Analysis of Behavior in Housing, Charity and Education

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics

by

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Abstracts

Chapter 1:
Data from Washoe County between 2006 and 2013 provides evidence for the loss aversion hypothesis. Homes that experienced a loss tend to have a higher asking price, a higher selling price and tend to stay on the market longer. Assembly Bill 284, a policy intervention intended to reduce the number of foreclosed homes, played an important role in how sellers responded to the loss in value of their homes. The bill magnified the effect of loss aversion, causing homes to stay on the market longer, exacerbating housing lock.

Chapter 2:
The standard public goods model for the financing of charity assumes voluntary private funding and tax financed public funding are equivalent. Here, the standard model is extended, so public financed charity can be discounted relative to private financing, capturing the possibility that publically financed charity is less efficient. When “public giving” is less efficient, “private giving” can be socially useful even when it does not generate a warm glow for the giver. Combining the discounting with the warm-glow further enhances the social usefulness of private giving.

Chapter 3:
This study reports the results of an experiment designed to test the hypothesis that segmenting a video will improve student performance. Two intermediate macroeconomic course sections were each provided 27 minutes of video instruction. One section was given the 27 minutes in one whole segment, while the other section was given the same video in three segments, each roughly nine minutes long. The nine minute segmenting, for the content delivered, did not enhance or detract from student performance based on
short-term and long-term measures. Those who watched the videos more did better on average, with diminishing returns.
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Chapter 1: Loss Aversion in the Housing Market: Evidence from an Exogenous Shock

1. Introduction

Loss aversion is a behavioral bias studied by behavioral economists. It is the tendency of people to prefer to avoid a loss over obtaining an equivalent gain. Given that the evidence for the existence of loss aversion is compelling, we would expect to find evidence of loss aversion in the housing market.

On October 1st, 2011 Nevada Assembly Bill 284 took effect. AB 284 made it more difficult to foreclose on houses in Nevada. The bill was intended to target the process known as robo-signing, where a representative of the mortgage holder signs foreclosure documents without reading them. After the bill took effect, notices of default required an affidavit based on the personal knowledge of the affiant setting forth certain information. This is intended to ensure that the signer of the notice has reviewed the foreclosure documents thus reducing invalid foreclosures. Figure 1 shows how much the number of notices of default decreased when the law took effect.
This paper examines the extent to which sellers in the Nevada housing market exhibited loss aversion during the aftermath of the Great Recession of 2008-2009, and it examines the impact of AB 284. Loss aversion is of interest because loss averse home owners tend to keep their homes on the market longer in an attempt to reduce their losses by attaining higher selling prices. This reduces labor mobility and tends to occur on a larger scale at a time when mobility is most needed.

2. Literature Review

In examining the housing market, there are several factors that must be considered. The primary factor of interest here is loss aversion. The effect that loss aversion has is determined in part by price expectations. The housing market also experiences a phenomenon known as equity constraint, when a home seller wants to use
equity as a down payment on the next home but lacks sufficient equity to do so. We will examine the literature on these three factors, loss aversion, price expectations, and equity constraint in turn before examining the expected effects in the housing market.

Loss aversion is a behavioral bias that occurs when people prefer to avoid a loss over obtaining an equivalent gain. Kahneman and Tversky (1979) developed “prospect theory” to explain certain behavioral biases they observed when conducting experiments under risk. This theory postulates a value function with a reference point and is much steeper in losses from the reference than in gains. An example of such a value function is presented in Figure 2. Referring to this value function, Kahneman and Tversky (1984) coined the term loss aversion.
Several behavioral phenomena can be explained by loss aversion. Kahneman, Knetsch, and Thaler (1990) demonstrated how loss aversion could account for the endowment effect, which occurs when a people value something more when they own it than they do when they do not own it. Thaler (1999) uses loss aversion to explain the sunk cost fallacy which occurs when people choose to continue a behavior or endeavor because they have already made an investment into it even if a better choice is available.

Dimmock and Kouwenberg (2010) found evidence that loss aversion kept many households out of equity markets and those households that chose to participate invested
less than expected. Pedace and Smith (2012) found evidence that loss averse managers were reluctant to fix their own mistakes, often requiring they be replaced, increasing managerial turnover. Boettcher (2004) even looked at the effect loss aversion had on international relations.

There are several effects of loss aversion we would expect to observe in the housing market. First, a loss averse seller would be expected to start with a higher asking price in the hope that the loss experienced would be reduced. Second, the final selling price a loss averse seller would accept would be expected to be larger. Finally, the time on market would be longer to get the higher price.

In order to be considered a loss, there must be some reference point. Genesove and Mayer (2001), Einiö, Kaustia and Puttonen (2008), and Anenberg (2011) all considered the reference to be the initial purchase price of the home. Framing is also an important part of determining that there is a loss (Tversky and Kahneman, 1981). In order for sellers to form an expectation of the loss they will incur on the sale of their home, they would need to form an expectation of the future selling price.

McCarthy and Mcquinn (2015) confirm that housing price expectations effect home owner consumption behavior. Taltavull and Mcgreal (2009) found that housing price expectations play an important role in determining the asking price of a home. Dröes and Hassink (2014) found that housing prices play a role because they effect the expected loan-to-value ratio of the home, which determines the degree to which the home owner is equity constrained.

An equity constraint can have similar impacts as loss aversion (Genesove and Mayer, 1997), so it is important to control for equity constraints when looking for loss
aversion impacts. An equity constraint occurs when the loan-to-value (LTV) ratio for a house is high. When the LTV is larger, the sale of the home provides less net proceeds to be used as a down payment on a new home. The effects of equity constraints have been examined by Anenberg (2011), Lee and Ong (2005), and Engelhardt (2003).

Genesove and Mayer (2001) looked at selling prices for condominiums in the Boston area between 1990 and 1997. They confirmed that equity constraint plays a role. They also found evidence to support the loss aversion hypothesis.

Einiö, Kaustia and Puttonen (2008) looked for evidence of loss aversion in the apartment market of the greater Helsinki area from 1987 to 2003. The housing market in this area suffered decline from 1989 to 1993, 1995 to 1996 and 2000 to 2002, giving good opportunities to find evidence of loss aversion. They also examined how investors responded to loss and found that the effects of loss aversion were much weaker for investment properties than for those owned by the occupant.

Anenberg (2011) developed a model using long panel data from the San Francisco Bay area from 1988 to 2005. He followed the same model as Genesove and Mayer (2001) but used a richer data set with more observations and additional explanatory variables. Anenberg found higher degrees of loss aversion than Genesove and Mayer. He also found that home owners with similar houses nearby exhibit less signs of loss aversion, which explains why his results are different from Genesove and Mayer. When there are close substitutes, a seller cannot demand as high a premium due to loss aversion. Genesove and Mayer examined condominiums, which tend to be similar to each other.
One reason for concern about the effects of loss aversion in the housing market is the effect on labor mobility due to housing lock. As Einiö, Kaustia and Puttonen (2008) points out, in an effort to reduce the loss, a loss averse seller will attempt to stay on the market longer to obtain a higher final sale price. This would tend to occur more often during economic downturn, the very time high labor mobility is desired.

The evidence for housing lock is mixed. Modestino and Dennett (2013), Ferreira, Gyourko, and Tracy (2012) and Chan (2001) found evidence for housing lock. Valletta (2013) found that home owners did not experience a significant difference in unemployment compared to renters, suggesting that the economic down turn did not keep home owners from relocating to find new employment. Schulhofer-Wohl (2012) was unable to find evidence of housing lock looking at homes with negative equity.

3. Methods

The model used is based on one developed by Genesove and Mayer (2001). All prices are logged. The asking price $L_{ist}$ of home $i$ purchased at time $s$ and listed at time $t$ is a function of the expected value $\mu_{it}$ of the home $i$ listed at time $t$ and the expected loss $LOSS_{ist}$ of home $i$ purchased at time $s$ and listed at time $t$:

$$L_{ist} = \alpha_0 + \alpha_1 \mu_{it} + \alpha_2 LOSS_{ist} + \epsilon_{it}.$$ 

The parameters of (1) are estimated using a two-step process. The first step consists of regressing the purchase price $P^0_{ls}$ of the home on a series of housing attributes $X_i$ and a set of dummy variables for the quarter of sale, $\delta_s$:

$$P^0_{ls} = \beta_s X_{is} + \delta_s + \epsilon_{is}.$$

This model (2) introduces time varying coefficients on the housing attributes. In contrast, Genesove and Mayer assumed the marginal impacts of housing characteristics were
constant over the entire time frame they examined. The estimated model (2) provides a predicted value of the home at the time $t$ when it is listed:

$$\hat{\mu}_{it} = \tilde{\beta}_t X_{it} + \delta_t.$$  

The predicted home value estimated from (3) is used to estimate the loss. $LOSS_{ist}$ is set to zero if there is no loss. If there is a loss, then $LOSS_{ist}$ is the difference between the original purchase price and the estimated value. Therefore, we can write

$$LOSS_{ist} = \max[0, (P^0_{is} - \hat{\mu}_{it})].$$

To account for the effect of equity constraint, Genesove and Mayer (2001) used (3) to estimate the loan-to-value ratio (LTV). They only considered the degree to which LTV was above 0.8. Below 0.8, the seller is not considered equity constrained, so the loan to value variable is set equal to zero. Specifically, the LTV variable is defined as

$$LTV_{ist} = \max \left[0, \frac{\text{Loan}_{ist}}{\hat{\mu}_{it}} - 0.8\right].$$

The complete model for the asking price is obtained by adding the LTV variable to the model (1), so we obtain

$$L_{ist} = \alpha_0 + \alpha_1 \mu_{it} + \alpha_2 LOSS_{ist} + \alpha_3 LTV_{ist} + \varepsilon_{it}.$$  

Since we are interested in the effect of AB284, (6) is extended to include a dummy for when AB284 takes place. This dummy will also be interacted with all the independent variables giving the final asking price equation

$$L_{ist} = \alpha_0 + \alpha_1 \mu_{it} + \alpha_2 LOSS_{ist} + \alpha_3 LTV_{ist} + D_{AB284} + \alpha_4 \mu_{it} \ast D_{AB284} +$$
$$\alpha_5 LOSS_{ist} \ast D_{AB284} + \alpha_6 LTV_{ist} \ast D_{AB284} + \varepsilon_{it}.$$  

Genosove and Mayer showed that the coefficient on $LOSS_{ist}$ is biased upward. This is because $\varepsilon_{is}$ in equation (2) is composed of the true error plus the overpayment or
underpayment at the time of purchase. Because $\mu_{it}$ is directly in equation (1), $\epsilon_{it}$ will also include the true error from the first stage. Since $LOSS_{lst}$ is the difference between the purchase price and the predicted value of the home, $LOSS_{lst}$ will also contain the true error term and thus be correlated with the second state error term.

By adding the residuals from the first stage regression, Genosove and Mayer show that the coefficient on $LOSS_{lst}$ will be biased downward. By including $\epsilon_{is}$, $\epsilon_{it}$ no longer includes the true error from the first stage. However the first stage residual contains the information about the overpayment or underpayment, which is also in $\mu_{it}$ and $LOSS_{lst}$. $\epsilon_{it}$ now contains a term from this overpayment or underpayment and again is correlated with $LOSS_{lst}$. This causes a bias in the opposite direction. Thus by estimating (7) once without the first stage residuals and once with the first stage residuals, we obtain an interval for the true value of $\alpha_2$ and $\alpha_5$.

The selling price $P_{lst}$ will tend to differ from the asking price $L_{lst}$, and the effect of experiencing a loss on the selling price may differ from its effect on the asking price. Consequently, a mode of the final sale price was also estimated. Incorporating the $LOSS_{lst}$ and $LTV_{lst}$ into the price regression yields

\begin{equation}
\begin{align*}
    P_{lst} = \gamma_0 + \gamma_1(\beta_t X_{it} + \delta_t) + \gamma_2 LOSS_{lst} + \gamma_3 LTV_{lst} + \epsilon_{it}.
\end{align*}
\end{equation}

The regression on price is no longer a linear model. The estimation for equation (8) simultaneously estimates the value of the home and the effect of loss on the final sale price.

Whereas the estimated coefficient $\alpha_2$ in (6) provides an indication of the impact of loss on asking price, the coefficient $\gamma_2$ in (8) provides an indication of the impact of
loss on selling price. In models (6) and (8), a price premium is expected for both a loss or a higher loan-to-value ratio (i.e., $\alpha_2 > 0$, $\alpha_3 > 0$, $\gamma_2 > 0$, and $\gamma_3 > 0$ are expected).

Since $LOSS_{ist}$ is the positive difference between the original purchase price of the home and the value of the home at time $t$, we can substitute $LOSS_{ist} = \max[0, P_{is}^0 - \beta X_{it} - \delta_t]$ into (8). Similarly with the loan-to-value ratio, we can substitute $LTV_{ist} = Loan_{ist}/e^{\beta X_{it} + \delta_t}$ into (8). The complete model for the sale price of the home is

$$
P_{ist} = \gamma_0 + \gamma_1(\beta X_{it} + \delta_t) + \gamma_2 \max[0, P_{is}^0 - \beta X_{it} - \delta_t] + \gamma_3 \frac{Loan_{ist}}{\exp(\beta X_{it} + \delta_t)} + \epsilon_{it}
$$

Since we are interested in the effect of AB284, (9) is extended to include a dummy for when AB284 takes place. This dummy will also be interacted with all the independent variables giving the final sale price equation

$$
P_{ist} = \gamma_0 + \gamma_1(\beta X_{it} + \delta_t) + \gamma_2 \max[0, P_{is}^0 - \beta X_{it} - \delta_t] + \gamma_3 \frac{Loan_{ist}}{\exp(\beta X_{it} + \delta_t)} + D_{AB284} + \gamma_4(\beta X_{it} + \delta_t) * D_{AB284} + \gamma_5 \max[0, P_{is}^0 - \beta X_{it} - \delta_t] * D_{AB284} + \gamma_6 \frac{Loan_{ist}}{\exp(\beta X_{it} + \delta_t)} * D_{AB284} + \epsilon_{it}
$$

Since equation (9) is not a linear equation, it will have to be estimated using non-linear least squares.

Finally, to account for the time on market a hazard function model was used,

$$
h(t) = h_0(t)e^{\theta Z}.
$$

The hazard function models the probability $h(t)$ that a given house will be sold in week $t$ based on housing attributes $Z$ which include the estimated value of the house, the expected loss and the loan-to-value estimates. The equation to be estimated in the hazard model is

$$
h(t) = h_0(t)e^{\theta_1 \mu_{it} + \theta_2 LOSS_{ist} + \theta_3 LTV_{ist} + D_{AB284} + \theta_4 \mu_{it} * D_{AB284} + \theta_5 LOSS_{ist} * D_{AB284} + \theta_6 LTV_{ist} * D_{AB284}}
$$

4. Data

To estimate the parameters of the model, data was obtained from multiple sources. Asking price data came from the Center for Regional Studies at the University
of Nevada, Reno. The selling price and housing attribute data were readily available from the Washoe County Assessor’s office. The data from the different sources were matched using the Parcel ID number for the property.

Figure 3 shows the median selling price and asking price by month. Asking price has a large spike at the beginning because houses enter the data set when they are sold, not listed. Homes with high asking prices would expect to take a longer time to sell. Selling prices go further back because prior selling prices are needed in order to estimate the loss.

**Figure 3. Selling and Asking Price**
Table 1 contains the median asking price for each quarter from 2006 to the first quarter of 2013.

