Spatial Variability of Hydraulic Properties in an Undisturbed Alluvial Soil

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Hydrology/Hydrogeology

by

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ABSTRACT

The soil hydraulic properties, steady-state hydraulic conductivity (K₀), steady-state moisture content (θ₀) and a proportionality coefficient (β) were measured at depths of 30, 61 and 91 cm at each of 40 sampling sites located within an 18000 m² area of undisturbed soil developed on an alluvial fan in the Frenchman Flat area of the Nevada Test Site. These properties are related to unsaturated hydraulic conductivity by \( K(\theta) = K_0 \exp(\beta(\theta-\theta_0)) \) and were estimated using an infiltration method which required that only moisture content be monitored during redistribution. Classical statistics revealed variability with depth while geostatistics showed that only three properties, \( \log(K_0) \) at 30 cm, \( \theta_0 \) at 30 cm and \( \log(K_0) \) at 61 cm were spatially dependent. Spatial variabilities were characterized by linear semivariograms with nugget effects. Additionally, \( \log(K_0) \) values at 30 cm were found to be anisotropic. Contour maps of the three spatially dependent properties were created by kriging techniques.
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INTRODUCTION

In the early 1900's, as relationships between soil characteristics and crop growth were realized, scientists became interested in quantifying the variability of various soil properties (Montgomery, 1913; Waynick, 1918; Waynick and Sharp, 1919). Much literature has been written on the subject, most within the past 15 years. Some has considered the variability of soil chemical parameters (Ball and Williams, 1968; Campbell, 1978; Burgess and Webster, 1980; Hajrasuliha et al., 1980; Yost et al., 1982a,b) but the majority has involved the variability of physical parameters and/or soil hydraulic properties.

Studies which endeavored to describe the variability of soil hydraulic properties were conducted on fields of various sizes by taking different numbers of samples in schemes ranging from completely random (Nielsen et al., 1973; Russo and Bresler, 1981b) to very structured (Vieira et al., 1981). Such investigations measured parameters, either in the laboratory or in situ, that affect water movement through soil, then applied statistical analysis to describe the variability observed. No studies, however, were undertaken in natural (undisturbed) soils.

The conventional statistical approach to describe variability in soil hydraulic properties treated the ob-
servations as being statistically independent regardless of spatial position (Nielsen et al., 1973; Biggar and Nielsen, 1976; Carvello et al., 1976; Baker, 1978; Cameron, 1978). A frequency distribution of observations was normally formulated from which could be calculated the number of samples necessary to estimate the mean value within some specific confidence interval. This method of analysis does not account for or attempt to describe the variability of hydraulic properties caused by differences in sample locations. Many believe (e.g., Russo, 1983; Vieira et al., 1981; Russo and Bresler, 1981b) that a more complete approach to the treatment of field variability should consider the positions within the study area where observations were taken and thus the separation distance between measurements.

Two ways of illustrating variability as a function of separation distance between observations are the autocorrelogram and the semivariogram. Autocorrelation expresses changes in a property over distance and the degree to which nearby observations depend on each other by comparing the (spatial) series of measurements to itself. An autocorrelogram is a mathematical function which, in the statistical sense, best describes the relationship between the calculated autocorrelation and distance between observations. This method has been used to determine at what separation between measurement locations various soil hydraulic properties, e.g., saturated hydraulic conductivity
(K_s), saturated moisture content (θ_s), residual moisture content (θ_r), sorptivity (s), were no longer dependent (Russo and Bresler, 1981b; Bresler and Green, 1982). Autocorrelation has also been used to describe spatial variability of several physical properties of a soil in Arizona by Gajem et al. (1981). A semivariogram, on the other hand, is a mathematical function describing the relationship between the semivariance and separation distance. Semivariance is calculated differently than autocorrelation, but it also describes the variability (or similarity) of a property as a function of distance. Whereas stationarity of order 2, which implies the existence of a finite variance of the measured values (Vieira et al., 1983), is assumed when using autocorrelation, this assumption may not be valid when determining semivariance. A semivariogram may also show a maximum correlation distance. This method was used in addition to autocorrelograms to describe spatial variability of soil properties by both Gajem et al. (1981) and Bresler and Green (1982). Variabilities of infiltration rates in an agricultural field (Vieira et al., 1981) and in mine tailings (Rogowski, 1980) were described by semivariograms. Russo (1983) used these to determine how K_s and α, known as the constant characteristic of the soil and defined as d(log K(h))/dh, varied over an agricultural field. In-
formation obtained from such applications of the semivariogram can be used to create accurate maps of soil properties by an operation known as kriging.

