis submitted in partial fulfilment of the requirements for the degree of Master of Science in Mining Engineering

by

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Abstract

In spite of the high energy cost, there may exist over-ventilation and uneconomic airflow distribution design in underground mines. It is difficult to intuitively reduce ventilation energy cost while maintaining health and safety.

This paper reviews the elements of ventilation optimization. The “best practices” should all be exercised first for optimization and meanwhile considering also their broader economic impact. The second step is to consider using booster fans instead of air regulators for air distribution control. The third step of ventilation optimization is to achieve the best match between the air flow in the ventilation network and the specified minimum air flow requirement for both safety and health for each air branch. The goal is to operate the ventilation network as close as possible to specified minimum air flow requirement in each branch at the mine and lower operating cost. This goal can be achieved by the assistance of ventilation model application algorithms. One specific algorithm is studied that systematically varies the adjustment parameters of the fans and air regulators until the best optimum solution for the objective functions is satisfied.

Numerical examples show that the algorithm can be used to find the best, optimal solution for a mine ventilation task.

Keywords: Ventilation, Optimization Algorithm, Air Distribution
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1 CHAPTER I. Introduction

A great portion of the energy consumption of a mine is spent on powering the ventilation system to provide a healthy and safe underground working environment. It is not easy to design a perfect ventilation system which can satisfy the health and safety requirements and meanwhile minimize cost. The existing problems for current mine ventilation systems include over-ventilation, high network resistance, high shock losses, high leakage, inefficient fan operation, and bad airflow distribution.

Ventilation optimization has three main components:

1. First, identify the high cost elements.

2. Second, try several different scenarios by modifying the high cost elements.

3. Third, compare and find the best combinations from these scenarios.

This method works well when the high cost elements are high network resistance and low operating efficiency. This would be called first stage optimization. While currently there is no algorithm to optimize a ventilation system by reducing bad airflow distribution and over-ventilation, developing an accurate and efficient tool to find the global optimal solution for these problems would be helpful.

This thesis reviews the most practical optimization methods. These optimization methods will be employed along with the tool/model “VnetPC” and “VentSim to analyse and optimize a few mine examples’ ventilation system. After the first stage optimization, this paper discusses a conditional optimal tool and algorithm which can find the optimal solution for the air distribution tasks based on the current improved ventilation network.
2 CHAPTER II. Literature Review

2.1 Introduction

Mine ventilation system serves as a life supply line for underground mining, providing fresh air and exhausting harmful contaminations for each working location. In modern mining, ventilation system typically contributes 30% of a mine’s annual total energy consumption (Chen, 2003). It is expected that more than 75% of the ventilation operation power cost may be reduced by reducing over-ventilation, preventing leakage, and adjusting uneconomic airflow distribution (Mulder, Fourie, & Stanton, 2014). In this chapter, the elements of ventilation optimization are examined for minimizing ventilation operation cost based on the basic mine ventilation air demand that ensures a healthy and safe underground working environment. Operational research method and its applications in ventilation optimization are also reviewed.

2.2 Ventilation Cost

Total underground mine ventilation cost consists of capital and operating cost. It can be expressed as an annual cost on the sum of the development and operating costs. A common target of the mine ventilation optimization is to minimize the combination of capital and operating cost. The equation below is used for annual ventilation cost calculation (Loomis, Ramirez, & Tarigan, 2014):

\[
EAC = \left[ \frac{k^{*}L^{*}per}{A^{3}} * Q^{3} * \frac{e}{1000\eta} * 24 * 365 \right] + \left[ \frac{P^{*}i^{*}}{1-(1+i)^{n}} \right] \quad (\$/year)
\]

Where \( EAC = \text{equivalent annual cost} \)

\( k = \text{Atkinson's friction factor} \ (kg/m^{3}) \)

\( L = \text{length} \ (m) \)
per = perimeter (m)

\[ A = \text{cross} - \text{sectional area} \ (m^2) \]
\[ Q = \text{volumetric flow rate} \ (m^3/s) \]
\[ e = \text{power cost} \ ($/kw - hr) \]
\[ \eta = \text{overall efficiency} \]
\[ i = \text{interest rate} \]
\[ n = \text{payback life} \]

\( P \) is the present sum of capital cost

2.2.1 Capital Cost

As part of the fixed underground ventilation capital costs, a substantial portion will be needed to purchase and install fans and regulators and develop ventilation shafts. Most of the time the capital cost is borrowed from bank and then paid back in equal installments per year. Even if it is not borrowed, same amount of fixed funds would have income from investment yearly. Including all these effects, the capital cost is expressed as equivalent annual cost (McPherson, 1993):

\[ EAC_c = \frac{P+i}{1-(1+i)^n} \ ($/year) \]

Where \( P \) is the present sum of capital cost

\[ i = \text{interest rate} \]
\[ n = \text{payback life} \]

2.2.2 Operating Cost

The total operating costs include consumable items, payments to ventilation engineers and workers, electricity power charging on fans, and maintenance fee. As the
personnel salary, consumable items, and maintenance fee are fixed and the cost of electricity power consumption on ventilation fans accounts for the largest component of the underground mine ventilation operating cost, to reduce the operating cost on the ventilation fans would considerably reduce the total cost. The fan operating cost basically is the fan power cost. Fan power supplied by main or booster fans, is depleted by frictional and shock loss. The air power can be expressed as:

\[ Air\ Power = \frac{P_f \times Q}{1000} \ (kW) \]  \hspace{1cm} (2.3)

Where \( P_f \) = \textit{total pressure loss} (Pa)

\[ Q = \textit{total airflow} \ (m^3/s) \]

Power losses on motor, transmission and impeller, are expressed by the overall efficiency. The fan power can be expressed as:

\[ Fan\ Power = \frac{P_f \times Q}{1000\eta} \ (kW) \]  \hspace{1cm} (2.4)

Where \( \eta \) = \textit{fan and motor overall efficiency}

Then the annual operating cost is:

\[ EAC_o = \frac{P_f \times Q}{1000\eta} \times e \times 24 \times 365 \ ($/year) \]

Or

\[ EAC_o = \frac{RQ^3}{1000\eta} \times e \times 24 \times 365 \ ($/year) \]  \hspace{1cm} (2.5)

Where \( e = \textit{power cost} \ ($/kw - hr) \)

\textbf{2.3 Airway Resistance}

Air resistance is the gas force opposing airflow. Airway resistance is very important in mine ventilation calculations. It can be evaluated by measurement using
anemometer and barometer readings, calculating volumetric airflow $Q$, and pressure
difference $p$ between airway entrance and exit. The airway resistance can be expressed
as:

$$R = \frac{p}{Q^2} \left( Ns^2/m^8 \right)$$ \hspace{1cm} 2.7

Where $p = \text{total airway pressure loss (Pa)}$

$Q = \text{volumetric flow rate (m}^3/\text{s})$

Pressure loss is due to two components: frictional pressure loss and shock loss.

Resistance $R$ can also be expressed as frictional and shock loss:

$$R_t = k_{1.2} (L + L_{eq}) \frac{per \ \rho}{A^{3.2}} \left( Ns^2/m^8 \right)$$ \hspace{1cm} 2.8

Where $R_t = \text{total resistance}$

$k_{1.2} = \text{Atkinson friction factor at standard air density of 1.2 kg/m}^3$

$per = \text{perimeter of the airway (m)}$

$A = \text{cross-section of the airway (m}^2)$

$L = \text{airway length (m)}$

$L_{eq} = \text{equivalent length (m), expressing the shock loss coefficient}$

$\rho = \text{air density (kg/m}^3)$

2.3.1 Atkinson’s Resistance

The Atkinson’s friction factor, $k_{1.2}$ is the frictional resistance of the airway. It
varies with the roughness of the wall, airway size, and air velocity. Atkinson’s resistance
can be expressed as:

$$R_f = k_{1.2} L \frac{per \ \rho}{A^{3.2}} \left( Ns^2/m^8 \right)$$ \hspace{1cm} 2.9
2.3.2 Shock Loss and Equivalent Length

Shock loss is the additional resistance which occurs when the airflow is forced to change its direction or velocity. It usually happens at obstructions, bends, and junctions, changes in cross-section area, entrance, and exit. In order to compare with frictional airway resistance, that additional resistance can be considered as an additional straight airway which would compensate for the shock loss. The resistance due to shock loss can be expressed as:

\[ R_s = k_{1,2}L_{eq}\frac{\rho}{A^{\frac{1}{2}}}\left(Ns^2/m^8\right) \]  

2.4 Optimization on Airway Resistance

The annual operating cost is expressed in Equation 2.5.

Accordingly, the annual operating cost is proportional to the total ventilation network resistance, \( R_t \):

\[ EAC_O \propto R_t \]

If the total network resistance can be reduced, then the operating cost will be reduced. Therefore, it is important to review the effects of the variable components in Equation 2.5.

2.4.1 Drift Size Optimization

Resistance is proportional to:

\[ R_t \propto \frac{per}{A^3} \]

It can be further written as:

\[ R_t \propto \frac{per}{A^{1/2}A^{5/2}} \]
\( \frac{\text{per}}{A^{1/2}} \) is called shape factor (SF) which is a constant that only varies with cross-section shape. The circular shape cross-section has the minimum possible shape factor:

\[
SF_{\text{circle}} = \frac{\text{per}}{A^{1/2}} = \frac{\pi d}{\sqrt{\pi/4d}} = 3.5449
\]

Where \( d = \text{airway diameter} \)

All other airways have a greater shape factor than this value. The relative shape factor can be used for comparison which is calculated by dividing the shape factors by 3.5449. Table 2.1 shows the relative shape factors comparison.

Table 2.1 Relative shape factors comparison (McPherson, 1993)

<table>
<thead>
<tr>
<th>Shape of Airway</th>
<th>Relative Shape Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular</td>
<td>1.00</td>
</tr>
<tr>
<td>Arched, upright legs</td>
<td>1.08</td>
</tr>
<tr>
<td>Arched, splayed legs</td>
<td>1.09</td>
</tr>
<tr>
<td>Square</td>
<td>1.13</td>
</tr>
<tr>
<td>Rectangular</td>
<td></td>
</tr>
<tr>
<td>width:height = 1.5:1</td>
<td>1.15</td>
</tr>
<tr>
<td>2:1</td>
<td>1.20</td>
</tr>
<tr>
<td>3:1</td>
<td>1.30</td>
</tr>
<tr>
<td>4:1</td>
<td>1.41</td>
</tr>
</tbody>
</table>

Since the circle has the lowest shape factor compared to other shapes, circular airway will be used for further study on airway size optimization. Circular airway resistance is proportional to:

\[
R_t \propto \frac{1}{d^5}
\]
The expression for $R_t$ shows that the size of the airway has a dominant effect on the airway resistance. Consequently, the operating cost is also dominated by airway size. With the airway size increasing, the main ventilation airway and infrastructure costs will significantly increase, which can be as high as 20-30% of the total capital cost (Loomis, Ramirez, & Tarigan, 2014). To balance the operating cost and capital cost, the optimum drift size must be evaluated.