**Table 1: Median asking price by quarter**

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Median</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006q1</td>
<td>949900</td>
<td>9</td>
</tr>
<tr>
<td>2006q2</td>
<td>439000</td>
<td>35</td>
</tr>
<tr>
<td>2006q3</td>
<td>436155</td>
<td>80</td>
</tr>
<tr>
<td>2006q4</td>
<td>320000</td>
<td>128</td>
</tr>
<tr>
<td>2007q1</td>
<td>319950</td>
<td>520</td>
</tr>
<tr>
<td>2007q2</td>
<td>329000</td>
<td>1008</td>
</tr>
<tr>
<td>2007q3</td>
<td>299900</td>
<td>862</td>
</tr>
<tr>
<td>2007q4</td>
<td>271000</td>
<td>558</td>
</tr>
<tr>
<td>2008q1</td>
<td>260000</td>
<td>858</td>
</tr>
<tr>
<td>2008q2</td>
<td>250000</td>
<td>1066</td>
</tr>
<tr>
<td>2008q3</td>
<td>220000</td>
<td>1017</td>
</tr>
<tr>
<td>2008q4</td>
<td>199900</td>
<td>1092</td>
</tr>
<tr>
<td>2009q1</td>
<td>188250</td>
<td>1268</td>
</tr>
<tr>
<td>2009q2</td>
<td>189900</td>
<td>1352</td>
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<td>2009q3</td>
<td>179900</td>
<td>1472</td>
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<td>2009q4</td>
<td>170600</td>
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<td>2010q1</td>
<td>174900</td>
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<td>2011q2</td>
<td>159900</td>
<td>1697</td>
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<td>2011q3</td>
<td>149900</td>
<td>1551</td>
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<td>2011q4</td>
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<td>2012q1</td>
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<td>2012q3</td>
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<tr>
<td>2012q4</td>
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<td>727</td>
</tr>
<tr>
<td>2013q1</td>
<td>214947.5</td>
<td>308</td>
</tr>
</tbody>
</table>

All the data required to ideally calculate the loan-to-value ratio does not exist, but a proxy can be constructed. Only the most recent loan information is available because, when a house is sold, the information about the prior loan is replaced with the new loan information. To estimate the loan associated with a home, the interest rate and down
payment information were obtained from Freddie Mac. You can see these in Figures 4 and 5. Interest rates are available as monthly averages and the down payment estimates are quarterly. Down payments seem a little high which may be in part because they are given as loan-to-value ratios where the value is the larger of the purchase price and the assessed value. Homes purchased for less than the assessed value would have a smaller loan-to-value and thus a larger estimated down payment.

**Figure 4. Interest rate averages from Freddie Mac**
The data only includes homes that actually sold. Home owners who have a larger loss aversion premium might be expected to ask too high a selling price and stay on the market so long that they are eventually taken off the market. Thus, there is reason to think that the estimate we obtain for the coefficient on $LOSS_{lst}$ underrepresents the degree of loss aversion that exists.

5. Results

Figure 6 shows the Loss vs Log Asking Price. The plot also includes a locally weighted scatterplot smoothing line “lowess”. The expectation is that there should be a discontinuity at a loss of zero. However, that does not appear to be what is shown in the
This could in part be due to the high variability in the asking price and loss only explaining a small portion of that variability. There does appear to be a discontinuity at a loss slightly greater than zero.

**Figure 6: Scatterplot of Loss vs Log Asking Price**

Table 2 shows regression results for models that predict the asking price. Columns (1) – (2) are the estimates of equation (6), before the dummy variable for AB284 was included. Columns (3) – (4) are the estimates of equation (7). To allow for a possible non-linearity, following Genosove and Mayer (2001), loss squared is included. Columns (2) and (4) contain the first-stage residuals.
Table 2. Asking price regression results

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>1.047***</td>
<td>0.980***</td>
<td>1.076***</td>
<td>1.020***</td>
</tr>
<tr>
<td></td>
<td>(0.0226)</td>
<td>(0.0181)</td>
<td>(0.0286)</td>
<td>(0.0281)</td>
</tr>
<tr>
<td>Value* D_{AB284}</td>
<td>-0.0511</td>
<td>-0.0665**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0321)</td>
<td>(0.0337)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D_{AB284}</td>
<td>0.763*</td>
<td>1.032**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.398)</td>
<td>(0.424)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loss</td>
<td>0.295***</td>
<td>0.0781***</td>
<td>0.359***</td>
<td>0.157***</td>
</tr>
<tr>
<td></td>
<td>(0.0330)</td>
<td>(0.0301)</td>
<td>(0.0422)</td>
<td>(0.0491)</td>
</tr>
<tr>
<td>Loss-sq</td>
<td>0.0796**</td>
<td>-0.114***</td>
<td>0.0406</td>
<td>-0.120***</td>
</tr>
<tr>
<td></td>
<td>(0.0366)</td>
<td>(0.0421)</td>
<td>(0.0408)</td>
<td>(0.0452)</td>
</tr>
<tr>
<td>Loss* D_{AB284}</td>
<td>-0.238***</td>
<td>-0.117*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0616)</td>
<td>(0.0656)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loss-sq* D_{AB284}</td>
<td>0.152</td>
<td>0.340*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.115)</td>
<td>(0.188)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LTV</td>
<td>-0.0479**</td>
<td>-0.0131</td>
<td>-0.0391*</td>
<td>-0.0102</td>
</tr>
<tr>
<td></td>
<td>(0.0201)</td>
<td>(0.0172)</td>
<td>(0.0212)</td>
<td>(0.0152)</td>
</tr>
<tr>
<td>LTV* D_{AB284}</td>
<td>-0.0494</td>
<td>-0.300*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0849)</td>
<td>(0.157)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D_{LOSS&gt;0}</td>
<td>-0.0162</td>
<td>-0.117***</td>
<td>-0.0199*</td>
<td>-0.104***</td>
</tr>
<tr>
<td></td>
<td>(0.0104)</td>
<td>(0.0132)</td>
<td>(0.0113)</td>
<td>(0.0130)</td>
</tr>
<tr>
<td>D_{LOSS&gt;0}* D_{AB284}</td>
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<td>-0.0230</td>
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<tr>
<td></td>
<td>(0.0256)</td>
<td>(0.0285)</td>
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<tr>
<td>Residuals</td>
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<td>0.310***</td>
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</tr>
<tr>
<td></td>
<td>(0.0443)</td>
<td></td>
<td>(0.0505)</td>
<td></td>
</tr>
<tr>
<td>Residuals* D_{AB284}</td>
<td></td>
<td></td>
<td>0.141*</td>
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</tr>
<tr>
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<td></td>
<td>(0.0760)</td>
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<tr>
<td>Constant</td>
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<td>-0.628*</td>
<td>-1.118***</td>
<td>-1.011***</td>
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<td>(0.281)</td>
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<td>18,167</td>
<td>18,167</td>
<td>18,167</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.768</td>
<td>0.788</td>
<td>0.774</td>
<td>0.795</td>
</tr>
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</table>

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Because all the prices are logged, the estimates for the coefficients on $LOSS_{lst}$ and $LOSS_{lst} \times D_{AB284}$ represent the elasticity of the asking price to the loss. From columns (3) and (4), the estimate of the coefficients on $LOSS_{lst}$ is positive which is the expected effect of loss on the asking price. The estimate of the coefficient on $LOSS_{lst} \times D_{AB284}$ is negative but smaller in magnitude than on $LOSS_{lst}$ which means the effect of
loss is smaller after AB284. The estimate for the coefficient on $LOSS_{ist}^2$ squared is negative in column (4), which means the elasticity decreases as the loss increases but not significant in column (3). In column (4), since the coefficient on $LOSS_{ist}^2 \times D_{AB284}$ squared is positive and larger in magnitude than the coefficient on $LOSS_{ist}$ squared, the elasticity of the asking price to the loss is increasing as loss increases after AB284.

Figure 7 shows the elasticity of the asking price with respect to loss. From this graph, the change is not expected to fall to 0% until the loss is 93%. Figure 8 shows the elasticity after AB 284.

**Figure 7: Elasticity of asking price with respect to loss before AB 284**
While coefficient on LTV are not the primary coefficients of interest, it is interesting to note that the estimates are negative. Since the equity constraint is expected to increase the asking price, these coefficients are expected to be positive. The estimated impact when the first stage residuals are not included of a 0.1 increase in the LTV above 0.8 is to decrease the expected asking price by 0.391% with no significant change after AB 284. From the estimates with the first stage residuals, the estimated impact of a 0.1 increase in the LTV above 0.8 is not significant before AB284 and is expected to decrease the expected asking price by 3% after AB284.
Figure 9 shows the relationship between LTV and the asking price. For the lowest loan to values, the relationship has a negative slope. This gradually becomes a positive slope which eventually flattens out due to a few outliers at very high LTV.

**Figure 9: Graph of loan-to-value vs asking price**

Table 3 shows results of the non-linear regressions conducted to predict the selling price. Columns (1) and (2) show the estimates for equation (9) while columns (3) and (4) are the estimates for equation (10). Columns (2) and (4) contain the first-stage residuals. From columns (3) and (4), the estimate of the coefficient on $LOSS_{lst}$ is positive which is the expected effect of loss on the selling price. The estimate of the coefficient on $LOSS_{lst} \times D_{AB284}$ is also positive which means that after AB 284 the effect of loss increased as expected.
### Table 3: Nonlinear final sale price regression results

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<td>1.551***</td>
<td>3.012***</td>
<td>2.021***</td>
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<td>(0.0273)</td>
<td>(0.0276)</td>
<td>(0.153)</td>
<td>(0.0636)</td>
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<tr>
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<td>-0.662***</td>
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<td>D&lt;sub&gt;AB284&lt;/sub&gt;</td>
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<td>7.902***</td>
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<td>(0.893)</td>
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<tr>
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<td>0.676***</td>
<td>0.663***</td>
<td>0.557***</td>
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<td>(0.0609)</td>
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<tr>
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<td>-0.630***</td>
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<td>0.313**</td>
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<td>(0.0796)</td>
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<tr>
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<td>D&lt;sub&gt;LOSS&gt;0&lt;/sub&gt;</td>
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<td>(0.174)</td>
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<td>0.0263</td>
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<td>(0.0731)</td>
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<tr>
<td>Residuals</td>
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<td>(0.00889)</td>
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<td>23859</td>
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<td>(0.361)</td>
<td>0.783</td>
<td>0.800</td>
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</table>

N = 23,859 23,859 23,859 23,859

R-sq = 0.779 0.796 0.783 0.800

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Figure 10 shows the elasticity of selling price with respect to the loss. With diminishing returns, the lower end of the range does not go below zero until the loss is 82%. Figure 11 shows the elasticity of selling price with respect to the loss after AB 284. The lower range goes below zero at a 38% loss while the upper range goes below zero at a 53% loss.
Figure 10: Elasticity of selling price with respect to loss before AB 284
The estimates for the coefficient on LTV are positive. This supports the conclusion that the equity constraint did play a part in the final selling price of the home. An increase in the LTV of 0.1 above the 0.8 threshold increased the selling price by 1.02% before AB 284 and 7.74% after.

Table 4 gives the results of the hazard analysis. The coefficient on loss is negative and significant in all regressions. This means, as the loss increased, the probability of a sale decreased resulting in an increased time to sell. None of the terms interacted with the dummy for AB284 were significant, indicating that the marginal effect of an increase in loss on the time to sale did not change significantly after AB284.
Table 4: Hazard model regression results

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</tr>
</thead>
<tbody>
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<td>-0.202***</td>
<td>-0.238***</td>
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<td>(0.0163)</td>
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<td>D_AB284</td>
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<td>-0.387***</td>
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<td>Loss-sq</td>
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<td>0.178**</td>
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<td>(0.0940)</td>
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<td>Loss*D</td>
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<td>-0.444***</td>
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<td>(0.150)</td>
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<tr>
<td>Loss-sq*D</td>
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<td>0.695***</td>
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<td>(0.294)</td>
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<tr>
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<td>-0.0512**</td>
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<td>-0.166***</td>
<td>-0.206***</td>
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<td>(0.0302)</td>
<td>(0.0330)</td>
<td>(0.0331)</td>
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<tr>
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<td>-0.0963</td>
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<tr>
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<td>(0.0831)</td>
<td>(0.0820)</td>
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</tr>
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<td>Residuals</td>
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<td>Residuals*D</td>
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<td>0.0872</td>
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</tr>
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</table>

| Observations | 20,467 | 20,467 | 20,467 | 20,467 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

The hazard results indicate the change in the probability of selling at time $t$. The estimates are the coefficients of the hazard model. To get the expected increase in time to sell, we take $1 - e^{\beta Z}$. The marginal effect on time to sale of a 10% increase in the loss is shown in figure 12. The marginal effect of a 10% increase in the loss after AB 284 is shown in figure 13. The increase in time to sell falls to zero at 66% at the lower end of the range and 82% and the upper end.
Figure 12: Graph of marginal effect of loss on time to sale before AB 284
The coefficient on the loss dummy is negative and significant. This increases the time to sale of homes that have a loss. The increase in time to sale due to just experiencing an expected loss is between 12.89% and 16.14% before AB284 and between 31.75% and 37.19% after AB284.

The estimated coefficient on LTV was negative, leading to the expected result that the equity constraint increased the time to sale. This coefficient was not significant when the first-stage residuals were included in column (4). The change in the effect of equity constraint due to AB284 was also not significant. This means the expected effect of a 0.1 increase in the loan-to-value ratio, we expect to see an increase in the time to sale of at most 0.58%.
6. Discussion

The results in tables 2-4 give the expected effect of loss aversion. However, in table 2, the coefficient on LTV is either negative or not significant. This is not consistent with prior analyses of the housing market. The expected effect of the equity constraint is to increase the asking price.

From figure 3, we can see that housing prices and thus the expected values fell significantly during this time period. It is possible that that falling home values resulted in many home having large loan-to-value ratios. With large LTVs, significantly greater than 1, equity constraint is not the problem as there is no equity. Much past an LTV of 1, any additional dollars from the sale of the home goes to pay the mortgage not to be used as a down payment. To test this hypothesis, a quadratic model of LTV and two nonparametric LTV models were analyzed. The first nonparametric model divided LTV into four groups (LTV less than .8, LTV between .8 and .9, LTV between .9 and 1, and LTV greater than 1). The second parametric model expanded on the first by separating the greater than 1 into LTV between 1 and 1.5 and greater than 1.5. For each of these models, all coefficients were still negative.

Due to the housing market crash, many homes were sold at short sale. Short sold homes would not be affected by the equity constraint as there is no equity in a short sold home. There are also other groups such as investors and bank owners that would not be expected to be affected by an equity constraint since they do not need equity to purchase a new home to live in.

The data includes information on who occupied the home and special conditions of sale. Specifically, it was possible to separate the data into owner occupied homes not
sold at short sale (OWN) from the rest of the data (OTHER). The disadvantage of doing this is that the special condition of sale did not start being reported until February 2009, which reduced the total number of observations from about 18,000 to 15,000.

Table 5 shows the results of splitting the data into two groups. While the coefficient on LTV for the OWN group is positive and significant as expected, the coefficient on $LOSS_{lst}$ is negative. This means that as the loss increased, the expected asking price decreases. The coefficient on the dummy for loss is positive for the OWN group, meaning having a loss resulted in a higher ask price. After AB 284, the effect of the equity constraint decreased while the effect of loss did not significantly change.