Kriging is an interpolation technique which produces estimates of values in unsampled areas without bias and with minimum and known variance. This technique was developed primarily to estimate ore reserves for mining operations (Matheron, 1963) but has been used extensively within the past five years in hydrology and soil science. Soil chemical properties have been contoured by Burgess and Webster (1980) and Yost et al. (1982b) while infiltration rates were displayed by this method by Vieira et al. (1980) and Rogowski (1980). Russo (1983) demonstrated that kriging could be used to design a trickle irrigation system for an agricultural field.

This study presents statistical analyses of the variabilities of parameters related to unsaturated hydraulic conductivity. These include steady-state hydraulic conductivity, \( K_0 \), steady-state moisture content, \( \theta_0 \), and a proportionality coefficient, \( \beta \). Spatial variability as well as variability with depth were investigated. Spatial distributions of these parameters are displayed using kriged contour maps. The possibility of relationships between observed variabilities and the soil fabric caused by depositional processes and/or the presence of soil horizons is evaluated.
The information presented in this investigation is unique in that 1) the study was performed in native, undisturbed desert alluvium, 2) an original sampling scheme was used, and 3) a new sampling method to evaluate the soil parameters was employed.

Figure 2 presents a more detailed map showing the location of the study site within the WTP. The enclosed section of Figure 2 is the Frenchman Flat area, the geology of which is illustrated in Figure 3. Frenchman-Flat is a closed basin in the southeast corner of the WTP with Frenchman Lake (a playa) occupying the lowest area.

Vegetation and Climate

The vegetation mosaic within the study area is dominated by creosote bush, Larrea tridentata. This is common for most areas of the Southern Nevada Test site below 1372 m (4450 feet) elevation (O'Farrell and Emery, 1976).

Precipitation in the Frenchman Flat area occurs primarily during winter months (November to March) but also to some extent during late summer (July and August). Winter storms originate over the Pacific Ocean and move southeast over the Sierra Nevada. Because most of the moisture is lost in the Sierra Nevada, such storms tend to be widespread
STUDY AREA

Regional Location

The study area is located on the Nevada Test Site (NTS) in southern Nevada approximately 70 miles northwest of Las Vegas (Figure 1). The NTS lies within the Basin and Range physiographic province which is typified by long north-south trending fault block mountains and adjacent valleys.

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Figure 1. Location of the Nevada Test Site (NTS). (French and Lombardo, 1984).
Figure 2. Location of the study site within the Nevada Test Site.
and only rarely intense (Case, et al., 1984). Summer precipitation occurs during local convective thunderstorms associated with moisture moving northward from the Gulf of California and the Southern Pacific Ocean (Case et al., 1984). These events can be quite intense. The mean annual precipitation found from observations made at a recording station located approximately five miles south of the study area is 11.68 cm (4.6 inches) (Case et al., 1984). The average annual daily minimum temperature observed at a station near Yucca Flat is 3°C (37°F) while the average annual daily maximum is 22°C (73°F) (O'Farrell and Emery, 1976). Yucca Flat is located approximately 10 miles northwest of the study site and 800 feet higher. Average midday temperature, in the upper 18 feet of soil, measured one half mile from the study area was found to be 18.4°C (65°F) (Panian, 1983).

Geology

Frenchman Flat is a structural basin formed primarily in late Pliocene times which has been subjected to additional faulting throughout the Tertiary and Quaternary Periods (Carr et al., 1975).

Figure 3 shows the distribution of rock types surrounding Frenchman Flat. In general, the rocks consist of Paleozoic sedimentary rocks, Tertiary volcanic rocks, and Quaternary alluvium. More detailed descriptions of rock types in the part of Figure 3 denoted as A will be discussed.
Figure 3. Geology surrounding the study area (from Kearl, 1982)
because they most directly affect the composition of alluvium upon which the study site is located. Knowledge of source rock material may also be helpful in describing soil development on this alluvium.

Paleozoic sedimentary rocks are essentially confined to the Buried Hills area (Figure 3). These are shown as carbonates, including both limestones and dolomites, but smaller outcrops of quartzite and shale are also present (Tschanz and Pampeyan, 1970b).

Clastic rocks, which are described as being largely cobble conglomerates and are believed to be Early Tertiary in age (Tschanz and Pampeyan, 1970a), also uphold portions of the Buried Hills area. Tertiary volcanic rocks dominate the lithology in the Massachusetts Mountain-Scarp and Nye Canyon areas. They consist primarily of Miocene to Pliocene welded and nonwelded ash-flow tuffs, zeolitized tuffs, tuffaceous sandstones, with some lava flows, claystones, and marls (Carr et al., 1975).

