University of Nevada, Reno

Three Essays in Macroeconomics

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics

by

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August, 2017
We recommend that the dissertation prepared under our supervision by

Mina Mahmoudi

Entitled

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Abstracts


This study investigates the direction and magnitude of the financial links between the European stock markets and the U.S. stock market before, during, and after the stock market crash of 2008, with an emphasis on the Central European countries. Our dataset consists of daily data for the S&P 500 and the EURO STOXX 50 indexes from January 2000 to May 2013, and also a new index which has been defined to measure the Central European countries stock market prices. The results show that there is an immediate response of the European markets to a price change in the U.S. stock market while the reverse relationship takes longer to develop. After the financial crisis, the bilateral relationship happens in a shorter period of time. We show that although the Central European stock markets are segmented from the U.S. market before the crisis, they become linked during the crisis and stay connected even after the crisis. Also, the quantitative results of the study show a significantly higher impact of the U.S. stock market on the Europe stock markets during the recent financial crisis, while this effect is decreasing after the crisis.

Chapter 2 - A Descriptive Growth Model with Unemployment

The standard descriptive growth model is modified in a straightforward way to incorporate what Keynes (1936) called the “essence” of his general theory. The essence is the idea that exogenous changes in investment cause changes in employment and
unemployment. We capture this idea by assuming the path for the capital growth rate is exogenous in the growth model. The result is a dynamic model comparable to the IS-model of static macro theory. Testing the model using post-WWII U.S. data, we show our model well explains both the long term growth trend and fluctuations in unemployment around the trend.

Chapter 3 - Bounded Rationality and Ambiguity

This paper examines the results of a preference experiment aimed at examining the ability of people to distinguish a better uncertain prospect from a worse uncertain prospect when the difference between the two is the probability distribution. This tests the extent to which human subjects perceive ambiguity because of limited cognitive capacity even though there is no ambiguity as ambiguity is normally defined. We found that subjects did, for the most part, place a higher value on better prospects – Cognitive ability to distinguish. However, an evidence of ambiguity was found due to the common and not rare valuation errors. By moving to ambiguity, bids were increased when max was high (more optimism) and decreased when max was low (more pessimism).
Dedicated

to my parents
for all their years of love, encouragement, and sacrifices

and

to my brothers
whose love and support inspire me...
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Chapter 1


Abstract

This study investigates the direction and magnitude of the financial links between the European stock markets and the U.S. stock market before, during, and after the stock market crash of 2008, with an emphasis on the Central European countries. Our dataset consists of daily data for the S&P 500 and the EURO STOXX 50 indexes from January 2000 to May 2013, and also a new index which has been defined to measure the Central European countries stock market prices. The results show that there is an immediate response of the European markets to a price change in the U.S. stock market while the reverse relationship takes longer to develop. After the financial crisis, the bilateral relationship happens in a shorter period of time. We show that although the Central European stock markets are segmented from the U.S. market before the crisis, they become linked during the crisis and stay connected even after the crisis. Also, the quantitative results of the study show a significantly higher impact of the U.S. stock market on the Europe stock markets during the recent financial crisis, while this effect is decreasing after the crisis.
1. Introduction

The Global Financial Crisis emerged by the bursting of the housing bubble which was followed by the banking crisis and led to a great recession in 2009. On September 15, 2008, Lehman Brothers, the fourth largest U.S. investment bank, collapsed and the stock markets tumbled in reaction to Lehman's failure. The S&P 500 price index lost 4.71% of its value which caused the stock markets in Europe to drop sharply as well (BBC News, 2008). At the same time, public debt problems spread out to many members of the European area and limited the ability of the European leaders to respond to the crisis.

The relationship of the stock markets in different countries, particularly the transmission of price shocks, has long attracted attention among economists, policymakers, and fund managers. The behavior of stock markets and the similarity of the price transmissions in different stock markets during the crisis can have important implications for asset pricing and portfolio allocation through using the diversification benefits of less financially linked markets.

The specific goal of this study is to examine the direction and magnitude of the financial links between the European stock markets and the U.S. stock market during the recent financial crisis. For this purpose, we use three different sample periods namely before crisis, during crisis, and after crisis periods. There is particular emphasis on the Central European countries which are recognized to be mostly segmented before the financial crises and have larger and more effective stock markets among European countries.
Considering the direction of the financial links as a goal of this study, the general idea based on the “World Systems Theory” is that the core countries’ markets tend to have more control over other financial markets. Based on this theory, the productivity dominance of the core countries can cause trade dominance which may lead to financial dominance over the periphery and semi-periphery countries (Wallerstein, 1980). According to Dunn, Kawana and Brewer (2000) and also Babones and Alvarez-Rivadulla (2007), the United States is considered as one of the most important core countries, while some of the countries in the Europe are considered as core and some as periphery. Therefore, the general thought based on the World Systems Theory is that the U.S. financial market tend to have more control over the European financial markets. However, this may not be the full story. It is conceivable that both through international trade links (see Eichengreen & O’Rourke, 2010) and widespread European debt problems, a feedback loop may have developed as to produce reverse causality from periphery to the core countries. This is what our paper investigates.

One contribution of our study is that the data that we use, allow us to estimate when the financial crisis started in the U.S. and the Europe area by affecting their stock markets. Using several Chow tests, we present the break points in our data, which show the start and end of the crisis, are summer 2006 and the end of 2009, respectively. We show that there is an immediate response of the European markets to a price change in the U.S. stock market while the reverse relationship takes longer to develop. Also, we show that the bilateral relationship happens in a shorter period of time after the financial crisis.

1 - Wallerstein (1974) developed “World Systems Theory” which divides the world into three types of countries: core, periphery, and semi-periphery. Core countries are considered as more dominant countries with high-skill labors and capital-intensive production, while periphery countries are more dependent on the core countries.
crisis and there is a significantly higher impact of the U.S. stock market on the European stock markets during the recent financial crisis, while this effect is decreasing after the crisis.

The rest of the paper is organized as follows. In the next section, we present relevant previous work. Section 3 describes the data including the price index which is built for the Central European countries and the methodology of the study. Section 4 reports the empirical results of the qualitative tests including the Granger causality test results. Section 5 provides the empirical results of the quantitative tests including both the variance decomposition analysis results and the impulse response analysis results. Finally, a summary is provided in section 6 together with a discussion of the results.

2. Literature Review

During the last several years, scholars have used different methods to examine the relationships between the stock markets of different countries during financial crises. Several studies have found that the magnitude of financial links between stock markets tends to be higher in times of crisis (Yang, Kolari, and Min, 2003; Yang, Hsiao, Li, and Wang, 2006), and also the contagion effect tends to run to smaller economies from larger economies (Breuss, 2011; Karunanayake, Valadkhani, and O’Brien, 2010; Kenourgiosa, Samitasb, and Paltalidis, 2011).

Nikkinen, Piljak, and Aijo (2011) presented the evidence that three emerging stock markets in the Baltic region were segmented before the financial crisis of 2008–2009, but they were highly linked to each other during the crisis. Using Granger causality tests and vector autoregressive analysis (VAR), they also showed that a large proportion of the forecast variance of the emerging stock markets can be explained by the developed European stock markets during the crisis. Similarly, Karunanayake, Valadkhani, and O’Brien (2010) used a multivariate DVECH model to examine the volatility transmission between Australia, Singapore, the UK, and the U.S. stock markets focusing on the Asian and global financial crises of 1997–1998 and 2008–2009. They found that both crises increased the stock return volatilities significantly across all of the four markets, and also that the U.S. stock market was the most crucial market impacting the volatilities of smaller economies.

Furthermore, Kenourgiosa, Samitasb, and Paltalidis (2011) used both a multivariate regime-switching Gaussian copula model and the asymmetric generalized dynamic conditional correlation (AG-DCC) approach, to capture non-linear correlation dynamics in four emerging equity markets (Brazil, Russia, India and China) and two developed markets (the U.S. and the UK) for recent financial crises during 1995–2006. Their results confirmed a contagion effect from the crisis country to all others, for each of the examined financial crises. Also, Wen, Wei, and Huang (2011) used the same copula method to measure the contagion between energy and stock markets of China and the U.S. during the financial crisis. They found a significantly increasing dependence between crude oil and stock markets during the financial crisis, thus supporting the
existence of contagion, and also the contagion effect was found to be much weaker for China than for the U.S.

A recent interesting study by Kotkatvuori-Örnberg, Nikkinen, and Äijö (2013) applied dynamic conditional correlation method to the data from 50 equity markets to examine the effect of 2008-2009 financial crisis on stock market correlations considering two significant banking events _ JP Morgan's acquisition of Bear Stearns and the Lehman Brothers' collapse. The results of this study showed a significant increase in the correlations of international stock markets by the Lehman Brothers' collapse, whereas the acquisition of Bear Stearns had negligible effects on global stock market interdependence.

This paper examines the direction and magnitude of the financial links between the European stock markets and the U.S. stock market during the 2008-2009 financial crisis, with an emphasis on the Central European countries. The next section presents the data including the price index which is built for the Central European countries and the methodology of the study.

3. Data & Methodology

The data used consists of: the EURO STOXX 50 index for Eurozone stocks, which is made up of fifty of the largest and most liquid European stocks; the S&P 500 index or the Standard & Poor's 500 for the U.S. stock market, which can be considered as one of the best representations of the U.S. stock market as it is based on the stocks of 500 large companies listed on the NYSE or NASDAQ; and also a price index that is built for the Central European countries which we call it the CE index.
The CE index consists of different stock price indexes for the Central European countries including Germany (GDAXI), Switzerland (SSMI), Poland (PTX.VI), Hungary (BUX), Austria (ATX), Slovakia (SAX), Slovenia (SBITOP) and the Czech Republic (PX). In order to build the CE index, we first normalize the price indexes so the last date for our data, which is May 31st, 2013, is given the value of 100. Then, based on the turnover ratio of the stocks traded in each country from the year 2000 to 2012 (the total value of shares traded during the period divided by the average market capitalization for the period; World Bank, 2013), an index weight is given to each country (Germany gets the highest weight between these 8 countries with 27.7% following by Switzerland with 19.8%) and a weighted average method is used to calculate a single stock price index for Central Europe.

We use daily data from January 2000 to May 2013, consisting of 3500 observations, both in price index level and return. Although most studies on the financial markets use return data for their analysis, we use the price index data first due to the fact that individuals acting in the stock markets are usually looking at the price charts and not the returns, thus making the price data a more reliable behavioral variable. However, we present the results of the return data as a robustness check. In order to calculate the return of the price index, we follow the Ding, Granger and Engle (1993) method, which defines the compounded return of a price index ($r_t$) as: $r_t = \ln P_t - \ln P_{t-1}$, where $P_t$ is the price index at time $t$.

Three periods of time are used in this study: before crisis, during crisis, and after crisis. For the before crisis period we use the data from January 2000 to June 2006, for the during-crisis period July 2006 to December 2009, and for the after-crisis period
January 2010 to May 2013. Chow test was applied to check if there are structural breaks occurring during the specified periods, indicating that our choices are reasonable. This simply tests for the presence of structural breaks by checking if the coefficients of two linear regressions on different time series are equal or not. The break points ultimately chosen were selected after also running the Chow test for other alternative break points. The results of this test is presented in section 3.

Figures 1, 2 and 3 show the changes in the U.S., Europe and Central Europe stock market price indexes from the year 2000 to May 2013. As can be seen, stock market prices follow a similar pattern in the U.S. and Europe especially before and during the crisis. For all three of the stock market price indexes, there is an upward trend at the start of the crisis period followed by a crash during the last months of 2008 and the start of 2009. Point A is not considered as the start of crisis because it is happening in years 2002-2003 which is not in our interest and also this period does not contain the global trough. Point B is not only a break in the price series according to the Chow test, but it also provides an early warning indicate of the impending crash, something that point C does not. In other words, B has predictive value as a break point while C does not. Finally, the end of the crisis is considered to be point E which shows a return to the midpoint value of ≈1100, which could be considered a return to normality. Point D, on the other hand, is the global trough, but the price series has not yet returned to a normal value.
Figure 1. The U.S. stock market price index

Figure 2. The Europe stock market price index
In Table 1 below, we see that the volatilities of the stock prices, measured by standard deviations, increase for all stock markets during the crisis, and decrease significantly after the crisis. Also, the calculated correlation of the data shows that the Central European stock markets are more segmented from the U.S. stock market before the crisis, while the correlation of their prices are significantly increasing during the crisis.²

### Table 1. Correlation and standard deviation of the data

<table>
<thead>
<tr>
<th>Standard Deviation</th>
<th>Before-crisis</th>
<th>During-crisis</th>
<th>After-crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>166.28</td>
<td>239.13</td>
<td>143.40</td>
</tr>
<tr>
<td>Europe</td>
<td>1088.79</td>
<td>1149.64</td>
<td>372.94</td>
</tr>
<tr>
<td>Central Europe</td>
<td>19.91</td>
<td>24.49</td>
<td>8.53</td>
</tr>
<tr>
<td>Correlation (U.S., Europe)</td>
<td>0.927</td>
<td>0.990</td>
<td>0.306</td>
</tr>
<tr>
<td>Correlation (U.S., Central Europe)</td>
<td>0.574</td>
<td>0.993</td>
<td>-0.008³</td>
</tr>
</tbody>
</table>

² - Correlation and not covariance is used to increase clarity, although the concept used in asset pricing models, such as the classical C-CAPM, is the covariance.

³ - Refer to figures 1 and 3. The first figure shows a post-crisis recovery along a monotonically increasing trend, whereas the Central European stock market shows a recovery along a broken, non-monotonic trend. This helps explain the negative correlation.
The first step in our study is to use Granger causality tests to determine which stock index forecasts the other. We say a time series data on S&P 500 Granger-causes a time series of EURO STOXX 50 if it can be shown that those S&P 500 values provide statistically significant information about future values of EURO STOXX 50. The point here is to test if the U.S. stock prices cause the changes in the Europe stock prices or vice versa. We also want to determine if bilateral causation exists, and if so, at what lag length it occurs.

In order to use the Granger causality tests, we need to know if our variables are stationary or not. There are several tests that can be used for this purpose. In general, the Augmented Dicky Fuller (ADF) and Philips Peron (PP) tests have low power against near unit root processes, meaning they cannot predict accurately whether the I(0) alternatives that are close to being I(1) are stationary or non-stationary. Also, the power of unit root tests decreases by including a constant and a trend in the test regression, rather than only including a constant. It may be useful to do a cross check by using a test with a non-stationary null hypothesis such as ADF together with a test with a stationary null hypothesis such as KPSS. In this study, we use the recent test suggested by Elliot, Rothenberg, and Stock (1996) because it is more powerful than the alternatives.

In the context of non-stationary data, the Granger causality test statistic does not follow its standard asymptotic chi-square distribution under the null. Therefore, we use the Toda-Yamamoto (1995) approach which is applicable whether the VAR is trend stationary, integrated or co-integrated of an arbitrary order.

In the Toda-Yamamoto Granger causality procedure, first we consider the maximum order of integration that we suspect might occur for the group of time series to
be $I_{max}$. On the second step, determining a lag length \( k \), we estimate a \( (I_{max} + k)^{th} \) order VAR. The important point here is that we do not include the coefficient matrices of the extra \( I_{max} \) lagged vectors in the model, since they are used just to follow the standard asymptotic theory. Therefore, we include the extra lags as exogenous variables to the model and test the restrictions only on the first \( k \) coefficient matrices (Toda & Yamamoto, 1995).

The next step in our study is to check the quantitative impacts of the U.S. stock market crash of 2008 on the European stock markets. The variance decomposition is used once VAR models are fitted in order to indicate the amount of information each variable contributes to the other variables in our regression. Also, the impulse response function of the model is a good instrument to analyze the dynamic effects of the system when the model receives a shock. The aim here is to see the persistence of the effect during the time by using the impulse responses.

4. Qualitative results on shocks’ transmission

In the first step, we use the Chow test to see if we have chosen reasonable break points. The break points ultimately chosen were selected after also running the Chow test for other alternative break points. Table 2 presents the results of this test on the stock markets price indexes. The null hypothesis in this test is that there is no structural break, and based on p-values, we can reject the null and conclude that structural breaks happen in our data as we mentioned according to the financial crises.
Using the unit root test proposed by Elliot, Rothenberg and Stock (1996), we find the result that all of the three variables (S&P 500, EURO STOXX 50 and CE) have unit roots in the price index while none of them have unit roots in the return data. The same result is obtained using both the ADF test and the KPSS test (results are not reported to save on space, but will be made available upon request). Therefore, we apply the Toda-Yamamoto Granger causality approach to the price index data and Granger causality Chi-square test to the return data. Tables 3(a) and 3(b) compare the results of these tests in the three different periods between price index and return data, respectively for the U.S. vs Europe and the U.S. vs Central Europe.