The results of table 5 suggest that home owners who are not selling at short sale are actually listing lower when they experienced a larger loss. Since the market was in a significant downturn during the majority of this study, this may indicate that sellers are trying to sell their homes faster to avoid a larger loss. If home owners see that the value of their home is falling, by choosing a lower asking price, they may be able to sell faster before the home value falls even farther.
Table 5. Asking price regression with data split into owner occupied, not selling at short sale and others

<table>
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<tr>
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<th>(1)</th>
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<th>(3)</th>
<th>(4)</th>
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<td></td>
<td></td>
</tr>
<tr>
<td>OWN</td>
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<td>(0.264)</td>
<td>(0.242)</td>
<td>(0.357)</td>
<td>(0.329)</td>
</tr>
<tr>
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<td>1.208***</td>
<td>1.225***</td>
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<td>(0.0246)</td>
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<td>(0.604)</td>
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<tr>
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<td>(0.117)</td>
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<tr>
<td>Loss-sq</td>
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<td>(0.0365)</td>
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<td>Loss*DAB284</td>
<td>-0.261***</td>
<td>-0.370***</td>
<td>(0.0602)</td>
<td>(0.0716)</td>
</tr>
<tr>
<td>Loss-sq*DAB284</td>
<td>-1.074***</td>
<td>-1.175***</td>
<td>(0.115)</td>
<td>(0.136)</td>
</tr>
<tr>
<td>LTV</td>
<td>0.0855***</td>
<td>0.136***</td>
<td>0.0927***</td>
<td>0.174***</td>
</tr>
<tr>
<td></td>
<td>(0.00919)</td>
<td>(0.0110)</td>
<td>(0.0103)</td>
<td>(0.0152)</td>
</tr>
<tr>
<td>LTV*DAB284</td>
<td>1.025***</td>
<td>1.192***</td>
<td>(0.0976)</td>
<td>(0.116)</td>
</tr>
<tr>
<td>DLOSS&gt;0</td>
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<td>-0.0715**</td>
<td>-0.0948***</td>
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<tr>
<td></td>
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<td>(0.0252)</td>
<td>(0.0261)</td>
<td>(0.0301)</td>
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<tr>
<td>DLOSS&gt;0*DAB284</td>
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<td>0.0439</td>
<td>(0.0331)</td>
<td>(0.0339)</td>
</tr>
<tr>
<td>Residuals</td>
<td>0.232***</td>
<td>0.242***</td>
<td>(0.00817)</td>
<td>(0.0111)</td>
</tr>
<tr>
<td>Residuals*DAB284</td>
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<td>-0.0341*</td>
<td>(0.0161)</td>
<td>(0.0161)</td>
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<tr>
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<td>R-squared</td>
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</table>

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Table 6 shows the results of the non-linear regression on the selling price with the
data separated into the OWN group and OTHER group. The estimates for the
coefficients on LTV and LTV after AB 284 are positive, consistent with the expected
effect of the equity constraint. The estimates of the coefficients on $LOSS_{ist}$ are either
negative or not significant. The coefficient on the loss-square term was negative. This
supports the idea that a loss averse seller tried to sell faster to avoid more future loss.
Table 6. Nonlinear final sale price regression results with data split into owner occupied, not selling at short sale and others

<table>
<thead>
<tr>
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<th>(1)</th>
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<tr>
<td>D&lt;sub&gt;own&lt;/sub&gt;</td>
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<td>2.990***</td>
<td>2.487***</td>
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<tr>
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<td>(0.357)</td>
<td>(0.329)</td>
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<td>1.227***</td>
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<td>1.208***</td>
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<td>(0.0254)</td>
<td>(0.0322)</td>
<td>(0.0315)</td>
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<td>D&lt;sub&gt;AB284&lt;/sub&gt;</td>
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<td>-0.113</td>
<td>(0.610)</td>
<td>(0.604)</td>
</tr>
<tr>
<td>Loss</td>
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<td>-0.171</td>
<td>-0.593***</td>
<td>-0.159</td>
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<td>(0.126)</td>
<td>(0.150)</td>
<td>(0.186)</td>
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<tr>
<td>Loss-sq</td>
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<td>-0.795***</td>
<td>-1.220***</td>
<td>-0.945***</td>
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<td>(0.180)</td>
<td>(0.193)</td>
<td>(0.225)</td>
<td>(0.273)</td>
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<tr>
<td>Loss* D&lt;sub&gt;AB284&lt;/sub&gt;</td>
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<tr>
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<td>(0.293)</td>
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</tr>
<tr>
<td>Loss-sq* D&lt;sub&gt;AB284&lt;/sub&gt;</td>
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<tr>
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<tr>
<td>LTV</td>
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<td>(0.297)</td>
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<td></td>
<td>(0.408)</td>
<td>(0.474)</td>
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<td>D&lt;sub&gt;LOSS&lt;/sub&gt;-&lt;sub&gt;o&lt;/sub&gt;</td>
<td>0.0886***</td>
<td>0.0510*</td>
<td>0.113***</td>
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<td>(0.0227)</td>
<td>(0.0218)</td>
<td>(0.0276)</td>
<td>(0.0302)</td>
</tr>
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<tr>
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<td>(0.0466)</td>
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<td>OTHER</td>
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</tr>
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<td>1.426***</td>
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<tr>
<td></td>
<td>(0.457)</td>
<td>(0.510)</td>
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</tr>
<tr>
<td>Loss</td>
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<td>0.832***</td>
<td>0.985***</td>
<td>0.876***</td>
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<tr>
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<td>(0.0281)</td>
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<tr>
<td>Loss-sq</td>
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<td>-0.455***</td>
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</tr>
<tr>
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<td>(0.0240)</td>
<td>(0.0262)</td>
<td>(0.0291)</td>
<td>(0.0365)</td>
</tr>
<tr>
<td>Loss* D&lt;sub&gt;AB284&lt;/sub&gt;</td>
<td>-0.261***</td>
<td>-0.370***</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(0.0602)</td>
<td>(0.0716)</td>
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<td></td>
</tr>
<tr>
<td>Loss-sq* D&lt;sub&gt;AB284&lt;/sub&gt;</td>
<td>-1.074***</td>
<td>-1.175***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.115)</td>
<td>(0.136)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LTV</td>
<td>0.0855***</td>
<td>0.136***</td>
<td>0.0927***</td>
<td>0.174***</td>
</tr>
<tr>
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<td>(0.00919)</td>
<td>(0.0110)</td>
<td>(0.0103)</td>
<td>(0.0152)</td>
</tr>
<tr>
<td>LTV* D&lt;sub&gt;AB284&lt;/sub&gt;</td>
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<td>1.192***</td>
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</tr>
<tr>
<td></td>
<td>(0.0976)</td>
<td>(0.116)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D&lt;sub&gt;LOSS&lt;/sub&gt;-&lt;sub&gt;o&lt;/sub&gt;</td>
<td>-0.0747**</td>
<td>-0.0715**</td>
<td></td>
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<td>(0.0232)</td>
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<td>(0.0261)</td>
<td>(0.0301)</td>
</tr>
<tr>
<td>D&lt;sub&gt;LOSS&lt;/sub&gt;-&lt;sub&gt;o&lt;/sub&gt;* D&lt;sub&gt;AB284&lt;/sub&gt;</td>
<td>0.0443</td>
<td>0.0439</td>
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<tr>
<td></td>
<td>(0.0331)</td>
<td>(0.0339)</td>
<td></td>
<td></td>
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<tr>
<td>Residuals</td>
<td>0.232***</td>
<td>0.242***</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.00817)</td>
<td>(0.0111)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residuals* D&lt;sub&gt;AB284&lt;/sub&gt;</td>
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<td>(0.0161)</td>
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</tr>
<tr>
<td>Constant</td>
<td>-6.196***</td>
<td>-5.672***</td>
<td>-5.898***</td>
<td>-5.676***</td>
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<td>(0.248)</td>
<td>(0.276)</td>
<td>(0.294)</td>
<td>(0.321)</td>
</tr>
<tr>
<td>N</td>
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<td>14968</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.787</td>
<td>0.797</td>
<td>0.789</td>
<td>0.800</td>
</tr>
</tbody>
</table>
Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The table 7 results shows the hazard model with the data split into OWN group and OTHER group. The coefficient on loss for the OWN group homes is positive but not significant. The coefficient on loss-square is positive indicating that as loss increased, the time to sale decreased. This supports the theory that loss averse home sellers are trying to sell their homes faster in a declining market to try to avoid further losses.
Table 7. Hazard regression results with data split into owner occupied, not selling at short sale and others

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
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<th>(4)</th>
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<td></td>
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<tr>
<td>$D_{Own}$</td>
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<td>6.322***</td>
<td>5.414***</td>
<td>5.395***</td>
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<td>(0.727)</td>
<td>(0.755)</td>
<td>(0.922)</td>
<td>(0.952)</td>
</tr>
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<td>Value</td>
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<td>-0.671***</td>
<td>-0.674***</td>
</tr>
<tr>
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<td>(0.0528)</td>
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<td>(0.0644)</td>
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<td>2.217*</td>
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<td>(1.331)</td>
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<td>(0.426)</td>
<td>(0.411)</td>
<td>(0.494)</td>
<td>(0.529)</td>
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<tr>
<td>Loss-sq</td>
<td>2.018***</td>
<td>2.106***</td>
<td>1.663***</td>
<td>1.702***</td>
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<td>(0.520)</td>
<td>(0.511)</td>
<td>(0.630)</td>
<td>(0.634)</td>
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<tr>
<td>$Loss \times D_{AB284}$</td>
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<td>-1.701*</td>
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<tr>
<td>$Loss-sq \times D_{AB284}$</td>
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<td>0.499</td>
<td>(1.088)</td>
<td>(1.091)</td>
</tr>
<tr>
<td>LTV</td>
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<td>-2.312***</td>
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<td>(0.572)</td>
<td>(0.583)</td>
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<td>(0.132)</td>
<td>(0.129)</td>
<td>(0.174)</td>
<td>(0.190)</td>
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<td>(0.262)</td>
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<td>(0.559)</td>
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<td>-0.193**</td>
<td>-0.108</td>
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<td>(0.0998)</td>
<td>(0.114)</td>
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<td>(0.149)</td>
</tr>
<tr>
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<td>-0.319**</td>
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<td>(0.160)</td>
</tr>
<tr>
<td>$Loss-sq \times D_{AB284}$</td>
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<td>0.433</td>
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<td>(0.334)</td>
</tr>
<tr>
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<td>(0.114)</td>
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<td>-0.263</td>
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</tr>
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<td>-0.111*</td>
<td>-0.122*</td>
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<td>-0.236**</td>
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<td>(0.106)</td>
</tr>
<tr>
<td>Residuals</td>
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<td>0.0337</td>
<td>0.0337</td>
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<tr>
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<td>(0.0444)</td>
<td>(0.0444)</td>
<td>(0.0444)</td>
<td>(0.0444)</td>
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<tr>
<td>Residuals* $D_{AB284}$</td>
<td>0.136</td>
<td>0.0394</td>
<td>(0.0946)</td>
<td>(0.0946)</td>
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</table>

Observations 14,956 14,956 14,956 14,956

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
7. Discussion

When looking at the market as a whole, evidence for the standard effect of loss aversion was found. As loss increases, home sellers had a higher asking price, a higher final sale price and stayed on the market longer.

The expected effect of Assembly Bill 284 was to increase the effect of loss aversion when the number of homes on the market was reduced. The change in the measured effect of loss was negative in the asking price regression and not significant in the hazard model. Only in the final sale price did we see that after AB 284 the effect of loss aversion increased.

Splitting the market into owner occupied, not sold at short sale homes and other homes results in a positive coefficient on LTV for the OWN group while the coefficient on LTV for the OTHER group was not significant. The coefficient on loss, however, became negative for the OWN group with the effect of AB 284 being positive but not significant. As mentioned before, it was expected that lacking the homes that were listed but not sold would bias the estimate on loss downward. However, a different possible explanation is that the data was during a severe decline in housing prices and loss averse home owners lowered their asking prices to sell faster to avoid future potential losses.

When looking at the market as a whole, the sale price regression results are as expected under the standard loss aversion model. Not only were the coefficients on loss and LTV positive and significant, the effect of loss on the final sale price of the home increase after AB 284 as originally predicted.

Once the market is separated into OWN group and OTHER group, the story changes. The coefficient on the loss dummy is positive while the coefficient on loss is
negative which indicates that smaller losses do result in a higher selling price but as the loss increases, the selling price goes down. This suggests that in the OWN group, a seller with a sufficiently large loss accepted a much lower final sale price. This is consistent with the prior conclusion that sellers with a loss were trying to sell the home faster so as to avoid experiencing a higher loss in a declining market.

The hazard model results looking at the market as a whole came out as predicted by the loss aversion model, the coefficients on loss and LTV are both negative, meaning as either the loss or LTV increases, the time to sale increases. The LTV results are not significant, however.

Once the market is split into the OWN group and OTHER group the results are unexpected. While the measured effect of loss is positive, meaning the home sells faster, the coefficient is not significant. The coefficient on loss-squared is positive indicating that as the loss increased, the time to sell decreased.

**Conclusion**

The expected effects of loss aversion were seen in the market as a whole but not when the market was separated. From figure 3, we can see that for the majority of the asking price data the housing market was on a decline. A person who is loss averse wants to avoid a loss. In a significantly falling market, that may mean having a lower asking price and selling faster to avoid potential future losses if the home is held.

This is similar to the effect seen in the stock market. When prices are rising, people are more hesitant to sell in the hopes of further gains or reduced losses by holding. When stocks are falling, there is a rush to sell to avoid losses. The housing market tends to have a longer scope than the housing market since stocks are more liquid. This means
a housing market downturn needs to be longer to see home owners trying to sell faster. The time frame of this paper provides such an opportunity.

There is another possible explanation for these results. Since mortgage data on
the homes was not available, a few assumptions had to be made in order to estimate the
loan-to-value ratio. It was assumed the buyer had an average down payment and got an
average interest rate at the time of sale, that payments were made consistently on time,
and that the buyer did not either refinance or take out a second mortgage.

If the owner had late or missing payments or took out a second mortgage, the amount of
the loan could be significantly higher. As home values rose, taking out a second
mortgage becomes more attractive. If the amount owed on the homes was larger than
expected, than the estimate for the coefficient on LTV would be biased downward. To
determine if this was simply an unusual time or if the results were due to the assumptions
needed would require analyzing a similar market during a downturn with actual mortgage
data.

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Chapter 2: The Implications of Discounting Publicly Financed Charity

1. Introduction

The earliest models of charity assumed charity is a public good. In particular, Warr (1982) and Roberts (1984) assumed givers in society care about the aggregate consumption of the poor, so they get utility from the total amount of charity available regardless of any one individual’s contribution. As a public good, charity would be expected to be underprovided as each individual gets to consume the giving of everyone else. This is the standard justification for public provision of public goods.

However, givers might also gain utility from the individual act of giving, which implies there may also be a private good aspect to charity. Andreoni (1989) coined the term “warm-glow” to refer to this second source of utility, the positive feeling a giver receives when giving, perhaps from “feeling I am doing my part” to help others or from “feeling I am contributing to a common goal.” Thus, in general, charitable giving has both a public good aspect to it and a private good aspect.

The primary purpose of charitable giving theory has been to examine the tradeoff that may exist between helping the poor using tax dollars, or “public giving,” versus using voluntary contributions, or “private giving.” Warr (1982) and Roberts (1984) showed, when charity only has a public good aspect it is best to help the poor through public giving. Thus, societal welfare is maximized when public giving totally crowds out private giving. Andreoni (1989) showed, when charity has a private good warm glow aspect, in addition to the public good aspect, then government should not totally crowd
out private giving. Rather, social welfare is maximized when some private giving and some public giving both exist.

This work shows a warm glow effect need not be introduced to make private charitable giving socially useful. To compare the different models of charitable giving, the War and Roberts model will be expanded to from a three person model to an $N$ person model. By introducing this more generalized form, this original model can be compared to the Andreoni model. The second part in the approach is to extend the standard charity model by recognizing private giving and public giving may differ either in their efficiency with regard to how they help the poor or in how they are viewed by givers. In these respects, Warr, Roberts and Andreoni each assumed public giving through taxation is equivalent to private charitable giving.