Quaternary deposits in the Fenchman Flat area are mainly alluvium deposited by fan processes as the structural basin was being formed. The study site lies on alluvium that originated from the Tertiary volcanics in Scarp Canyon, while alluvium to the east was derived from the Paleozoic sedimentary rocks. A large deposit of dune sand has collected on the western flank of Buried Hills which was undoubtedly derived from airborne particles originating from alluvial fans and Frenchman Playa to the west. Tuffaceous
sandstone and gravel make up a large portion of the source area for Scarp Canyon fan. Also within the areas of Nye and Scarp Canyons are Quaternary basalt dikes and sills.

Alluvial Environment

Introduction

Hydraulic conductivity, as well as other soil hydraulic parameters, depends upon matrix and fluid properties. It is directly related to the intrinsic permeability of the matrix which is a function of grain-size distribution, grain shape, tortuosity, specific surface, and porosity. It is also directly related to fluid density while inversely related to fluid viscosity. Both of these fluid properties are temperature dependent, viscosity being much more so than density. Throughout the course of an infiltration experiment, which would yield steady-state hydraulic conductivity values, the temperature of the infiltrated water would remain relatively constant. This effect of temperature changes on hydraulic conductivity can be considered negligible. Any variation in the hydraulic conductivity then can be attributed to variation from place to place in the intrinsic permeability of the soil matrix. Expectation of variability of the soil matrix properties in space is justified when considering the depositional environment, the alluvial fan, and the changes the alluvial deposit(s) may undergo after deposition, namely soil development.
Alluvial Fans

An alluvial fan is formed by the accumulation of detritus originating from watersheds in adjacent highlands. It has the shape of a low cone the apex of which is coincidental with the point of sediment delivery by the stream channel issuing from the mountain watersheds. Generally, alluvial fans form as a result of base-level lowering of the depositional area relative to the source area. These base-level lowerings may either be erosional or tectonic in character. Many fans produced at the same mountain front may coalesce into a geomorphic feature known as a bajada. When this occurs the original fan shape is lost and replaced by a more or less uniform feature.

Alluvial fans can be considered as erosional/depositional systems and attempts have been made to interpret the roles such of processes play in the stability or instability of fans. Some authors view fans as steady-state forms where processes and morphology are in a state of continual dynamic equilibrium (Denny, 1967; Hooke, 1967). In their view, a fan is a disequilibrium landform while it is growing until a balance is attained between sediment deposition and erosion when it increases in size. Beatty (1970) envisions fans as actively growing while Hunt and Mabey (1966) envision them as being dissected, processes which indicate that fans have not yet reached some sort of equilibrium. Still others believe that alluvial fans never attain steady-state
conditions (Bull, 1976) or that fan characteristics can be explained by climatic changes (Lustig, 1965).

Deposition on the fan surface occurs when a flow exits from a trunk channel and the hydraulic geometry suddenly changes. When the flow becomes unconfined the channel width increases dramatically, which decreases the depth and velocity of flow to such levels that the load can no longer be transported. The types of flows which transport sediment may range from highly viscous debris or mud flows to normal water flows. The type of flow depends on the magnitude of precipitation and the lithology of the source area (Ritter, 1978).

Commonly, large fans are dissected near the mountain front by channels entrenched in the alluvium. This shifts the loci of deposition from the geometric apex of the fan to a point farther down gradient. Fanhead trenching may result from fault displacement of the alluvium or as a normal consequence of large variations in flood discharge (Denny, 1967). The upper section of the fan thus is abandoned and becomes subject to erosion and weathering. Fanhead trenching may occur numerous times throughout the development of a fan, pirating runoff and abandoning areas which were once depositional surfaces.

The study site is located on the western flank of Scarp Canyon fan which has its source area to the north of
Frenchman Flat basin in Scarp Canyon (Figure 4). The entire watershed, comprised of fan and source area has an area of 90.7 square miles (Case et al., 1984).

Recent flow and sediment deposition has occurred in two areas which are shown as hatched areas (Figure 4). One area is located below the fanhead trench which extends approximately four miles down the fan. The other is on the west side of the fan where small channels carry runoff originating from Scarp Canyon and the larger channel which forms the boundary between Scarp Canyon fan and the smaller watershed directly to the north. These small channels are not presently incised deeply into the alluvium. With time, individual channels may meander over the area, depositing sediment, thus contributing to the particle size variation within the study area. It will be seen whether this variation is reflected in the variation of hydraulic properties measured in this study.

Soil Description

A soil profile is formed by the weathering of parent material which takes place at the ground surface and is due to processes occurring at relatively shallow depths. Such processes cause the physical and chemical mutation of original mineral matter, and include the downward movement of water, macro- and microbiological activity, gas exchange
Figure 4. Topographic map showing location of study site and soil profiles on Scarp Canyon fan (from Frenchman Lake Quadrangle, 15 Minute Series, U.S.G.S., 1952)
with the atmosphere, and heat flow. As these processes continue, initial materials are differentiated into horizons. The complexity and character of the horizons are functions of how long and/or how intense the processes operate. A substantial review of soil horizonation can be found in Buol et al. (1973).