Considering the price relationship between the U.S. and Europe stock markets (table 3(a)) using the price index data, a one-way relationship occurs at one lag (one day) where the U.S. stock index forecasts the European stock index but not the reverse. By increasing the number of lags to four days, a bilateral relationship occurs between the two stock indexes. However, for the after-crisis period, the bilateral relationship occurs at

### Table 2. Results of the Chow test

<table>
<thead>
<tr>
<th>Chow Breakpoint Test: 6/30/2006</th>
<th>F-statistic</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>659.369</td>
<td>0.000</td>
</tr>
<tr>
<td>Log likelihood ratio</td>
<td>544.616</td>
<td>0.000</td>
</tr>
<tr>
<td>Europe</td>
<td>605.639</td>
<td>0.000</td>
</tr>
<tr>
<td>Log likelihood ratio</td>
<td>507.105</td>
<td>0.000</td>
</tr>
<tr>
<td>Central Europe</td>
<td>483.127</td>
<td>0.000</td>
</tr>
<tr>
<td>Log likelihood ratio</td>
<td>393.087</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chow Breakpoint Test: 12/31/2009</th>
<th>F-statistic</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>219.068</td>
<td>0.000</td>
</tr>
<tr>
<td>Log likelihood ratio</td>
<td>204.354</td>
<td>0.000</td>
</tr>
<tr>
<td>Europe</td>
<td>263.261</td>
<td>0.000</td>
</tr>
<tr>
<td>Log likelihood ratio</td>
<td>242.322</td>
<td>0.000</td>
</tr>
<tr>
<td>Central Europe</td>
<td>196.319</td>
<td>0.000</td>
</tr>
<tr>
<td>Log likelihood ratio</td>
<td>127.821</td>
<td>0.000</td>
</tr>
</tbody>
</table>
only one lag, which is shorter than the number of lags during and before the financial crisis.

Considering the return data, the bilateral relationship happens at three lags before and during the crisis and likewise the price index data, it happens at just one lag after the crisis. Therefore, we can get the same result that the bilateral relationship happens in a shorter period of time after the crisis than either before or during the crisis.

| Null Hypothesis | Price Index Data (Toda-Yamamoto): | | | Return Data: |
|-----------------|---------------------------------|-----------------|-----------------|
|-----------------|---------------|---------------|--------------|---------------|---------------|--------------|
| U.S. → Europe   | 1 172.4 0.000 | 1 180.23 0.000 | 1 28.97 0.000 | 1 169.61 0.000 | 1 184.75 0.000 | 1 27.89 0.000 |
| Europe → U.S.   | 0.055 0.815 | 1.01 0.315 | 2.89 0.089 | 0.25 0.616 | 1.46 0.227 | 4.02 0.045 |
| U.S. → Europe   | 4 208.97 0.000 | 4 193.48 0.000 | 4.02 0.045 | 3 209.49 0.000 | 3 203.39 0.000 | 4.02 0.045 |
| Europe → U.S.   | 14.25 0.007 | 9.46 0.048 | 4.02 0.045 | 6.31 0.098 | 11.3 0.01 | 4.02 0.045 |

- Chi-squares values that are statistically significant at the 10% level are presented in bold face.

Table 3(a) presents the results of Granger causality tests in different lags (The U.S. vs. Europe). Table 3(b) presents the results of testing how the stock index of Central European countries relates to the U.S. stock index. Considering both the price index and return data series, a one-way relationship happens at one lag where the U.S. stock index forecasts the Central European stock index. A bilateral relationship can be found by expanding the number of lags to 14 days for the before-crisis period and by increasing the lags to eight days for the after-crisis period. However, for the during-crisis period, the results of the price index and return data series are not the same which may be caused by the nature of
the return data and how we build it. The price index data shows that during the financial crisis, only the U.S. stock index forecasts the Central European stock index in one day and only by expanding the lags to 16, a bilateral relationship can be found. On the other hand, the return data shows that a bilateral relationship happens at only one day during the crisis.

Table 3(b). Results of Granger causality tests (The U.S. vs. Central Europe)

<table>
<thead>
<tr>
<th>Price Index Data (Toda-Yamamoto):</th>
<th>Before-crisis</th>
<th>During-crisis</th>
<th>After-crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null Hypothesis</td>
<td>lags</td>
<td>Chi-sq</td>
<td>P-Value</td>
</tr>
<tr>
<td>U.S. CE</td>
<td>1</td>
<td>118.96</td>
<td>0.000</td>
</tr>
<tr>
<td>CE U.S.</td>
<td>0.08</td>
<td>0.776</td>
<td>1.47</td>
</tr>
<tr>
<td>U.S. CE</td>
<td>14</td>
<td>139.21</td>
<td>0.000</td>
</tr>
<tr>
<td>CE U.S.</td>
<td>24.77</td>
<td>0.037</td>
<td>29.44</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Return Data:</th>
<th>Before-crisis</th>
<th>During-crisis</th>
<th>After-crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null Hypothesis</td>
<td>lags</td>
<td>Chi-sq</td>
<td>P-Value</td>
</tr>
<tr>
<td>U.S. CE</td>
<td>1</td>
<td>3.09</td>
<td>0.079</td>
</tr>
<tr>
<td>CE U.S.</td>
<td>0.23</td>
<td>0.634</td>
<td>2.77</td>
</tr>
<tr>
<td>U.S. CE</td>
<td>14</td>
<td>36.13</td>
<td>0.001</td>
</tr>
<tr>
<td>CE U.S.</td>
<td>22.92</td>
<td>0.062</td>
<td></td>
</tr>
</tbody>
</table>

- Chi-squares values that are statistically significant at the 10% level are presented in bold face.

Consistent with the previous results, we see a bilateral relationship occurs in a shorter period of time after the crisis compared to the before-crisis period. However, it occurs with a much longer lag structure between Central Europe and the U.S. than between Europe and the U.S.

5.1. Quantitative Results: Variance Decomposition Analysis

For both the price index and the return series of data, VAR models are estimated between the S&P 500 and the EURO STOXX 50 and also between the S&P 500 and the
CE variables, all in levels, in the different periods. We use the Schwarz information criterion to find the optimum number of lags for each model. The variance decomposition is used to determine how much the forecast error variance of each variable can be explained by exogenous shocks to the other variables. Tables 4(a) and 4(b) present the results of variance decomposition during different periods of before-crisis, during-crisis, and after-crisis respectively for the U.S. vs Europe and the U.S. vs Central Europe.

The results of the Granger causality test showed a bilateral relationship between the U.S. and Europe, and also between the U.S. and Central Europe after a few days during different periods of time. However, the results of variance decomposition provide clear evidence of the independence of the U.S. market versus both the European market and the Central European countries in all periods, using both price index and return data, as its forecast variance is only caused by its own innovations.

The U.S. shows a significantly higher impact on the European stock markets during the crisis period when we use the price index data. As can be seen in Table 4(a), the S&P 500 index explains about 33-60% of the EURO STOXX 50 index during five days for the period of before-crisis, but this percentage range increases to 48-74% during the crisis, and then decreases to 52-63% after the crisis. This increasing impact during the crisis compared to before the crisis is much more significant for Central European countries. Based on the results of Table 4(b), the U.S. stock index explains about 17-35% of the Central European stock index during five days for the period of before-crisis, which increases to 39-70% during the crisis, and then decreases to 44-59% after the crisis. These findings further suggest that a larger proportion of the forecast variance of the Europe stock markets can be explained by the U.S. stock market during the crisis.
Table 4(a). Results of Variance Decomposition tests (The U.S. vs. Europe)

### Price Index Data:

<table>
<thead>
<tr>
<th>Period</th>
<th>Variance Decomposition of U.S.</th>
<th>Variance Decomposition of Europe</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>100.000</td>
</tr>
<tr>
<td>2</td>
<td>0.022</td>
<td>99.978</td>
</tr>
<tr>
<td>3</td>
<td>0.151</td>
<td>99.849</td>
</tr>
<tr>
<td>4</td>
<td>0.158</td>
<td>99.842</td>
</tr>
<tr>
<td>5</td>
<td>0.153</td>
<td>99.847</td>
</tr>
</tbody>
</table>

Variance Decomposition of Europe

| 1 | 66.600 | 33.400 | 11.043 | 51.437 | 48.563 | 17.795 | 47.160 | 52.840 | 13.399 |
| 3 | 42.190 | 57.810 | 18.701 | 26.474 | 73.526 | 38.736 | 36.591 | 63.409 | 22.342 |
| 5 | 39.339 | 60.661 | 23.948 | 25.156 | 74.844 | 36.118 | 36.591 | 63.409 | 28.538 |

### Return Data:

<table>
<thead>
<tr>
<th>Period</th>
<th>Variance Decomposition of U.S.</th>
<th>Variance Decomposition of Europe</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>100.000</td>
</tr>
<tr>
<td>2</td>
<td>0.010</td>
<td>99.990</td>
</tr>
<tr>
<td>3</td>
<td>0.188</td>
<td>99.812</td>
</tr>
<tr>
<td>4</td>
<td>0.210</td>
<td>99.790</td>
</tr>
<tr>
<td>5</td>
<td>0.211</td>
<td>99.789</td>
</tr>
</tbody>
</table>

Variance Decomposition of Europe

| 1 | 64.779 | 35.221 | 0.011 | 48.724 | 51.276 | 0.017 | 46.574 | 53.426 | 0.011 |
| 2 | 62.472 | 37.528 | 0.011 | 50.235 | 49.765 | 0.017 | 46.003 | 53.997 | 0.011 |
| 3 | 62.467 | 37.533 | 0.011 | 49.677 | 50.323 | 0.017 | 45.985 | 54.015 | 0.011 |
| 4 | 62.437 | 37.563 | 0.011 | 49.522 | 50.478 | 0.017 | 45.984 | 54.016 | 0.011 |
| 5 | 62.430 | 37.570 | 0.011 | 49.469 | 50.531 | 0.017 | 45.983 | 54.017 | 0.011 |

Considering the price return data, although the effect of the U.S. stock market on the European markets increases significantly during the crisis compared to before the crisis, it even increases a few percent or stays about the same after the crisis. The return data confirms the more significant increase in the effect of the U.S. stock market on the Central European markets during the crisis compared to the whole European market. It's interesting that considering the return data, Central European countries are almost independent from any changes in the S&P 500 returns before the crisis, while this result changes significantly during and after the collapse of stock markets as shown in the bottom panel of table 4(b).
Table 4(b). Results of Variance Decomposition tests (The U.S. vs. Central Europe)

<table>
<thead>
<tr>
<th>Price Index Data:</th>
<th>Before-crisis</th>
<th>During-crisis</th>
<th>After-crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance Decomposition of U.S.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Period</td>
<td>CE</td>
<td>U.S.</td>
<td>S.E.</td>
</tr>
<tr>
<td>1</td>
<td>0.000</td>
<td>100.0</td>
<td>0.655</td>
</tr>
<tr>
<td>2</td>
<td>0.003</td>
<td>99.997</td>
<td>0.977</td>
</tr>
<tr>
<td>3</td>
<td>0.004</td>
<td>99.996</td>
<td>1.212</td>
</tr>
<tr>
<td>4</td>
<td>0.006</td>
<td>99.994</td>
<td>1.408</td>
</tr>
<tr>
<td>5</td>
<td>0.008</td>
<td>99.992</td>
<td>1.580</td>
</tr>
</tbody>
</table>

Variance Decomposition of CE

| 1 | 82.555 | 17.445 | 11.187 | 60.407 | 39.593 | 17.801 | 55.930 | 44.070 | 13.411 |
| 2 | 69.731 | 30.269 | 15.671 | 38.465 | 61.535 | 23.348 | 44.481 | 55.519 | 18.348 |
| 4 | 65.384 | 34.616 | 21.963 | 31.184 | 68.816 | 30.328 | 41.555 | 59.012 | 28.430 |
| 5 | 64.609 | 35.391 | 24.460 | 29.142 | 70.858 | 33.411 | 40.988 | 59.012 | 28.430 |

| Return Data:          |              |              |              |
| Variance Decomposition of U.S. |              |              |              |
| 1 | 0.000 | 100.0 | 0.038 | 0.000 | 100.00 | 0.014 | 0.000 | 100.00 | 0.010 |
| 2 | 0.012 | 99.98 | 0.038 | 0.059 | 99.941 | 0.015 | 0.177 | 99.823 | 0.010 |
| 3 | 0.020 | 99.98 | 0.038 | 0.096 | 99.904 | 0.015 | 0.186 | 99.814 | 0.010 |
| 4 | 0.135 | 99.86 | 0.052 | 0.097 | 99.903 | 0.015 | 0.187 | 99.813 | 0.010 |
| 5 | 0.148 | 99.85 | 0.052 | 0.097 | 99.903 | 0.015 | 0.187 | 99.813 | 0.010 |

Variance Decomposition of CE

| 1 | 97.75 | 2.244 | 0.011 | 54.16 | 45.836 | 0.017 | 51.00 | 48.991 | 0.011 |
| 2 | 97.25 | 2.746 | 0.011 | 48.93 | 51.069 | 0.018 | 48.64 | 51.351 | 0.011 |
| 3 | 97.23 | 2.768 | 0.011 | 48.32 | 51.673 | 0.018 | 48.54 | 51.452 | 0.011 |
| 4 | 98.49 | 1.501 | 0.011 | 48.07 | 51.922 | 0.018 | 48.54 | 51.459 | 0.011 |
| 5 | 98.22 | 1.773 | 0.011 | 48.02 | 51.978 | 0.018 | 48.54 | 51.459 | 0.011 |

5.2. Quantitative Results: Impulse Response Analysis

Due to the fact that the price index data are non-stationary, we just apply the Impulse Response Analysis to the return data. Figure 4 shows the response of the Europe and Central European stock indexes to a Cholesky-factorized one standard deviation change in the U.S. stock index during a 10 day window. It is interesting that the effects of the U.S. stock market on the Central European countries is moving around the zero line during the whole 10 days and the same result can be obtained by extending the window to 30 days. This shows that the Central European stock markets are segmented from the U.S. market before the crisis, while they become linked during the crisis and stay
connected even after the crisis. Also, there is a peak for the first few days of all periods which shows the immediate response of the European markets to a price change in the U.S. stock market. The peak is higher during the crisis and significantly lower after the crisis which suggests the higher impact of the U.S. stock market on the European stock markets during the crisis while it’s decreasing after the crisis.

**Figure 4. Response of the Europe and Central Europe stock price returns to one std. dev. shock in the U.S. stock price returns**

In order to see the persistency of the effects, we use the accumulated response of the Europe and Central Europe stock markets to the price changes in the U.S. stock market which is presented in Figure 5. It can be seen that the impact of the U.S. stock market stays persistent 10 days after the price shock. The same result can be obtained by extending the window to 30 days.
6. Summary and Discussion

The purpose of this study is to examine the direction and magnitude of the financial links between the European stock markets and the U.S. stock market before, during, and after the 2008 financial crisis. The results show the immediate response of the European markets to a price change in the U.S. stock market while the reverse relationship takes longer to develop. This confirms that the World Systems Theory is partially validated by the data, is that the original shock stems from the U.S. (the core economy) and spread to the Europe (semi-periphery economy). However, the data also shows reverse causation, thus highlighting the importance of trade and debt links.

The economic crisis had been transmitted to the European countries with a delay as the leaders of Europe’s largest economies were attempting to avoid the recession in different ways. Key agencies such as the Federal Reserve and the European Central Bank tried to help stabilize the financial markets in different ways by cutting interest rates and
increasing the money supply (Guillén, 2012). These adjustments could also explain a feedback effect from the collapse in the European countries to the U.S. economy with a delay.

The results also show that for both the European markets vs. the U.S. market and the Central European countries vs. the U.S. market, the bilateral relationship happens in a shorter period of time after the financial crisis. This could be rational due to the fact that after experiencing a crisis, the stock markets pay more attention to the price changes in the other markets and respond faster. In other words, they may became more inter-related as it is resulted clearly for the Central European stock markets and the U.S. market.