Here, it is shown that, if public giving is discounted, then private giving may be socially useful even when the giver does not experience a warm glow. An increase in public giving through taxation does not fully crowd out the private giving of utility maximizers and should not. Combining the discounting with the warm-glow effect further increases the social usefulness of private giving. Though the main focus in this paper is on discounting, the opposite, placing a premium on public giving, is also possible.

The paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents the original model of charity using a framework that can be generalized, and the original results are reviewed. Section 4 introduces the distinction of private and public funding and examines how the results of the original model are changed. Section 5 adds a warm glow for private giving, to examine how distinguishing
private from public funding influences the results in the warm glow model. Section 6 concludes.

2. Literature Review

The key insight is that givers have different views on the effectiveness of government spending. Givers may be too distant from government giving to see it as effective while they may give to and work with private charities directly. Government may also face diseconomies of scale so that public giving is wasteful and possibly even harmful.

Currie (2005) lists five reasons she believes people support cutting welfare spending to reduce the federal deficit. Two of these are possible explanations for why tax financing a charitable public good might be discounted. The first is that programs are sometimes ineffective and the second is that some programs are rife with waste and fraud. Therefore, givers may well view these types of government spending as less effective in helping the poor compared to their private counterparts.

Another problem to consider is that welfare programs could work against the poor who have self-control problems or discount hyperbolically. Beaulier and Caplan (2007) examine the negative effects of welfare programs from a behavioral perspective. They claim that the poor suffer from self-control problems to a larger degree than the general population. Welfare programs that provide an immediate benefit for avoiding menial low paying work prevent people from gaining valuable job skills that would make them better off in the future. If givers view government programs as discouraging people from getting valuable job skills, they may view private charity that helps the poor learn jobs skills as more effective.
Public housing may also work against the poor. Griffiths and Tita (2009) conclude that public housing developments are hotbeds of violence. They also found that those on housing assistance are twice as likely to be victims of gun violence. The Pruitt-Igoe housing development in New Orleans suffered from major design flaws and was eventually demolished (Bristol, 1991). Schill (1993) examined the plans by the National Commission on Severely Distressed Public Housing determined that demolishing existing public housing and switching to a demand oriented subsidies such as voucher programs would improve the benefits. The failures of these large scale government housing projects may convince givers that public aid is wasted and that private charity may do a better job.

There may also be religious motivations behind believing that private charity is better than public giving. The Jewish Rabi Moses Maimonides (trans. 2003) listed eight ways for charitable giving each greater than the next. The first is to help someone become independent, for example by teaching them a trade or giving them a loan to start a business. The next six involve giving directly to the poor voluntarily and vary depending on whether either the giver or recipient knows the other. The last and least best way to give is to give unwillingly as in through taxation. Under the teachings of Maimonides voluntary private giving was better than compulsory public giving.

Early models of charity (Warr 1982, Roberts 1984) analyzed charity as a form of altruism, where the utility of givers is a function of their personal consumption as well as the consumption of the recipient. Since utility is dependent only on how much is given irrespective of the source, government contributions are perfect substitutes for private giving and complete crowd-out is expected. Free riding is also a potential problem.
because the giving of person A contributes to the utility of person B. The conclusion reached, is that government should take over giving in order to eliminate free-riding and to reach the Pareto optimal level of charity.

Rather than being completely crowded-out as predicted, as government became more involved in charitable giving, private giving underwent a transformation (Roberts 1984). The focus of private charity changed from the alleviation of poverty to health and social services. According to Roberts, data from the New Deal Era supports the conclusion that private transfer payments to the poor fell almost to zero. As the basic needs of the poor were being met by government programs, charitable spending was redirected to other areas. While government anti-poverty programs did not completely crowd-out charitable giving, it did change the way in which it manifested itself.

Andreoni (1993) provided the most favorable circumstances for complete crowd-out in a public goods experiment, but was still only able to find partial crowd-out.

Ferris and West (2003) attempted to explain why only partial crowd-out is found. They proposed that the cost of giving is different for private donors than for the government. Private donors have different ideas about who is worthy and how they can best be assisted. They find that even if private and public giving are perfect substitutes, the differences in the costs of giving can account for the partial crowd-out.

Some studies failed to find crowd-out. Khanna, Posnett and Sandler (1995) examined 159 of the most prominent charities in the UK from 1983 to 1990 and found that government contributions crowded-in giving. Khanna and Sandler (2000), with a slightly different model, used data on 159 UK charities from 1983 to 1990 and found modest crowd-in. Manzoor and Straub (2005) looked at donation data to a public radio station from 1996 and failed to find evidence for either crowd-out or crowd-in.

Most studies have only looked at the possibility of government crowding-out private giving. However, if government decreased contributions to a charity in response to increased private giving, the analysis would support the conclusion that private contributions crowd out government contributions. Heutel (2009) used instrumental variables analysis to determine the direction of causality and found that government contributions crowd-in private giving but that private contributions actually crowd-out government grants. The implication is that government funding acts as a signal to private donors where as private giving signals that the charity is not in need and results in reduced government contributions.
While the original theory of public giving shows that crowd-out should be complete, often this is not the case. The assumption made by Warr and Roberts was that givers get utility from other people’s consumption. If givers get satisfaction from the act of giving, a new model is needed to account for this warm-glow effect. Andreoni’s (1990) impure altruism or warm-glow model adds private giving to the utility function in addition to total charitable giving. The result of his model suggests that crowd-out should be partial.

Because of specification issues, measurements of crowd-out may not be correct (Kingma 1989). Measures of crowd-out are based on aggregating both private giving and government giving at the highest level. To get a true measure of crowd-out, Kingma (1989) examined the individual contributions to a public radio station. He was able to find crowd-out as consistent with the impure altruist model.

Some experiments have been conducted to test the warm-glow hypothesis. Tonin and Vlassopoulos (2010) conduct an experiment at the University of Southampton with existing college students. Using three separate conditions ordered randomly, the participants were asked to allocate money between themselves and either the experimenter or a charity. The results of the experiment lend support to the model of impure altruism.

Other motivations play a part in how an individual decides to allocate charitable giving. Rotemberg (2011) applied two assumptions from psychology to explain how givers give. The first assumption is that givers are happier when they learn that there is more agreement with their point of view. The second is that they have warmer feelings towards, and are more willing to help, individuals whom they perceive as sharing their
beliefs or, more generally, individuals who are more similar to themselves. Under different circumstances, his model can predict both crowd-out and crowd-in.

Giving can also be accounted for by how worthy the giver views the receiver. Fong (2007) conducted an experiment using actual welfare recipients. Participants in the study were given $10 and could decide how much to share with the person on welfare. When the recipient was viewed as worthy, i.e. his condition was due to bad luck rather than laziness, those who scored high on a humanitarianism measure gave a median amount of $5 while those who scored low gave $1. When the recipient just appeared lazy, the median donation for participants with high measures was no different from those with a low score, $1.

3. The Original Model of Charity

Warr (1982) and Roberts (1984) developed models of charity which are similar. Here, a representation of these models is presented which will be called the Original Model of Charity.

There are $N$ individuals in the economy, and $\rho$ is the proportion of the population that is poor.\(^1\) The total level of charity provided as a public good is $Y$. Thus, $Y / [\rho N]$ is the charity level provided per poor person.

The utility of decision maker $i$, who will be referred to as DM when convenient, is a function of the private consumption level $x_i$ and the charity per poor person $Y / [\rho N]$, so $U_i = U(x_i, Y / [\rho N])$. Each individual’s budget is constrained by individual real wealth, $w_i$. This wealth is used to finance private consumption $x_i$, private charitable

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\(^1\) Warr and Roberts each considered a 3 person model with 2 rich givers and 1 poor recipient of charity. Here the model has been recast to include $N$ individuals and the poor proportion $\rho$ to be consistent with later models and consistent with the extensions of interest in this paper.
giving $g_l$ and the tax $t_i$ paid to government, so $w_i = x_i + g_l + t_i$. Utility increases at a decreasing rate in both $x_i$ and $Y/\rho N$, implying the restrictions $\frac{\partial u}{\partial x_i} = U_x > 0$, $\frac{\partial u}{\partial x_i} = U_x < 0$, $\frac{\partial u}{\partial y} = U_y > 0$ and $\frac{\partial u}{\partial y} = U_y < 0$.

Charity is a public good financed by private giving $G$ and taxation $T$, so $Y = G + T$. Total private giving is given by $G = \sum_{n=1}^{(1-\rho)N} g_n$ and total taxation is given by $T = \sum_{n=1}^{(1-\rho)N} t_n$. It will often be convenient to write total private giving as the sum of DM’s private giving $g_l$ and the private giving of others, $G_{-l}$, so $G = g_l + G_{-l}$. Similarly, total tax funding can be written as $T = t_i + T_{-i}$.

DM’s decision problem is to choose individual consumption and giving levels that maximize personal satisfaction. Formally, the problem is

$$\max_{x_i, g_l} U_i = U(x_i, Y/\rho N),$$

subject to $w_i = x_i + g_l + t_i$ and $Y = g_l + G_{-l} + t_i + T_{-i}$.

The first order condition for utility maximization determines the optimal level of private giving:

$$U_x \left( w_i - g_l^* - t_i, \frac{g_l^* + G_{-l} + t_i + T_{-i}}{\rho N} \right) = \frac{1}{\rho N} U_y \left( w_i - g_l^* - t_i, \frac{g_l^* + G_{-l} + t_i + T_{-i}}{\rho N} \right).$$

The optimal level of private giving $g_l$ depends upon DM’s individual tax payment $t_i$, the tax payment of others $T_{-i}$, the giving of others $G_{-i}$, the wealth level $w_i$, the proportion $\rho$ who are poor, and the population level $N$.

The comparative statics of historical interest have been those examining how a change in taxation impacts private giving. For simplicity, assume all givers have identical taxes, wealth, and tastes and preferences. This implies $t_i = t$, so total taxation
is $T = (1 - \rho)Nt$. It also implies $g_i^* = g^*$ for all optimizing givers, and total giving is $G = (1 - \rho)Ng^*$. The first order condition result (2) can be rewritten as

$$U_x \left( w - g^* - t, \frac{(1-\rho)Ng^* + (1-\rho)Nt}{\rho N} \right) = \frac{1}{\rho N} U_Y \left( w - g^* - t, \frac{(1-\rho)Ng^* + (1-\rho)Nt}{\rho N} \right).$$

As shown in the Appendix, the marginal impact of a tax increase on the giving of DM is $\frac{dg}{dt} = -1$. The intuition is that the increased funding of charity through taxation increases the marginal utility of private consumption and decreases the marginal utility of the charity. Each giver reduces giving in response, and the two marginal utilities move toward each other. Because private giving and public giving (through taxation) are perfect substitutes for providing the public good, the only way the two marginal utilities can be equated is for each giver, including DM, to maintain the same level of consumption as before the tax change. For each giver, including DM, additional forced public giving through taxation crowds out private giving dollar for dollar.

Moving on, should government completely crowd out private giving? To consider this question, let $t^c$ denote that level of taxation at which public charity just crowds out private charity. That is, $g > 0$ will optimally hold when $t < t^c$, but $g = 0$ will be optimal when $t \geq t^c$. It follows that, when $t \geq t^c$, the utility of the representative giver is

$$U(x, Y/[\rho N]) = U \left( w - t, \frac{(1-\rho)N}{\rho N} t \right).$$

For $t > t^c$, the marginal utility of taxation for DM is given by

$$\frac{\partial u}{\partial t} = -U_x (w - t, \frac{(1-\rho)N}{\rho N} t) + \frac{(1-\rho)}{\rho} U_Y \left( w - t, \frac{(1-\rho)N}{\rho N} t \right),$$

and (5) holds in the limit as $t$ approaches $t^c$ from above. When $t = t^c$, the first order condition (2) becomes
Substituting (6) into (5) yields

\[
\frac{\partial U}{\partial t} = \left( \frac{(1 - \rho)N - 1}{\rho N} \right) \cdot U_Y \left( w - t^c, \frac{(1 - \rho)N}{\rho N} t^c \right).
\]

The derivative (7) is positive when \((1 - \rho)N > 1\), meaning there is more than 1 giver. That is, as long as there is more than one giver, government can increase the utility of givers by increasing taxation beyond the minimum tax level required for total crowd-out. This is the same conclusion that was reached by both Warr and Roberts.

Why should government crowd-out private giving? The intuition is as follows. When charity is entirely funded through private giving, a one dollar increase in private giving by a single individual leads to a one dollar increase in the total level of charity. However, when the charity is funded exclusively through public financing, the total level of the charity increases by \((1 - \rho)N\) dollars when the tax increases by one dollar because the tax is increased on each individual. DM’s personal giving does not create a positive externality, but the same sacrifice for DM associated with a tax increase in essence creates a positive externality because everyone else is also forced to give more.

It is worth illustrating these ideas using a Cobb-Douglas utility function. Let the utility of DM under a uniform tax rate be given by

\[
U(x, Y/(\rho N)) = x^\alpha \left( Y/(\rho N) \right)^\beta = \left( w_l - g_l - t \right)^\alpha \left( \frac{g_l + G_{-l} + (1 - \rho)Nt}{\rho N} \right)^\beta.
\]

The first order condition of the Cobb-Douglas utility function is

\[
\frac{\alpha}{w_l - g_l - t} = \frac{\beta}{g_l^* + G_{-l} + (1 - \rho)Nt}
\]

This first order condition implies the following utility maximizing level of giving by DM
\( g_i^* = \frac{\beta}{\alpha+\beta} w_i - \frac{\alpha}{\alpha+\beta} G_i - \frac{\alpha(1-\rho)N+\beta}{\alpha+\beta} t. \)

If all givers have identical taxes, wealth, and tastes and preferences, then \( x_i = x, w_i = w \) and \( g_i^* = g^* \), so (10) simplifies to

\( g^* = \frac{\beta}{\alpha(1-\rho)N+\beta} w - t. \)

From (11) it is easy to see that \( \frac{dg}{dt} = -1. \)

When government sets the taxation level at or above the minimum tax required for total crowd-out, \( t^c \), the utility function for each decision maker can be written as

\( U(x, \frac{y}{\rho N}) = x_i^\alpha \left( \frac{y}{\rho N} \right)^\beta = (w - t)^\alpha \left( \frac{(1-\rho)}{\rho} t \right)^\beta \)

Due to complete crowd-out, (8) is constant when the tax is between zero and the total crowd-out level, \( 0 \leq t \leq t^c \). At \( t = t^c \), (8) and (12) are equal so government should totally crowd-out private giving if (12) has a maximum at \( t > t^c = \frac{\beta}{\alpha(1-\rho)N+\beta} w. \) Taking the derivative of (12) with respect to the tax level and solving the first-order condition for \( t \), the utility maximizing tax is

\( t = \frac{\beta}{\alpha+\beta} w. \)

This utility maximizing tax level will be greater than the minimum tax required for total crowd-out \( t^c = \frac{\beta}{\alpha(1-\rho)N+\beta} w \) if and only if \( (1 - \rho)N > 1 \). This is consistent with the result obtained from equation (7) for the general utility function case.

To summarize, in the Original Model of Charity, government should totally crowd-out all private giving when the number of givers is greater than one. When the tax for charity is set below the socially optimal level, such that the representative giver optimally gives some positive amount, an increase in private giving by a single giver is not optimal
because a one dollar increase in giving implies only a one dollar increase in charity and the gain in utility from helping the poor is more than offset by the loss of individual consumption. Alternatively, a one dollar increase in taxation implies more than a one dollar increase in charity because all are forced to give. This significant help for the poor increases the utility of the representative giver more than enough to compensate for the loss of individual consumption. That is, in the Original Model of Charity, the problem of providing sufficient charity is equivalent to the problem of sufficiently providing a pure public good.