The operating cost can be expressed by the first part of Equation 2.1. Capital cost consists of two parts – fixed costs and variable costs. The variable costs can be expressed as functions of the length and shaft diameter. Total capital cost can be written as (McPherson, 1993):

$$P_c = C_f + aL + bV (\$) \tag{2.11}$$

Where $C_f = fixed costs$

$$a = airway\ length\ cost\ efficiency\ (\$/m)$$

$$b = airway\ size\ cost\ efficiency\ (\$/m^3)$$

$$V = volume\ excavated, A * L (m^3)$$

Then with equation 2.2, $EAC_c$ can be calculated.

Operating cost is shown in equation (1) the first part

$$EAC_O = \left[ \frac{k^{\star}L^{\perp}}{A^3} \ast Q^3 \ast \frac{e}{1000} \ast 24 \ast 365 \right] \tag{2.12}$$

After all required parameters are evaluated for Equation 2.11 and 2.12, EAC for capital cost, operating cost, and total cost from Equation 2.1, 2.2 and 2.3 can be established. Then, these three functions can be plotted in one figure to find the optimal airway size. For example, Case A is illustrated in Figure 2.1 below:
In Case A, we can see that 4.5m is the best shaft diameter size which has the lowest total cost. With the excavation cost varying, operating cost curve would not change while capital cost would be different. Case B has high excavation cost compared to the case A.
For Case B, the minimum total cost achieved were the shaft diameter was equal to 3.7m. Depending on variations in the excavation cost, electricity cost and interest rate, the curves could be different. Therefore, it is important to collect all required parameters and build the function based on local situation. With the limitation on maximum air velocity and the requirement on minimum air quantity, seeking an optimal total cost result may require additional drifts. A large scale underground mine case, with peak production of 100,000 tonnes per day requiring 3,000 m³/s of intake fresh, is studied in the project “Optimizing Ventilation Drift Size as Capital Infrastructure, a Current Perspective” (Loomis, Ramirez, & Tarigan, 2014). The results are shown in Figure 2.3 below:
The overall lowest cost was achieved using four large cross-section drifts in this example.

In some cases, especially in large scale underground mining, the development may involve several stages. Case studies are published regarding these scenarios by Loomis, Ramirez and Tarigan at 2014. In another case study, five developing methods were compared regarding cost, shown in Figure 2.4 below.
The optimization example in Figure 2.4 indicated that approximate 10% overall ventilation cost may be saved. However, the authors pointed out that other factors, such as geology, and working efficiency should also be considered when comparing different designs and mining methods.

2.4.2 Ventilation System Optimization

Duct Friction Factor

Ventilation cost must also be considered in auxiliary systems in air ducts.

Resistance is proportional to:

\[ R_t \propto k \]

That means if we can reduce the friction factor from 0.0037 (typical steel duct) to 0.002 (smooth plastic duct), we can reduce 45% of the airway frictional resistance, which is
approximately 45% reduction on power usage (Mulder, Fourie, & Stanton, 2014). With balancing the operating cost and capital cost, optimal duct material should be selected.

**Booster Fan and Regulator**

Booster Fan:

It is installed underground and used to divert air in a specific location and provide sufficient pressure. It is necessary to control airflow distribution. The advantage of using booster fans is that it would not increase network resistance. The disadvantage is the high capital cost and maintenance cost.

Regulator:

It is also used for network airflow distribution. It varies the airflow by artificially increasing or decreasing the resistance of an airway. The advantage of using regulators is the low capital cost and being easy to install. Its disadvantage is that it creates additional resistance and consequently increases operating cost. There is an example in the study “Safety, Health and Ventilation Cost Benefit Optimization with Simulation and Control” – National Institute for Occupational Safety and Health (NIOSH) project. Figure 2.5 shows the basic information about this simple ventilation network: \( q_1 = 85.64 \), \( q_2 = 236.02 \), \( q_3 = 38.63 \). If a minimum air demand is defined in all three branches as \( q_D = 120 \text{ kg/s} \). The minimum airflow is not satisfied in two out of three branches. Thus, \( q_1 \) and \( q_3 \) need to be increased to satisfy the minimum requirement.
In Case A, it is assumed that no air regulators or booster fans are installed. Only the main fan can be adjusted to meet the minimum air demand. The solution shows that the system will be significantly over-ventilated in branches 1 and 2 as shown in Figure 2.6 below. The changes in pressure, air flow rates, and power are shown in Table 2.2.
Figure 2.6 Air Flow Distribution for Case A (Danko & Bahrami, 2014)

Table 2.2 Pressure Change, Airflow Rates, and Power for Case A

<table>
<thead>
<tr>
<th>CASE2</th>
<th>BASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch</td>
<td>Pressure Change (Pa)</td>
</tr>
<tr>
<td>Main fan</td>
<td>19315.2</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Power (W)</td>
<td></td>
</tr>
<tr>
<td>Main Fan</td>
<td>18026894.8</td>
</tr>
</tbody>
</table>

In Case B, air regulators are used together with the adjustment of the main fan.

The airflow can be balanced in this case, eliminating over-ventilation. The results are shown in Figure 2.7 and Table 2.3.
Figure 2.7 Air Flow Distribution for Case B (Danko & Bahrami, 2014)

Table 2.3 Pressure Change, Airflow Rates, and Power for Case B

<table>
<thead>
<tr>
<th>Case 3</th>
<th>Doors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch</td>
<td>Pressure Change (Pa)</td>
</tr>
<tr>
<td>Main fan</td>
<td>5080.84</td>
</tr>
<tr>
<td>Door1</td>
<td>2939.3</td>
</tr>
<tr>
<td>Door2</td>
<td>3587.11</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Power (W)</td>
<td></td>
</tr>
<tr>
<td>Main Fan</td>
<td>1525564.5</td>
</tr>
</tbody>
</table>
In Case C, the combined usage of booster fans and adjusted the main fan is demonstrated to balance the air distribution and eliminate over-ventilation. The results are shown in Figure 2.8 and Table 2.4.

Figure 2.8 Air Flow Distribution for Case C (Danko & Bahrami, 2014)
Table 2.4 Pressure Change, Airflow Rates, and Power for Case C

<table>
<thead>
<tr>
<th>Branch</th>
<th>Pressure Change (Pa)</th>
<th>Flow Rate (kg/s)</th>
<th>Power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>fan</td>
<td>1545.65</td>
<td>360</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>635.7</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>120.8</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3505.63</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>Fan</td>
<td>463695</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>63570</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>350563</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>877828</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The Case C has the lowest operating cost, where it has 0.88 MW power consumption. The Case B with regulators has 1.52 MW power consumption, but much better than the Case A with 18 MW power consumption without having booster fan and regulators. It is shown that using booster fans for airflow control would have a lower cost on operation. However, the investigators did not compare the capital cost spending on booster fan or regulator installation.

2.5 Optimization on Air Quantity

John Marks said in his acceptance of the Hartman Award in 2008, “I guess that without the occasional complaint, your mine is probably over-ventilated” (Marks, 2008). To reduce over-ventilation is critical to cost and should be part of our mine ventilation optimization. Since operating cost is proportional to $Q^3$, if the airflow can be reduced by 20%, fan power consumption can be reduced by a factor of $0.8^3 = 0.51$ that is, by 51%.
In the air quantity optimization process, first current airflow distribution, $Q_{\text{air}}^M$, has to be known. Second, the minimum airflow demand $Q_{\text{air}}^D$ for each branch needs to be determined. Then, a tool and an algorithm is needed to calculate the optimal result for air distribution. The target to minimize is $\sum (Q_{\text{air}}^M - Q_{\text{air}}^D)$, with the condition of $Q_{\text{air}}^M \geq Q_{\text{air}}^D$. For all $i=1 \ldots N$ branches.

2.6 Air Demand Determination

As mine keeps advancing, the total and particular airflow demands vary due to the changes on mine working schedule and mine layouts. To find out proper real-time air demand on critical branches becomes essential for optimization procedure.

The procedure to determine the minimum airflow demand is essential but very difficult. Various factors needed to be considered. These factors include contaminate gases, heat, dust, and DPM, as well as various contaminate sources such as people, mining equipment, hot rock, and strata. The demand for each person is $0.01 \ m^3/s$ which is negligible compared to the air demands from other sources. Thus, it is only necessary to compare the contaminate sources of equipment, mined ore and the strata.
Generally, contaminate gases derive from two sources. Firstly, strata gases have been produced over geological time and can escape into airway. Secondly, contaminants generated from diesel engines and toxic gases resulting from explosives spread into ventilation system. The current ventilation design criteria are based on the diesel equipment emission. In the United States, threshold limit values (TLV) are based on the recommendations from the American Conference of Governmental Industrial Hygienists (ACGIH) and the National Institute of Health and Safety (NIOSH). Some of time-weighted average (TWA) in which workers may be exposed over an 8-hours shift are shown as follow:
• Mine air shall not contain less than 19% oxygen
• Typical TLV, 0.5% (5,000ppm) CO₂/8hrs
• Typical TLV, 0.0035% (35ppm) CO/8hrs
• Typical TLV for NO: 0.0025% (25ppm) NO/8hrs
• Typical TLV for NO₂: 0.0003% (3ppm) NO₂/8hrs
• Typical TLV: 0.0002% (2ppm) SO₂/8hrs (McPherson, 1993)

Methane is not toxic, but it can be extremely explosive when it properly mixes with air. The explosive range of methane in the air is from 5% to 15%.

Calculation of airflow requirement for gas concentration:

\[ Q = \frac{100E_g}{C_g} \]

Where \( Q \) = required airflow \( \left( \frac{m^3}{s} \right) \)

\( E_g \) = gas emission rate \( \left( \frac{m^3}{s} \right) \)

\( C_g \) = gas concentration (%)  

(McPherson, 1993)

Typically, metal mines calculate airflow requirement from the dilution of gases from diesel engine. However, other mines (such as coal mine) with serious gas issues, must consider gas emission rates in the calculation of air demand.

**DPM**

It is necessary to apply a ventilation rate for how much air flow needed to dilute particulate contaminant emissions that are generated by diesel engines. This is usually done by engine tests which calculate the airflow requirement to dilute DPM. But it widely varies in different mines and different countries. From subsurface ventilation and
climate control (McPherson, 1993), it suggests that many ventilation planners employ 6 to 8 \( m^3/s \) of airflow over machine for each 100 kW of rated diesel power, all equipment being cumulative in any one air split. The Mine Safety and Health Administration (MSHA) also calculated a “Particulate Index” (PI) for each approved engine, based on a weighted average of the 8-mode tests. PI is to dilute the particulate emissions of an engine to 1 mg/m\(^3\) which is much higher than 0.16 mg/m\(^3\) MSHA SWA PEL for DPM. Also, MSHA PI ventilation rate is just for informational purposes but not an enforced law.