The quantitative results of the study show a significantly higher impact of the U.S. stock market on the European stock markets during the 2008 financial crisis, while this effect is decreasing after the crisis. This shows that using portfolio diversification by investing in multiple international stock markets would not be much useful in case of a financial crisis due to the more powerful financial links between the affected markets. Also, the increasing impact of the U.S. stock market during the crisis compared to before the crisis is much more significant for Central European countries as these special countries were more segmented before the crisis.

Future possible work could include investigating the direction of financial links between core and periphery countries through other important crisis periods and also measuring the magnitude of these links more in depth such as considering price elasticities or volatility. One possibility is to examine the financial links between the United States and major European countries such as Germany, France, and the United Kingdom before, during, and after the Great Depression. Another possibility is to study
the direction of the 1997 Asian financial crisis considering Thailand and other major Asian countries. By looking at other episodes, the qualitative results of this study would be more reliable and the next step would be digging more through the quantitative effects of the financial crises around the world. Also, there is possible implication in behavioral economics comparing the periods of normal shocks and the periods of rare shocks, investigating how much the stock market behavior change in regards of price taking from other major markets in different defined periods.

References


Chapter 2

A Descriptive Growth Model with Unemployment

Abstract

The standard descriptive growth model is modified in a straightforward way to incorporate what Keynes (1936) called the “essence” of his general theory. The essence is the idea that exogenous changes in investment cause changes in employment and unemployment. We capture this idea by assuming the path for the capital growth rate is exogenous in the growth model. The result is a dynamic model comparable to the IS-model of static macro theory. Testing the model using post-WWII U.S. data, we show our model well explains both the long term growth trend and fluctuations in unemployment around the trend.
1. Introduction

Growth theory has primarily become a tool used to examine long term trends for full employment economies, but the first growth models were developed to examine whether full employment can be maintained as the economy grows. Harrod (1939) said his work in “trade cycle theory” is what motivated him to develop his seminal growth model. He sought to understand whether the “income generating function” of investment could be compatible with its “capacity generating function.” Similarly, Domar (1946, p. 139) emphasized the “dual character” of investment, noting, “because investment in the Keynesian system is merely an instrument for generating income, the system does not take into account the extremely essential, elementary and well-known fact that investment also increases productive capacity.” In his model, where investment had a dual character, generating income and generating productive capacity, Domar (1946, p. 143) found “the failure of the economy to grow at the required rate creates unused capacity and unemployment.”

Solow (1956, p. 65) noted, “The characteristic and powerful conclusion of the Harrod-Domar line of thought is that, even for the long run, the economic system is at best balanced on a knife-edge of equilibrium growth.” Solow’s (1956) “contribution to the theory of economic growth” was to show the Harrod-Domar knife-edge stems from an overly restrictive modeling assumption. Solow (1956) and Swan (1956) showed full employment on a balanced growth path is possible under a wide variety of conditions if labor and capital are substitutable in production rather than being restricted to a fixed ratio as assumed by Harrod and Domar.
Understanding the economy will tend toward full employment when wages are flexible, growth theorists moved away from the coordination questions of Harrod and Domar toward understanding how various factors influence the full employment growth path. Solow (1956) himself obtained the most important result. “The permanent rate of growth of output per unit of labor is independent of the saving (investment) rate and depends entirely on the rate of technological progress” (Solow, 1988, p.309). Growth accounting, perhaps best represented by Denison (1985), was developed to examine how growth theory explains long term growth trends, and did so by fitting models to data. Cass (1965) and Koopmans (1965) extended the work of Solow (1956) and Ramsey (1928) to characterize optimal economic growth. Romer (1986) and Lucas (1988) constructed models that endogenized the rate of technical change, so “endogenous growth theory” could explain why technological change occurs and consequently why all countries do not converge to the same rate of economic growth.

Yet, in his Nobel address, Solow (1988) laments the focus of growth theory on full employment. He summarizes the history saying, “Growth theory was invented to provide a systematic way to talk about and to compare equilibrium paths for the economy,” but … “in doing so, it failed to come to grips adequately with an equally important and interesting problem: the right way to deal with deviations from equilibrium growth” (Solow, 1988, p. 311). A primary message of his Nobel lecture was the “theory of equilibrium growth badly needed—and still needs—a theory of deviations from the equilibrium growth path” Solow (1988, p309).
This paper seeks to address this need. The approach is to directly apply what Keynes (1936, Chapter 3) called the “essence” of his general theory. We first carefully show this essence is the idea that exogenously determined investment demand determines the level of output and employment. We then imbed Keynes’ essence into the standard descriptive growth model by assuming the path for capital is exogenous. Also, following Keynes (1936, Chapter 3), we relax typical growth theory assumption that the nominal wage adjusts to equate labor supply and labor demand. It is this relaxed assumption that makes the theory a more general theory, for it allows unemployment to occur. Fitting the model to data for the U.S. from 1947 to 2015, we show our growth model with a sticky wage can well explain both the long term growth trend and fluctuations in unemployment around this trend.

Theoretically, our model is of interest because it is a dynamic version of the static IS model. We carefully show the essence of Keynes’ (1936, Chapter 3) general theory amounts to an IS equation and a production function. The presence of an IS equation indicates the capital market clears. Capital market clearing is standard in a growth model, so we also impose it. One critique of the IS Model is it does not capture the capacity generating function of investment as it is added to capital, but our growth model does not have this problem. Like the IS model, though, our growth model is a partial equilibrium model, not imposing labor market clearing. When the standard growth model imposes labor market clearing, it forces the income generating function of investment to match the capacity generating function which supports full employment. By not imposing labor market clearing, the income generated from investment in our growth model can fall short of that need to produce full employment, just as it can in the
IS model. In this situation, additional investment demand, or additional aggregate demand generated by government through a tax cut or a spending increase, can increase employment and reduce unemployment.

Despite its deficiencies, researchers are still finding the static IS-LM model useful, in part because of its simplicity and in part because it allows a less than full employment economy to be examined. More recently, researchers have used this static framework to examine the macroeconomic consequences of safe asset scarcity (Caballero et al, 2016), examine the consequences of recognizing saving as hoarding money as opposed to thrift (Rowe, 2016), debate Keynes’ view of how the economy works (O’Donnell and Rogers, 2016), derive optimal fiscal policy rules (Correani et al. 2014), endogenize business cycle fluctuations (Bella et al, 2013), and provide a dynamic model of inflation (Guirao et al, 2012). Our modified growth model can be similarly used, the advantage being the model is dynamic in the sense of explicitly capturing the dual nature of investment emphasized by Domar (1946).

We should, however, also note our model’s relative weaknesses. Like the standard descriptive growth model, it is an aggregative model without much of a microfoundation. Second, like the partial equilibrium the IS-LM model, the rigidity of the nominal wage is left unexplained. We do not discount efforts to explain unemployment using more disaggregated models, or models with more explicit micro-foundations, or models that explain unemployment by explaining wage rigidities.

For example, Hickman (1987) obtains unemployment in a model that, like ours, is a modification of the Solow model. He does so by assuming imperfect product market
competition, which leads to product price stickiness. Aghion and Howitt (1994) introduce a search theory micro-foundation to obtain a relationship between growth and unemployment, an approach which has since been widely used. Parello (2009) uses an efficiency-wage, which implies a wage rigidity, as an alternative to search frictions, to produce a relationship between the rate of economic growth and the rate of unemployment. Galí et al. (2011) obtain a wage rigidity from assumed labor market power, which allows them to explain unemployment fluctuations as arising from demand shocks. Casares et al. (2014) illustrate the use of an approach that assumes wages are rigid because producers use rules to set wages until they obtain signals from the market, and they show this creates a relationship between growth and unemployment. Other studies have obtained an impact of economic growth on unemployment through technical change (Mortensen and Pissarides, 1995; Acemoglu, 1997; Postel-Vinay, 1998; Cerisier and Postel-Vinay, 1998; Mortensen, 2005) or through institutional constraints (Cahuc and Michel, 1996; Van Schaik and De Groot, 1998; Daveri and Tabellini, 2000; Peretto 2000). This type of work has added depth to our understanding primarily by identifying mechanisms by which demand shocks can be transmitted and transformed into unemployment.

Rather than extending the micro-foundation of growth theory, our work re-emphasizes the Keynesian idea that unemployment may largely be explained by exogenous shocks to investment demand. Because our model retains the simplicity of the standard descriptive growth model, other researchers may find our model useful for examining issues in a less than full employment economy, just as researchers are still
finding it useful to extend the IS-LM model. As Van den Berg (2013) stresses, there is value in continuing to explore the coordination issues of interest Harrod and Domar and not just assume the economy naturally settle to an efficient, full employment state.

The rest of the paper is organized as follows. In the next section, we present a basic canonical growth theory model. Section 3 describes how we alter the canonical model to incorporate the essence of Keynes’ general theory. In section 4, we describe the data we use to examine the model’s explanatory power. Section 5 examines the extent to which our model fits the data. Section 6 concludes.

2. A Standard Growth Model

Assume the economy’s production level $Y$ depends on the employment (or labor demand) level $L^d$, capital level $K$, and level of technology $A$, so

$$Y = F(AL^d, K).$$

Assume technology grows exogenously at the rate $a$ so the change in technology is

$$A' = \alpha A.$$ 

Technological change is embodied in labor, so $AL^d$ is the economy’s effective labor level. Effective labor and capital are productive, meaning $F_{AL^d} > 0$ and $F_K > 0$, but are subject to diminishing returns, meaning $F_{AL^d, AL^d} < 0$ and $F_{KK} < 0$. Production exhibits constant returns to scale, so $\lambda Y = F(\lambda AL^d, \lambda K)$, where $\lambda$ is a scalar.

---

4 See Li and Wu (2015) for an approach to adjusting the IS-LM the model to include a microeconomic foundation, and see Dos Santos Ferreira (2014) for an approach to adjusting the IS-LM model to include other elements of the New Keynesian perspective.
The economy is competitive, with producers taking as given the nominal wage level $W$, the nominal capital rental rate $R$, and the price level $P$. Profit is maximized only if the labor demand level satisfies

$$\frac{w}{p} = F_{AL^d} (AL^d, K)$$

and only if the level of capital employed satisfies

$$\frac{r}{p} = F_K (AL^d, K).$$

The labor supply $L^s$ grows exogenously at the rate $n$ so

$$L^s' = nL^s.$$

We introduce an unemployment measure $U$, given by the ratio

$$U = L^s/L^d.$$

The standard growth model does not include condition (6) because the standard model assumes the nominal wage $W$ it adjusts to equate labor supply and labor demand, so

$$L^d = L^s,$$

which would imply $U$ in equation (6) would always equal 1. That is, the standard model assumes labor market clearing, so there is no need to distinguish labor demand from labor supply and no need to measure unemployment. A labor surplus, or the “involuntary unemployment” described by Keynes, is ruled out by assumption.

Households are assumed to save the constant fraction $s$ of the real income generated from the sale of the output produced, so

$$S = sY.$$

This supply of private saving finances private sector investment $I$, the public budget deficit $G - T$, and net exports $X$, so
(9) \[ S = I + [G - T] + X. \]

The rental rate \( R \) on capital is assumed to adjust, to eliminate a surplus or shortage of saving, so condition (9) holds. That is, like the labor market, the capital market is assumed to clear and remain in equilibrium over time. Capital depreciates at the rate \( \delta \), so when investment and depreciating capital are each considered, the change in the capital level is given by

\[ K' = I - \delta K. \]

Data on the government budget deficit and net exports, which is presented below, suggest that their levels remain roughly proportionate to the level of production. This will be modeled by assuming the level of taxes \( T \) is the fraction \( t \) of real income, the level of government purchases \( G \) is the fraction \( g \) of the real income, and the level of net exports \( X \) is the fraction \( x \) of income, so

\[ (11) \quad T = tY, \]

\[ (12) \quad G = gY, \]

and

\[ (13) \quad X = xY. \]

Together, equations (1)-(13) sequentially determine the paths followed by 13 variables: \( Y, A, L^d, I, L^e, U, W, S, R, K', T, G, \) and \( X \). The variables \( K, L^d, A \) are predetermined, meaning the model begins to unfold given initial conditions for these three variables. The exogenous variables driving the system are \( a, n, s, \delta, t, g, x, \) and \( P \). The nominal price level \( P \) is not determined in this standard model because the model is expressed in real terms. Adding money to the model would allow the price level to be
determined, but it complicates the model unnecessarily for our purposes, so the price level will be left exogenous.

The implications of this standard model can be derived by reducing it to a single dynamic equation that describes the path of a single core state variable for the reduced form model. In the Appendix, it is shown that by defining \(k^d = K/\left[AL^d\right]\) the basic dynamic equation for this standard growth theory model is

\[
(14) \quad k^d' = (s + t - g - x)f(k^d) - (\alpha + n + \delta)k^d,
\]

which describes the path of the core state variable \(k^d\). In the Appendix, it is also shown equations (1)-(13) imply the following three auxiliary equations for the reduced form model:

\[
(15) \quad y = f(k^d),
\]

\[
(16) \quad r = f'(k^d),
\]

\[
(17) \quad w = f(k^d) - f'(k^d)k^d,
\]

where the real wage is given by \(w = \frac{w}{p}\), the real rate of return on capital is given by \(r = \frac{r}{p}\), and the output level per effective unit of labor is given by \(y = Y/[AL^d]\). With the initial value of \(k^d\) given, equations (14)-(17) determine the paths for the variables \(k^d, y, r, \) and \(w\).

There are two steady-state values, \(\bar{k}_1^d = 0\) and \(\bar{k}_2^d > 0\) for the state variable \(k^d\). These values satisfy \(k^d' = 0\) in (14), or \((s + t - g - x)f(\bar{k}^d) = [\alpha + n + \delta]\bar{k}^d\). The positive steady state \(\bar{k}_2^d\) is of primary interest, which is shown in the Appendix to be
stable. Conditions (15)-(17) then imply \( \bar{y} = f(\bar{k}_{2}^{d}), \bar{r} = f'(\bar{k}_{2}^{d}), \) and \( \bar{w} = f(\bar{k}_{2}^{d}) - f'(\bar{k}_{2}^{d})\bar{k}_{2}^{d}. \)

In the Appendix, calculations are presented for a number of results related to the steady state. First, if the economy remains in the steady state, then both the capital to labor ratio \( K/L^{d} \) and per capita output level \( Y/L^{d} \) each grow at the rate of technological improvement \( \alpha \). That is, the model indicates only technological improvement can explain consistent increases in living standards.

Second, if there is no technical change (i.e., \( \alpha = 0 \)), one time increases in the steady state per output level \( Y/L^{d} \) occur with an increase in the savings rate \( s \) or the tax rate \( t \). One time decreases occur with increases in the government spending rate \( g \), the net exports rate \( x \), the labor supply growth rate \( n \), or the depreciation rate \( \delta \). The intuition behind these results is as follows. An increase in the savings rate or tax rate provide more savings to flow into invest and increase the steady state capital to labor ratio, which increases labor productivity, which increases per capita output. Increases in the rate of government spending or net exports do the opposite. An increase in the labor supply growth rate or depreciation rate each reduce the steady state capital to labor ratio by diluting the per person investment which savings provides.

When the economy is not in a steady state, the stability of the positive steady state \( \bar{k}_{2}^{d} \) implies the economy is headed toward \( \bar{k}_{2}^{d} \). As \( k^{d} \) approaches \( \bar{k}_{2}^{d} \), there is never any unemployment, and the direction of the changes in the real rate of return on capital and real wage per effective labor unit depend upon whether the steady state is approached
from below or above. When \( k^d < \bar{k}_2^d \), \( r \) decreases to \( \bar{r} \) and \( w \) increases to \( \bar{w} \) as \( k^d \) increases to \( \bar{k}_2^d \). The converse occurs when \( k^d > \bar{k}_2^d \).

In summary, standard growth theory provides an explanation of long term economic trends. The nation’s output level grows because labor and capital grow, and because technology improves. The average living standard depends upon average labor productivity. If the level of technology does not change, the capital to labor ratio for the economy will converge to a particular level, and that capital to labor ratio determines the living standard by determining the level of labor productivity. Changes in the savings rate, population growth rate, capital depreciation rate, tax rate, government spending rate, and net export rate can all provide one time improvements in the living standard by providing one time increases in the capital to labor ratio. Only steady technological improvement, which steadily improve labor productivity, can explain long term steady improvements in the living standard.