4. **Distinguishing Private Giving from Public Giving**

Now, let’s innovate and extend the Original Model of Charity by relaxing the assumption that private giving and public giving are equally efficacious. Specifically, assume DM perceives public good consumption as \( Y_i = G + \delta_i T \), where \( \delta_i \) is a discount factor, representing the extent to which DM perceives public giving through taxation as less effective than private giving. While \( \delta_i < 1 \) might be most natural to assume, this restriction need not be imposed. The situation \( \delta_i > 1 \) would be one where DM perceives public giving as being more efficacious than private giving.

With the distinction between private and public giving, DM’s decision problem becomes

\[
\max_{g_i, x_i} U_i = U(x_i, Y_i/(\rho N)),
\]

subject to \( w_i = x_i + g_i + t_i \) and \( Y_i = g_i + G_{-i} + \delta_i t_i + \delta_i T_{-i} \).

The first order condition which determines the optimal level of private giving becomes

\[
U_x \left( w_i - g_i^* - t_i, g_i^* + G_{-i} + \delta_i t_i + \delta_i T_{-i} \right) = \frac{1}{\rho N} U_Y \left( w_i - g_i^* - t_i, g_i^* + G_{-i} + \delta_i t_i + \delta_i T_{-i} \right).
\]
When all givers have identical taxes, wealth, and tastes and preferences, the first order condition can be rewritten as

\[(16) \quad U_x \left( w - g - t, \frac{(1-\rho)Ng + \delta(1-\rho)Nt}{\rho N} \right) = \frac{1}{\rho N} U_y \left( w - g - t, \frac{(1-\rho)Ng + \delta(1-\rho)Nt}{\rho N} \right).\]

The impact of a tax increase on private giving is given by

\[(17) \quad \frac{dg}{dt} = -\frac{u_{xx}^{1+\delta(1-\rho)N}u_{xy}^{\delta(1-\rho)}u_{yy}^{1-\rho}}{u_{xx}^{1+\delta(1-\rho)N}u_{xy}^{1-\rho}u_{yy}^{1+\rho N}}.\]

When \(\delta = 1\), the multiplier is \(-1\) as in the Original Model. However, when \(\delta < 1\), the multiplier is greater than \(-1\), or closer to zero, which means crowd-out is not complete.

Consider now the intuition. As in the Original Model, a tax increase reduces private consumption and causes an increase in the marginal utility of the private good relative to the public good for DM. DM optimally responds by decreasing private giving to restore some of the lost private consumption. When funding the public good through taxation is discounted, there is no longer a one for one tradeoff between private giving and public giving. Private giving is more potent, so a smaller decrease in private giving is needed to offset the increase in public giving obtained through the tax increase. Because private giving does not decrease as much as public giving increases, DM will decrease private consumption in response to the increase in tax funding. For this reason, DM’s utility will decrease in response to a tax increase, which is not true in the Original Model.

To examine the effect of the tax on utility more carefully, consider again the case where all givers have identical taxes, wealth, and tastes and preferences so \(t_i = t\), \(g_i = g\), \(w_i = w\) and \(\delta_i = \delta\) for all \(i\). The marginal utility of taxation when every giver maximizes utility is
(18) \[ \frac{\partial U}{\partial t} = -(1 + \frac{dg}{dt}) \cdot U_x \left( w - g - t, \frac{g + \delta(1 - \rho)Nt}{\rho N} \right) + \left( \frac{(1 - \rho)N + \frac{dg}{dt}}{\rho N} \right) \cdot U_y \left( w - g - t, \frac{g + \delta(1 - \rho)Nt}{\rho N} \right). \]

Since \( \frac{dg}{dt} = (1 - \rho)N \frac{dg}{dt} \), combining (15) and first order condition (18) one obtains

(19) \[ \frac{\partial U}{\partial t} = [\delta(1 - \rho)N - 1 + ((1 - \rho)N - 1) \cdot \frac{dg}{dt}] \cdot U_x \left( w - g - t, \frac{g + \delta(1 - \rho)Nt}{\rho N} \right). \]

For an increase in taxation to increase utility, (19) must be positive. This occurs when

(20) \[ \delta > \frac{1}{(1 - \rho)N} - \frac{1 - (1 - \rho)N - 1 \cdot \frac{dg}{dt}}{1 - (1 - \rho)N}. \]

Substituting (17) into (20) one obtains

(21) \[ \delta > \frac{1}{(1 - \rho)N} - \frac{1 - (1 - \rho)N - 1}{(1 - \rho)N} \left( -\frac{U_{xx}}{U_{xx} + \delta(1 - \rho)N U_{xy} + \frac{(1 - \rho)}{\rho N} U_{yy}} - \frac{U_{xy}}{U_{xx} + \delta(1 - \rho)N U_{xy} + \frac{(1 - \rho)}{\rho N} U_{yy}} \right). \]

Solving for \( \delta \) yields

(22) \[ \delta > 1. \]

This means that utility is only increasing in taxes when tax funding of the public good is more effective than private funding. When (22) is satisfied, total crowd-out will be welfare maximizing.

If (22) is not satisfied, tax funding may still be welfare enhancing if utility increases with increased taxes past the point of total crowd-out and the maximum utility past the crowd-out point is greater than the utility at \( t = 0 \). To examine this possibility, set the tax rate equal to the minimum tax required for total crowd-out, \( t = t^c \) so that \( g_l = 0 \) is optimal for all givers. This implies the marginal utility of the tax with no private funding of the public good is

(23) \[ \frac{\partial U}{\partial t} = -U_x \left( w - t^c, \delta \frac{(1 - \rho)N}{\rho N} t^c \right) + \delta N U_y \left( w - t^c, \delta \frac{(1 - \rho)N}{\rho N} t^c \right). \]
When the tax rate is at the minimum tax level required for total crowd-out, the welfare maximizing level of private funding to the public just becomes zero and equation (16) still holds. Substituting (16) into (23) yields

\[
\frac{\partial u}{\partial t} = (\delta (1 - \rho)N - 1) \ast U_x \left( w - t, \delta \frac{(1 - \rho)N}{\rho N} t \right)
\]

Equation (24) is positive if \(\delta (1 - \rho)N > 1\).

Note first that the condition \(\delta (1 - \rho)N > 1\) reduces to \(\delta > 1\) when \(N = 1\). That is, the primary result for the extended model reduces to the primary result for the Original Charity Model when public giving is not discounted. As a reminder, this primary result is that it is socially optimal for government to entirely crowd out private giving, so that charity is entirely financed through taxation.

However, when \(\delta < 1\), matters become more interesting.

From the previous discussion surrounding the \(dg/dt\) multiplier (17) it was shown that DM’s utility decreases as the tax rate is increased from zero and DM optimally responds by reducing private giving. That is, until total crowd out occurs, DM experiences maximal utility when \(t = 0\) and charity is only financed with private giving. For now, call this private giving level \(g^*\). From condition (24), if \(\delta (1 - \rho)N < 1\), or \(\delta < 1/[(1 - \rho)N]\), then DM’s utility continues to decrease as the tax level increases beyond the level \(t_c\) where private giving is totally crowded out. Thus, if government financed charity is ineffective enough (i.e. \(\delta\) small enough), then it is socially optimal that all charity be privately financed with the private giving level equal to \(g = g^*\).

If \(\delta (1 - \rho)N > 1\), then condition (24) alternatively indicates utility will start increasing once the taxation level surpasses \(t_c\). Thus, it is still possible that financing
charity with taxation is socially optimal, with private charity being totally crowded out. When \( t > t_c \), the optimal level of taxation is \( t^* > t_c \) which sets \( \frac{\partial U}{\partial t} \) in condition (23) equal to zero. If \( U \left( w - t^*, \delta \frac{(1-\rho)N}{\rho N} t^* \right) > U \left( w - g^*, \frac{(1-\rho)}{\rho} g^* \right) \), then total crowd-out will be optimal with charity being entirely financed publically. Otherwise, no public financing is best and charity should be entirely financed with private giving.

This discussion indicates there exists a discount level \( \delta^* \) such that private financing and public financing are equal in terms of maximizing social utility. To consider this more formally, let

\[
(25) \quad \Psi(\delta) = U \left( w - g^*, \frac{(1-\rho)}{\rho} g^* \right) - U \left( w - t^*, \delta \frac{(1-\rho)N}{\rho N} t^* \right).
\]

Assuming \( U(w - t^*, 0) = 0 \), so that no public good implies no utility, then \( \Psi(0) = U \left( w - g^*, \frac{(1-\rho)}{\rho} g^* \right) > 0 \). That is, if public financed charity does no good, then private financed charity is superior to public financed charity. From this starting point of \( \delta = 0 \), the function \( \Psi(\delta) \) is strictly decreasing in \( \delta \). \( \delta = 1 \) is the original charity model where \( U \left( w - t^*, \frac{(1-\rho)N}{\rho N} t^* \right) > U \left( w - g^*, \frac{(1-\rho)}{\rho} g^* \right) \) as long as \( (1-\rho)N > 1 \). Thus, assuming \( (1-\rho)N > 1 \), \( \Psi(1) < 0 \). Since the function \( \Psi(\delta) \) is continuous and strictly decreasing, there must then be a unique level of \( \delta \), call it \( \delta^* \), such that \( \Psi(\delta^*) = 0 \).

To summarize, even when government financed charity is not as efficacious as private financed charity, it may still be true that it is best that private charity be crowded out by publically financed charity, but it need not be true. Because public charity is inferior when it is less efficacious, using it in small amounts is not socially useful. Substituting public charity for private charity diminishes well-being up to a point.
However, if public charity is not too inefficacious, the public good aspect of increasing
taxes on all to finance charity which individuals appreciate implies it will still be best for
government to totally crowd out private charity and finance a socially optimal level of
charity with tax dollars.

It is useful to illustrate the extended model using the Cobb-Douglas Utility
function, where now utility is given by

\[ U(x_i, Y_i/(\rho N)) = x_i^\alpha \left( \frac{Y_i}{\rho N} \right)^\beta = (w_i - g_i - t)^\alpha \left( \frac{g_i + g_i + \delta_i (1 - \rho) Nt}{\rho N} \right)^\beta. \]

The first order condition for this Cobb-Douglas Utility function is

\[ \frac{\alpha}{w_i - g_i - t} = \frac{\beta}{g_i + g_i + (1 - \rho) N\delta t}. \]

Solving the first order condition for the utility maximizing level of giving by DM yields

\[ g^* = \frac{\beta}{\alpha(1 - \rho) N + \beta} w - \frac{\alpha \delta (1 - \rho) N + \beta}{\alpha(1 - \rho) N + \beta} t. \]

From here, it is easy to see how an increase in the tax affects the level of giving, \( \frac{dg}{dt} = \)

\[ \frac{\alpha \delta (1 - \rho) N + \beta}{\alpha(1 - \rho) N + \beta}. \]

Solving (28) for \( g = 0 \), private giving will be totally crowded-out when \( t = \)

\[ \frac{\beta}{\alpha \delta (1 - \rho) N + \beta} w. \] Past this minimum tax level required for total crowd-out, the utility
function can be written as

\[ U(x, \frac{Y}{\rho N}) = x^\alpha \left( \frac{Y}{\rho N} \right)^\beta = (w - t)^\alpha \left( \delta \frac{(1 - \rho)}{\rho} t \right)^\beta \]

Since (26) and (29) are equal at \( t = \frac{\beta}{\alpha \delta (1 - \rho) N + \beta} w \), if (29) has a maximum at

\[ t^* > \frac{\beta}{\alpha \delta (1 - \rho) N + \beta} w \] and \( U(w - t^*, \delta \frac{(1 - \rho)}{\rho} t^*) > U(w - g^*, \frac{(1 - \rho)}{\rho} g^*) \) government should
totally crowd-out private giving. Maximizing (29) with respect to the tax and solving the first order condition for \( t \), the potential utility maximizing tax is

\[
t^* = \frac{\beta}{\alpha + \beta} \omega.
\]

Consistent with the work using the general utility function above, this potentially utility maximizing tax is greater than the minimum tax level for total crowd-out as long as

\[
\delta > 1/[(1 - \rho)N].
\]

Figure 1 presents a number of cases, under the assumption \( 1/[(1 - \rho)N] < \delta < 1 \). The value \( \delta = \delta^* \) is particularly interesting because private financing is best for \( \delta < \delta^* \) while public financing is best for \( \delta > \delta^* \). Solving (28) for when \( t = 0 \) and substituting into (26) then substituting (30) into (29), total crowd-out is welfare enhancing when

\[
\delta > \delta^* = \frac{\alpha + \beta}{\alpha(1 - \rho)N + \beta} \left( \frac{(\alpha + \beta)(1 - \rho)N}{\alpha(1 - \rho)N + \beta} \right)^\alpha / \beta
\]

Figure 1 was created with \( \alpha = 0.4, \beta = 0.8, N = 5, \rho = 0.3 \) and \( \omega = 1000 \) so that \( \delta^* = .7537 \). With any amount of discounting, an increase in the tax rate from zero to publically finance charity reduces utility as DM responds (and all others respond) by reducing the undiscounted private giving. However, once private giving is entirely crowded out, utility increases with further tax increases because DM (and each other) experiences a public good externality. DM (and each other) loses utility from the tax increase, but this is more than offset by the increase in the utility coming from the increase in charity provided by the publically financed charity coming from all those being taxed. As long as \( \delta > \delta^* = .7537 \), it is best for government to totally crowd out private giving so only government helps the poor. However, if government charity is
discounted enough, so $\delta < \delta^* = .7537$, then it is best that government not be involved in providing charity. If $\delta = 0.2857$, utility is always decreasing in the tax.

**Figure 1: Impact of Discounting Public Financing**

![Graph showing impact of discounting public financing](image)

5. **Warm Glow**

Andreoni (1989) developed a model of impure altruism or warm glow giving. The Andreoni model can be considered an extension of the Original Model in that the giver receives utility through the public good. However, the giver also receives utility directly from his individual private giving, a warm glow. Adding a warm-glow to the model of discounted public financing, the formal problem can be written as

$\max_{g, x_i} U_i = U(x_i, Y_i/(\rho N), g_i)$. 
subject to \( w_i = x_i + g_i + t_i \) and \( Y_i = g_i + G_{-i} + \delta_i t_i + \delta_i T_{-i} \).

The first order condition for utility maximization is

\[
(33) \quad U_x \left( w_i - g_i^* - t_i, \frac{g_i^* + G_{-i} + \delta_i t_i + \delta_i T_{-i}}{\rho N}, g_i^* \right) =
\]

\[
\frac{1}{\rho N} U_Y \left( w_i - g_i^* - t_i, \frac{g_i^* + G_{-i} + \delta_i t_i + \delta_i T_{-i}}{\rho N}, g_i^* \right) +
\]

\[
U_g \left( w_i - g_i^* - t_i, \frac{g_i^* + G_{-i} + \delta_i t_i + \delta_i T_{-i}}{\rho N}, g_i^* \right)
\]

When all givers have identical taxes, wealth, and tastes and preferences, the first order condition can be rewritten as

\[
(34) \quad U_x \left( w - g - t, \frac{g + [(1 - \rho) N - 1] g + \delta (1 - \rho) N t}{\rho N}, g \right) =
\]

\[
\frac{1}{\rho N} U_Y \left( w - g - t, \frac{g + [(1 - \rho) N - 1] g + \delta (1 - \rho) N t}{\rho N}, g \right) +
\]

\[
U_g \left( w - g - t, \frac{g + [(1 - \rho) N - 1] g + \delta (1 - \rho) N t}{\rho N}, g \right)
\]

The impact of a tax increase on private giving is given by

\[
(35) \quad \frac{dg}{dt} = - \frac{U_{xx} + \delta^{1 - \rho} u_{xx} + \frac{1}{\rho N} u_{xy} - \frac{1}{\rho N} u_{xg} + \delta^{1 - \rho} u_{yg}}{U_{xx} + \frac{1 - \rho}{\rho N} u_{yy} + \frac{1 - \rho}{\rho N} u_{gg} - \frac{1 - \rho}{\rho N} u_{xy} - \frac{1 - \rho}{\rho N} u_{xg} + \delta^{1 - \rho} u_{yg}}
\]

As DM’s private giving increases, one might expect him to feel better about his own consumption which implies \( U_{xg} > 0 \). If total contributions to the public good decrease, one might expect that DM would feel better about his own contributions since it shows he cares more than others which implies \( U_{gy} < 0 \). From these assumptions and

(35) it can be concluded that \(-1 < \frac{dg}{dt} < 0\) when \( \delta \leq 1 \).