(DieselNet, 2012).

Table 2.5 Historic Ventilation Rates for Approved MSHA Engines (Haney, 2012)

<table>
<thead>
<tr>
<th>EPA Tier</th>
<th>Number of Engines Tested</th>
<th>Gaseous Vent Rate ( m^3/s/kW )</th>
<th>PI ( m^3/s/kW )</th>
<th>PI X 5 ( m^3/s/kW )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non EPA Compliant</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&lt;73 kW</td>
<td>21</td>
<td>0.05±0.057</td>
<td>0.119±0.088</td>
<td>0.595±0.438</td>
</tr>
<tr>
<td>Non EPA Compliant</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&gt;73 kW</td>
<td>41</td>
<td>0.038±0.008</td>
<td>0.059±0.024</td>
<td>0.297±0.19</td>
</tr>
<tr>
<td>Tier I/II&lt;73 kW</td>
<td>73</td>
<td>0.03±0.0095*</td>
<td>0.041±0.015</td>
<td>0.206±0.076</td>
</tr>
<tr>
<td>Tier I/II&gt;73 kW</td>
<td>141</td>
<td>0.035±0.008*</td>
<td>0.012±0.01</td>
<td>0.098±0.047</td>
</tr>
<tr>
<td>Tier III&lt;73 kW</td>
<td>27</td>
<td>0.032±0.004**</td>
<td>0.028±0.015</td>
<td>0.139±0.071</td>
</tr>
<tr>
<td>Tier III&gt;73 kW</td>
<td>47</td>
<td>0.025±0.003**</td>
<td>0.025±0.009</td>
<td>0.123±0.046</td>
</tr>
<tr>
<td>Tier IV</td>
<td>2</td>
<td>0.025±0.003**</td>
<td>0.002***</td>
<td>0.01***</td>
</tr>
</tbody>
</table>

*Based on NO  **Based on \( CO_2 \)  ***Based on a PI of 0.01 gm/hp-hr
From that table we can see the highest ventilation rate for gases is 0.038 m³/s/kW, and the highest PI is 0.059 m³/s/kW.

**Heat**

There are variations in heat sources in underground ventilation systems, and the most common three are heat from auto-compression, strata heat, and heat generated by mining equipment. From subsurface ventilation and climate control (McPherson, 1993), the equation is as follows:

\[ Q = \frac{q_{12}}{\rho(S_2 - S_1)} \]  \hspace{1cm} (2.14)

Where \( \rho = \text{mean density of the air} \left( \frac{kg}{m^3} \right) \)

\( S_1 = \text{sigma heat of the air at inlet} \)

\( S_2 = \text{highest sigma heat of the air at outlet} \)

\( q_{12} = \text{total heat flux into the air} \)

(McPherson, 1993)

In order to find the real time \( S_1, S_2, \) and \( q_{12}, \) a mine dynamic ventilation-thermal-humidity (V-T-H) model is necessary to determine underground climatic conditions. Ventsim, Climsim, VUMA and MULTIFLUX can be applied in mining system in order to meet the requirements. That kind of underground thermal environment simulation is of great importance, especially for the rapidly developed modern underground mines.

Since the heat produced by diesel equipment is different from other sources, part of this heat is latent heat. It is important to discuss how to determine airflow requirement from diesel equipment.

\[ Q_T = \frac{f_c C_{diesel}}{3600} \]  \hspace{1cm} (2.15)
\[ Q_l = \frac{v_{H_2O}l_{H_2O}}{3600} \]  
\[ Q_s = Q_T - Q_l \]

Where: \( Q_T = \text{total heat (kW)} \)

\[ f_c = \text{fuel consumption \( \frac{\text{litres}}{\text{hr}} \)} \]

\[ C_{\text{diesel}} = \text{heat content of diesel \( \frac{\text{kJ}}{\text{litre}} \)} \]

\[ Q_l = \text{latent heat (kW)} \]

\[ V_{H_2O} = \text{volume of water production \( \frac{\text{litres}}{\text{hr}} \)} \]

\[ l_{H_2O} = \text{latent heat of vaporisation of water \( \frac{\text{kJ}}{\text{kg}} \)} \]

\[ Q_s = \text{sensible heat (kW)} \]

By using these equations we can find total sensible heat generated by diesel equipment.

Then, required mass flow of air can be calculated from equation below

\[ m_{\text{air}} = \frac{Q_s}{\Delta T \cdot C_p} \]

Where \( \Delta T = \text{temperature change (K)} \)

\[ Q_s = \text{sensible heat (kW)} \]

\[ m_{\text{air}} = \text{mass flow rate of air \( \frac{\text{kg}}{\text{s}} \)} \]

\[ C_p = \text{specific heat of dry air \( \frac{\text{kJ}}{\text{kgK}} \)} \]

It is easy to convert mass flow of air to volume flow with air density.

Then with the equation below ventilation rate for heat generated by diesel equipment can be calculated

\[ VR = \frac{v_{\text{air}}}{p_{\text{machine}}} \]
Where $VR = \text{ventilation rate } \left( \frac{m^3}{s} \text{ per kW} \right)$

$v_{air} = \text{ventilation rate } \left( \frac{m^3}{s} \right)$

$P_{machine} = \text{machine power (kW)}$ (Stinnette & De Souza, 2013)

**Dust**

Dust is one of the serious and worldwide issues for underground mining which can lead to silicosis of lungs, pneumoconiosis of lungs, and cancer. Up to date, a number of approaches (such as water infusion, water sprays, and air filtration systems) have been implemented to control mineral dust. An equation is given for determination of dust particles (less than 5um diameter) dilution by airflow.

$$Q = \frac{E_d}{C_d} \times \frac{P}{3600}$$

(2.20)

Where $E_d = \text{the emission rate of respirable dust } \left( \frac{mg}{\text{tonne}} \right)$

$P = \text{rate of mineral production } \left( \frac{\text{tonnes}}{h} \right)$

$C_d = \text{allowable increase in the concentration of respirable dust } \left( \frac{mg}{m^3} \right)$

However, larger particles (particles with diameter greater than 10um) cannot be diluted proportionally to the increased airflow. Dust concentration-air velocity relationship is shown in Figure 2.10:
Figure 2.10 Dust Concentrations at Various Air Velocities (McPherson, 1993)

Based on Figure 2.10’s data, 2.2 m/s is the optimal air velocity for producing lowest dust concentration. However, as previously mentioned that there are several dust controlling methods which may be used simultaneously. The 2.2 m/s airflow may not be enough for clean DPS and other contaminated gases. As a result, velocity ranging from 1 to 4 m/s may be used for recommendation air velocity (McPherson, 1993).

**Current Ventilation Design Criteria**

Currently, the air requirement of ventilation design is based on the diesel exhaust dilution. By considering and comparing all ventilation rate from gas, DPM, heat, and dust, the most widely used ventilation rate is $0.063\frac{m^3}{s}/kW$. But, as mentioned previously,
ventilation rate varies in different places and different mines. Table 2.6 shows various ventilation rates in different countries.

Table 2.6 Ventilation Rate in Different Countries (Gangal, 2012)

<table>
<thead>
<tr>
<th>Location</th>
<th>Statutory Ventilation Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.06 m³/s per kW minimum</td>
</tr>
<tr>
<td>Canada</td>
<td>varies by province from 0.045-0.092 m/s per kW minimum-most commonly 0.06 m/s per kW</td>
</tr>
<tr>
<td>Chile</td>
<td>0.063 m³/s per kW</td>
</tr>
<tr>
<td>China</td>
<td>0.067 m³/s per kW</td>
</tr>
<tr>
<td>South Africa</td>
<td>0.063 minimum (based on “best practice”)</td>
</tr>
<tr>
<td>USA</td>
<td>Based on MSHA certificate</td>
</tr>
</tbody>
</table>

For most countries, the ventilation rate used is around 0.063 m³/s/kW.

Figure 2.11 shows that at different types of mines the ventilation rate may have significant imparity.

![Average Quantity per Mine Diesel Power](image)

Figure 2.11 Ventilation Rates for Various Types of Mines (Wallace, 2012)
Table 2.6 and Figure 2.11 indicate that even in the same country but in different mines, different ventilation rates are used.

The U.S Environmental Protection Agency (EPA) enforces all non-road diesel-powered vehicles (as mining equipment) to meet “Tier IV” emissions standards by 2014. With the new standards, by the time the rule is fully implemented in 2014, toxic emissions of non-road diesel engines should be reduced by approximately 90%. It may lead to enormous change on mine ventilation rate. A direct engine test was done by Natural Resources Canada (NRC) on mining equipment LHD. Results show as in Table 2.7 below:

Table 2.7 Comparison of Methods for Calculating Required LHD Airflow

<table>
<thead>
<tr>
<th>Method of determining airflow</th>
<th>Total Airflow</th>
<th>Ventilation Rate</th>
<th>% of Greatest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Engine Testing</td>
<td>5.9</td>
<td>0.21</td>
<td>18%</td>
</tr>
<tr>
<td>Empirical Derivation</td>
<td>18</td>
<td>0.63</td>
<td>55%</td>
</tr>
<tr>
<td>Proposed Method: Gases</td>
<td>8</td>
<td>0.28</td>
<td>25%</td>
</tr>
<tr>
<td>Proposed Method: DPM</td>
<td>3.1</td>
<td>0.11</td>
<td>10%</td>
</tr>
<tr>
<td>Proposed Method: Heat</td>
<td>21.4</td>
<td>0.75</td>
<td>66%</td>
</tr>
<tr>
<td>Proposed Method: Dust</td>
<td>32.5</td>
<td>N/A</td>
<td>100%</td>
</tr>
</tbody>
</table>

(Stinnette & De Souza, 2013)

Even though the Tier IV engine harmful gases emission is reduced by 90%, due to other parameters like heat and dust which take precedence, the overall ventilation rate does not significantly change. Therefore, in the rest of the study, the 0.063 m³/s per kW will be used as default ventilation rate. However, there are not only minimum airflow...
requirement, but also maximum airflow limitation for some specific mines such as coal mines (which need more consideration on gases) and hot deep mines (which needs a geothermal model to analyse the air-demand on heat). Table 2.8 below shows the recommended maximum flow velocity.

Table 2.8 Maximum velocities

<table>
<thead>
<tr>
<th>Airway</th>
<th>Maximum Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working Faces</td>
<td>4</td>
</tr>
<tr>
<td>Conveyor Drifts</td>
<td>5</td>
</tr>
<tr>
<td>Main Haulage Routes</td>
<td>5</td>
</tr>
<tr>
<td>Main Ventilation Drifts</td>
<td>8</td>
</tr>
<tr>
<td>Hoisting Shafts</td>
<td>10</td>
</tr>
<tr>
<td>Ventilation Shafts</td>
<td>20</td>
</tr>
</tbody>
</table>

(McPherson, 1993)

2.7 Fan Performance Management

After air demand is determined, the control of the fan must be considered. One way to achieve this is to control the fan speed (RPM) through Variable Speed Drives [VSDs]. VSDs have gained wide acceptance and in some cases have been applied to control main mine fans (du Plessis & Marx, Main fan energy management, 2008). Experiment results also show that changing the fan RPM would not influence fan efficiency. Another method is to use Inlet Guide Vanes [IGV] control, which involves introducing specially designed adjustable vanes to the air stream entering the fan inlet, to generate a swirl of air in the direction of the impeller rotation, and then the effect and work done by the impeller is lowered. IGV control is very effective and can be easily applied on existing fans. However, for this study, we will use VSDs for two reasons. The primary one is that IGV cannot increase fan performance to exceed full load, while VSDs
can. Another reason is that using IGV will drop fan efficiency when reducing fan duty.