What the standard growth theory model cannot well explain are short term economic fluctuations, and it cannot at all explain involuntary unemployment. Deviations from a long term growth trend (caused by the constant rate of technical change) can be explained by variations in exogenous variables (i.e., the capital depreciation rate, population growth rate, tax rate, rate of government spending, rate of net export spending). However, because labor demand is assumed to be equal to the exogenously growing labor supply, the standard growth theory model cannot explain fluctuations in unemployment, and fluctuations in employment cannot be caused by fluctuations in investment.
In the next section, we modify the standard growth model by relaxing the assumption of market clearing and by assuming investment is exogenously determined. When labor demand does not automatically adjust to labor supply through a nominal wage adjustment, exogenous changes in investment can cause changes in employment. We show this model can explain economic fluctuations in unemployment reasonably well.

3. Modifying the Growth Model to Incorporate the Essence of Keynes’ General Theory

When he converted the words of Keynes’ (1936) “general theory” into a mathematical model, Hicks (1937) created what has become known as the IS-LM model. The IS-LM model captures the Keynesian notion that the economy, in the short term, is driven primarily by fluctuations in aggregate demand. Much of the debate about what Keynes meant arises from the fact that he described most of his theory using words rather than equations. However, in Chapter 3, Keynes (1936) describes the “essence” of his general theory using three equations, so we are able to derive precisely what we meant.

In his original notation, Keynes posited that the investment demand level $D_2$ is exogenous, and defined aggregate demand as $D = D_1 + D_2$. Keynes presented a “propensity to consume” function $D_1 = \chi(N)$, which postulates the consumption demand level $D_1$ depends upon the employment level $N$. Keynes presented an “aggregate supply function” $\phi(N)$, which maps the employment level $N$ into output, and presented $D = \phi(N)$ as an equilibrium condition, where the aggregate demand for output equals the aggregate supply. Substituting the first two equations into the third yields
\( x(N) + D_2 = \phi(N) \). This last equation indicates the equilibrium level of employment depends upon the level of investment.

Using our growth theory notation, where we suppress capital, our aggregate supply function is the production function \( Y = F(L^d) \). We can replicate the propensity to consume function of Keynes by introducing the consumption function \( C = C(Y) \). Replacing the output level with the production function, we have a function \( C = C(F(L^d)) \) which maps employment to consumption demand as Keynes did. Using our investment level \( I \) instead of the \( D_2 \) used by Keynes, aggregate demand can be written \( D = C + I \). Notice then that, if we recast the equilibrium condition as \( D = Y \) and replace both aggregate demand and consumption, we have the IS equation \( C(Y) + I = Y \). This IS equation, together with the production function \( Y = F(L^d) \), yields \( C(F(L^d)) + I = F(L^d) \), which like Keynes’ condition in the previous paragraph indicates that the equilibrium level of employment \( L^d \) depends upon the level of investment \( I \).

That is, the essence of Keynes’ general theory is an IS equation and a production function. No LM equation, or money market, is needed to capture the essence. Moreover, the equations representing the essence indicate fluctuations in the economy’s employment level can arise from exogenous fluctuations in investment demand. One primary assumption driving this result is the assumption that there is at least a component of investment demand that is exogenous, driven by what Keynes called “animal spirits.” If involuntary unemployment is permitted, which Keynes did permit, a second implicit assumption is that the nominal wage is sticky, not automatically adjusting to set labor demand equal to labor supply.
We now recast the growth theory represented by equations (1)-(13) so it captures the essence of Keynes’ general theory. We begin by not imposing the labor market equilibrium condition (7). In response to losing this condition, we reclassify the nominal wage $W$ as exogenous, capturing the Keynesian idea that the nominal wage is “sticky.” The second change is to assume $K'$ is exogenous. With $K'$ exogenous, the entire path for the capital level $K$ is exogenous. This captures the Keynesian idea that investment choices are driven by animal spirits more so than by the real interest rate level or some other equilibrating force.

The Keynesian story for how the economy works can then be told as follows. Given the predetermined capital level $K$, the exogenous change in capital $K'$ determines the investment level $I$ through equation Error! Reference source not found.. The capital market equilibrium condition (9) then determines the effective demand level for output $Y$. The effective demand for output $Y$ obtained from equation (9) determines the levels of $S$, $T$, $G$, and $X$ through equations (8), (11), Error! Reference source not found., and (13). It also determines the “derived demand” for labor $L^d$ through the production function (1). This labor demand level, combined with the predetermined levels of capital $K$ and technology $A$, determines the price level $P$ through equation Error! Reference source not found. and the capital rental rate $R$ through equation (4). The labor demand level $L^d$, combined with the predetermined labor supply level $L^s$, also determines the unemployment level $U$ through equation (6). Equations (2) and (5) determine the changes in technology $A'$ and labor supply $L^s'$. With capital, technology, and labor supply at their new predetermined levels, the variable levels for the economy in the next
instant of time are prepared to be determined in the same manner as described in this paragraph for the previous instant.

The driving force of this Keynesian version of the growth model is the assumption that the path for capital is exogenously determined, and this driving force can be recognized in a intensive form for the model by introducing the variable \( b = K'/K \), the growth rate of capital. If \( K' \) is exogenously determined and \( K \) predetermined for the levels model above, then the variable \( b \) is exogenously determined for the intensive form model that will now be derived. Eliminating the variables in equation (9) using equations (8), Error! Reference source not found., (11), Error! Reference source not found., and replacing the variable \( Y \) using equation (1) yields

\[
K' + \delta K = (s + t - g - x)F(AL^d, K).
\]

Dividing by \( AL^d \) and remembering the definition

\[
k^d = K/AL^d,
\]

this latter equation becomes

\[
[K'/K]k^d + \delta k^d = (s + t - g - x)[F(AL^d, K)]/AL^d.
\]

The constant returns to scale assumption and the definition \( F(1, k^d) = f(k^d) \) implies \( F(AL^d, K)/[AL^d] = f(k^d) \), so using the definition \( b = K'/K \), we have

\[
(18) \quad [b + \delta]k^d = [s + t - g - x]f(k^d).
\]

Equation (18) provides a relationship between the intensive capital demand level \( k^d \), the endogenous variable determined by the equation, and six other variables. Five of the six variables are the original exogenous variables \( s \), \( t \), \( g \), \( x \), and \( \delta \). The sixth variable is the newly defined capital growth rate variable \( b \), which is an additional exogenous variable. Noticeably missing is the technology growth rate variable \( a \), an indication that
the rate of technical change does not impact the employment level through the variable \( k^d \).

Figure 1 demonstrates how a change in the capital growth rate \( b \) can change the level of \( k^d \) and the corresponding unemployment level \( U \). Since \( U = L^s / L^d \), it also follows that

\[
U = k^d / k^s.
\]

There is involuntary unemployment when \( L^s > L^d \), which implies \( k^d > k^s \). In the initial situation show in Figure 1, the intensive capital demand level supported by the capital growth rate \( b_1 \) through equation (18) is \( k^d_1 = k^s \). This yields the initial unemployment level \( U_1 = k^d_1 / k^s = 1 \), meaning there is no involuntary unemployment. A decrease in the capital growth rate from \( b_1 \) to \( b_2 \) increases the equilibrium intensive capital demand level to \( k^d_2 \). (Figure 1 is drawn under the assumption that the change in \( b \) will not change \( k^s \), which is a reasonable assumption in the short run since \( k^s = K / [AL^s] \) and the levels of \( K, A, \) and \( L^s \) are predetermined). This implies the unemployment level increases to \( U_2 = k^d_2 / k^s > 1 \) as shown in Figure 1.
Figure 1. Effect of a Decrease in the Rate of Capital Growth

If the capital growth rate does not support full employment, then fiscal policy can be used to move the economy back toward full employment, as shown in Figure 2. In the initial equilibrium, the capital growth rate $b$ is not sufficient to provide full employment. An increase in the government spending rate $g$ from $g_1$ to $g_2$ shifts the $(s + t - g - x)f(k^d)$ curve down, and the economy shown in Figure 2 moves to full
employment. It is evident that a cut in the tax rate $\tau$ could also be used to move the economy to full employment. The model also indicates an increase in the savings rate $s$, a decrease in the net export rate $x$, or a decrease in the depreciation rate $\delta$ will move the economy away from full employment. All of these results are consistent with standard Keynesian theory.

**Figure 2. Effect of an Increase in the Government Spending**

If the other exogenous variables of the model remain constant, then changes in the employment level and changes in the unemployment level will be related to changes in the growth rate of capital. When the capital growth rate $b$ changes, the change in the change in the growth rate $b'$ is not equal to zero. In response, the Keynesian view is that
the employment level $L^d$ would change, causing a change in the intensive capital demand level $k^d$; implying $k^{d'}$ is not equal to zero. Differentiating condition (18), we obtain the following relationship between $b'$ and $k^{d'}$:

$$(20) \quad k^{d'}/k^d = b'/[(s + t - g - x)f'(k^d) - (b + \delta)].$$

The imposition of the Inada conditions implies, for any given capital growth rate $b$, that $[(s + t - g - x)f'(k^d) > (b + \delta)]$ holds, as shown in figures 1 and 2. Thus, consistent with comparative static analysis shown in Figure 1, condition (20) indicates an increase in the capital growth rate (i.e., $b' > 0$) implies a decrease in the intensive capital demand level (i.e., $k^{d'} > 0$).

The change in the unemployment level depends not only on the change in intensive capital demand but also the change in the intensive capital supply. Differentiating condition (19), we obtain $U'/U = k^{d'}/k^d - k^{x'}/k^d$, which is same as

$$(21) \quad U'/U = k^{d'}/k^d - [b - [a + n]].$$

Together, equations (20) and (21) determine the values of the endogenous variables $k^{d'}$ and $U'$. The variables $k^d$ and $U$ are predetermined. The variables $s, t, g, x, a$, and $\delta$ are exogenous as in the original model. The change in the growth rate of capital $b'$ is exogenous along with the growth rate level $b$.

When the capital growth rate is constant, conditions (20) and (21) indicate unemployment level is changing at a constant rate. Specifically, $b' = 0$ implies through (20) that $k^{d'} = 0$. In this situation, condition (21) reduces to $U'/U = a + n - b$. This
condition indicates the unemployment rate will be increasing when the capital growth rate is relatively low and decreasing when the capital growth rate is relatively high.

There is no change in the unemployment level (i.e., $U' = 0$) when $b = a + n$. In this special case, the demand for labor generated by the additional investment is just sufficient to absorb the additional labor supply and additional production capability provided by technological improvement. Interestingly, $U' = 0$ does not imply $U = 1$. In the standard growth model, $U = 1$ or full employment is assumed, and the economy converges to a steady state where the capital growth rate $b$ is equal to the sum $a + n$. What our work here shows is the standard model is a special case, a full employment special case, a special case comparable to that which led Keynes to call his theory and “general” theory. In our modified model, $b = a + n$ can hold with $U > 1$, implying a labor surplus (i.e., $L^s > L^d$). The unemployment in our growth model here is comparable to the unemployment that can occur in an IS-LM model when the capital market is in equilibrium (i.e. on the IS curve). The investment level is supporting a particular level of spending, but this is not providing full employment because the real wage is too high and sticky. If wages cannot fall to provide more employment, more employment can be obtained through an increase in demand.

When the capital growth rate is not constant, the growth rate of the unemployment level fluctuates. Using condition (21) to eliminate $k^{d'}/k^d$ from condition (20), we obtain

$$
\frac{U'}{U} = a + n - b + \frac{b_l}{[s+t-g-x]f'(k^d)-[b+\delta]}.
$$

Condition (22) indicates
The equality \( b' = \left[ b - [\alpha + n] \right] [s + t - g - x] f'(k^d) \) from condition (23) is plotted in Figure 3 for three different cases. Thinking of \( b' \) as a function of \( b \) in this equation, the function is a quadratic. The equation indicates \( b' = 0 \) when \( b = [\alpha + n] \) and when \( b = [s + t - g - x] f'(k^d) - (b + \delta) \). The three cases arise depending upon whether \( [s + t - g - x] f'(k^d) \) is greater than, equal to, or less than \( \delta \), as shown in the figure.

**Figure 3. The Change in Unemployment and the Path for the Capital Growth Rate**

In any of the cases, the \( b' = \left[ b - [\alpha + n] \right] [s + t - g - x] f'(k^d) \) curve in Figure 3 separates the \( U' < 0 \) region from the \( U' > 0 \) region. When the \( (b, b') \) combination is above the given curve, \( U' < 0 \), and vice versa. When \( b \) is smaller than
\[ a + n, \quad k^{s'}/k^{z} = b - [a + n] \] is negative and this is adding to the growth rate of unemployment, which is \( U'/U = k^{d'}/k^{d} - k^{s'}/k^{z} \). In this situation, \( b' > 0 \) is necessary in any of the three cases to keep the unemployment rate from increasing. The difference in the three cases has to do with the capital depreciation rate. The capital growth rate must increase faster to maintain a given level of unemployment when the depreciation rate is higher.

To relate the model to data, it is useful to restate condition (22). Using equation (18), the value \( b + \delta \) is equal to \( [s + t - g - x] f(k^{d}) \). Using this latter condition to replace \( b + \delta \) in equation (22), we obtain \( \frac{U'}{U} = a + n - b + \frac{b'}{[s + t - g - x][f(k^{d})/k^{d}]} \). The definition of the real wage in (17) then implies \( \frac{U'}{U} = a + n - b - \frac{b'}{[s + t - g - x][w/k^{d}]} \). This is the same as

\[ \frac{U'}{U} = a + n - b - \frac{b'}{[s + t - g - x][WAL^{d} / PY][Y/K]} \]

In (24), \( WAL^{d} / PY \) is the share output paid to labor, and \( Y/K \) is the output to capital ratio.

4. Data Description

The model presented in the last section allows for unemployment and provides theoretical relationships between changes in unemployment and other variables. We seek to examine how well this model explains changes in unemployment. This section presents relevant data for the U.S. economy over the 1947-2015 period, collected from the U.S. Bureau of Economic Analysis and the U.S. Department of Labor.
Figure 4 shows gross private domestic investment $I$ as a percent of nominal gross domestic product $Y$, a measure of the investment rate. The average the investment rate, which is 17.2 percent, is also presented. Notice, there is no long term trend, either up or down, in the investment rate. Rather, the investment rate fluctuates around the 17.2 percent average.

![Figure 4. Investment Rate](image)

Figure 5 presents data on the savings rate $s$, taxation rate $t$, government spending rate $g$, and net export rate $x$. Data for these variables were obtained by dividing the nominal values for gross private domestic savings $S$, net taxes $T$, government consumption expenditures and gross investment $G$, and net exports of goods and services $X$ by the nominal GDP. National income accounting implies $s + t - g - x$ is equal to the investment rate presented in Figure 4, which as noted fluctuates around a 17.2 percent average. However, there are some interesting movements in the components that make up this net national saving quantity. For example, the net tax rate $t = T/Y$ and the
saving rate $s = S/Y$ clearly move in opposite directions in more recent years. Also, the net tax and net export rates have trended downward for some time.

Figure 6 presents, in real terms, gross private domestic investment, the private consumption of fixed capital, and net private domestic investment. The consumption of fixed capital is a depreciation measure. Net private domestic investment is gross private domestic investment minus the private consumption of fixed capital. It provides a measure of the net addition to capital. Because the consumption of fixed capital is not volatile, net and gross private domestic investment vary similarly. Thus, the variation in net investment provides a measure of the variation in the demand for investment goods that is roughly the same as that provided by gross investment.
A capital stock series is presented in Figure 7. The capital stock for a given year is equal to that of the previous year plus the net private domestic investment level for the given year.

Figure 8 shows the output to capital ratio for the economy. This ratio fluctuates around the average value of 0.46, or 46%, which is also shown. The initial capital stock
level (for 1946) used to create the Figure 7 capital stock series was chosen to make the output to capital ratio is roughly constant over time, a fact which has been repeatedly documented and repeatedly presented in empirical examinations of growth theory. The data in Figure 8 show that the output to capital ratio does remain relatively stable, with output ranging between 42 and 49 percent of capital.