The warm glow and discounting of public giving reinforce each other. When there is no discounting (\( \delta = 1 \)), the warm glow of private giving reduces the impact of
taxation on giving. An increase in taxation for public giving does still crowd out private giving. However, condition (35) indicates the crowd out is not one for one because DM receives a warm glow from the giving. Showing that incomplete crowd occurs because giving provides a warm glow is the contribution of Andreoni. The contribution here is to show incomplete crowd out will also occur when the decision maker discounts the funding of charity through taxation. Specifically, condition (35) indicates the level of crowd-out gets closer to zero as $\delta$ decreases from 1 toward 0, indicating the degree of crowd out is getting further from being complete.

Since warm glow giving directly enters the utility function, total crowd-out will not occur if DM has a typical utility function. Thus, it is no longer of interest to ask whether government should totally crowd out private giving. However, it is of interest to ask whether some public financing should occur. When DM is maximizing utility with respect to private giving, the marginal utility of taxation is

\[
\frac{\partial U_i}{\partial t} = \left( -1 - \frac{dg}{dt} \right) * U_X \left( w - g - t, \frac{(1-\rho)Ng + \delta (1-\rho)Nt}{\rho N}, g \right) + \frac{1-\rho}{\rho} \left( \frac{dg}{dt} + \delta \right) *
\]

\[
U_Y \left( w - g - t, \frac{(1-\rho)Ng + \delta (1-\rho)Nt}{\rho N}, g \right) + \frac{dg}{dt} * U_g \left( w - g - t, \frac{(1-\rho)Ng + \delta (1-\rho)Nt}{\rho N}, g \right)
\]

Combining (36) and (33) one obtains

\[
\frac{\partial U}{\partial t} = \left[ \left( \frac{1-\rho}{\rho} - \frac{1}{\rho N} \right) \frac{dg}{dt} + \left( \frac{1-\rho}{\rho} \delta - \frac{1}{\rho N} \right) \right] * U_Y \left( w - g - t, \frac{(1-\rho)Ng + \delta (1-\rho)Nt}{\rho N}, g \right) -
\]

\[
U_g \left( w - g - t, \frac{(1-\rho)Ng + \delta (1-\rho)Nt}{\rho N}, g \right).
\]

The sign of (37) is ambiguous, meaning an increase in taxation may be welfare enhancing, but may not. In order for taxes to be welfare enhancing, it must be true that \( \frac{\partial U}{\partial t} > 0 \) when \( g \geq 0 \) for some \( t > 0 \). Solving for \( \delta \), \( \frac{\partial U}{\partial t} > 0 \) when
\[ \delta > \left( \frac{1-\rho}{\rho N} \right) \left( U_{XX} - \frac{1}{\rho N} U_{XY} - U_{Xg} \right) + \left( U_{XG} + \frac{1}{\rho N} U_{YY} + U_{g} - \frac{1-\rho}{\rho N} U_{XY} - 2U_{Xg} + \left( \frac{1-\rho}{\rho N} \right) U_{g} + \frac{1-\rho}{\rho N} U_{Yg} \right) \]

The case where \( \delta = 1 \) is of particular interest because that is the Andreoni case.

Substituting \( \delta = 1 \) into (37) results in

\[ \frac{\partial U}{\partial t} = \left[ \left( \frac{1-\rho}{\rho N} \right) - \frac{1}{\rho N} \right] \frac{dg}{dt} + \left( \frac{1-\rho}{\rho N} - \frac{1}{\rho N} \right) \] 

\[ U_g \left( w - g - t, \frac{(1-\rho)N+g+(1-\rho)Nt}{\rho N}, g \right) \]

From (39) the government should not tax finance charity when

\[ \left[ \left( \frac{1-\rho}{\rho N} \right) + \frac{1}{\rho N} \right] \left( 1 + \frac{dg}{dt} \right) \] 

\[ U_g \leq U_g \]

This condition is not surprising. Since the tax negatively affects giving, if the marginal utility of the warm glow is high enough, the tax will negatively affect utility.

This means that even without discounting, there is a condition where there should be no public financing of the charitable good.

To further illustrate this point, consider a Cobb-Douglas utility function

\[ U \left( x_i, \beta, g_i \right) = x_i^\alpha \left( \frac{y_i}{\rho N} \right)^\beta g_i^\gamma = \left( w_i - g_i - t \right)^\alpha \left( \frac{g_i + G - (1-\rho)Nt}{\rho N} \right)^\beta g_i^\gamma \]

The first order condition for utility maximization is

\[ \frac{\alpha}{w - g - t} + \frac{\beta}{(g + \delta t)(1-\rho)N} + \frac{\gamma}{g} = 0 \]

From the first order condition, solve for the utility maximizing level of private giving by DM,
(43) \[ g = \frac{[-\alpha(1-\rho)N\delta t + \beta(w-t) + \gamma(1-\rho)N(w-t(1+\delta))] \pm \sqrt{[-\alpha(1-\rho)N\delta t + \beta(w-t) + \gamma(1-\rho)N(w-t(1+\delta))]^2 + 4[\alpha(1-\rho)N + \beta + \gamma(1-\rho)N]\gamma(w-t)(1-\rho)N\delta t}}{2[\alpha(1-\rho)N + \beta + \gamma(1-\rho)N]} \]

Solving for how an increase in the tax affects the private giving of DM,

(44) \[ \frac{dg}{dt} = -\frac{g[-\alpha(1-\rho)N\delta - \beta - \gamma(1-\rho)N(1+\delta)] + \gamma w(1-\rho)N\delta - 2\gamma(1-\rho)N\delta t}{2g[-\alpha(1-\rho)N\delta - \beta - \gamma(1-\rho)N] + [-\alpha(1-\rho)N\delta t + \beta(w-t) + \gamma(w-t)(1-\rho)N - \gamma(1-\rho)N\delta t]} \]

If \( \frac{du}{dt} \leq 0 \) for all \( t \geq 0 \), then tax funding of the charity will reduce utility.

Assuming givers continue to set their giving to the utility maximizing level given the tax,

(45) \[ \frac{du}{dt} = -\alpha \left( 1 + \frac{dg}{dt} \right)(w - g^* - t)^{\alpha-1} \left( \frac{(1-\rho)(g^* + \delta t)}{\rho} \right)^{\beta} (g^*)^\beta + \beta \frac{1-\rho}{\rho} \left( \frac{dg}{dt} \right)^{\beta-1} (g^*)^\beta + \gamma(w - g^* - t)^{\alpha} \left( \frac{(1-\rho)(g^* + \delta t)}{\rho} \right)^{\beta} (g^*)^\gamma - 1. \]

Since DM is setting private giving to the utility maximizing level given the tax, the first order condition for utility maximization (42) still holds. Substituting (42) into (45) and simplifying, increasing the tax will not be welfare enhancing when

(46) \[ \frac{\gamma}{\beta} \geq \frac{\gamma(1-\rho)N\frac{dg^*}{dt} + \delta - \left( \frac{dg^*}{dt} + 1 \right)}{-\frac{dg^*}{dt}(1-\rho)N(g^* + \delta t)} \]

Solving for when \( t = 0 \), taxes will never be welfare enhancing when

(47) \[ \frac{\gamma}{\beta} \geq \frac{(1-\rho)N\frac{dg^*}{dt} + \delta - \left( \frac{dg^*}{dt} + 1 \right)}{-\frac{dg^*}{dt}(1-\rho)N} = \left[ \frac{\gamma}{\beta} \right]^* \]
The condition (47) demonstrates that it is the relationship between DM’s preference for the total charity and the warm-glow that determines if taxes are welfare enhancing. Setting $\delta = 1$, the standard warm-glow model,

$$\left[ \frac{\gamma}{\beta} \right]^* = \frac{[(1 - \rho)N - 1][\frac{dg^*}{dt} + 1]}{-\frac{dg^*}{dt}(1 - \rho)N}$$

Condition (48) is positive as long as $(1 - \rho)N > 1$, meaning as long as there is more than one giver. This means that it is possible under Andreoni’s warm-glow model that taxes are not welfare enhancing. Funding a public good with taxation is needed to overcome the free rider problem. When the warm-glow is sufficiently large to overcome the free rider problem, taxes will not be welfare enhancing.

If the number of givers is exogenous and private giving is set to maximize utility given the tax, then the only parameter left to determine $\left[ \frac{\gamma}{\beta} \right]^*$ is $\delta$. When $\delta = \delta^*$, $\frac{du}{dt} = 0$ at $t^* = 0$.

Solving (45) when $t^* = 0$ yields

$$\delta^* = \frac{a(\frac{dg}{dt} + 1)g + \rho \frac{dg}{dt}(w - g) + \gamma(w - g)}{\beta(w - g)}$$

Figure 2 presents a number of cases. The value $\delta = \delta^*$ is particularly interesting because private financing is best for $\delta < \delta^*$ while some public financing is best for $\delta > \delta^*$. The parameters chosen to produce Figure 2 are $\alpha = 0.3$, $\beta = 0.5$, $\gamma = 0.2$, $N = 10000$, $\rho = 0.3$ and $w = 1000$ so that $\delta^* = 0.8003$. Because of the incomplete crowd-out, when $\delta > \delta^*$ as the tax increases utility increases. At some point, negative effect of the crowding-out of private giving is no longer off-set by the increase in the tax. As $\delta$ decreases, this maximum utility point occurs closer to $t = 0$. 
6. **Conclusion**

This paper proposes that private giving and public giving might not be viewed the same by givers. To capture this insight, the original model of charity by Warr and Roberts was first extended from a three person model to an N person model. This generalized original model was examined and crowd-out was shown to be complete, meaning a tax increase of one dollar reeducated the optimal level of individual private giving by one dollar. That is, the primary Warr and Roberts result was shown to hold for the generalized original model.
Warr and Roberts demonstrated that charitable giving tends to create a public goods problem. This problem can be effectively addressed by increasing taxes past the point where giving drops to zero. Specifically, the utility of givers can be increased by entirely crowding out private giving so all charitable giving is public and financed with taxation. This result was also confirmed in our generalized version of the original model of charity.

The primary innovation here is to allow public giving, or the tax financing of charity, to be discounted. The effect of discounting is to reduce the degree of crowd-out, meaning a one dollar increase in the tax financing of public giving reduces private giving by less than a dollar. Importantly, this indicates that the observed incomplete crowd-out in the real world can be explained by the discounting of public giving. There need not be a warm glow associated with charitable giving in order to explain incomplete crowd out.

The discounting has a negative effect on the amount of the charitable public good provided. If the degree of discounting is small enough, then the standard Warr and Roberts result holds that government can increase the well-being of givers by increasing taxes beyond the point where private giving is entirely crowded out. However, if the degree of discounting is above a threshold, then the ability of tax funding to address the free rider problem is not sufficient to overcome the discounting and it is best for government to let all giving be private and use not tax funding of charity.

The Original Model of Charity was extended to include Andreoni’s (1989) warm glow for giving. Without any discounting of public giving, the addition of a warm glow implies public financing will no longer crowd out private giving dollar for dollar, and it also implies it will never be optimal for government to entirely crowd out private giving.
Indeed, the warm glow can be so strong that it is best for government to leave all giving to the private sector. The cases of interest are those where the warm glow are not too strong. In such cases, the addition of discounting enhances the warm glow effect. That is, as the degree of discounting increases, an increase in taxation to finance public giving has less of an impact on private giving. Also, as the degree of discounting increases, it is as though the warm glow from private giving increases, making it better for government to leave giving to the private sector. There is a degree of discounting that is large enough that government should leave all giving to the private sector, because the warm glow and the discounting outweigh the good public financing can do by addressing the public goods problem.
References


Appendix

A. Comparative statics for Original Model of Charity

(50) \[ \max_{g_i, x_i} U_i = U(x_i, Y_j / (\rho N)), \]
subject to \( w_i = x_i + g_i + t_i \) and \( Y = g_i + G_i + t_i + T_i \).

The first order condition of utility maximization is

(51) \[ U_x \left( w_i - g_i^* - t_i, \frac{g_i^* + G_i + t_i + T_i}{\rho N} \right) = \frac{1}{\rho N} U_Y \left( w_i - g_i^* - t_i, \frac{g_i^* + G_i + t_i + T_i}{\rho N} \right). \]

Set \( t_i = t \) and \( w_i = w \) for all \( i \) so that \( g_i = g \) and rewrite (51) as

(52) \[ U_x \left( w - g - t, \frac{g^* (1 - \rho N - 1) + g^* (1 - \rho N t)}{\rho N} \right) = \frac{1}{\rho N} U_Y \left( w - g - t, \frac{g^* (1 - \rho N - 1) + g^* (1 - \rho N t)}{\rho N} \right). \]

Take the total differential of (52) to obtain

(53) \[ -(d g - dt) U_{xx} - \frac{1}{\rho N} (1 - \rho) N (d g + dt) U_{xy} + \frac{1}{\rho N} (-d g - dt) U_{yx} + \frac{1}{(\rho N)^2} (1 - \rho) N (d g + dt) U_{yy} = 0. \]

Collect terms to obtain

(54) \[ d g \left( U_{xx} - \left[ \frac{1 - \rho}{\rho^2} + \frac{1}{\rho N} \right] U_{xy} + \frac{1 - \rho}{\rho N} \right) + dt \left( U_{xx} - \left[ \frac{1 - \rho}{\rho^2} + \frac{1}{\rho N} \right] U_{xy} + \frac{1 - \rho}{\rho N} U_{yy} \right) = 0. \]

Solve for \( d g_i / d t_i \), to obtain

(55) \[ \frac{d g_i}{d t_i} = -1. \]

B. Comparative statics for discounted public giving in the Warr and Roberts model.

(56) \[ \max_{g_i, x_i} U_i = U(x_i, Y_i / (\rho N)), \]
subject to \( w_i = x_i + g_i + t_i \) and \( Y_i = G + \delta_i T \).

The first order condition of utility maximization is

(57) \[ U_x \left( w_i - g_i^* - t_i, \frac{g_i^* + G_i + \delta_i t_i + \delta_i (T_i)}{\rho N} \right) = \frac{1}{\rho N} U_Y \left( w_i - g_i^* - t_i, \frac{g_i^* + G_i + \delta_i t_i + \delta_i (T_i)}{\rho N} \right). \]

Set \( t_i = t, w_i = w, \) and \( \delta_i = \delta \) for all \( i \) so that \( g_i = g \) and rewrite (57) as

(58) \[ U_x \left( w - g - t, \frac{g^* (1 - \rho N - 1) + g^* (1 - \rho N N t)}{\rho N} \right) = \frac{1}{\rho N} U_Y \left( w - g - t, \frac{g^* (1 - \rho N - 1) + g^* (1 - \rho N N t)}{\rho N} \right). \]

Take the total differential of (52) to obtain
(59) \[-(dg - dt)U_{xx} - \frac{1-\rho}{\rho N} (dg + \delta dt)U_{xy} + \frac{1}{\rho N} (-dg_i - dt_i)U_{Yx} + \frac{1-\rho}{\rho^2 N} (dg + \\
\delta dt)U_{YY} = 0\]

Collect terms.