Figure 2.12 shows fan efficiency reduction by IGV.

![Figure 2.12 IGV Angle Effect on Efficiency](image)

Figure 2.12 IGV Angle Effect on Efficiency

(du Plessis & Marx, Main fan energy management, 2008)

### 2.8 Operational Research

Operational research is a sub-field of mathematics and it is a discipline that deals with advanced analytical method to find the mathematical optimal solution; under certain constraints, how to make the best decisions to arrive optimal or near-optimal solutions. It is widely used in engineering design, program management, scientific experiments, military science, and daily life. The objective of operational research is to help determine the maximum (of profit, performance) or minimum (of loss, risk, or cost) results.
Mathematical Modeling

Operational research model includes two parts, objective function and constraints.

(1) Objective Function

Objective function is the function to express and calculate the target. It can be chosen as the minimum ventilation power in ventilation optimization.

(2) Constrains

A standard form of operational research problem is as follows:

\[
\begin{align*}
\min f(X) \\
\text{s.t. } & g_i(X) = 0, \ i \in E \\
& g_i(X) \geq 0, \ i \in I
\end{align*}
\]

Where \( X = (x_1, x_2, ..., x_n)^T \) is decision variable vector; \( f(X) \) is objective function; \( g_i(X) \) is constrains.

If objective function \( f(X) \) and all constrains \( g_i(X) \) are linear functions, then the problem is linear programming (LP). If the objective function \( f(X) \) is quadratic function, all constrains \( g_i(X) \) are linear functions, then we call it quadratic programming (QP); If \( f(X) \) is not linear or quadratic function, or not all \( g_i(X) \) are linear function, then it is a nonlinear programming.

**Operational Research Problem Solving Methods**

Programming problem not always exists in optimal solutions, but may exist in multiple optimal solutions. Depending on different mathematic models, different solving methods will be used.

(1) Linear Programming: Simplex Method

(2) Quadratic Programming: Paired Method or Piecewise-linear Method
(3) Nonlinear Programming: Generalized Reduced Gradient Method or Genetic Algorithm

2.8.1 Jacobian Sensitivity Matrix

The Jacobian generalizes the gradient of a scalar-valued function of multiple variables, which itself generalizes the derivative of a scalar-valued function of a single variable. Suppose we have $n$ functions and $m$ variables $F_1(x_1, \ldots, x_m), \ldots, F_n(x_1, \ldots, x_m)$, then we have relationship below:

$$
\begin{bmatrix}
\frac{\partial F_1}{\partial x_1} \\
\vdots \\
\frac{\partial F_n}{\partial x_1}
\end{bmatrix} = [J] \cdot 
\begin{bmatrix}
\frac{\partial x_1}{\partial x} \\
\vdots \\
\frac{\partial x_m}{\partial x}
\end{bmatrix}
$$

From that relationship, the Jacobian matrix can be expressed as:

$$
J = 
\begin{bmatrix}
\frac{\partial F_1}{\partial x_1} & \ldots & \frac{\partial F_1}{\partial x_m} \\
\vdots & \ddots & \vdots \\
\frac{\partial F_n}{\partial x_1} & \ldots & \frac{\partial F_n}{\partial x_m}
\end{bmatrix}
$$

2.8.2 Linear Programming (LP)

In linear programming, constrains define a feasible region. The optimal task is to find out at which point the given objective function can achieve its max/min value.

A standard form of LP:

$$
\begin{align*}
& \text{min } c_1 x_1 + \cdots + c_n x_n \\
& \text{s. t. } \\
& a_{11} x_1 + \cdots + a_{1n} x_n = b_1 \\
& \vdots \\
& a_{m1} x_1 + \cdots + a_{mn} x_n = b_m \\
& x_1, \ldots, x_n \geq 0
\end{align*}
$$

Or in matrix format
\[
\min C^T x \\
\text{s.t.} \\
Ax = b \\
x_1, \ldots, x_n \geq 0
\]

Where \( \min C^T x \) is objective function, \( Ax = b \) is constrain. All variables \( x_1, \ldots, x_n \) are non-negative.

2.8.3 Least Squares Fitting (LSQ)

"Least squares" means that the overall solution minimizes the sum of the squares of the errors made in the results of every single equation. It can be expressed as equation below:

\[
\text{Min} \sum_{i=1}^{n} (x'_i - x_i)^2
\]

Where \( x'_i = \text{Fitting result} \)

\( x_i = \text{actual value} \)

An LSQ curve-fitting to scattered data is illustrated in Figure 2.13.

![Figure 2.13 General picture of an LSQ method](image)
2.9 Mine Air Distribution Optimization

Operational research can be applied in ventilation network design to solve problems, such as, how to satisfy all required airflow with lowest capital and operational cost on ventilation system development, and how to schedule ventilation developing procedure to maximize labor utilization while minimize expenditure.

For an existing mine ventilation system, operational research methods can be used to operate the fans optimally. The relationship between airflow distribution and fan RPM can be expressed using a Jacobian matrix (Danko & Bahrami, 2014). Then, by using LP or Least Squares Fitting (LSQ) method, an optimal setting on fan can be determined. This optimal fan setting can be achieved by using VSDs.

Figure 2.14 Overview of Ventilation Optimization with Network Model (Danko & Bahrami, 2014)
Jacobian matrix can be expressed as below:

\[
\begin{bmatrix}
    dQ_1 \\
    dQ_2 \\
    \vdots \\
    dQ_n
\end{bmatrix} = \begin{bmatrix}
    a_{11} & \cdots & a_{1(m)} \\
    \vdots & \ddots & \vdots \\
    a_{n1} & \cdots & a_{n(m)}
\end{bmatrix} \times \begin{bmatrix}
    dR_1 \\
    \vdots \\
    dR_m
\end{bmatrix}
\]

Where \( dQ = \text{branch airflow change} \)

\[
dR = \text{fan RPM change}
\]

\[
\begin{bmatrix}
    a_{11} & \cdots & a_{1(m)} \\
    \vdots & \ddots & \vdots \\
    a_{n1} & \cdots & a_{n(m)}
\end{bmatrix} = \text{Jacobian sensitivity matrix}
\]

Jacobian matrix can be determined by find \( dQ \) and \( dR \). In order to apply it to LP, the condition equations and the objective function have to be determined. Because operating cost is \( P = RQ^3 \), and with limited fan RPM change the network resistance would not change substantially; the total P is only related to \( Q^3 \). Minimizing the total intake air Q can be used as the optimization target, which is the objective function. The mine ventilation problem can be described as follows:

\[
\text{min } c_1x_1 + \cdots + c_nx_n
\]

s. t.

\[
a_{11}x_1 + \cdots + a_{1n}x_n \geq b_1 \\
\vdots \\
a_{m1}x_1 + \cdots + a_{mn}x_n \geq b_m
\]

\[
lb \leq x_1, \ldots, x_n \leq ub
\]

Where \( x = \text{the speed ratio change on fan's RPM} \)

\[
A = \text{the sensitivity matrix (Jacobian)}
\]

\[
b = \text{the minimum airflow change requirement, constrain}
\]
\[ \text{lb} = \text{the lower boundary for fans RPM change} \]
\[ \text{ub} = \text{the upper boundary for fans RPM change} \]
\[ C^T = \text{coefficient of total intake airflow change by fan RPM change} \]

To solve that problem, three steps will be performed:

1. Investigate all branches, find out the required minimum airflow on every branch \((Q_D)\), then identify the current existing airflow distribution on all branches \((Q_M)\).
   \[ b = Q_D - Q_M, \text{positive b means unsatisfied branch, negative b means over-} \]
   \[ \text{ventilated branch.} \]

2. A mine ventilation model to determine the Jacobian. MULTIFLUX is an accurate
   \[ \text{mine ventilation modeling software which can create and update the Jacobian} \]
   \[ \text{directly.} \]

3. The LP Simplex method will be used to determine the optimal solution.

   Another way to solve for optimum is to use a weighted LSQ method (Danko &
   \[ \text{Bahrami, 2014). For LSQ process, first two steps are the same as using LP method. The} \]
   \[ \text{third step is to compare the model result } Q_M \text{ and minimal air demand } Q_D \text{ to find the best} \]
   \[ \text{matching result (fan setting).} \]
   \[ \text{Min } \sum_{i=1}^{n} (Q'_i - Q_i)^2 \]  \[ 2.24 \]
   \[ \text{Where } Q'_i = \text{Fitting result} \]
   \[ Q_i = \text{minimal air demand} \]

   In real-world practice, some air demand points are hard to achieve. Therefore, a
   \[ \text{weight factor is added to LSQ method. Equation can be rewritten as below:} \]
   \[ \text{Min } \sum_{i=1}^{n} a_i(Q'_i - Q_i)^2 \]  \[ 2.25 \]
Where \( a_i = \text{weight factor for branch } i \)

### 2.10 Mine Ventilation Optimization Concept

For most of the mine optimization studies, the investigators did not have a general tool and algorithm to find the optimal ventilation situation. Most of the optimization study cases only compared several scenarios and then picked the best one among those scenarios. It might be the local optimal result, but may not be a global optimal solution. For example, in the study “Mine Ventilation System Optimization Considering Optimal Energy, Health and Safety” (J.J.L. dsu Plessis & W.M. Marx, 2014), the investigators provided 7 scenarios for the ventilation and booster fan optimization. Then it calculated the fan input power and airflow reduction for each scenario, and just picked the best one out of the seven. It might be just an improvement rather than optimization. Another case is the study “Energy Saving Optimization of Auxiliary Ventilation Systems in Hard Rock Development Headings” (G.E. Mulder, J.W. Fourie & D.Stanton, 2014). The investigators achieved some savings on new design. However, in their future work, they claimed that “an algorithm to optimize the balance between VSD speed and regulator position would further enhance the energy savings”. This claim is consistent with the methodology employed in this study.