Figure 8. Output to Capital Ratio

Figure 9 presents the actual capital growth rate, our variable $b$, calculated as the percentage change from the previous year in the capital level plotted in Figure 8. The average for the series of 3.1% is also presented. As shown in the figure, however, the actual growth rate does not just fluctuate around the average. Rather, the actual growth rate follows a decreasing trend. Along the trend shown in Figure 9, the growth rate decreases 0.04 percentage points per year, so that the rate of growth decreases by one full percentage point over roughly 30 years.
Because it facilitates building the model we presented above, we measure unemployment as the ratio of the labor supply $L^s$ to labor demand $L^d$. We use the U.S. civilian labor force as a measure of the labor supply and civilian employment as a measure of labor demand, both obtained from the U.S. Department of Labor. Figure 10 shows the unemployment level $U$ fluctuating around the average value of 1.062.
Figure 11 presents $U'/U$, the growth rate of the unemployment ratio $U$, along with the capital growth rate $b$. The inverse relationship between these two growth rates is evident. Keynes theorized that cause and effect ran from the capital growth rate to the change in unemployment. Our Keynesianized growth model captures this cause and effect relationship. We now turn to fitting the model to the data.

5. Fitting the Model to the Data

For the model presented in section 3, equation (24) relates the changes in the unemployment rate to the changes in the capital growth rate. The growth rate of technology $a$ is assumed constant. However, it may not be constant, (and it turns out that a changing rate best fits the data). To allow for a changing rate of technical change, assume the level of technology is given by $A = e^{a_0 + a_1 t + a_2 t^2 + a_3 t^3}$ so the rate of technical change is $a = a_1 + 2a_2 t + 3a_3 t^2$. Replacing the variable $a$ in equation (24) and rearranging terms, we obtain
Combining data for the variables on the left side of (25), we obtain a variable we can regress on the time variables and \( b' \) to obtain an estimate for \( \frac{1}{[s+t-g-x][WAL_2/\Pi][\bar{Y}/K]} \). The theoretical discussion surrounding Figure 3 explains why we expect this estimate to be negative. Intuitively, a higher rate of capital growth rate increases the demand for output, which increases the demand for labor, which curbs unemployment. Regressing \( \frac{U_t}{U} - n + b \) on \( t, t^2 \), and \( b' \), we obtain\(^5\)

\[
\frac{U_t}{U} - n + b = 0.023 + 0.00018t - 0.000008t^2 - 0.596b' \quad R^2 = 0.61
\]

\[
(0.0027)^{***} (0.00019) \quad (0.000003)^{***} (0.117)^{***}
\]

Using (26), we can construct the predicted paths for both the rate of change in unemployment \( \frac{U_t}{U} \) and the rate of technical change \( a \). The significant estimate on the \( t^2 \) variable indicates the rate of technical change is not constant over time. Rather, our technical change indicates the rate of technical change follows the path \( a = 0.023 + 0.00018t - 0.000008t^2 \). The model \( \frac{U_t}{U} \) path is plotted in Figure 12, along with the actual path, and the model rate of technical change \( a \) is plotted in Figure 13.

\(^5\) The standard error for each estimated coefficient is presented in parenthesis underneath the coefficient. The asterisks indicate the level of significance at which the coefficient is significantly different than zero. Three asterisks (***), indicates a 1% level of significance, two asterisks (**), indicates 5%, one asterisk (*), indicates 10%, and no asterisks indicates less than 10%.
The path for technical change shown in Figure 13 shows an estimated negative rate of technical change for the present. While a declining rate of technical change is conceivable, a negative rate of change for the present is not realistic. To move to a more realistic model, we drop the $t^2$ variable and estimate a linear path for the rate of technical change. Regressing the dependent variable on $b'$ and $t$ and we obtain
(27) \[ \frac{\dot{U}}{U} - \dot{n} + b = 0.029 - 0.00038t - 0.585b', \quad R^2 = 0.55 \]

\[(0.002)^{***} (0.00005)^{***} (0.125)^{***}\]

In this case, the rate of technical change follows the path \( a = 0.029 - 0.00038t \), and that path is plotted in Figure 13. The path for the unemployment growth rate \( \frac{\dot{U}}{U} \) for this model is presented in Figure 12.

The model (27) indicates the rate of technical change as slowed over time, from 2.9 percent in 1949 to 0.4 percent in 2015. This path, along with the decreasing growth rate for capital shown in Figure 9, explains why the growth rate of output for the U.S. economy has decreased over time. That is, this model can explain the long term growth rate of the U.S. economy, like the standard growth theory model. However, as shown in Figure 12, this model effectively explains variations in unemployment.

6. Conclusion

We have shown standard growth theory can be adjusted in a natural way to capture the essence of Keynes’ general theory, which hypothesizes fluctuations in employment and unemployment are caused by exogenous changes in investment. Compared to the standard model, our modified growth model is unique in that it specifies the path for capital as being exogenously determined. It is also relaxes the assumption of full employment by assuming a sticky wage.

Comparable to Keynes general theory, our growth model provides a more general macroeconomic theory. It reduces to the standard growth theory, when labor supply is restricted to be equal to labor demand. When the equality is not enforced, ours is a
partial equilibrium model comparable the IS model of static macro theory. As in the standard growth model, an investment equals saving condition is imposed in our model, which in static theory implies the economy is on the IS curve. In static theory, being on the IS curve does not imply full employment, and the same is true in our model. In particular, if we impose \( b = a + n \) in our model, so the exogenous path for the capital growth rate is constant and equal to the sum of the technology and labor supply growth rates, then the unemployment level \( U \) will be constant. However, as in the IS model, this unemployment level is not necessarily associated with full employment. In summary, our model can be thought of as the partial equilibrium IS model for growth theory.

In the empirical section, we showed our model explains unemployment movements relatively well. The capital growth and an increase in the capital growth rate each have a significant and positive impact on the change in the unemployment level. The marginal impact of \( b' \) (the change in the capital growth rate) on \( U'/U \) (the unemployment growth rate) is -0.58, meaning an increase in the capital growth rate by one percent reduces the growth rate of unemployment by 0.58 percentage points.

A byproduct of our structural estimation is an estimate of the rate of technological improvement. Our model indicates the rate was 2.9 percent in 1949 but has decreased to just 0.4 percent in 2015. Our data also indicates the growth rate of capital has decreased by 0.04 percentage points per year on average, (see Figure 9), or one full percentage point over roughly 30 years. Thus, our model not only allows short term fluctuations to be explained, but also long term growth. In particular, our model indicates the observed long term decrease in the rate of economic growth (for output) in the U.S. is partly due to
a decrease in the rate at which technology improves and partly due to a decrease in the rate at which capital accumulates.

Empirically, we have focused on examining how fluctuations in the capital growth rate impact unemployment because this was what Keynes emphasized. Yet, as shown in section 3, our model can be used to examine additional causes (i.e., changes in savings, tax, government spending, depreciation, or net export rates). The model could be used in additional empirical research to examine how factors other than changes in capital influence unemployment.

References


Hicks, J.M., 1937. Mr. Keynes and the Classics. Econometrica. 5, 74-86.


Appendix

Deriving the reduced form of the standard growth theory model

Considering equation (10):

\[ K' = I - \delta K \]

\[ K' = S - [G - T] - X - \delta K \]

\[ K' = sY - gY + tY - xY - \delta K \]

\[ K' = (s - g + t - x)Y - \delta K \]

\[ K' = (s - g + t - x)F(AL^d, K) - \delta K \]

Dividing both sides by \( AL^d \):

\[ \frac{K'}{K} \cdot \frac{K}{AL^d} = (s - g + t - x)F\left(1, \frac{K}{AL^d}\right) - \delta \frac{K}{AL^d} \]

Defining \( k^d = K/[AL^d] \):

\[ \frac{K'}{K} \cdot k^d = (s - g + t - x)f(1,k^d) - \delta k^d \]

Note that: \( k^d = K/[AL^d] \)

\[ \frac{k'^d}{k^d} = \frac{K'}{K} \cdot \frac{1}{A^d} \cdot \frac{L^d}{L^d} \cdot \frac{A^d}{A^d} \]

\[ \frac{K'}{K} = k^d + \frac{1}{A^d} + \frac{L^d}{L^d} \]

\[ (k^d + \frac{1}{A^d} + \frac{L^d}{L^d}) \cdot k^d = (s - g + t - x)f(k^d) - \delta k^d \]

\[ k'^d = (s + t - g - x)f(k^d) - (a + n + \delta)k^d \]

Deriving the reduced form of the production level

Consider equation (1):

\[ Y = F(AL^d, K) \]

Dividing both sides by \( AL^d \):

\[ \frac{Y}{AL^d} = F\left(1, \frac{K}{AL^d}\right) \]

Defining \( y = Y/[AL^d] \) and \( k^d = K/[AL^d] \):
\[ y = f(k^d) \]

**Deriving the reduced form of the nominal wage level**

Consider equation (3):

\[
\frac{W}{P} = F_{AL^d}(AL^d, K)
\]

\[ w = \frac{W}{P} \]

\[ w = F_{AL^d}(AL^d, K) \]

Assuming a Cobb-Douglas production function as \( Y = [AL^d]^{1-\alpha}K^\alpha \) and a profit function as \( \pi = PY - W[AL^d] - RK \):

\[
\pi = P[AL^d]^{1-\alpha}K^\alpha - W[AL^d] - RK
\]

\[
\frac{\sigma \pi}{\sigma AL^d} = (1 - \alpha)P[AL^d]^{-\alpha}K^\alpha - W
\]

\[
\frac{\sigma \pi}{\sigma [AL^d]} = 0
\]

\[
(1 - \alpha)P\left(\frac{K}{AL^d}\right)^\alpha - W = 0
\]

\[
(1 - \alpha)\left(\frac{K}{AL^d}\right)^\alpha = \frac{W}{P}
\]

\[
(1 - \alpha)(k^d)^\alpha = w
\]

\[
(k^d)^\alpha - \alpha(k^d)^\alpha = w
\]

We also have:\[
\frac{Y}{[AL^d]} = [AL^d]^{-\alpha}K^\alpha
\]

\[
y = K^\alpha/[AL^d]^\alpha
\]

\[
y = (k^d)^\alpha
\]

\[
f(k^d) = (k^d)^\alpha
\]

\[
f'(k^d) = \alpha(k^d)^{\alpha-1}
\]

\[
f'(k^d)/(k^d)^{\alpha-1} = \alpha
\]

Therefore:\[
w = f(k^d) - f'(k^d)k^d
\]

**Deriving the reduced form of the nominal capital rental rate**
Consider equation (4):

\[ \frac{R}{P} = F_K(AL^d, K) \]

\[ r = \frac{R}{P} \]

\[ r = F_K(AL^d, K) \]

Assuming a Cobb-Douglas production function as \( Y = [AL^d]^{1-\alpha}K^\alpha \) and a profit function as \( \pi = PY - W[AL^d] - RK \):

\[ \pi = P[AL^d]^{1-\alpha}K^\alpha - W[AL^d] - RK \]

\[ \sigma \pi / \sigma K = \alpha P[AL^d]^{1-\alpha}K^{\alpha-1} - R \]

\[ \sigma \pi / \sigma K = 0 \]

\[ \alpha (K^{\alpha-1}/[AL^d]^{\alpha-1}) = R/P \]

\[ \alpha (k^d)^{\alpha-1} = r \]

Therefore: \( r = f'(k^d) \)

**Steady State Analysis:**

- **Slope:** \( \bar{\pi} = f'(\bar{k}_2^d) \)
- **\( \bar{w} = f(\bar{k}_2^d) - f'(\bar{k}_2^d)\bar{k}_2^d \)**
- **\( f(\bar{k}_2^d) \)**
- **\( [a + n + \delta]\bar{k}_2^d \)**
- **\( (s + t - g - x)f(\bar{k}_2^d) \)**

\[ y = Y/[AL^d] \]
\[ y = \frac{Y}{L^d} \cdot \frac{1}{A} \]
\[ \ln y = \ln \left( \frac{Y}{L^d} \right) \cdot \ln \left( \frac{1}{A} \right) \]

Differentiate with respect to time we get:
\[ g_y = g_{\frac{y}{L^d}} - g_A \]

In steady state \( y = \bar{y} \) and so \( g_y = 0 \), therefore:
\[ g_{\frac{y}{L^d}} = g_A \]
\[ g_{\frac{y}{L^d}} = \alpha \]

Same for capital, we have \( k^d = K/\left[AL^d \right] \)
\[ k^d = \frac{K}{L^d} \cdot \frac{1}{A} \]
\[ \ln k^d = \ln \left( \frac{K}{L^d} \right) \cdot \ln \left( \frac{1}{A} \right) \]

Differentiate with respect to time we get:
\[ g_{k^d} = g_{\frac{k}{L^d}} - g_A \]

In steady state \( k^d = \bar{k^d} \) and so \( g_{k^d} = 0 \), therefore:
\[ g_{\frac{k}{L^d}} = g_A \]
\[ g_{\frac{k}{L^d}} = \alpha \]
Chapter 3

Bounded Rationality and Ambiguity

Abstract

This paper examines the results of a preference experiment aimed at examining the ability of people to distinguish a better uncertain prospect from a worse uncertain prospect when the difference between the two is the probability distribution. This tests the extent to which human subjects perceive ambiguity because of limited cognitive capacity even though there is no ambiguity as ambiguity is normally defined. We found that subjects did, for the most part, place a higher value on better prospects – Cognitive ability to distinguish. However, an evidence of ambiguity was found due to the common and not rare valuation errors. By moving to ambiguity, bids were increased when max was high (more optimism) and decreased when max was low (more pessimism).
1. Introduction

Subjective expected utility (SEU) theory (Savage, 1954) is the benchmark theory for how people make decisions under uncertainty. As Einhorn and Hogarth (1985, p.433) note, this theory assumes “probability, which is a measure of one’s degree of belief, can be operationally defined via choices amongst gambles” … [so that] … “if two gambles have identical payoffs but one is preferred to the other, it follows that the probability of winning is greater for the chosen alternative.” Here, we examine this assumption, seeking to understand the extent to which cognitive limitations make uncertainty ambiguous to the decision maker even when the decision environment suggests there is no ambiguity.

Ellsberg (1961) used the word ambiguity to distinguish what Frank Knight called “unmeasurable uncertainty” versus “measureable uncertainty.” Uncertainty is measurable when a single probability distribution is known to exist over the possible states. Frank Knight, Daniel Ellsberg, and researchers since Ellsberg have used the word risk to characterize this type of uncertainty. “The amount of ambiguity [or unmeasurable uncertainty] is an increasing function of the number of distributions that are not ruled out (or made implausible) by one's knowledge of the situation (Einhorn and Hogarth, 1985, p.435).

Ellsberg (1961, pp. 660-661) recognized that the degree of ambiguity is “a subjective variable,” but he went on to say, “it should be possible to identify 'objectively' some situations likely to present high ambiguity, by noting situations where available information is scanty or obviously unreliable or highly conflicting; or where expressed
expectations of different individuals differ widely; or where expressed confidence in estimates tends to be low.” Ellsberg specifically contrasted coin-flipping and roulette, where the random processes generating the outcomes are stable, with “the results of research and development, or the performance of a new President, or the tactics of an unfamiliar opponent. Einhorn and Hogarth (1985, p.455) comment that “when coins are “fair” or random drawings are taken from urns with known p, there is no second-order uncertainty,” meaning no ambiguity.

Why might the uncertainty be ambiguous to the decision maker even when the random processes generating outcomes are stable, meaning the single probability distribution is known? Herbert Simon (2000, p. 247) gave us a reason when he noted, “The practical empirical limits to computation typically come into play long before the logical and mathematical limits do.” Going further he said, “Once one introduces into the SEU maximization Eden the snake of boundedness, it becomes difficult to find a univocal meaning of rationality, hence a unique theory of how people will, or should, decide” (Simon, 2000 p. 251).

The main idea of the designed experiments in this study is the fact that what appears to be risk may be ambiguity because of our limited cognitive capacity. Apparent risk-seeking behavior may not be a preference for taking risk. It may instead be optimism, which under ambiguity can overcome risk aversion.

Two experiments is designed one under risk (no-ambiguity) and one under total-ambiguity. Under the risk, uncertainty that is measurable (Knight, 1921) and a single probability distribution is known to exist over the possible states. Risk aversion attribute
of the individuals implies that for two prospects with the same expected value, the lower variance bet will be preferred which arises from diminishing marginal utility. However, under ambiguity, there is a room for pessimism and optimism behavior. When the probability distribution is unknown, pessimism assumes the applicable distribution is one that puts more weight on the worse outcome(s). When the probability distribution is unknown, optimism assumes the applicable distribution is one that puts more weight on the better outcome(s). Therefore, apparent risk seeking behavior may not be risk seek, but may rather be risk averse behavior overcome by optimism.