(60) \[dg \left( U_{xx} - \left[\frac{1-\rho}{\rho} + \frac{1}{\rho N}\right]U_{xy} + \frac{1-\rho}{\rho^2 N} U_{YY} \right) + dt \left( U_{xx} - \left[\frac{1}{\rho} + \frac{1}{\rho N}\right]U_{xy} + \\
\delta \frac{1-\rho}{\rho^2 N} U_{yy} \right) = 0\]

(61) \[\frac{dg}{dt} = - \frac{U_{xx} - \frac{1+\delta(1-\rho)N}{\rho N}U_{xy} + \frac{\delta(1-\rho)}{\rho^2 N} U_{YY}}{U_{xx} - \frac{1+\delta(1-\rho)N}{\rho N}U_{xy} + \frac{1-\rho}{\rho^2 N} U_{yy}}\]

C. Comparative statics for discounted public giving adding Andreoni’s Warm Glow.

(62) \[\max_{g_i, x_i} U_i = U(x_i, Y_i/(\rho N), g_i), \]

subject to \(w_i = x_i + g_i + t_i\) and \(Y_i = G + \delta_i T\).

The first order condition of utility maximization is

(63) \[-U_x \left( w_i - g_i^* - t_i, \frac{g_i^* + G + \delta_i T + \delta_i T_i}{\rho N}, g_i \right) + \\
\frac{1}{\rho N} U_y \left( w_i - g_i^* - t_i, \frac{g_i^* + G + \delta_i T + \delta_i T_i}{\rho N}, g_i \right) + \\
U_g \left( w_i - g_i^* - t_i, \frac{g_i^* + G + \delta_i T + \delta_i T_i}{\rho N}, g_i \right) = 0.\]

Set \(t_i = t, w_i = w,\) and \(\delta_i = \delta\) for all \(i\) so that \(g_i = g\) and rewrite (63) as

(64) \[-U_x \left( w - g - t, \frac{g + [(1-\rho)N-1]g + \delta(1-\rho)T}{\rho N}, g \right) + \\
\frac{1}{\rho N} U_y \left( w - g - t, \frac{g + [(1-\rho)N-1]g + \delta(1-\rho)T}{\rho N}, g \right) + \\
U_g \left( w - g - t, \frac{g + [(1-\rho)N-1]g + \delta(1-\rho)T}{\rho N}, g \right) = 0.\]

Take total differential of (64) to get

(65) \[-(dg - dt)U_{xx} - \frac{1-\rho}{\rho} (dg + \delta dt)U_{xy} - U_{xg} dg + \frac{1}{\rho N} (-dg - dt)U_{Yx} + \\
\frac{1-\rho}{\rho^2 N} (dg + \delta dt)U_{yy} + \frac{1}{\rho N} U_{yg} dg + (-dg - dt)U_{gx} + \frac{1-\rho}{\rho} (dg + \delta dt)U_{gy} + \\
U_{gg} dg = 0\]

Collect terms.
\[
\begin{align*}
(66) \quad dg & \left( U_{xx} + \frac{1-\rho}{\rho^2 N} U_{YY} + U_{gg} - \left[ \frac{1-\rho}{\rho} + \frac{1}{\rho N} \right] U_{XY} - 2U_{xg} + \left[ \frac{1-\rho}{\rho} + \frac{1}{\rho N} \right] U_{gY} \right) + \\
& \quad dt \left( U_{xx} + \delta \frac{1-\rho}{\rho^2 N} U_{YY} - \left[ \delta \frac{1-\rho}{\rho} + \frac{1}{\rho N} \right] U_{XY} - U_{gx} + \delta \frac{1-\rho}{\rho} U_{gY} \right) = 0 \\
(67) \quad \frac{dg}{dt} &= - \frac{U_{xx} + \delta \frac{1-\rho}{\rho^2 N} U_{YY} - \left[ \delta \frac{1-\rho}{\rho} + \frac{1}{\rho N} \right] U_{XY} - U_{gx} + \delta \frac{1-\rho}{\rho} U_{gY}}{U_{xx} + \frac{1-\rho}{\rho^2 N} U_{YY} + U_{gg} - \left[ \frac{1-\rho}{\rho} + \frac{1}{\rho N} \right] U_{XY} - 2U_{xg} + \left[ \frac{1-\rho}{\rho} + \frac{1}{\rho N} \right] U_{gY}}
\end{align*}
\]
Chapter 3: Video Segmenting and Student Performance

1. Introduction

Educational videos are a ubiquitous part of today’s learning landscape. There are formal educational videos offered within blended and online academic courses, massive open online courses (MOOCs), and the likes of Khan Academy, Lynda.com or YouTube. The increase in online educational video offerings, comprising half of all videos watched (Purcell, 2013) is significant. As of December 2013, 188.2 million world-wide internet users watched some of the 52.4 billion videos available, a jump of over 14 million viewers and 20 billion videos from four years earlier (comScore, 2010, 2014).

Videos are now readily available because they are now easy to create relative to the past, but creating a video that effectively enhances learning is challenging. More effective videos consider multiple facets, including theory (e.g., Mayer, 2009), design (e.g., Alessi & Trollip, 2001; White, 1956), and techniques (e.g., Hausman & Palombo, 1993). One issue of general interest is deciding how long to make the video. Should an educational video be about a half hour long, mimicking the time students typically spend in a classroom? Or, should they be less than one minute, mimicking the typical television and radio commercials?

This study examines the video length issue by conducting a controlled experiment using two sections of an undergraduate intermediate level economics course which were comparable in size and student composition. Vigilance decrement theory (Parasuraman, 1986), as based on James’ (1890/1981) concept of attention through spacing or

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2 This paper is the result of joint work with David J. Harrison; Instructional Designer; University of Nevada, Reno; davidharrison@unr.edu; and Mark Pingle; Professor of Economics; University of Nevada, Reno; pingle@unr.edu.
segmenting (Bjork, Dunlosky, & Kornell, 2013), suggests that segmenting a longer video will enhance learning. This is the theory we test. One section received content by watching a whole 27-minute online video, while another section received the same content via three segments of roughly nine minutes in length.

2. Theoretical background

In 1890, William James (1890/1981) defined attention as comprising focus, concentration, and consciousness on a desired stimulus, with the simultaneous withdrawal of the same from competing stimuli. Twentieth century vigilance decrement studies measured attention, showing that, when faced with unpredictable and infrequent events, humans are unable to keep focused attention on a task for an extended period of time. According to Parasuraman (1986), this was first discovered by the English Royal Air Force, Coastal Command during World War II in those who watched the shoreline for signs of enemy submarines. Their accuracy in differentiating actual versus “ghost” events significantly dropped in the second half hour of a session. Subsequent research determined that significant decreases in accurate attention occur at the first fifteen-minute mark (Parasuraman, 1986, p. 43:5), and can occur after only eight minutes in visual impairment situations (Parasuraman, et al., 2009). Self-reports of conscious states of boredom and monotony correlate highly with decreasing vigilance (Parasuraman, 1986, pp. 43:6-43:7).

Caution must be exercised when connecting vigilance decrement research with educational videos because vigilance tasks are defined by “a low probability of a critical event and a requirement for sustained observation” (Parasuraman, et al., 2009). In contrast, educational videos can be and often are dynamic, offering movement, changes
in information, and differing visual and auditory stimuli. It is conceivable that vigilance decrement is applicable to videos of lectures. The lecture format, whether delivered by video or live, involves information processing of a passive nature (Poh, Swenson, & Picard, 2010) so one might expect decreasing attention as time on task increases (Johnstone & Percival, 1976; Smith, 2006; Parasuraman, 1986).

Young, Robinson, and Alberts (2009, p. 43) note “there is a good deal of overlap between applied vigilance research in ergonomics, and the problem of student concentration in the pedagogical domain.” In fact, a loss of student concentration during lectures mirrors the loss of attention when workers monitor automated equipment (Young et al., p. 52). Lapses in attention to various lecture styles can first occur between ten and eighteen minutes after instruction begins, with decreasing spans of attention falling “to three or four minutes towards the end of a standard lecture” (Johnstone & Percival, 1976, p. 50). Mid-lecture breaks that interrupt this cycle of inattention, utilizing various techniques ranging from rest to interactive activities, “reset” the attention clock and allow students to maintain attention (Young, et al., 2009; Smith, 2006). Active engagement activities, even as short as two minutes, appear to re-energize students for the next fifteen to twenty minutes (Middendorf & Dalish, 1995). Such research suggests that online videos be presented in shorter lengths with breaks between them. This technique is known as “segmenting,” as opposed to a continuous stream of video known as “massing.”
Generally, spacing elements apart from each other increases learning outcomes, such as temporal separation of study times (Bjork, Dunlosky, & Kornell, 2013), practice times (Romiszowski, 1993, p. 149), spatial separation of groups of paintings by various artists (Kornell & Bjork, 2008), and segmenting animations in computer based education (Mayer & Chandler, 2001; Lusk et al., 2009; Mayer, Dow, & Mayer, 2003). Mayer’s research (e.g., Mayer & Chandler, 2001; Mayer, Dow, & Mayer, 2003) concerns segments of media much shorter than those under consideration here, indicating that the current research regarding segmenting and length examines a different research question than one examined by Mayer (R. Mayer, personal communication, February 21, 2013).

Other segmenting research uses static material to physically separate instructional elements. For example, Mautone and Mayer (2007) with static PowerPoint slides and Ayres (2006) with paper-based mathematical equations, and Lee, Plass, and Homer (2006) with multiple computer screens. Such studies examine static instruction, not temporality transient information that are to be processed in a linear fashion as is the case with video, audio, and live lecture instruction.

There is limited research on videos whose length approaches attentional boundaries. Ibrahim, Antonenko, Greenwood, and Wheeler (2012) combined segmentation of a 32-minute video into approximately six-minute segments with other interventions, including a bulleted list signaling important concepts and the removal of entertaining but inessential sections of the original video. While the findings of Ibrahim et al. (2012) support segmentation, the presence of other confounding variables obscures the applicability of the results to segmentation alone.

Historic research examining educational reel films, the medium most closely
related to modern videos, offers guidance but is inconclusive (Hoban, 1946; Ash, 1950).
While temporally transient presentation of material is present in these studies, such
historic film research is also not directly applicable to modern videos. They utilized reels
that required manual replacement as the linear footage of film ran out, introducing natural
pauses as the projectionists changed reels. They indicated such actions may have reset
learner attentional clocks and provided for cognitive pauses. However, the inconclusive
nature of their findings and other findings indicate there is a need to more carefully
examine video length and segmenting.

3. Methods

The method used was to compare learning outcomes of two groups of students,
differentiated by whether they were delivered content by one whole video or by a
segmented video. Specifically, one group was provided online access to a 27-minute
video, while the other was provided the same video divided into three segments.
Learning outcomes were measured using three assessments: (1) A quiz for measuring
short-term retention, test questions on a mid-term for measuring moderate-term retention,
and test questions on a final exam for measuring long-term retention.

The 27 minute video as a whole included two types of content. One type
involved the professor standing before a whiteboard discussing the content, which
involved analyzing economic data using linear and non-linear models. The other type of
content involved screen captured demonstrations of regression analysis within a
spreadsheet.

The data was collected from 92 college students, 65 males and 27 females,
who self-selected their enrollment in one of two sections of an intermediate economics
course at the University of Nevada, Reno. The “full-length” video group included 47 participants, 38 males and 9 females. The “segmented” video group included 45 participants, 27 males and 18 females.

On Monday in class, both sections were directed to watch the video content to be prepared for a quiz over that content that would be given in the next class on Wednesday. The full-length group had opportunity to go online and watch the whole 27-minute video. The segmented group had opportunity to go online and watch the video content segmented into the following lengths (minutes:seconds): 7:46, 10:13, and 9:43. The segmented group could control when they moved from one video to the next. Any participant in either group could pause, stop, rewind, fast-forward, or replay any part of the video.

The segmenting into roughly 9 minute segments was selected because of the previous research reported above. Nine minutes is long enough to present significant content, but according to previous research is roughly where we might expect a more significant decrease in the degree of attention. Certainly, we might expect such a decline after 18 minutes, so that by the end of the 27 minute video the degree of attention should be less.

A much vetted national examination of economic knowledge, the Test of Economic Literacy (Walstad & Rebeck, 2001a; 2001b), was administered the first day of class to both sections to provide a baseline assessment. The test results did impact the student’s course grade, so each student had an incentive to perform as well as possible. The test directly measures economic knowledge, but it provides a proxy for other differences among the participants (e.g., differences in raw talent, differences is work
It is reasonable to expect stronger students to perform more effectively on any quiz or test. The baseline result from the Test of Economic Literacy was used as a control variable for individual student quality, so differences related to video segmenting versus not could be more readily deciphered.

One section was offered at 11AM, and the other section was offered at 1PM. Because these are both prime times to take a class, it is not perceived that one particular type of student would self-select into one of the sections versus another. The 11AM section was the full video group, while the 1PM section was the segmented group. Because the 11AM section did contain a significantly higher percentage of males than the 1PM section, gender was also recorded and used as a control variable.

Self-report data on video viewing was also collected from participants. Specifically, participants categorized the number of times they remembered pausing the video: 0 times, 1 time, 2-3 times, 4-5 times, and 6 or more times. Rewinding behavior was similarly assessed.

Short term learning was measured using a quiz containing fifteen multiple-choice questions. Medium-term retention was measured with three specific mid-term exam questions about three weeks after the initial video viewing. Long-term retention was measured using four specific final exam questions conducted about two months after the video viewing.

To confirm equality of distributions for the performance measures, means tests and standard deviations tests were used. The distributions were determined to be non-normal based on Shipiro-Wilks tests. The results of these tests can be found in the Appendix. Because of this non-normality, the Mann-Whitney test was used to confirm
equality of means while a Levene test was used to confirm homogeneity of standard deviation. Table 1 contains the results of the Levene tests used to confirm non-Normality.

Table 1: Table of Levene tests

<table>
<thead>
<tr>
<th>Metric</th>
<th>Shapiro-Wilks</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEL Full-Length</td>
<td>z=1.46 p=.073</td>
<td>-.75</td>
<td>3.66</td>
</tr>
<tr>
<td>TEL Segmented</td>
<td>z=2.05 p=.020</td>
<td>-1.1</td>
<td>4.86</td>
</tr>
<tr>
<td>Quiz Full-Length</td>
<td>z=1.52 p=.064</td>
<td>-.65</td>
<td>3.37</td>
</tr>
<tr>
<td>Quiz Segmented</td>
<td>z=.71 p=.239</td>
<td>-.71</td>
<td>4.14</td>
</tr>
<tr>
<td>Midterm Full-Length</td>
<td>z=2.04 p=.021</td>
<td>.23</td>
<td>1.7</td>
</tr>
<tr>
<td>Midterm Segmented</td>
<td>z=.68 p=.248</td>
<td>-.23</td>
<td>1.96</td>
</tr>
<tr>
<td>Final Full-Length</td>
<td>z=1.23 p=.110</td>
<td>-.49</td>
<td>2.01</td>
</tr>
<tr>
<td>Final Segmented</td>
<td>z=3.22 p=.000</td>
<td>-.99</td>
<td>2.94</td>
</tr>
</tbody>
</table>

4. Results

The Test of Economic Literacy

In spite of the fact that students had a grade incentive to perform well, three participants were clearly outliers, scoring unusually low on the Test of Economic Literacy. There were 40 multiple choice questions on this test, with four possible answers for each question. Thus, a student randomly selecting an answer would be expected to get 10 of 40 correct. All but three students scored more than 14 correct. Three students who took the test scored only 3 correct out of 40, and two of these three did not finish the course. The data for these three participants were removed from the study.