Another major consideration in mine ventilation optimization is to maximize the usage of the existing infrastructure (Wallace, 2006). That is also our major consideration for this study and the biggest advantage from using Jacobian matrix tool. The optimization concept followed in the thesis is shown in Figure 2.15.
### Figure 2.15 Optimization concept (Danko & Bahrami, 2014)

**Analysis**

Analyze Results – Look for critical air flow requirements for safety and health:

- Air flow requirements are calculated by the presence of vehicles and persons in particular locations as well as by limits for harmful parameters.
- Air flow requirements will be satisfied in optimized solution.
Analyze Results – Look for high cost elements:

- Check high power loss branches and fans efficiencies.
- Consider replacing high efficiency fans.
- Reducing high shock loss branches for improvements.
- Consider to reduce high friction resistance.
- Consider whether airway diameter can be increased with lower total cost.

Analyze Results – Look for over-ventilation.

- Compare air flow requirements with current air flow distribution, and find the over-ventilated branches.
- Optimal solution will be given with best reduce on over-ventilation while all air demand be satisfied.

Analyze Air Distribution System

- Analyze reasons for low air flow areas.
- Consider about ventilation configuration.
- Can booster fans be used instead of air regulators?

Variation

- Determine increase in flow rates where needed
- Change fan curves in the model
- Follow best practice and change resistance in the model
- Determine decrease in air flow where needed
- Change configuration in vent model
Optimum Modification with Tools and Results

- Build Jacobian sensitive relationship between changes on fans’ RPM and changes on certain branches’ air flow.
- Build objective function to relate fan’s RPM changing with total input fan power.
- Use the Jacobian matrix to find the optimal solution which can satisfy all air demand with minimum cost.

2.11 Software Support

2.11.1 VnetPC

VnetPC is commonly used software for ventilation operation. It uses volumetric flow balance but does not include air density. Due to the variations of ventilation air flows and pressures, the VnetPC model will go off calibration since everything is density dependent. Therefore, it cannot be used for air distribution optimization. Consequently, VnetPC is used only for cases on a booster fan and regulator comparison in this study.

2.11.2 VentSim

VentSim uses mass balance, including total pressure balance and natural ventilation. Its unique feature as a ventilation drift size optimization tool can be used to balance capital cost and operating cost and to provide recommended airway size for entire ventilation network. In this study, VentSim is used as a tool for drift size optimization.

2.11.3 MULTIFLUX

MULTIFLUX is a qualified thermal, hydrologic, and airflow model and software, developed at University of Nevada, Reno by Dr. George Danko. It can be employed to
solve the flow of heat, moisture, and air in and around an underground opening.

MULTIFLUX can correctly reflect the airflow and pressure change with the changes on the ventilation setting such as fan RPM adjustment and closing-opening regulators. Also, MULTIFLUX can generate the flow-fan RPM Jacobian matrix. It is also easy to apply LP or LSQ method with MULTIFLUX. For the rest of this study, it will be used for air distribution optimization calculation and modeling.
3 CHAPTER III. Booster Fan and Regulator Optimization with VnetPC

3.1 Introduction

Optimizing an existing mine ventilation system can be operated in three aspects: reducing network resistance, decreasing redundant airflow, and improving total efficiency. The study of NIOSH project “Safety, Health and Ventilation Cost Benefit Optimization with Simulation and Control” final report (Danko, 2014) shows that 50% of operating cost could be saved by using booster fan compared with by using regulators. In this study, a small mine model (Example 1) has been used to optimize its total ventilation cost by adjusting the fan performance and using regulators and booster fans with VnetPC software. We hypothesize that the original airflow would be the minimum air demand and could not be further reduced. Also, we assume that it would be unnecessary to replace the fan with a high efficiency fan. The fan performance will be controlled by adjusting fan RPM.

3.2 Background

This mine model has 60 branches, 52 junctions, 2 fans, and 4 regulators. Regulator 1, 2, 3 and 4 are installed on level 1, 2, 3, 4 to control the air distribution and to ensure all the levels to meet their minimum air demand. Air demand is shown in Table 3.1. Mine layout is shown in Figure 3.1. Parameters used for EAC calculation are shown in Table 3.2. Six scenarios will be discussed. The original mine model is used as Scenario 1. Then all the regulators will be removed from the original model. Scenario 0 is the case without using any regulators. Scenario 3 is optimization on fan RPM and regulators’ resistance. In Scenario 4, both of the regulators and booster fans are used. Scenario 5 only use booster fans. Scenario 6 lets the booster fans share more duty with main fans. A
comparison of the total cost among the 6 scenarios will be made, and optimization results will be summarized.

Figure 3.1 2D Layout of Mine Example 1 – Scenario 1

Table 3.1 Air Demands on Mine Example 1 – Scenario 1

<table>
<thead>
<tr>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q demand (m³/s)</td>
<td>35.4</td>
<td>23.6</td>
<td>23.6</td>
</tr>
</tbody>
</table>

Table 3.2 Parameters for Cost Calculation

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Working days</td>
<td>365</td>
</tr>
<tr>
<td>Working hours per day</td>
<td>24</td>
</tr>
<tr>
<td>Total efficiency</td>
<td>65%</td>
</tr>
<tr>
<td>Cost of power ($/kWhr)</td>
<td>0.065</td>
</tr>
</tbody>
</table>
3.3 Optimization Scenarios

3.3.1 Scenario 1 – Original Mine Model

This scenario is the one currently existing. Each of the level 1, 2, 3 and 4 has been installed with one regulator. There is no airflow redundancy, as all airflow just meets its minimum air demand. Fans’ working points are measured as shown in Table 3.3 below. Levels’ current air flow and regulators’ resistance are shown in Table 3.4 below. Mine layout is shown in Figure 3.1.

Table 3.3 Fans’ Working Point – Scenario 1

<table>
<thead>
<tr>
<th></th>
<th>Main fan 1</th>
<th>Main fan 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (kPa)</td>
<td>1.633</td>
<td>1.495</td>
</tr>
<tr>
<td>Q (m$^3$/s)</td>
<td>102.81</td>
<td>22.64</td>
</tr>
</tbody>
</table>

Table 3.4 Current Flow Rate and Regulators’ Resistance

<table>
<thead>
<tr>
<th></th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q (m$^3$/s)</td>
<td>35.4</td>
<td>23.6</td>
<td>23.6</td>
<td>35.4</td>
</tr>
<tr>
<td>regulator resistance</td>
<td>0.85015</td>
<td>1.8405</td>
<td>1.80237</td>
<td>0.79332</td>
</tr>
</tbody>
</table>

Results of Financial Assessment:

- Expected annual operating cost: US$ 176,720
3.3.2 Scenario 0 – No Regulators & No Booster Fans

This scenario is the worst scenario situation. This scenario would show how inefficient it is if no regulators or boosters are used to control air distribution. Regulators are removed from the level 1, 2, 3 and 4. The mine layout is shown in Figure 3.2. Fans’ working points are measured as shown in Table 3.5 below. Levels’ air flow and regulators’ resistance are shown in Table 3.6 below.

![Figure 3.2 2D Layout of Mine Example 1 – Scenario 0](image)

<table>
<thead>
<tr>
<th></th>
<th>Main fan 1</th>
<th>Main fan 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (kPa)</td>
<td>1.535</td>
<td>1.495</td>
</tr>
<tr>
<td>Q (m³/s)</td>
<td>110.22</td>
<td>104.15</td>
</tr>
</tbody>
</table>

Table 3.6 Current Flow Rate and Regulators’ Resistance
Results of Financial Assessment:

- **Expected annual operating cost**
  
  US$ 284,605

### 3.3.3 Scenario 2 – Lower Fan RPM and Lower Regulators’ Resistance

This scenario adjusts the fan RPM and regulators’ resistance base on the scenario 1. Both of the fans’ and regulator’s power are reduced to ensure equivalent airflow delivered to each level. Mine layout is the same as scenario 1 as shown in Figure 3.1. Fans’ working points are measured as shown in Table 3.7 below. Levels’ air flow and regulators’ resistance are shown in Table 3.8 below.

#### Table 3.7 Fans’ Working Point – Scenario 2

<table>
<thead>
<tr>
<th></th>
<th>Main fan 1</th>
<th>Main fan 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (kPa)</td>
<td>0.57</td>
<td>0.55</td>
</tr>
<tr>
<td>Q (m³/s)</td>
<td>64</td>
<td>58.39</td>
</tr>
</tbody>
</table>

#### Table 3.8 Current Flow Rate and Regulators’ Resistance

<table>
<thead>
<tr>
<th></th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q (m³/s)</td>
<td>35.4</td>
<td>23.6</td>
<td>23.6</td>
<td>35.4</td>
</tr>
<tr>
<td>regulator resistance</td>
<td>0.06881</td>
<td>0.08773</td>
<td>0.05075</td>
<td>0.01971</td>
</tr>
</tbody>
</table>
Results of Financial Assessment:

- Expected annual operating cost: US$ 60,088

### 3.3.4 Scenario 3 – Regulators & Booster Fans

In this scenario, improvement is made based on Scenario 2. This scenario would use both booster fans and regulators. Regulators are removed from level 3 and 4, and booster fans are installed on level 3 and 4. Mine layout is shown in Figure 3.3. Fans’ working points are measured as shown in Table 3.9 below. Levels’ air flow and regulators’ resistance are shown in Table 3.10 below.

![Figure 3.3 2D Layout of Mine Example 1 – Scenario 3](#)
Table 3.9 Fans’ Working Point – Scenario 3

<table>
<thead>
<tr>
<th></th>
<th>Main fan 1</th>
<th>Main fan 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (kPa)</td>
<td>0.534</td>
<td>0.5</td>
</tr>
<tr>
<td>Q (m³/s)</td>
<td>68.29</td>
<td>53.93</td>
</tr>
</tbody>
</table>

Table 3.10 Current Flow Rate and Regulators’ Resistance

<table>
<thead>
<tr>
<th></th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q (m³/s)</td>
<td>35.4</td>
<td>23.6</td>
<td>23.6</td>
<td>35.4</td>
</tr>
<tr>
<td>regulator resistance</td>
<td>0.03375</td>
<td>0.00854</td>
<td></td>
<td></td>
</tr>
<tr>
<td>booster fan pressure (kPa)</td>
<td></td>
<td></td>
<td>0.016</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Results of Financial Assessment:

- Expected annual operating cost US$ 56,548

3.3.5 Scenario 4 – Booster Fans Only

In this scenario, all regulators are removed, and four booster fans are installed to the four levels (one in each level). Mine layout is shown in Figure 3.4. Fans’ working points are measured as shown in Table 3.11 below. Levels’ air flow and regulators’ resistance are shown in Table 3.12 below.
Figure 3.4 2D Layout of Mine Example 1 – Scenario 4

Table 3.11 Fans’ Working Point – Scenario 4

<table>
<thead>
<tr>
<th></th>
<th>Main fan 1</th>
<th>Main fan 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (kPa)</td>
<td>0.462</td>
<td>0.45</td>
</tr>
<tr>
<td>Q (m³/s)</td>
<td>61.04</td>
<td>60.9</td>
</tr>
</tbody>
</table>

Table 3.12 Current Flow Rate and Regulators’ Resistance

<table>
<thead>
<tr>
<th></th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q (m³/s)</td>
<td>35.4</td>
<td>23.6</td>
<td>23.6</td>
<td>35.4</td>
</tr>
<tr>
<td>regulator resistance</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>booster fan pressure (kPa)</td>
<td>0.017</td>
<td>0.055</td>
<td>0.075</td>
<td>0.078</td>
</tr>
</tbody>
</table>
Results of Financial Assessment

- Expected annual operating cost US$ 54,344

### 3.3.6 Scenario 5 – Booster Fans Share More Duty

In this scenario, adjustment has been made on the fan based on Scenario 4. More duty would be put on four booster fans in Scenario 5, where we assume that the booster fans have the same efficiency as the main fans. Mine layout is shown in Figure 3.5. Fans’ working points are measured as shown table 3.13 below. Levels’ air flow and regulators’ resistance are shown in Table 3.14 below.