The remainder of the paper unfolds as follows. Section 2 presents relevant previous work, and leads to the logic behind the preference experiment performed. Section 3 describes the experimental design. Section 4 presents theory related to our experiment. Section 5 explains the results in both no-ambiguity and total-ambiguity cases. Section 6 concludes by summarizing the results and commenting upon their implications.

2. Relevant Previous Work

Heiner (1983, p. 562) contends “The presence of a C-D (competence-difficulty) gap will introduce uncertainty in selecting most preferred alternatives.” In the decision theory offered by Heiner, environmental variables determine the difficulty of the decision problem and variables internal to the decision maker determine the decision maker’s competence. Cognitive limitations imply a gap between competence and difficulty. Standard models like the SEU model implicitly assume the competence of the decision maker arises to meet the problem difficulty. However, Heiner (1983, p. 561)
alternatively proposes imperfect behavioral rules or heuristics “arise because of uncertainty in distinguishing preferred from less-preferred behavior.”

People may behave “as if” they subjectively construct a probability distribution over states and then maximize expected utility, as SEU theory postulates. However, Archibald, Simon, and Sameulson (1963) critiqued as-if theory by arguing, “The unreality of premises is not a virtue in scientific theory but a necessary evil---a concession to the finite computing capacity of the scientist.” Herbert Simon (1955, p. 99) sought to “replace the global rationality of economic man with a kind of rational behavior that is compatible with the access to information and computational capacities that are actually possessed”

The assumption that people behave as if they solve a known optimization problem is useful because the solution to the problem gives insight as to how the decision maker will optimally respond to changes in the environment. This standard approach loses its usefulness either when the problem does not sufficiently characterize the real world or when the decision maker does not actually optimize, so the prediction of the model does not well explain real world behavior. It therefore follows that the value of exploring how cognitive limitations impact decision-making under uncertainty lies in being able to better predict behavior. Simon (2000 p. 251) noted, “Perhaps the simplicity we should look for, in place of unattainable classical rationality, will come as we study empirically and concretely … how human beings actually adapt to the very severe limitations on their computational powers.”
Previous research provides insight about how cognitive limitations influence decision making under uncertainty.

Simon’s (1967) presented the idea that emotions redirect cognitive resources in uncertain situations. Lowenstein, Weber, Hsee, and Welch (2001) develop this idea by recognizing emotions often conflict with cognitive evaluations. While cognition can over-ride emotion, emotional responses are more universally exhibited by all animals, serving to alert the animal to danger and risk of the unknown. Camerer, Loewenstein, and Prelec (2005) present evidence that emotion in the form of fear is traceable to the amygdala in the brain, and present evidence that emotion in the amygdala can be over-ridden by cognitive “cortical inputs.” Nonetheless, the physiological and psychological tendency to respond emotionally rather than cognitively to uncertainty may cause the SEU model of uncertainty to fail.

Jeske and Ute (2008) present neurological evidence that people distinguish outcomes from probabilities when they make decisions under uncertainty. Neural functioning can be categorized as cognitive or affective. Cognition involves conscious deliberation, while affect involves an emotional response. The cognitive system seems to be more sensitive to probabilities, while the affective system seems to be more sensitive to outcomes. Outcomes and probabilities also seem to activate different areas of the brain. Functional magnetic resonance imaging has documented that the subcortical nucleus accumbens is activated by an anticipated outcome and the cortical mesial prefrontal cortex is activated by an anticipated probability. “The brain is much more responsive to changes in gain size than to equivalent changes in probability” (Jeske and
Ute, 2008, p.52). Jeske and Ute (2008, p.62) go further and note, “Information about probability might be useless without the corresponding information on outcome, but not necessarily vice versa. This inherent asymmetry alone might explain the overall finding that participants spent more time looking at outcomes.”

Probability and probability theory have been devised by we humans to represent uncertainty. Crovelli (2009, p. 10) defines probability as “a numerical measure of uncertainty, … a subjective numerical statement of man’s beliefs about the operant causes in the world.” The theory of probability, he notes, provides a methodology for evaluating probabilities. Mulligan (2013, p. 314) describes a probability as a heuristic, much like a market price, providing the decision maker with high quality information at much lower cost than would have to be incurred if complete knowledge were pursued.

Gilboa, Leiberman, and Scheidler (2009) describe three approaches to representing uncertainty with probability. The “Classical Approach” implements the “Principle of Insufficient Reason” and presumes each outcome is equally likely. The “Frequentist Approach” presumes the likelihood of an event can be represented by the past empirical frequency of the event. The “Subjective Approach” presumes the numerical probability is a measure of a degree of belief, constrained to satisfy certain conditions. Our primary interest will be in whether cognitive limitations introduce subjectivity into situations where the Classical Approach applies, where the decision maker should be able to objectively recognize and then apply the relevant probabilities.

Falk and Wilkening (1998) review experiments with children, and they demonstrate probability requires higher level cognition. Children age 4 or 5 years old
exhibit only “a glimmer of probability understanding.” A common error, which is almost completely eliminated by age 11, is for the child to prefer a prospect with more chances over a prospect with fewer chances but higher probability.

Barron and Giovanni (2013) report a difference in the response to risk when the information is described versus experienced. People tend to overweight small probabilities in decisions from description, while they tend to underweight small probabilities in decisions from experience. The underweighting from experience is perceived to occur because experience generates a small sample, which causes the bias.

Karelitz and Budescu (2004) examine how probabilities are communicated verbally. They find people prefer to express uncertainties verbally, use diverse language to describe uncertainty, and vary in their numerical interpretations of linguistic terms. This implies likelihoods are often not well communicated. Karelitz and Budescu (2004, p.26) conclude “Except in very special cases, all representations of uncertainties are vague to some degree in the minds of the originators and in the minds of the receivers.” Yet, they note decision makers must resolve this vagueness in some way because “one can have imprecise opinions but cannot take imprecise actions.”

Weber (1994) notes that, to be able to explain anomalies, more recent models of decision making under uncertainty have deviated from the assumption that the subjective probabilities a decision maker associates with outcomes are independent of the outcomes. Weber labels these models “configural,” meaning the weight the decision maker gives to an outcome is not just the probability of the outcome itself but the weight is dependent upon the rank of the outcome in the configuration of possible outcomes. Lowenstein,
Weber, Hsee, and Welch (2001, p.276) note that “One of the most robust observations in the domain of decision making under uncertainty is the overweighting of small probabilities, particularly those associated with extreme outcomes.”

Cumulative prospect theory (Tversky and Kahneman, 1992) can be considered a culmination of the effort to modify subjective expected utility to be able to account for behaviors which subjective expected utility cannot explain (e.g. loss aversion, equity premium puzzle, why gamblers also buy insurance). As Weber (1994, p. 234) explains, cumulative prospect theory kept the original value function of prospect theory, which is concave for gains and convex and steeper for losses, but replaced the prospect theory decision weighting function with Quiggin-Yaari rank dependent transformation of cumulative probabilities. The rank dependent transformation allows optimism and pessimism to enter the decision.

Tversky and Kahneman (1974) present three basic reasons why people systematically error in subjectively forming probability judgments: Representativeness, availability, and anchoring and adjustment. While our focus is on a situation where an objective formation of probability is expected, it is still useful to review these ideas. Representativeness is characterized by the statement, “The probability that process B generates event A is evaluated by the degree to which A is representative of B” (Tversky and Kahneman, 1974, p.1124). Availability describes using a cue for assessing probability because it is more easily brought to mind. Because people search using adjustment compared to a reference point, a reference point may act as an anchor and receive more weight than other alternatives.
Medin (1989, p. 1469) defines categorizing as “treating two or more distinct entities as in some way equivalent in the service of accessing knowledge and making predictions.” The classical view of categorization distinguishes categories using lists of features or properties that individually are necessary for category membership and collectively are sufficient to determine category membership. The probabilistic view of category structure is an alternative. In the probabilistic view, categories are fuzzy, being organized around a set of correlated attributes that are only characteristic or typical of category membership. The prototype view presents a single summary representation around which a category is formed. The exemplar view denies that there is a single summary representation and instead claims that categories are represented by means of examples. Medin offers theory as an additional way to construct categories, noting that people construct categories because they allow some theory to be applied, thus the appropriate categorization will depend upon the theory being applied.

Peters and Par (2015) emphasize people perceive numeric magnitudes inexactly. The ability to discriminate numeric magnitudes develops with age and with education. People who are better able to discriminate numeric magnitudes (having a more exact approximate sense of numerical magnitudes) better at applying math. People who are worse tend to be more risk averse. Peters and Par also note that a number has little affective meaning if there is not another number to compare it with, and the attention paid to a number can depend upon the comparison made.

Moyer & Landauer (1967) show people are better able to discriminate far-apart values (5 vs. 9) than close-together values (5 vs. 6), and Parkman (1971) shows is easier
for people to distinguish a difference of the same magnitude when the numbers are smaller (e.g., 5 vs. 15, 90 vs. 100). These outcomes are theorized to arise because people do not represent numbers in an exact way in their minds. Specifically, Moyer & Landauer theorize people place numbers on an imaginary continuum in their mind, so they can compare them in the same way they would compare the length of two lines or other physical phenomena. The primary alternative to this “analog” representation is a “digital” representation, where the person places the numbers (say 5 and 9) in a set (say \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}), so that the comparison of two numbers (say 5 and 9) involves incrementing through numbers in the set (say from 5 through the numbers 6, 7, and 8 to the number 9). Parkman (1971) notes there is evidence of the digital representation and also presents the “Principle of Lexical Marking” as a distinction idea. This idea is people use adjectives like larger versus smaller in an asymmetric way so a method of distinction can be implemented. For example, it seems people tend to identify the larger of two numbers first in their minds and then compare the smaller to the larger rather than doing the opposite.

3. Experimental Design

If people perceive numeric magnitudes inexactly, as Peters and Par (2015) emphasize, then they will perceive likelihoods inexactly as they gauge the attractiveness of prospects. The availability heuristic suggests people might use particular likelihoods as a reference points. We examine the possibility that the units commonly used to divide money may be reference points. That is, people may more easily understand the following probabilities:
Probabilities close to these but are less familiar and therefore more cognitively challenging to apply are:

- 5/9: Better odds than a coin flip, but harder to apply.
- 5/18: Better odds than one quarter, but harder to apply.
- 5/45: Better odds than a dime to a dollar, but harder to apply.
- 5/450: Better odds than a penny to a dollar, but harder to apply.

It is conceivable that availability leads people to categorize the less familiar probabilities in the same likelihood category as the familiar probabilities. It is also conceivable that a prospect associated with the less familiar probability will be discounted because it is cognitively more demanding to apply the less familiar probability.

A group of 60 students at the University of Nevada, Reno were conducted to test for this purpose. The participants were randomly selected from University of Nevada business school students. Six sessions were conducted in two treatment orders of A (No-Ambiguity, Total-Ambiguity) and B (Total-Ambiguity, No-Ambiguity). Table 1 shows the experimental design used for the No-Ambiguity case. At the beginning of the experiment, subjects had to do a simple “earnings task” to earn an endowment of $10. The simple task was to do 10 squats. We offered an alternative activity that was not
physical activity in case that a subject was physically unable to do the squats or felt significantly uncomfortable about doing them. The subjects’ task was to use the $10 to decide how much they are willing to bid to play a lottery. After reading a set of instructions with the experiment administrator, and after playing 4 practice games, participants played a total of 9 games where they were required to express their preferences for lottery prospects by bidding to play each given lottery. As shown in Table 1, the design involves four pairs of simple lotteries and one extra lottery included to test for an additional item of interest.

### Table 1. Experimental Design: No-Ambiguity

<table>
<thead>
<tr>
<th>Game</th>
<th>Probability Label</th>
<th>Outcome</th>
<th>Probability</th>
<th>Expected Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>High</td>
<td>Low</td>
<td>P(High)</td>
</tr>
<tr>
<td>Game 8</td>
<td>&quot;5/9&quot;</td>
<td>5</td>
<td>1</td>
<td>0.556</td>
</tr>
<tr>
<td>Game 4</td>
<td>&quot;1/2&quot;</td>
<td>5</td>
<td>1</td>
<td>0.500</td>
</tr>
<tr>
<td>Game 1</td>
<td>&quot;5/18&quot;</td>
<td>9</td>
<td>1</td>
<td>0.278</td>
</tr>
<tr>
<td>Game 6</td>
<td>&quot;1/4&quot;</td>
<td>9</td>
<td>1</td>
<td>0.250</td>
</tr>
<tr>
<td>Game 5</td>
<td>&quot;5/45&quot;</td>
<td>21</td>
<td>1</td>
<td>0.111</td>
</tr>
<tr>
<td>Game 2</td>
<td>&quot;1/10&quot;</td>
<td>21</td>
<td>1</td>
<td>0.100</td>
</tr>
<tr>
<td>Game 3</td>
<td>&quot;5/450&quot;</td>
<td>201</td>
<td>1</td>
<td>0.011</td>
</tr>
<tr>
<td>Game 7</td>
<td>&quot;1/100&quot;</td>
<td>201</td>
<td>1</td>
<td>0.010</td>
</tr>
<tr>
<td>Game 9</td>
<td>&quot;1/9&quot;</td>
<td>21</td>
<td>1</td>
<td>0.111</td>
</tr>
</tbody>
</table>

The first column in Table 1 shows the order in which the subjects played the games. The order was mixed so subjects would not play two of the lottery pairs one after the other, making it difficult for the subject to readily compare the designed pairs. In each simple lottery, the low outcome is a gain of one dollar. It is the high outcome that varies. Let $p$ denote the probability of the high outcome for the lottery. The probability $p$ and the high outcome were varied as shown in columns two and three. The second lottery in each pair is designed to be cognitively less challenging to evaluate, while the first in the
pair is designed to be cognitively more challenging. The probabilities and high outcomes are set so the cognitively more challenging lottery has a higher expected value. The primary question of interest is whether the average subject will ever value a cognitively less challenging lottery significantly higher than a cognitively more challenging lottery even though it is clearly inferior.

Subjects were not given information on the expected values of the game, but rather were given the probability and outcome information in a format designed to minimize any ambiguity (See the instructions in the Appendix). The lottery was conducted using red and blue beads drawn from an urn. Using Game 1 as an example, subjects were shown a picture of 18 beads on a piece of paper. Five of the beads shown were blue, and the remaining 13 beads were red, allowing the subject to clearly see the proportion of the more desirable blue beads to the less desirable red beads. Understanding that one dollar is always gained when a red bead is drawn from the urn, the subject was shown a graphic illustrating the number of dollars gained if the preferable blue bead is drawn. The subject was then asked to bid on the lottery by circling a number 1, 2, 3, 4, 5, or 6.

To encourage subjects to provide their willingness to pay as their bid, a Becker-DeGroot-Marcshak (BDM) method was used (Becker, DeGroot, Marschak, 1964). Subjects were told they should be willing to bid at least one dollar to play the lottery because by bidding one dollar they cannot lose anything but could gain. A maximum bid of 6 dollars was allowed. To determine whether the bid was sufficient to play the game, the subject had to roll a six sided die. If the die was greater than the subject’s bid, then
the subject’s bid was deemed insufficient to play the lottery. Alternatively, if the die was less than or equal to the subject’s bid, then the subject’s bid was sufficient.

Game 9 was included to compare with Game 5. Notice the probability of the high outcome is the same in each game, since \( \frac{1}{9} = \frac{5}{45} \). The only difference in the games is that there are 5 blue beads that provide the larger gain in game 5, but only 1 in game 9. We constructed each of the more cognitively challenging games with 5 opportunities to obtain the large gain. It is possible that subjects favor these lotteries because there are 5 blue beads and not just one, irrespective of the probability. By comparing behavior in Game 9 to Game 5, we can test whether subjects were significantly influenced by the number of preferable beads independent of the probability.

We also conducted another design with all the subjects, which we label as Total-Ambiguity case. Subjects in the Total-Ambiguity design played the 9 games in the same order shown in Table 1. However, in the Total-Ambiguity case, subjects only knew how many beads were in the urn. They knew at least one was red and at least one was blue, but they did not know how many of each color. They knew they would receive one dollar for drawing a red bead, and they knew the gain shown in Table 1 for drawing the preferable blue bead. Knowing nothing of the probabilities, the task for subjects in the Total-Ambiguity design was to value the lottery under total ambiguity. They were also motivated using the same BDM die roll mechanism.

To obtain within subject comparisons, as mentioned before, there were two treatment orders of A (No-Ambiguity, Total-Ambiguity) and B (Total-Ambiguity, No-Ambiguity). This A-B block design also allowed us to examine whether there is some
“priming” that occurs. It is of interest to see whether experiencing total ambiguity first leads subjects in the Total-Ambiguity design to play differently when they know the probabilities, and vice versa.