The outcomes for the two groups on the Test of Economic Literacy are displayed in Table 2. The small reported difference in performance between the two groups is not statistically significant. The Mann-Whitney test confirms that the means
are statistically equivalent ($z = -1.20 \ p = .231$). There was less variation in performance within the segmented group, but a Levene test indicates this variation is not quite statistically significant ($F = 2.22, \ p = .140$). In summary, using the Test of Economic Literacy as a measure, there was no significance between the two groups regarding their prior economic knowledge. Using this test performance as a proxy for other factors that might impact student quality, the Test of Economic Literacy results indicate the average student quality of the two groups was the same.

Table 2: Performance on the Test of Economic Literacy

<table>
<thead>
<tr>
<th>Group</th>
<th>Participants</th>
<th>Mean Score</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-Length</td>
<td>44</td>
<td>31.3</td>
<td>5.68</td>
</tr>
<tr>
<td>Segmented</td>
<td>40</td>
<td>32.8</td>
<td>4.63</td>
</tr>
</tbody>
</table>

**Short-Term Retention.**

In total, 81 participants took the 15 question quiz. With one point for each correct answer, the 41 participants taking the quiz in the full length group scored a mean of 9.02, while the mean for the 40 participants in the segmented group was 9.60. The Mann-Whitney test indicates this difference between the means is not statistically significant ($z = - .895, \ p = .371$). The standard deviation in the quiz score was 2.62 for the full length group and 2.80 for the segmented group. The Levene test indicates this difference in the degree of deviation from the mean is not significant ($F = .00 \ p = .992$). Summarizing, the standard measures of central tendency and variation indicate that segmenting the video did not significantly affect student performance.

Figure 1 presents the distribution of scores on the quiz. A difference between the two groups is evident. The distribution for the segmented group is close to a normal
distribution but is skewed so that there are relatively fewer poor performers. In contrast, the distribution for the full length group is close to a uniform distribution, with roughly the same percentage of people performing very poorly and very well as the percentage performing at an average level.

**Figure 1: Distribution of Quiz Scores**

The distributions in Figure 1 suggest that segmenting may have improved the performance of the poorer students. One important difference found between the groups is that the students in the segmented group who scored 8 or less on the quiz watched the video 1.67 times on average which is significantly less than the students who scored 9 or more who watched the video 2.32 time (z=2.28 p=.023). The full-length group, the students who score 8 or less watched the video 2.36 times on average which is not significantly different from the students who scored 9 or more who watched the video 2.85 times on average (z=2.28 p=.269).

The following equation, estimated using ordinary least squares (OLS)
regression, offers some insight into student quiz performance:

\[
QUIZ = 1.17 + .35 \ SEG + .20 \ TEL + 1.14 \ VID - .11 \ VIDSQ, R^2 = 0.28.
\]

(1.85)  (.54)  (.05)**  (.31)**  (.03)**

The variable \(QUIZ\) is the quiz score of the participant. SEG is a dummy variable, where SEG = 1 represents the segmented group and SEG = 0 represents the full-length group. TEL is the score the student received on the Test of Economic Literacy. For the full-length group, VID is the actual number of times the full-length video was accessed while for the segmented group VID represents an average number of views rounded to the nearest whole number. VIDSQ is the variable VID squared, included to allow the examination of diminishing returns.\(^3\)

Regressions including other explanatory variables and interactions between variables were tried but not included because they were not found to significantly impact the quiz score. For example, a variable for gender was tried, but no significant difference between male and female performance was found. Also an interaction between TEL and VID was tried, as was an interaction between the TEL and SEG to see if student quality significantly changed the impact of either video viewing or segmenting. We also tried an interaction with SEG and VID due to the result found when looking at Figure 1. None of the interaction terms was significant.

The interesting result in the regression is the significant and positive effect of video viewing on the quiz score. More time spent viewing the video resulted in a better

\(^3\) The standard errors of estimated coefficients are shown in parentheses under the coefficients. Asterisks are used to indicate statistical significance. One asterisk indicates significance at the 5% level (.01 < \(p\) ≤ .05), two indicate significance at the 1% level (.001 < \(p\) ≤ .01), and three indicate significance at the 0.1% level (\(p\) ≤ .001).
quiz score. However, the negative coefficient on VIDSQ indicates there are diminishing returns to video watching, which is what we might expect. In particular, the regression indicates viewing the video more than five or six times actually decreases performance on the quiz.

Including VIDSQ in the regression allows for diminishing returns, but it may be overly restrictive. A quadratic is symmetric, and diminishing returns may not arise in a symmetric form. One way to allow for asymmetry is to introduce dummy variables which categorize video viewing. Replacing VID and VIDSQ with a set of dummy variables, the estimated regression equations becomes:

\[
\text{QUIZ} = 0.98 + 0.37 \text{SEG} + 0.21 \text{TEL} + 1.60 \text{VID1} + 0.97 \text{VID2} + 2.77 \text{VID3} + 2.48 \text{VID45} + 4.34 \text{VID67} + 2.48 \text{VID8P}, \quad R^2 = 0.22
\]

\[
(1.89) (0.56) (0.05)*** (0.83) (0.83) (0.89)** (1.10)* (1.35)** (1.24)
\]

\text{VID1} = 1 indicates the participant viewed the video exactly 1 time (20 observations).

\text{VID2} = 1 indicates the participant viewed the video exactly 2 times (18 observations).

\text{VID3} = 1 indicates the participant viewed the video exactly 3 times (17 observations).

\text{VID45} = 1 if the participant viewed the video 4 or 5 times (7 observations). \text{VID67} = 1 if the participant viewed the video 6 or 7 times (4 observations). \text{VID8P} = 1 if the participant viewed the video 8 or more times (5 observations). We tried a model that separated 8 or more group into two separate groups, one for if the participant viewed the video 8 or 9 times and one if the participant viewed the video 10 or 11 times. The coefficients for both groups were not significantly different from zero so we decided to use just one group.

Figure 2 shows the effect of video viewing on performance, comparing the quadratic model compares to the dummy variable model. There is an important difference between these two models. The quadratic model suggests a symmetric peak at five
viewings. Alternatively, the dummy variable model indicates relatively consistent marginal improvement up to 6 or 7 viewings followed by a steep and asymmetric decrease in performance.

The least squares models used do not support a difference between the two treatments. Figure 1 suggests that there might be a relationship at the lower end of the distribution. The full length group is disproportionately represented in scores of 6 or 7. Quantile regressions were done for each decile from 10% to 90%. The only variables found to be significant were still the Test of Economic Literacy and the number of times the videos were accessed.

Given the relationship between video watching and performance, if segmenting the videos increased the video watching, then segmenting may have had an impact. The Mann-Whitney test \((z = .480, p = .63)\) showed no significant differences between the video watching of the full-length group \((N = 41, M = 2.68, SD = 2.72)\) and the segmented group \((N = 40, M = 2.13, SD = 2.10)\).
Retention

There were three items from the first midterm and four items from the final that related to content covered in the experiment. The midterm was administered three weeks after the quiz, and the final three months after the mid-term. Equality of means and variances were again confirmed using Mann-Whitney and F tests.

The Mann-Whitney test \( z = -0.823, p = 0.41 \) showed no significant differences between the summed scores on the midterm items by the full-length group \( N = 41, M = 20.3, SD = 5.57 \) and the segmented group \( N = 40, M = 21.3, SD = 5.86 \). The Mann-Whitney test \( z = 0.031, p = 0.976 \) also showed no significant difference between the
summed scores on the final items by the full-length group (\(N = 39, M = 25.9, SD = 3.30\)) and the segmented group (\(N = 37, M = 24.84, SD = 5.26\)).

Table 3 reports regressions performed to explain performance on the Midterm and Final. The primary indicator for student performance is student quality, as measured by the score on the Test of Economic Literacy TEL. Video viewing was not found to have a significant impact on performance, in contrast to the result for the quiz. This suggests video viewing may have a short-term positive impact on performance but not a lasting impact. Controlling for significant student quality differences did not change the primary result for the question of interest: We find no significant difference in performance between the full-length and segmented groups in terms of their ability to learn and retain course content.

**Table 3: Explanation of Midterm and Final Performance**

<table>
<thead>
<tr>
<th></th>
<th>Midterm</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>Items</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TEL</td>
<td>0.56</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>(.11)***</td>
<td>(.08)***</td>
</tr>
<tr>
<td>SEG</td>
<td>0.38</td>
<td>-1.56</td>
</tr>
<tr>
<td></td>
<td>(1.1)</td>
<td>(0.86)</td>
</tr>
<tr>
<td>VID</td>
<td>1.15</td>
<td>-0.44</td>
</tr>
<tr>
<td></td>
<td>(0.63)</td>
<td>(0.49)</td>
</tr>
<tr>
<td>VIDSQ</td>
<td>-0.15</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(.07)*</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Constant</td>
<td>1.56</td>
<td>13.07</td>
</tr>
<tr>
<td></td>
<td>(3.7)</td>
<td>(2.86)***</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.29</td>
<td>0.31</td>
</tr>
<tr>
<td>(N)</td>
<td>83</td>
<td>78</td>
</tr>
</tbody>
</table>

Standard errors in parentheses  
* \(p < .05\). ** \(p < .01\). *** \(p < .001\)

The coefficient on VIDSQ is significant at the 5% level but coefficient on VID is not. Given that video watching was significant on the quiz, it is possible that VID and
VIDSQ are jointly significant. A test of joint significance ($F(2,78) = 2.86, p = .06$) rejects the hypothesis that they are jointly significant at the 5% level.

**Pausing and Rewinding Self-Reports**

The variables for the number of times students paused and rewound the videos is based on survey data. The values are ordinal and not interval. A Mann-Whitney test confirms that there is no significant difference in the distributions of the number of times paused between the 2 classes ($z=1.59$ $p=.111$). We also confirm that there is no significant difference in the distributions of the number of times the video was rewound between the 2 classes ($z=.92$ $p=.359$).

5. **Discussion**

Unlike recent segmenting and spacing research (e.g. Bjork, Dunlosky, & Kornell, 2013; Kornell & Bjork, 2008; Mayer & Chandler, 2001; Mayer, Dow, & Mayer, 2003), the current study did not find significant differences in immediate, moderate or long-term retention between the full-length video and the same video segmented into three smaller videos. The study did, however, affirm Ash (1950) in that longer videos are just as efficacious as the same content divided into shorter videos.

The means tests indicate there is no significant difference between the two classes. This is confirmed by the regression results for short, moderate and long-term measures. The Test of Economic Literacy, the proxy for prior knowledge, remained the best predictor of student performance.

We did find that increased viewing was associated with better performance in the short term with diminishing returns. These findings explain the research of McTavish (1949) who similarly found that two views of educational videos offered the most value
for the task, with a fifth approaching zero benefit – similar results as our research. He concluded that “for factual films of the kind used in this study, showing them twice results in appreciably more learning; showings after the first two contribute little more to learning, and the drop-off is very rapid” (as cited in Hoban and van Ormer, 1970, p. 8-35).

While the real-world situation of this study and use of YouTube as a hosting service did not allow us to examine user control of the videos, we did find that pausing and rewinding self-report behaviors across the groups was the same. It is possible that this behavior fits the Parasuraman (1986) heuristic, in that, as learning outcomes were equivalent overall, the provision of learner control may be an efficacious technique to combat fatigue, boredom, and decreased attention in longer videos. Learner-controlled pauses are efficacious (Mayer & Chandler, 2001), as are active pauses containing assessment pieces (Cheon, Crooks, & Chung, 2014). Significant increases in learning can be obtained for students possessing low prior knowledge when material is segmented (Ayres, 2006), but learner-control requires high pre-existing self-regulating learning strategies to be effective (Young, 1996). Existing studies, however, do not use videos as instructional materials, as in the current study, but rather paper (Ayres, 2006) and computer based instruction utilizing animations lasting less than 200 seconds (Mayer & Chandler, 2001; Lusk et al., 2009; Cheon, Crooks, & Chung, 2014; Hatsidimitris & Kalyuga, 2012).

While we considered the moderate and long-term measures as the collective performance on the relevant items on the midterm and the final, we also examined the performance on individual items.
The analysis of the quiz indicated that there were diminishing returns to watching the video. When looking at the individual items on the midterm and the final, the same diminishing returns relationship can be seen for item 7 of the midterm. This question was with regards to interpreting regression results and presenting it in a professional manner. This is a topic that students are likely to have seen for the first time in the video, so this correlation makes sense. Item 1 on the final was also with regards to interpreting regression results and presenting it in a professional manner but did not show any relationship to video access times. If this indicates that students can overcome diminishing returns through some factor we did not control for, such as through study habits, is a matter for future research.

While neither of the long-term measures of retention showed a significant difference between the two groups, item 3 on the final did show a significant difference at the 5% level with the segmented group scoring .55 points less on average out of 5 than the non-segmented group. The question was to interpret a regression coefficient. This difference is surprising since it suggests that segmenting the video may have been detrimental to a student’s ability to interpret a regression coefficient. Other factors we did not control for, like differences in study habits between the groups, may better explain this difference. Item 8 on the midterm was also to interpret a regression coefficient but showed no significant difference between the two groups.

Other results of interest

The video files were uploaded to YouTube. Using the YouTube Analytics we can determine which parts of the videos were watched most often. Figure 3 shows a comparison of the video watching rates for both groups. The top graph shows how
participants watched the full length video. The bottom three graphs represent the three videos for the segmented group.

**Figure 3: YouTube Analytics of video usage**

While we do not have the data points for how the graphs were generated, we know that higher levels correspond to the parts of the videos the students watched more often. There are two peaks in the second segmented video, Video B, that correspond to peaks in the full length video. These parts of the videos were procedural, where the professor showed the students how to use certain functions in Microsoft Excel, required for completion of the assignments.

There were also two peaks in the third segmented video, Video C, that corresponded to peaks in the full length video. These were also procedural, showing students how to professionally report regression results.

The YouTube Analytics suggest that videos are most beneficial for procedural information. The advantage of having the video is that a student can access the relevant portion of the video when attempting to perform the required procedure. Whelden (1954)
found that films can efficiently teach procedures, but that guided practice immediately following the videos significantly increases learning outcomes.

6. Conclusion

In this study, we tested the hypothesis that segmenting a 27 minute video in three approximately nine minute videos will improve student performance. Our results reject this hypothesis for the content delivered. This held true for short-term, moderate-term and long-term measures of learning retention.

It could be the segmenting is fruitful, in spite of our findings. In our environment, participants could pause and rewind the videos, meaning subjects in the full video group could effectively segment their longer video in a self-controlled manner. From Figure 3 we can see that there are multiple locations in the videos that students watched more than others. These sections are only a few minutes each. It is also possible certain types of content, e.g., declarative, procedural, and conceptual, are better suited to presenting through online video. The effectiveness of videos with different content types warrants further investigation.

Our results affirmed Ash’s (1950) findings that dispel the advantages of dividing video content into shorter segments, in contradistinction to current thinking on the topic (e.g. Bjork, Dunlosky, & Kornell, 2013; Kornell & Bjork, 2008; Mayer & Chandler, 2001; Mayer, Dow, & Mayer, 2003).

Diminishing returns of video viewing found in our study affirms McTavish’s (1949) research that indicated a lessening of educational benefit once students approached five to six viewings.
Because of the limitation in this study in which participants would watch the videos at any time of the day in any location of their choosing, future research should consider a controlled environment, eliminating the learners’ abilities to pause, stop, and rewind, or keep track of the user’s use of pausing, stopping, rewinding as covariates.
References