![Figure 3.5 2D Layout of Mine Example 1 – Scenario 5](image-url)
Table 3.13 Fans’ Working Point – Scenario 5

<table>
<thead>
<tr>
<th></th>
<th>Main fan 1</th>
<th>Main fan 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (kPa)</td>
<td>0.314</td>
<td>0.3</td>
</tr>
<tr>
<td>Q (m³/s)</td>
<td>61.13</td>
<td>60.06</td>
</tr>
</tbody>
</table>

Table 3.14 Current Flow Rate and Regulators’ Resistance

<table>
<thead>
<tr>
<th></th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q (m³/s)</td>
<td>35.4</td>
<td>23.6</td>
<td>23.6</td>
<td>35.4</td>
</tr>
<tr>
<td>regulator resistance</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>booster fan pressure (kPa)</td>
<td>0.166</td>
<td>0.203</td>
<td>0.224</td>
<td>0.227</td>
</tr>
</tbody>
</table>

Results of Financial Assessment:

- Expected annual operating cost: US$ 53,613

3.4 Study Results

Scenario 0 has high power consumption and inefficient airflow distribution, where the level 1 is seriously over-ventilated, and the level 2 and 4 have flow redundancy, and the level 3 is far away from the minimum air demand. This model not only has the highest annual total cost, but also cannot meet the design criteria.

Scenario 1 is the fundamental case, where all branches satisfy the minimum air demand and no redundancy. Also annual operating cost reduces 38%.

In Scenario 2, significant improvement has been shown in annual operating cost, and the design criteria are easy to achieve. No infrastructure change or new fan needs to
be installed. No additional capital cost needs to be spent to gain a 66% saving on the operating cost.

Scenario 3 has a combined usage of booster fans and regulators. Due to the low cost of the low power fans, it has 5.8% less saving on the annual operating cost compared to Scenario 2. But, because of using booster fans, the capital cost need to be considered.

Scenario 4 uses booster fans to replace all the regulators. It saves 3.8% annual operating cost compared to the scenario 3 in the long-term. While this scenario has more booster fans, the capital cost further increased.

For Scenario 5, we still have 1% saving on annual operating cost compared to Scenario 4. This scenario separates the fan duty, where the booster fans share duty with the main fans. Since the booster fans require more working power than the previous scenario, capital cost keeps increasing.

Scenario 5 has the lowest annual operating cost, it would be the best solution. But we need to compare this saving with the extra capital cost. Also, in the real world, we need to consider other aspects such as availability to buy or install such a booster fan. Also, variation in the cost of power and fan would influence the capital and operating cost, consequently influence the total cost. As a result, the scenario 2, 3, 4 and 5 could be considered to be the feasible optimal solutions.

Operating cost comparison is shown in Figure 3.6.
3.5 Conclusion and Recommendation

- By comparing Scenario 0 and 1, we find that it is necessary to use booster fans or regulators for air distribution control.

- By comparing Scenario 1 and Scenario 2 & 3, we can conclude that high resistance regulators need to be identified. Lower fan setting, lower regulators resistance setting, or booster fans can be used to replace the high resistance regulators while achieving the same air distribution.

- Using booster fan for air distribution would yield lower operating cost than using regulators. Capital cost and other aspects need to be considered when using booster fan to substitute regulator.

- Share or separate fan duty would be effective and efficient not only because of economic consideration, but also safety issues. In other word, instead of using one big fan, two small fans would provide same flow and pressure while performing efficiently and being safe. Firstly, two small fans together would be cheaper than
a big fan. Secondly, once one fan is off, the other one can make up its duty by increasing RPM.
4 CHAPTER IV. Drift Size Optimization with VentSim

4.1 Introduction

Previously we discussed that drift size has significant influence on mine ventilation operating cost and capital cost. Operating cost will significantly decrease with drift size increases. Also lowering the power setting on fan may result in additional saving from fan purchasing. VentSim ventilation software, with a drift size optimization tool, allows us to optimize the drift size, as well as the cost. As we correctly set the mine life, interest rate, power cost and other costing parameters, this software can provide us with optimal size for all airways by comparing the total cost in different sizes. One mine model will be tested in two scenarios, supposing one scenario has lower mining cost and higher power cost, the other has higher mining cost and lower power cost. From the difference between two scenarios, we can detect how mining cost and power cost influence drift size determination.

4.2 Background

Mine example 2 network is shown in Figure 4.1. This mine model has 832 airways, 16 fans, and 4 regulators. The total intake airflow is 300 m³/s, and the network efficiency is 57.7%.
4.3 Optimization Scenarios

4.3.1 Scenario 1 – Low Mining Cost, High Power Cost

Mining cost and power cost parameters are shown in Figure 4.2 below. Suppose airways capital cost is based on airway length and size. Horizontal airway has $800/m fixed cost plus $130/m³ variable cost base on drift size. Shaft has $1600/m fixed cost plus $180/m³ variable cost. Other vertical airways have the same fixed and variable cost as shaft. Mine life is 20 years, power cost is 0.1 $/kWh. Fan purchase cost increases as fan operating power increases. With 1kW increase on fan power, fan price will increase 1000$. Optimized result is shown in Figure 4.3 below. Table 4.1 shows the recommended size for these 32 airways. For these 32 branches, each has its own recommended size, but all need to increase their airway size. -1,452,991 saving on mine capital cost means that
an increased amount of expense on mine capital cost. 7,738,725 is the total cost saving for 20 years.

Figure 4.2 Mining and Power Cost Parameters – Scenario 1

Figure 4.3 Optimization Results – Scenario 1
Table 4.1 Recommended Airway Size – Scenario 1

<table>
<thead>
<tr>
<th>Branch ID</th>
<th>Draft diameter (m)</th>
<th>Branch ID</th>
<th>Draft diameter (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>2.4</td>
<td>171</td>
<td>1.2</td>
</tr>
<tr>
<td>2793</td>
<td>4.5</td>
<td>277</td>
<td>3</td>
</tr>
<tr>
<td>109</td>
<td>4*4</td>
<td>6811</td>
<td>3*3</td>
</tr>
<tr>
<td>106</td>
<td>3</td>
<td>4538</td>
<td>3</td>
</tr>
<tr>
<td>199</td>
<td>2.4</td>
<td>4537</td>
<td>3.6</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>179</td>
<td>2.4</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>4541</td>
<td>3.6</td>
</tr>
<tr>
<td>41</td>
<td>3</td>
<td>2699</td>
<td>2.4</td>
</tr>
<tr>
<td>4540</td>
<td>3.6</td>
<td>15</td>
<td>4.5</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>169</td>
<td>4</td>
</tr>
<tr>
<td>315</td>
<td>2.4</td>
<td>2634</td>
<td>3.6</td>
</tr>
<tr>
<td>308</td>
<td>3</td>
<td>2586</td>
<td>3.6</td>
</tr>
<tr>
<td>2784</td>
<td>1.2</td>
<td>324</td>
<td>4.5</td>
</tr>
<tr>
<td>2783</td>
<td>1.2</td>
<td>2592</td>
<td>3.6</td>
</tr>
<tr>
<td>2785</td>
<td>1.2</td>
<td>2794</td>
<td>4.5</td>
</tr>
<tr>
<td>326</td>
<td>1.2</td>
<td>168</td>
<td>4</td>
</tr>
</tbody>
</table>

One of those 32 branches is used for further analysis and comparison with Scenario 2. Branch 22’s current diameter is 2.4m and the recommended diameter is 4.1m.
Total cost saving is shown in Figure 4.4 below. Figure 4.5 shows capital cost and operating cost in different airway size. Figure 4.6 plots the airways size versus cost for capital cost and operating cost. With airway diameter increasing, capital cost will steadily increase, while operating cost will rapidly decrease.

Figure 4.4 Optimization Information for Branch 22 – Scenario 1
Figure 4.5 Mining Cost and Operating Cost Comparison in Different Airway Sizes for Branch 22 – Scenario 1

Figure 4.6 Airway Size versus Cost for Branch 22 – Scenario 1
4.3.2 Scenario 2 – High Mining Cost, Low Power Cost

Mining cost and power cost parameters are shown in Figure 4.7 below. Suppose airways capital cost is based on airway length and size. Horizontal airway has 1000$/m fixed cost plus 200$/m\(^3\) variable cost based on drift size. Shaft has 1800$/m fixed cost plus 300$/m\(^3\) variable cost. Other vertical airways have 1800$/m fixed cost plus 250$/m\(^3\) variable cost. Mine life is 20 years, and power cost is 0.07 $/kWh.

![Figure 4.7 Mining and Power Cost Parameters – Scenario 2](image)

Optimized result is shown in Figure 4.8 below. Table 4.2 shows the recommended size for these 19 airways. For these 19 branches, each has its own recommended size, but all need to increase their airway size. -1,270,776 saving on total capital cost means that
an increased amount of expense on capital cost. 3,961,875 is the total cost saving on 20 years.

Figure 4.8 Optimization Results – Scenario 2

Table 4.2 Recommended Airway Size – Scenario 2

<table>
<thead>
<tr>
<th>Branch ID</th>
<th>Draft diameter (m)</th>
<th>Draft diameter (m)</th>
<th>Branch ID</th>
<th>Draft diameter (m)</th>
<th>Draft diameter (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current</td>
<td>New</td>
<td></td>
<td>Current</td>
<td>New</td>
</tr>
<tr>
<td>22</td>
<td>2.4</td>
<td>3.5</td>
<td>4538</td>
<td>3</td>
<td>3.2</td>
</tr>
<tr>
<td>109</td>
<td>4.4</td>
<td>4.7*4.7</td>
<td>315</td>
<td>2.4</td>
<td>2.7</td>
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<td>2793</td>
<td>4.5</td>
<td>5.4</td>
<td>1</td>
<td>3</td>
<td>4.3</td>
</tr>
<tr>
<td>106</td>
<td>3</td>
<td>4.3</td>
<td>6811</td>
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<tr>
<td>199</td>
<td>2.4</td>
<td>3</td>
<td>2784</td>
<td>1.2</td>
<td>1.7</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>4.3</td>
<td>2783</td>
<td>1.2</td>
<td>1.7</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>4.3</td>
<td>2785</td>
<td>1.2</td>
<td>1.7</td>
</tr>
<tr>
<td>41</td>
<td>3</td>
<td>4.3</td>
<td>171</td>
<td>3</td>
<td>3.2</td>
</tr>
<tr>
<td>4540</td>
<td>3.6</td>
<td>3.7</td>
<td>277</td>
<td>3</td>
<td>3.1</td>
</tr>
</tbody>
</table>
The same branch is used for further analysis as in Scenario 1. Branch 22’s current diameter is 2.4m. In Scenario 2, the recommended diameter is 3.2m. Total cost saving is shown in Figure 4.9 below. Figure 4.10 shows capital cost and operating cost in different airway sizes. Figure 4.11 plots the airways size versus cost for capital cost and operating cost.