To avoid wealth effects, we had subjects value all 18 lotteries before any subject actually had the opportunity to play a lottery and win money. After valuing all the lotteries, each subject took two draws from a hat containing 18 numbers, 1 through 18. The numbers drawn were associated with the 18 lotteries that had been valued. For those two lotteries, subjects then rolled the die to determine whether they were played. Subjects played the lotteries when the value of the die was less than or equal to the bid, but not otherwise.

The choice of 3 for the expected value was intentional. When a subject bids 2, it indicates risk aversion. When a subject bids 3, it indicates rough risk neutrality. When a subject bids more than 3, it indicates risk seeking behavior. Thus, the design of the experiment allows subjects to reveal their risk preferences to an extent.

4. Theory

Economic analysis of decision making under risk or uncertainty has been explained by the “expected utility” (EU) theory of Von Neumann and Morgenstern (1944) for a long time, which was followed by the subjective expected utility (SEU) theory of Savage (1954). Quiggin (1982) suggested a more general theory trying to cover the choices violating the EU axioms. He called his theory as “anticipated utility” (AU) theory due to the use of decision weights in forming the utility function rather than mathematical expectation. In 1993, Quiggin offered a more general description of AU known as the
generalized expected utility theory which can explain many interesting applications in lottery choices, portfolio choices, gambling, insurance, and the behavior of individuals under risk. In the generalized expected utility theory, outcomes are transformed by a von Neumann-Morgenstern utility function which equalizes a function \( V \) on \( y \) to the sum of each utility multiplied by its probability i.e. \( V(y) = \sum_i p_i U(x_i) \). Moreover, the cumulative probabilities are transformed by a pessimistic or concave weighting function.

The related model is able to show that risk attitudes do not depend on the marginal utility of wealth by reflecting wealth risk aversion.

Hurwicz (1951) introduced a simple max-min model to analyze decision making under ambiguity - or in his own words, under ignorance - where the probabilities are not known. In his original max-min model \( \sigma(M_a, m_d) = \alpha M_d + (1 - \alpha)m_d \), choices are made based on the degree of optimism or pessimism related to the size of outcome. In Hurwicz’s words, the max-min principles are “open to the charges of excessive (systematic) conservatism and recklessness”. He believes that these charges, in the decision making process under ambiguity or ignorance, cannot be explained by the Principle of Insufficient Reason or \( 1/2 \) -optimality which assumes if there is no reason to think otherwise, outcomes are equally likely.

The Alpha Minimax Expected Utility (\( \alpha - MMEU \)) model, developed by Gilboa and Schmeidler (1989), considers both risk and ambiguity in the decision making process by combining the Expected Utility Model and the Hurwicz Model. The Expected Utility Model assumes that an optimal choice by decision maker (DM) will maximize expected utility when the probability of each possible state is known and decision maker
experiences pure risk with no ambiguity. Alternatively, the Hurwicz Model assumes a total ambiguity case where decision maker will evaluate the uncertain prospect based upon the sizes of the outcomes. Consider the following version of the $\alpha - MMEU$ model (Melkonyan and Pingle, 2010), where the value of a prospect to DM is given by

$$V = [1 - \lambda][pU(X_{max}) + (1 - p)U(X_{min})] + \lambda[U(X_{max}) + (1 - \alpha)U(X_{min})],$$

where $\lambda$ measures the degree of ambiguity, $\alpha$ measures the degree of optimism, $p$ is the probability of obtaining the $X_{max}$ outcome when there is no ambiguity, and $1 - p$ is the probability of obtaining the lower $X_{min}$ outcome. When $\lambda = 0$, the model reduces to the Expected Utility Model and DM faces pure risk. Alternatively, when $\lambda = 1$, the model reduces to the Hurwicz Model and decision maker faces total ambiguity. When $\alpha = 0$, DM possesses the highest possible degree of pessimism or ambiguity intolerance. When $\alpha = 1$, DM possesses the highest possible degree of optimism or ambiguity tolerance.

Suppose DM seeks to compare two prospects $(X_{max}, p_1; X_{min}, 1 - p_1)$ and $(X_{max}, p_2; X_{min}, 1 - p_2)$. Assume $p_2 > p_1$, so the probability of obtaining the better outcome $X_{max}$ is greater for the second prospect. If there is no ambiguity (i.e., $\lambda_1 = \lambda_2 = 0$), then DM must strictly prefer the second prospect to the first, for

$$V_2 - V_1 = [p_2U(X_{max}) + (1 - p_2)U(X_{min})] - [p_1U(X_{max}) + (1 - p_1)U(X_{min})]$$

reduces to

$$V_2 - V_1 = [p_2 - p_1][U(X_{max}) - U(X_{min})] > 0.$$

Using the $\alpha - MMEU$ model (1) to compare two prospects, we find

$$V_1 - V_2 = [U(X_{max}) - U(X_{min})][\lambda_2[p_2 - \alpha_2] - \lambda_1[p_1 - \alpha_1] - [p_2 - p_1]].$$
We are interested in the case where DM values prospect 1 more highly than prospect 2 (i.e., $V_1 > V_2$) even though the max outcome occurs with higher probability for prospect 2 (i.e., $p_2 > p_1$). From condition (2), $V_1 > V_2$ implies

$\lambda_2[p_2 - \alpha_2] - \lambda_1[p_1 - \alpha_1] > [p_2 - p_1]$.  

We are interested in the hypothesis that DM’s bounded rationality may generate ambiguity and may also impact ambiguity tolerance. For example, while $p_2 = 5/9 > p_1 = 1/2$, it is cognitively more challenging to evaluate the prospect with $p_2 = 5/9$. This increased cognitive challenge may imply DM perceives more ambiguity when applying $p_2 = 5/9$, so $\lambda_2 > \lambda_1$. The increased challenge may also make DM less ambiguity tolerant, so $\alpha_2 < \alpha_1$.

To examine possibilities, it is useful to assume $\lambda_2 = \lambda_1 + \varepsilon$ and let $\alpha_1 = \alpha_2 + \varepsilon$. This assumption captures the idea that a probability that is cognitively more challenging to evaluate both creates more perceived ambiguity and creates less ambiguity tolerance. The parameter $\varepsilon$ is a measure of the degree to which this impact occurs. Under this assumption, condition (3) reduces to

$\lambda_1 + \frac{[p_2 - \alpha_2 + \lambda_1]}{[p_2 - p_1]} \varepsilon > 1$

Condition (4) helps us understand when DM might value a prospect higher even when it is strictly inferior. Let’s call this a decision error. First, note condition (4) will more likely hold when the ambiguity level $\lambda_1$ is higher. A positive value for $\lambda_1$ indicates DM perceives ambiguity even for the probability $p_1$ that is cognitively more manageable,
so we learn a decision error is more likely when more of this type of ambiguity is perceived. Second, condition (4) will more likely hold when \( p_2 > \alpha_2 \). This indicates a decision error is more likely when DM is more pessimistic (small \( \alpha_2 \)) and when the event is more likely (large \( p_2 \)). Third, when \( p_2 - \alpha_2 + \lambda_1 > 0 \), condition (4) will more likely hold when \( p_2 - p_1 \) is small. For our experiment, \( p_2 - p_1 \) is smaller for low probability events. Combining these last two observations, condition (4) suggests decision error in our experiment may be most likely for intermediate probability prospects, where \( p_2 \) is large enough that \( p_2 > \alpha_2 \) may hold but small enough to make \( p_2 - p_1 \) relatively small.

We can also gain insight by considering the special case \( \lambda_1 = 0 \). In this case, DM does not perceive ambiguity when the probability of winning is the less cognitively demanding \( p_1 \) but perceives some ambiguity when the probability is the more cognitively demanding \( p_2 \). In our experiment, for example, there might not be ambiguity in the case where DM faces a probability of 1/2 because DM might understand that 1/2 relates to a coin flip or to 50 cents on the dollar, but DM might perceive ambiguity when facing the probability 5/9 because there is no simple and natural way to conceive of 5/9. With \( \lambda_1 = 0 \), (3) becomes

\[
(5) \, \lambda_2 [p_2 - \alpha_2] > [p_2 - p_1]
\]

Even when \( \lambda_1 \) is not equal to zero, as long as \( \lambda_1 \) is near zero, condition (5) roughly holds. For condition (5) to hold, \( p_2 > \alpha_2 \) must hold. Thus, we learn that a high level of optimism (large \( \alpha_2 \)) will rule out decision error while a high level of pessimism (small \( \alpha_2 \)) makes decision error more possible. Condition (5) indicates a higher probability
event makes decision error more likely, for then \( p_2 > \alpha_2 \) is more possible. However, condition (5) is also more likely to hold when \( p_2 - p_1 \) is small, which for our experiment will occur for lower probability events. Consequently, again, for our experiment, we have a theoretical prediction that intermediate level probably events have more potential for decision error.

A final special case that can provide some insight is that where \( \alpha_2 = 0 \). In this case, DM is entirely pessimistic toward probabilities that are cognitively challenging to evaluate, and condition (3) reduces to

\[
(6) \quad \lambda_2 p_2 - \lambda_1 p_1 + \lambda_1 \alpha_1 > [p_2 - p_1].
\]

Since we might expect \( \lambda_2 > \lambda_1 \) because the probabilities are cognitively more challenging in case 2, and since \( p_2 > p_1 \) by assumption, we would expect the left side of condition (6) to be positive, so it is possible that (6) holds. For the lower probability pairs shown in Table 1, \( p_2 - p_1 \) is smaller, so it is reasonable to think a decision error is more likely for the lower probability pairs. However, risk aversion might discourage subjects from bidding high on all low probability lotteries, so it is again conceivable that a lottery with an intermediate probability of obtaining the high outcome would be more apt to generate a decision error.

5. Experimental Results

Figure 1 presents the expected changes in the mean value of the bids for both no-ambiguity and total-ambiguity games. Considering the no-ambiguity case, a risk averse person is expected to bid lower as the probability of winning the higher outcome is
decreasing. However, in the actual graphs on the mean bids of the no-ambiguity games (Figure 2), there is a tendency for higher bids through the end. This could be either due to more sensitivity of neural functioning to the changes in the size of outcomes (Jeske and Ute, 2008) or due to some possible ambiguity created through the end.

There are two ambiguity hypotheses we can consider in this experiment: the Principle of Insufficient Reason which assumes if there is no reason to think otherwise, outcomes are equally likely; and the Principle of Proportionality which assumes a better outcome is proportionately less likely. The expected changes in the mean value of the bids for total-ambiguity games is formed in Figure 1 based on both of these hypotheses. In treatment A, subjects were primed for proportionality, since they had previously participated in the risk experiment. In Figure 1 (total-ambiguity – treatment A), the average bid is decreasing as the high outcome increases significantly to $201. Yet, the larger outcomes had significant pull, indicating the principle of proportionality was not typically applied.

**Figure 1. Expected Changes in Mean of the Bids**
Figure 2 presents the mean of bids for both no-ambiguity and total-ambiguity games considering treatments A and B. In the no-ambiguity histograms, the horizontal axis shows the order of games from the highest probability label (5/9) to the lowest (1/100), or similarly from the lower level of risk to the higher level of risk. When the probabilities were known, for most of the pairwise comparisons, the mean bid was higher when the probability of the high outcome was higher, an indication that subjects on average were able to distinguish the better game. The only exception was in the second paired games where subjects bid more on average for a lower expected value prospect when the probability of the high outcome was 1/4 than when it was the higher probability of 5/18. This indicates the average subject was irrational in the sense of not recognizing the 5/18 game was strictly better than the 1/4 game. The probabilities in this paired games can be considered in an intermediate probability category (not too high or too low) and based on our theory described in section 4, a lottery with an intermediate probability of obtaining the high outcome would be more apt to generate a decision error.

The average bids for the 1/2 and 5/9 games are really close to each other which may show that distinguishing 1/2 and 5/9 is more difficult comparing to the other paired games. The reason could be that the quantities 1/2 and 5/9 might be viewed as being equally representative of a high probability event and valued equally. The exemplar theory might indicate a person lumps together different high probabilities into a high probability category, and different low probabilities into a low probability category, rather than making fine distinctions.
Considering the total-ambiguity case, not knowing the odds leads to greater bids when the high outcome gets bigger. This is more sensible in treatment B when we started with the total-ambiguity lotteries. There is another pattern in the total-ambiguity games of treatment A: Not knowing the odds, for a pair of games with the same payoff, the average bid is higher for the smaller stakes of each paired game.

**Figure 2. Mean of the Bids**
We used different Mann–Whitney U-tests to check whether the mean bid for the lotteries with higher expected value (3.22) is higher than the mean bid for the lotteries with lower expected value (3.00). Table 2 reports the results of these tests. The only significant difference can be seen in the mean bids for the paired games of 1/10 vs. 5/45 in both treatments, and also 1/10 vs. 1/9 in treatment B. Subjects can easier distinguish 1/10 from 5/45 (or 1/9), valuing the 1/10 game much lower comparing to the other paired games. Also, the probability of bidding higher on the lower expected value game is much higher for the paired games of 1/4 vs. 5/18 specially in treatment A.

Table 2. Results of the Mann–Whitney U-Test for the Difference in the Mean Bids

<table>
<thead>
<tr>
<th>Treatment A</th>
<th>Ho</th>
<th>z</th>
<th>P-value</th>
<th>Prob(3.00&gt;3.22)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2 vs. 5/9</td>
<td>diff = 0</td>
<td>-0.491</td>
<td>0.6231</td>
<td>no diff</td>
</tr>
<tr>
<td>1/4 vs. 5/18</td>
<td>diff = 0</td>
<td>0.948</td>
<td>0.3433</td>
<td>no diff</td>
</tr>
<tr>
<td>1/10 vs. 5/45</td>
<td>diff = 0</td>
<td>-2.077</td>
<td>0.0378*</td>
<td>diff &lt; 0</td>
</tr>
<tr>
<td>1/10 vs. 1/9</td>
<td>diff = 0</td>
<td>-1.557</td>
<td>0.1196</td>
<td>no diff</td>
</tr>
<tr>
<td>1/9 vs. 5/45</td>
<td>diff = 0</td>
<td>-0.440</td>
<td>0.6598</td>
<td>no diff</td>
</tr>
<tr>
<td>1/100 vs. 5/450</td>
<td>diff = 0</td>
<td>-1.290</td>
<td>0.1969</td>
<td>no diff</td>
</tr>
</tbody>
</table>

Treatment B

| 1/2 vs. 5/9  | diff = 0 | -0.209  | 0.8347       | no diff         | 0.483           |
| 1/4 vs. 5/18 | diff = 0 | -0.520  | 0.6031       | no diff         | 0.457           |
| 1/10 vs. 5/45| diff = 0 | -3.175  | 0.0015*      | diff < 0        | 0.241           |
| 1/10 vs. 1/9 | diff = 0 | -2.656  | 0.0079*      | diff < 0        | 0.284           |
| 1/9 vs. 5/45 | diff = 0 | -0.419  | 0.6750       | no diff         | 0.466           |
| 1/100 vs. 5/450 | diff = 0 | -0.550  | 0.5826       | no diff         | 0.454           |

The hypothesis of interest is that limited cognitive capacity leads people to perceive ambiguity in situations where there actually is none, so they cannot make decisions as precisely as implied by expected utility theory. One manifestation of this bounded rationality might be to mistakenly value a prospect more when it is strictly inferior to another prospect. Table 3 presents the percentage of the participants make
such mistakes for the four different pairs of no-ambiguity games. We say that a participant errs if they bid more for the 1/2 game than the 5/9 game, or bid more for the 1/4 game than the 5/18 game, or bid more for the 1/10 game than the 5/45 game, or bid more for the 1/100 than the 5/450 game. The highest percentage of errors were made when the probability of the high outcome was 1/4 as compared to 5/18. In this pair, 26.7 percent bid more for the 1/4 game than the 5/18 game. The probability pairing of 1/10 vs. 5/45 led to the fewest errors. Only 5 percent made the error of valuing the 1/10 game higher than the 5/45 game. This was the only pairing where the average subject bid more for the better prospect.

Table 3. Errors in Valuing Prospects (No-Ambiguity)

<table>
<thead>
<tr>
<th></th>
<th>1/2 vs. 5/9</th>
<th>1/4 vs. 5/18</th>
<th>1/10 vs. 5/45</th>
<th>1/100 vs. 5/450</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid More for Better Game</td>
<td>%31.7</td>
<td>%26.7</td>
<td>%60</td>
<td>%35</td>
</tr>
<tr>
<td>Bid Same for Each Game</td>
<td>%48.3</td>
<td>%46.6</td>
<td>%35</td>
<td>%50</td>
</tr>
<tr>
<td>Error: Bid More for Worse Game</td>
<td>%20</td>
<td>%26.7</td>
<td>%5</td>
<td>%15</td>
</tr>
</tbody>
</table>

Table 4 compares the results from no-ambiguity or risk experiment with the results from the total-ambiguity experiment. As can be seen, there is more gambling under risk than ambiguity when max outcome is lower and more gambling under ambiguity than risk when max outcome is higher.
Table 4. Comparing Risk and Ambiguity

<table>
<thead>
<tr>
<th>Game</th>
<th>Probability Label</th>
<th>Max Outcome</th>
<th>Bid of 2 or Less</th>
<th>Bid of 4 or More</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Risk</td>
<td>Ambiguity</td>
</tr>
<tr>
<td>Game 8</td>
<td>&quot;5/9&quot;</td>
<td>5</td>
<td>28%</td>
<td>67%</td>
</tr>
<tr>
<td>Game 4</td>
<td>&quot;1/2&quot;</td>
<td>5</td>
<td>28%</td>
<td>43%</td>
</tr>
<tr>
<td>Game 1</td>
<td>&quot;5/18&quot;</td>
<td>9</td>
<td>27%</td>
<td>43%</td>
</tr>
<tr>
<td>Game 6</td>
<td>&quot;1/4&quot;</td>
<td>9</td>
<td>32%</td>
<td>25%</td>
</tr>
<tr>
<td>Game 5</td>
<td>&quot;5/45&quot;</td>
<td>21</td>
<td>25%</td>
<td>25%</td>
</tr>
<tr>
<td>Game 2</td>
<td>&quot;1/10&quot;</td>
<td>21</td>
<td>60%</td>
<td>18%</td>
</tr>
<tr>
<td>Game 3</td>
<td>&quot;5/450&quot;</td>
<td>201</td>
<td>47%</td>
<td>23%</td>
</tr>
<tr>
<td>Game 7</td>
<td>&quot;1/100&quot;</td>
<td>201</td>
<td>58%</td>
<td>18%</td>
</tr>
<tr>
<td>Game 9</td>
<td>&quot;1/9&quot;</td>
<td>21</td>
<td>40%</td>
<td>8%</td>
</tr>
</tbody>
</table>

6. Conclusion

A summary of the results of this study shows that subjects did, for the most part, place a higher value on better prospects – Cognitive ability to distinguish. There is no evidence that 1/2, 1/4, 1/10, and 1/100 were particularly attractive relative to 5/9, 5/18, 5/45, and 5/450. However, an evidence of ambiguity was found in our experiment due to the common and not rare valuation errors. The valuation errors were more common for the intermediate prospects of 1/4 vs. 5/18 where probability of winning and max outcome were moderate. By moving to ambiguity, bids were increased when max was high (more optimism) and decreased when max was low (more pessimism). Comparing the results from the risk experiment with the results from the total-ambiguity experiment, there is more gambling under risk than ambiguity when max outcome is lower and more gambling under ambiguity than risk when max outcome is higher.

Based on our results, there is a possibility that the degree of optimism might grow with the size of the max which might provide a partial explanation for gambling. Also,
there are possible implication for entrepreneurship. One could be the fact that ambiguity is ubiquitous, but especially prevalent for entrepreneurs. A fundamental characteristic of entrepreneurs may not be more willingness to take risk, but rather may be more optimism.

Future possible work could include measuring the optimism of gamblers relative to others, measuring the optimism of entrepreneurs relative to others, and considering the valuation of prospects in other relative contexts.

References


Appendix

Game Instructions: Treatment A

You have been asked to participate in an experiment. The experiment has begun with the reading of these instructions. **I ask that you do not communicate at all** from this point forward because any comment you make out loud, or any question you ask, may influence how others behave. If you have a question, raise your hand and I will come listen to your question privately. I will not be able to answer your question if you are seeking information beyond what these instructions provide. I can only answer clarifying questions. These instructions should provide all the information you need to play this game, so please pay attention as I read through them with you.

To be able to play the game, you first must complete a task to earn an endowment of funds. The simple task is to do 10 squats. This will earn you $10, or $1 per squat. If you are physically unable to do the squats, or if feel significantly uncomfortable about doing them, we can offer an alternative activity that is not physical activity.

(Break from the instructions and earn some money.)

Congratulations! You have earned $10 which you can now use to play a game we will call the “bid game.” Your task in the *bid game* is to decide how much you are willing to bid to play a lottery.

An example lottery is now presented below. There is an “urn” which contains 6 beads. As illustrated below, 5 of the beads are red and 1 is blue.
6 Beads (5 Red, 1 Blue)

If you play this lottery, you will draw a bead from the urn containing the 6 beads. If you draw a red bead, you will win 1 dollar. If you draw a blue bead, you will win 13 dollars. Your potential winnings are summarized in the following picture:

**Winnings**

<table>
<thead>
<tr>
<th>If Red Bead</th>
<th>If Blue Bead</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
</tr>
</tbody>
</table>

Notice, you will win at least 1 dollar no matter what. Thus, you should be willing to bid at least 1 dollar to play this lottery. However, you can bid more. When you bid on a lottery, you will be asked to circle one of the options shown in the following table:

<table>
<thead>
<tr>
<th>My bid for this lottery (circle the number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid</td>
</tr>
</tbody>
</table>

Because you will pay the amount you bid to play the lottery, it is sensible to bid as little as possible. In the real world, you would likely be bidding against others for the opportunity to play. To provide a form of competition in this lab experiment, we will have you roll a die after you circle your bid. If the value on the die is less than or equal to your bid, then you will be able to play the lottery. However, if the value on the die is higher than your bid, the competition essentially wins and you will not be able to play the
lottery. Given these game rules, you will be more likely to play the lottery when you bid a larger amount. The die roll encourages you to pay as much as you are willing to pay to play the game, but not more.

Let’s play this illustration game. Please circle, in the table above, the amount you are willing to bid to play the lottery above.

Now, in this illustration game, we will not actually have your roll a die. However, when you roll the die in the experiment, you will circle the value of your roll in a table like the table below.

<table>
<thead>
<tr>
<th>My die roll for this lottery (circle the number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Die Roll</td>
</tr>
</tbody>
</table>

Together, let’s think about some different possible bids.

Suppose you bid 6.

- When you roll the die, what is the chance that you will get to play the game?
- When you play the game, what is the best outcome you can expect?
- When you play the game, what is the worst outcome you can expect?
- Note, with a bid of 6, you will surely play the lottery, but you risk losing 5 dollars.

Suppose you bid 3.

- When you roll the die, what is the chance that you will get to play the game?
- If you play the game, what is the best outcome you can expect?
- If you play the game, what is the worst outcome you can expect
• Note, with a bid of 3, you have a 3 out of 6 chance of getting to play, but you risk losing 2 dollars.

Suppose you bid 1.

• When you roll the die, what is the chance that you will get to play the game?
• If you play the game, what is the best outcome you can expect?
• If you play the game, what is the worst outcome you can expect?
• Note, with a bid of 1, you do not risk losing any dollars, but you will have a 1 in 6 chance of playing the lottery.

After going through these three bid examples, would you want to change the bid you circled? Don’t answer this question out loud, but think about it.

Just for practice, let’s consider three more lotteries

For the lottery presented below, there is an urn which contains 36 beads. As illustrated, 31 of the beads are red and 5 are blue.

36 Beads (31 Red, 5 Blue)

If you play this lottery, you will draw a bead from the urn. If the bead you draw is red, you will win 1 dollar. If the bead you draw is blue, you will win 17 dollars. Your potential winnings are summarized in the following picture:
Winnings

If Red Bead  
If Blue Bead  

1  
17  

Now, circle your bid for this lottery.

| Bid for this lottery (circle the number) |
| Bid | 1 | 2 | 3 | 4 | 5 | 6 |

For the bid you have circled, answer the following questions:

- When you roll the die, what is the probability that you will get to play the game? 
  __________ of 6 chance.

- If you play the game, what is the best outcome you can expect?  Gain _____

- If you play the game, what is the worst outcome you can expect? Lose ____

Consider the new lottery presented below. There is an urn which contains 50 beads. As illustrated below, 49 of the beads are red and 1 is blue.

50 Beads (49 Red, 1 Blue)
If you play this lottery, you will draw a bead from the urn. If the bead you draw is red, you will win 1 dollar. If the bead you draw is blue, you will win 101 dollars. Your potential winnings are summarized in the following picture:

**Winnings**

<table>
<thead>
<tr>
<th></th>
<th>If Red</th>
<th>If Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>101</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now, circle your bid for this lottery.

<table>
<thead>
<tr>
<th>Bid for this lottery (circle the number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid</td>
</tr>
</tbody>
</table>

For the bid you have circled, answer the following questions:

- When you roll the die, what is the probability that you will get to play the game? __________ of 6 chance.
- If you play the game, what is the best outcome you can expect? Gain _____
- If you play the game, what is the worst outcome you can expect? Lose _____

Consider the last practice game presented below. There is an urn which contains 300 beads. As illustrated below, 295 of the beads are red and 5 are blue.
300 Beads (295 Red, 5 Blue)

If you play this lottery, you will draw a bead from the urn. If the bead you draw is red, you will win 1 dollar. If the bead you draw is blue, you will win 135 dollars. Your potential winnings are summarized in the following picture:

**Winnings**

If Red | If Blue
---|---
1 | 135

Now, circle your bid for this lottery.
Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>

For the bid you have circled, answer the following questions:

- When you roll the die, what is the probability that you will get to play the game? __________ of 6 chance.
- If you play the game, what is the best outcome you can expect? Gain _____
- If you play the game, what is the worst outcome you can expect? Lose _____

For the real experiment, you will now bid on a number of different lotteries. However, we cannot afford to have you play all the lotteries. What will happen is that there will be random draw to select one lottery for you to play for real money. This process encourages you to play each game as those it is the only game you will play. That is, you should play each game as though you will play it because it may end up being the game you play for real money.

We will now begin the experiment, unless any of you have a question. Raise your hand if you have a question. Any questions?

OK. Here is the first lottery opportunity.
Subject ID _______________

Game #1

18 Beads (13 Red, 5 Blue)

Winnings

If Red If Blue

Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>
Subject ID _____________

Game #2

10 Beads (9 Red, 1 Blue)

Winnings

If Red  If Blue

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Subject ID _____________

Game #3

450 Beads (445 Red, 5 Blue)
Winnings

If Red (R)                     If Blue (B)

<table>
<thead>
<tr>
<th>1</th>
<th>201</th>
</tr>
</thead>
</table>

Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Subject ID ______________

Game #4
2 Beads (1 Red, 1 Blue)

Winnings
If Red  If Blue
1  5

Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Subject ID _____________

Game #5

45 Beads (40 Red, 5 Blue)

Winnings

If Red                      If Blue

1                                                  21

Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Subject ID _____________

Game #6

4 Beads (3 Red, 1 Blue)

R R
R B

Winnings

If Red R R R R If Blue B B

1 9

Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>
Subject ID _____________

Game #7
100 Beads (99 Red, 1 Blue)

Winnings
If Red                      If Blue

Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>
Subject ID ______________

Game #8

9 Beads (4 Red, 5 Blue)

Winnings

<table>
<thead>
<tr>
<th>If Red</th>
<th>If Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>
Subject ID _____________

Game #9

9 Beads (8 Red, 1 Blue)

Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>

Winnings

If Red  If Blue

Bid 21
Now, you will be asked to bid on a set of lotteries where you do not know how many beads of each color are in the urn.

An example is presented below. There is an “urn” which contains 6 beads. Some are red and some are blue. However, you do not know how many are red, nor do you know how many are blue. Below we show you the 6 beads to illustrate just how many are in the urn.

6 Beads

If you play this lottery, you will draw a bead from the urn containing the 6 beads. If you draw a red bead, you will win 1 dollar. If you draw a blue bead, you will win 13 dollars. Your potential winnings are summarized in the following picture:

Winnings

<table>
<thead>
<tr>
<th>If Red Bead</th>
<th>If Blue Bead</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="1.png" alt="Red Bead" /></td>
<td><img src="13.png" alt="Blue Bead" /></td>
</tr>
</tbody>
</table>

Your task, as with the previous set of lotteries you bid on, is to circle the amount you are willing to bid to play this lottery. Again, you will have to roll a die less than or equal to your bid in order to play the lottery. Please circle the amount you are willing to bid for the lottery.

<table>
<thead>
<tr>
<th>My bid for this lottery (circle the number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid</td>
</tr>
</tbody>
</table>

![Red Bead](1.png) ![Blue Bead](13.png)
For practice with this type of lotter, let’s consider three more lotteries of this type.

For the lottery presented below, there is an urn which contains 36 beads. Some are red and some are blue. You do not know how many are red, nor do you know how many are blue. Below we show you the 36 beads to illustrate just how many are in the urn.

**36 Beads**

![36 Beads Diagram]

If you play this lottery, you will draw a bead from the urn. If the bead you draw is red, you will win 1 dollar. If the bead you draw is blue, you will win 17 dollars. Your potential winnings are summarized in the following picture:

**Winnings**

<table>
<thead>
<tr>
<th>If Red Bead</th>
<th>If Blue Bead</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17</td>
</tr>
</tbody>
</table>

Now, circle your bid for this lottery.

| Bid for this lottery (circle the number) |
|---|---|---|---|---|---|---|
| Bid | 1 | 2 | 3 | 4 | 5 | 6 |

Consider the new lottery presented below. There is an urn which contains 50 beads. Some are red and some are blue. You do not know how many are red, nor do you know
how many are blue. Below we show you the 50 beads to illustrate just how many are in the urn.

**50 Beads**

If you play this lottery, you will draw a bead from the urn. If the bead you draw is red, you will win 1 dollar. If the bead you draw is blue, you will win 101 dollars. Your potential winnings are summarized in the following picture:

**Winnings**

<table>
<thead>
<tr>
<th>If Red</th>
<th>If Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>101</td>
</tr>
</tbody>
</table>

Now, circle your bid for this lottery.

<table>
<thead>
<tr>
<th>Bid for this lottery (circle the number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid</td>
</tr>
</tbody>
</table>

Consider the last practice game presented below. There is an urn which contains 300 beads. Some are red and some are blue. You do not know how many are red, nor do you know how many are blue. Below we show you the 300 beads to illustrate just how many are in the urn.
300 Beads

If you play this lottery, you will draw a bead from the urn. If the bead you draw is red, you will win 1 dollar. If the bead you draw is blue, you will win 135 dollars. Your potential winnings are summarized in the following picture:

**Winnings**

If Red  If Blue
2 \[\text{R}\] 135 \[\text{B}\]

Now, circle your bid for this lottery.

| Bid for this lottery (circle the number) |
|---|---|---|---|---|---|---|
| Bid | 1 | 2 | 3 | 4 | 5 | 6 |
Thank you for bidding on these practice lotteries of this type. We will now move to having you bid on a number of lotteries of this type for real.
Subject ID ____________

Game #1
18 Beads

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<td></td>
</tr>
</tbody>
</table>

Winnings
If Red 6  If Blue 8

Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tr>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Subject ID ______________

Game #2

10 Beads

Winnings

If Red 6 If Blue 8

1 2 21

Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>
Subject ID ______________

Game #3

450 Beads
Winnings

If Red (R)       If Blue (B)

1

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Subject ID _____________

Game #4
2 Beads

○ ○

Winnings
If Red □ If Blue ■

1 5

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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</thead>
<tbody>
<tr>
<td>Bid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Subject ID _____________

Game #5
45 Beads

Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>

Winnings
If Red  R  If Blue  B

1


21
Subject ID _____________

Game #6
4 Beads

Winningste@
If Red  |  If Blue

Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Subject ID _____________

Game #7

100 Beads

Winnings

If Red  R  If Blue  B

Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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</thead>
</table>
Subject ID _____________

Game #8
9 Beads
○ ○ ○ ○ ○ ○ ○ ○ ○

Winnings
If Red 6  If Blue 8
1 5

Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>
Subject ID ______________

Game #9

9 Beads

Winnings

If Red  If Blue

Bid for this lottery (circle the number)

<table>
<thead>
<tr>
<th>Bid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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