Figure 4.9 Optimization Information for Branch 22 – Scenario 2
Figure 4.10 Mining Cost and Operating Cost Comparison in Different Airway Sizes for Branch 22 – Scenario 2

Figure 4.11 Airway Size versus Cost for Branch 22 – Scenario 2
4.4 Conclusion

- By comparing Scenario 1 and Scenario 2, we can conclude that if mining cost increases and power cost reduces, the optimal drift size will be smaller.

- Drift size optimization is better done during the designing stage. For existing ventilation network, more aspects need to be considered, and the methods shown above may be insufficient to determine the optimal scheme.

- VentSim drift size optimization tool only reports that the airway size of branches needs to be increased. However, for a designing consideration, the results of both increased and decreased size optimization should be reported.
CHAPTER V. Conditional MULTIFLUX Jacobian Optimization

5.1 Introduction

Mine ventilation design is based on current peak value of airflow demand, while the airflow demand fluctuates in the real situation. This is why more and more mines have been developing their Ventilation on Demand (VOD) system. Also, with the development of infrastructure, working schedule airflow demand may increase. It is necessary to develop a tool which can monitor airflow demand fluctuation and calculate correct optimal fan and regulator settings. In this study, we focus on the optimal solution calculation. Jacobian matrix can be used as the tool, and the LP simplex method and the LSQ method are algorithms used to solve that problem. This paper only discusses airflow distribution with fan setting. Airflow with regulator setting will be investigated in the future.

Suppose a mine model has \(n\) branches and \(m\) fans. Airflow function for \(k^{th}\) branch can be written as \(f_k(x_1, x_2, ..., x_m)\), where \(x_1, ..., x_m\) variables are fan setting for \(m\) fans. Assume current mine model is the base case, then any changes of fan settings will cause changes in airflow in that \(k^{th}\) branch, where \(dQ_k = df_k = a_{11}dx_1 + a_{12}dx_2 + \cdots + a_{1m}dx_m\). This model has \(n\) branches, so we have \(n\) equations. If we replace \(x\) with \(dR\) and write all equation in a matrix format; then it is the equation 2.22:

\[
\begin{bmatrix}
    dQ_1 \\
    dQ_2 \\
    \vdots \\
    dQ_n
\end{bmatrix}
= 
\begin{bmatrix}
    a_{11} & \cdots & a_{1(m)} \\
    \vdots & \ddots & \vdots \\
    \vdots & \ddots & \ddots \\
    a_{n1} & \cdots & a_{n(m)}
\end{bmatrix}
\times
\begin{bmatrix}
    dR_1 \\
    \vdots \\
    dR_m
\end{bmatrix}
\]

First, we do the test on Jacobian to see if it can correctly reflect relationship between airflow and fan settings. Suppose we know changes on fan RPM, we will get
\[ dQ_i \ (i=1,\ldots,n) \] from equation 2.2. New \( Q = \text{original} \ Q + dQ \). Also, we would know new fan \( RPM = \text{original} \ R + dR \). Then we put this new fan RPM setting into model, and get result of airflow from this model. Then, we compare new airflow from Jacobian with new airflow from this model.

If the Jacobian optimization can be proved to correctly express airflow changes with fan RPM change, it could be applied with the LP simplex method and LSQ method in different cases.

5.2 Background

Model layout (Mine Example 3) is shown in Figure 5.1 below. This ventilation model has 233 branches, 174 junctions, 4 main surface intake fans and 3 booster fans. Three cases will be tested. Case 1 is the one with only a few critical branches having air demand. Both LP simplex and LSQ methods will be applied on it and be compared. In other two cases, each has air demand in every branch. Suppose in Case 2, it has -10% airflow change everywhere. Then, we compare the two optimal results from the LP simplex and LSQ method, verify solution with MULTIFLUX model, and analyze results, and make conclusions. In Case 3, an artificial target will be made with additional booster fan. Seven fan Jacobians will be used to achieve that target, then eight fan Jacobians (include the extra fan) will be used to find the optimal solution. Two methods and approaches will be compared, and then the conclusion will be made.
5.3 Jacobian Matrix Error Test

Five branches at different levels were picked as critical branches. Critical branch and fan locations are shown in Figure 5.2. Jacobian matrix is the foundation of these five branches. Suppose we increase Fan 1 & 2 RPM by 5%, decrease Fan 3 & 4 RPM by 10%, increase Fan 5 & 6 RPM by 20%, and decrease Fan 7 RPM by 20%. Fan 1 & 2 are in Jacobian matrix and results are shown in Table 5.1. Since Fan 1 & 2 are in parallel, it would be more efficient to run the same RPM, as well as for the fan S & 4. So, with only five variables, combining Fan 1 & 2 to be $dR_1$, Fan 3 & 4 to be $dR_2$; and Fan 5 is $dR_3$, Fan 6 is $dR_4$ and fan 7 is $dR_5$. Comparison of results between Jacobian and the model are shown in Table 5.2. All errors are shown within 1%. Also after finding an optimal result each time, result of the Jacobian will be compared with the results from the model. If the error is greater than 1%, the current setting will be used as base case to create new
Jacobian function to find out the optimal solution to original target. After 2 or 3 times iteration a correct and accurate optimal solution would be determined.

Figure 5.2 Critical Branch and Fan locations
Table 5.1 Jacobian Matrix and Calculated Results

<table>
<thead>
<tr>
<th>dQ</th>
<th>Jacobian</th>
<th>d RPM ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.98583</td>
<td>8.3235 13.16 -6.46 5.75 9.72</td>
<td>* 0.05</td>
</tr>
<tr>
<td>3.87491</td>
<td>16.4782 42.31 -0.61 13.29 -23.73</td>
<td>-0.1</td>
</tr>
<tr>
<td>-3.30288</td>
<td>-3.8576 30.76 -0.04 -0.33 -0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>-3.18798</td>
<td>-4.5595 28.9 -0.1 -1 -0.75</td>
<td>0.2</td>
</tr>
<tr>
<td>-0.50932</td>
<td>15.4208 7.9 -0.6 -3.4 -1.5482</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

Table 5.2 Comparison between Airflow from Jacobian and the Model

<table>
<thead>
<tr>
<th>Bracnch</th>
<th>From</th>
<th>To</th>
<th>Jacobian Q</th>
<th>Model Q</th>
<th>error</th>
<th>error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>182</td>
<td>162</td>
<td>156 27.838</td>
<td>27.735</td>
<td>-0.1024</td>
<td>-0.37%</td>
</tr>
<tr>
<td>2</td>
<td>209</td>
<td>404</td>
<td>400 53.301</td>
<td>53.077</td>
<td>-0.2247</td>
<td>-0.42%</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
<td>115</td>
<td>116 24.727</td>
<td>24.561</td>
<td>-0.1655</td>
<td>-0.67%</td>
</tr>
<tr>
<td>4</td>
<td>219</td>
<td>413</td>
<td>412 19.742</td>
<td>19.7</td>
<td>-0.0420</td>
<td>-0.21%</td>
</tr>
<tr>
<td>5</td>
<td>29</td>
<td>76</td>
<td>81 17.660</td>
<td>17.697</td>
<td>0.0371</td>
<td>0.21%</td>
</tr>
</tbody>
</table>

5.4 Optimization Cases

5.4.1 Jacobian Optimization Case 1 – Few Critical Demand

This case has same 5 critical branches as the error test has. There are no limitations on other branches. Our optimal objective target is to minimize operating cost. Therefore, first we leave capital cost. On the other hand, with only limited RPM changing the capital cost would not be high.
Since we only change the fan RPM, the \( k, L, \text{per}, A^3 \) and \( \eta \) would not change or would be within ignorable scale. Then, our objective function would be the minimized total mine intake airflow. \( EAC \propto Q^3 \)

First, we need to identify current status of the critical branches. Information is shown in Table 5.3 below:

### Table 5.3 Current Airflow on Critical Branches

<table>
<thead>
<tr>
<th>Branch</th>
<th>From</th>
<th>To</th>
<th>Current Q (kg/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>182</td>
<td>162</td>
<td>156</td>
<td>35.15</td>
</tr>
<tr>
<td>209</td>
<td>404</td>
<td>400</td>
<td>32.81</td>
</tr>
<tr>
<td>70</td>
<td>115</td>
<td>116</td>
<td>28.43</td>
</tr>
<tr>
<td>219</td>
<td>413</td>
<td>412</td>
<td>22.9</td>
</tr>
<tr>
<td>29</td>
<td>76</td>
<td>81</td>
<td>19.17</td>
</tr>
</tbody>
</table>

Now we need to analyze what is the new air demand on these critical branches. Results are shown in Table 5.4.
Table 5.4 Air Demand Determination

<table>
<thead>
<tr>
<th>Branch ID</th>
<th>Total kw</th>
<th>Ventilation Rate</th>
<th>Total Air demand (m³/s)</th>
<th>Total required mass flow (kg/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>182</td>
<td>453</td>
<td>0.063</td>
<td>28.54</td>
<td>34.23</td>
</tr>
<tr>
<td>209</td>
<td>453</td>
<td>0.063</td>
<td>28.54</td>
<td>34.23</td>
</tr>
<tr>
<td>70</td>
<td>332</td>
<td>0.063</td>
<td>20.92</td>
<td>25.1</td>
</tr>
<tr>
<td>219</td>
<td>242</td>
<td>0.063</td>
<td>15.25</td>
<td>18.26</td>
</tr>
<tr>
<td>29</td>
<td>242</td>
<td>0.063</td>
<td>15.25</td>
<td>18.26</td>
</tr>
</tbody>
</table>

By comparing the current status with the new design criteria, we would know how much more airflow we need to supply to certain airway or where the redundant is.

Table 5.5 Prepare dQ for Jacobian

<table>
<thead>
<tr>
<th>Branch ID</th>
<th>From</th>
<th>To</th>
<th>Current Q (kg/s)</th>
<th>New air demand</th>
<th>d Q (kg/s)</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>182</td>
<td>162</td>
<td>156</td>
<td>35.15</td>
<td>34.23</td>
<td>-0.92</td>
<td>Over-ventilated</td>
</tr>
<tr>
<td>209</td>
<td>404</td>
<td>400</td>
<td>32.81</td>
<td>34.23</td>
<td>1.42</td>
<td>Unsatisfied</td>
</tr>
<tr>
<td>70</td>
<td>115</td>
<td>116</td>
<td>28.43</td>
<td>25.1</td>
<td>-3.33</td>
<td>Over-ventilated</td>
</tr>
<tr>
<td>219</td>
<td>413</td>
<td>412</td>
<td>22.9</td>
<td>18.26</td>
<td>-4.65</td>
<td>Over-ventilated</td>
</tr>
<tr>
<td>29</td>
<td>76</td>
<td>81</td>
<td>19.17</td>
<td>18.26</td>
<td>-0.91</td>
<td>Over-ventilated</td>
</tr>
</tbody>
</table>

Except the branch 209, other critical branches have redundant airflow. Now we use LP simplex and LSQ methods to calculate it.
LP Simplex Method

To satisfy all the air requirement is our algorithm’s first priority. Therefore, minimum flow requirement is constrained. After requirements are satisfied, a most efficient power saving result will be selected as our optimal solution. Here the objective target is to minimize total intake airflow. Objective functions and constrains are shown in equations below:

\[
\begin{align*}
\text{min} & \quad 160.56dR_1 + 179dR_2 + 0.48dR_3 + 0.48dR_4 + 0.72dR_5 \\
\text{s.t} & \quad 9.728dR_1 + 16.334dR_2 - 6.125dR_3 + 5.765dR_4 + 8.527dR_5 \geq -0.92 \\
& \quad 14.172dR_1 + 37.952dR_2 + 0.24dR_3 + 11.41dR_4 - 32.307dR_5 \geq 1.42 \\
& \quad -4.203dR_1 + 33.268dR_2 - 0.24dR_3 - 0.48dR_4 - 0.36dR_5 \geq -3.33 \\
& \quad -4.564dR_1 + 29.425dR_2 + 0.12dR_3 - 0.961dR_4 - 0.841dR_5 \geq -4.65 \\
& \quad 15.613dR_1 + 8.767dR_2 - 0.601dR_3 - 3.363dR_4 - 1.561dR_5 \geq -0.92 \\
& \quad -1 \leq dR_1, \ldots, dR_5 \leq 1.5 
\end{align*}
\]

As solved above LP problem by simplex method, we get the optimal fan RPM setting and flow distribution as shown in Table 5.6. Fan 1 & 2 need to decrease fan RPM by 10%, Fan 3 & 4 need to decrease fan RPM by 7.5%. No changes on other fans are needed. By using that fan setting all airflow on critical branch will satisfy their minimum requirements.
Table 5.6 Fan RPM and Airflow Result from LP Simplex Method

<table>
<thead>
<tr>
<th>RPM ratio</th>
<th>Calculated dQ</th>
<th>Demand dQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1005</td>
<td>0.2452</td>
<td>≥</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.9248</td>
</tr>
<tr>
<td>0.07487</td>
<td>1.4172</td>
<td>≥</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.4172</td>
</tr>
<tr>
<td>0</td>
<td>2.9133</td>
<td>≥</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-3.3268</td>
</tr>
<tr>
<td>0</td>
<td>2.6618</td>
<td>≥</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-4.6479</td>
</tr>
<tr>
<td>0</td>
<td>-0.9128</td>
<td>≥</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.9128</td>
</tr>
</tbody>
</table>

**LSQ Method**

LSQ will find the best fitting result with minimum airflow rate. No need to use objective function; LSQ method will achieve a result with \( \min \sum (Q_{air}^M - Q_{air}^D) \) where \( Q_{air}^M \) is the airflow from LSQ method, \( Q_{air}^D \) is the minimum demand which means best power saving result. There are two defects in the LSQ method. One is that it cannot identify which branch has higher priority. This defect can be solved by adding weigh factor. A weighted LSQ method can be written as equation below:

\[
\min \sum a_i (Q_{air}^M - Q_{air}^D)^2
\]

5.1

Where \( a_i = weight \ factor \ for \ ith \ branch \)

The other defect is that it cannot ensure \( Q_{air}^M \geq Q_{air}^D \). In order to solve that problem, a comparing process needs to be applied. Each time LSQ result will be compared with flow requirement; if \( Q_{\text{air}}^M < Q_{\text{air}}^D \) for it branch, the difference between \( Q_{\text{air}}^M \) and \( Q_{\text{air}}^D \) will be added to \( Q_{\text{air}}^D \) to create a new higher artificial air demand on ith
branch. New optimal result will be calculated and compared with original air demand until all air demand are satisfied.

For this case, airflow results are shown in Figure 5.3. The Red points are the minimum air demand on critical branches, and the blue line is air distribution given by LSQ method.

![Figure 5.3 Airflow Distribution Result from LSQ Method](image)
Table 5.7 Fan RPM and Airflow Result from LSQ Method

<table>
<thead>
<tr>
<th>d RPM ratio</th>
<th>Calculated dQ</th>
<th>Demand dQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.167</td>
<td>≥</td>
</tr>
<tr>
<td>0.002</td>
<td>1.41</td>
<td>≈</td>
</tr>
<tr>
<td>0</td>
<td>0.062</td>
<td>≥</td>
</tr>
<tr>
<td>0.017</td>
<td>0.068</td>
<td>≥</td>
</tr>
<tr>
<td>-0.035</td>
<td>0.01</td>
<td>≥</td>
</tr>
</tbody>
</table>

Results of Financial Assessment:

- Expected annual operating cost – Original case US$240,931
- Expected annual operating cost – LP result US$236,241
- Expected annual operating cost – LSQ result US$238,968

LP simplex method gives a better power saving solution for this case, while since this case only has 5 critical branches which is not close to real situation, so case 2 has been tested.

5.4.2 Jacobian Optimization Case 2 – All branches Have Requirements

We assume that the changed airflow on all branches due to working schedule can be reduced by 10%.

LP Simplex Method

Simple method gives us a result as above, increased Fan 1 RPM by 6.29%, decreased Fan 2 RPM by 3.8%, decreased Fan 3 RPM by 1.33%, decreased Fan 4 RPM by 1.7%, and increased Fan 5 RPM by 0.26%.
Table 5.8 Fan RPM change determined by LP simplex method

<table>
<thead>
<tr>
<th></th>
<th>Fan 1</th>
<th>Fan 2</th>
<th>Fan 3</th>
<th>Fan 4</th>
<th>Fan 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>dRPM</td>
<td>0.0629</td>
<td>-0.038</td>
<td>-0.0133</td>
<td>-0.017</td>
<td>0.0026</td>
</tr>
</tbody>
</table>

All -1 < dRPM < 1.5

Under this RPM setting airflow distribution is shown in Figure 5.4. All branches meet their minimum demand with some branches showing increased flow rate.

![Optimization result](image.png)

**Figure 5.4 Air Distribution Comparison between LP Simplex Result and Target**

**Minimum Flow**

**LSQ Method**

LSQ method find that airflow will reduce 10% everywhere by reducing 10% RPM on all fans. Fan RPM changes are shown in Table 5.9.
Table 5.9 Fan RPM change determined by LSQ method

<table>
<thead>
<tr>
<th></th>
<th>Fan 1</th>
<th>Fan 2</th>
<th>Fan 3</th>
<th>Fan 4</th>
<th>Fan 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>dRPM</td>
<td>-0.1</td>
<td>-0.1</td>
<td>-0.1</td>
<td>-0.1</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

All \(-1<dRPM<1.5\)

Figure 5.5 Air Distribution Comparison between LSQ Result and Target Minimum Flow

Results of Financial Assessment:

- Expected annual operating cost – Original case US$240,931
- Expected annual operating cost – LP result US$219,414
- Expected annual operating cost – LSQ result US$175,721

5.4.3 Jacobian Optimization Case 3 – Artificial Target and Additional Fan Source

In this case, the required airflow distribution is created by putting an additional fan on branch 77-456. The target fan setting change is shown in Table 5.10. Fan 0 is the extra fan. The required flow change is shown in Figure 5.6. First, suppose we do not have that additional source, only current fans will be used to build Jacobian matrix and
calculate the optimal result. Then, suppose we have that fan and know the location, can we find the correctly optimal fan setting?

Table 5.10 Artificial Target Fan RPM Change

<table>
<thead>
<tr>
<th></th>
<th>Fan 0</th>
<th>Fan 1</th>
<th>Fan 2</th>
<th>Fan 3</th>
<th>Fan 4</th>
<th>Fan 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>dRPM</td>
<td>-0.1</td>
<td>0.1</td>
<td>-0.05</td>
<td>-0.05</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Figure 5.6 Required Flow Change for Case 3

No Extra Fan Jacobian

With this Jacobian, LP simplex method cannot find any feasible solution. LSQ method also cannot find out the result with all air demand being satisfied. After enough times of iteration, we still get an unacceptable result from LSQ method. Result is shown in Figure 5.7. Green stars are the required flow change, red points are LSQ result. Some points are still far lower, which means unsatisfied flow rate, than the target.
Figure 5.7 LSQ Result without Extra Fan

**Jacobian with Extra Fan**

With this Jacobian, LP simplex method still cannot find any feasible solution. LSQ method finds out that artificial target, and the fan setting given by LSQ method is almost the same as our target fan setting. Predict flow distribution also matches with model target. LSQ fan setting result is shown in Table 5.11, LSQ air distribution result is shown in Figure 5.8.
Table 5.11 Comparison of fan RPM Setting between LSQ Result and Target

<table>
<thead>
<tr>
<th>LSQ fan RPM setting</th>
<th>Target fan RPM setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90039</td>
<td>0.9</td>
</tr>
<tr>
<td>1.0997</td>
<td>1.1</td>
</tr>
<tr>
<td>0.95011</td>
<td>0.95</td>
</tr>
<tr>
<td>0.95011</td>
<td>0.95</td>
</tr>
<tr>
<td>1.1007</td>
<td>1.1</td>
</tr>
<tr>
<td>1.0999</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Figure 5.8 LSQ Result Compared with Target Flow Requirement

5.5 Result Analysis and Conclusion

- The Jacobian matrix has acceptable errors. After iteration calculation, the error can be kept within 1% or lower (depending on accuracy requirements).
• The LP simplex method is so strict to meet all flow demand conditions, that it may not be able to find feasible solution. If the Jacobian has even a very small (less than 1%) error, the method may be unstable. If the LP simplex method cannot find a feasible solution, iteration cannot be applied to eliminate error. The LSQ method always gives a solution which can be constrained by conditions using the weighting method of Danko and Bahrami (2014).

• The results from the LSQ method will not yield an exact optimal power saving result. Since weighted LSQ equation \( \min \sum a_i(Q_{air}^M - Q_{air}^D)^2 \) does not equal to \( \min \sum (Q_{air}^M - Q_{air}^D) \).

• This is also true for conditional optimization of the objective function of the LP method. When conflicting conditions and demands are set, compromise must be accepted.
6 CHAPTER VI. Reference