The soil on which this study was conducted is termed an Aridisol. A general description of an Aridisol is a soil that has developed in an arid environment and remains dry throughout most of the year. Quite commonly Aridisols will develop pedogenic horizons resulting from the translocation and accumulation of salts, carbonates, or silicate clays or of cementation by silica or carbonates (Soil Survey Staff, 1975). These horizons occur within 1 m of the surface.

The soil at the study site does not exhibit a well defined pedogenic horizon. Instead, extensive coating by carbonates occurs on all cobbles and pebbles. Because of the temperature and moisture regime of the soil and because of the amount of carbonate present and the absence of an argillic (clay-containing) horizon, this soil would be classified in the Calciorthid great group (Soil Survey Staff, 1975).

The morphology of an Aridisol is inherited from the parent material (Buol et al., 1973). This is particularly so when considering soil development on fluvial deposits. Aridisol profiles developed on alluvial fan deposits should
exhibit remnant stratification due to the sorting of material caused by the manner in which the alluvium is deposited. Weathering and alteration may be well underway if the deposited sediment is reworked alluvium.

Numerous studies have been conducted in the northern Frenchman Flat area to describe soil profile characteristics. It should be noted that all textural classes presented in this study are from the classification scheme of the U.S. Department of Agriculture (1951). Kearl (1982) characterized soil samples at three depths, the deepest at 0.9 meters, and found each to have particle size distributions indicative of a sandy loam textural class. In another study performed at the same site, Kautsky (1984) determined that soil textural classes began as sandy loams near the surface, then graded to loamy sands and fine sands with depth. The location of both research efforts is denoted as KK in Figure 4. Romney et al. (1973) described soil profiles at two locations near the present study area. These are shown as R42 and R43. Horizon descriptions for the two profiles are contained in Appendix A. Case et al. (1984) described soil profiles at locations D1 and D2 (Figure 4). These profile descriptions are also contained in Appendix A.

In summary, the soils that have developed on the alluvium at and around the study site contain abundant sand and
are alkaline in nature. Horizonation is evident in each described profile. Particle sizes may range from gravel (e.g. gravelly sand) to silt (e.g. loamy sand). Horizon thickness ranges from 2 cm to 56 cm with an average thickness of 24 cm.
FIELD METHODS

The objectives of this study require that many samples be taken within the area of interest. The method used here to determine soil properties related to unsaturated hydraulic conductivity was chosen because it had recently been shown by Jones and Wagenet (1984) to produce much data in a minimal amount of time. The method also yields in situ values from undisturbed soil at numerous depths.

Theory

Libardi et al. (1980) described a field technique for determining steady-state hydraulic conductivity values, \( K_0 \), which requires that only volumetric moisture content, \( \Theta \), be monitored through time. The impetus for their research was to devise a simple infiltration method of estimating \( K(\Theta) \) relationships without using tensiometers and complex mathematical curve fitting technique.

Estimating \( K(\Theta) \) at a given depth, \( z \), calls for the integration of Richard's equation which yields

\[
\int_0^z \frac{3\Theta}{3z} \, dz' = K(\Theta) \left. \frac{3H}{3z} \right|_{z'=z} - \left. K(\Theta) \frac{3H}{3z} \right|_{z'=0}
\]

where \( H \) is the hydraulic head (cm), \( \frac{3\Theta}{3z} \) is the time rate of change of moisture content, and \( \frac{3H}{3z} \) is the hydraulic...
gradient. The above equation assumes that at $t=0$ the soil water flux throughout the profile ($0<z<L$) is constant and that for $t>0$ the flux at the soil surface is zero and all changes in moisture content within the profile are solely due to redistribution and drainage. This requires that the last term in equation (1) be zero.

Because the first term in equation (1) represents the time rate of change of stored moisture in the profile to depth $z$, it reduces to

$$z \frac{\partial \theta^*}{\partial t} = K(\theta) \frac{\partial H}{\partial z} \bigg|_{z'=z}$$

where $\theta^*$ is the average soil water content to depth $z$. The average water content from $z'=0$ to $z'=z$ is related to the water content, $\theta$, observed at $z$ by the assumed linear equation

$$\theta^* = a \theta + b$$

where $a$ and $b$ are constants. A linear regression of $\theta^*$ on $\theta$ will yield the constants.

Unsaturated hydraulic conductivity is commonly expressed as an exponential function of $\theta$ and this format is
adopted here. This relationship can be mathematically described as

\[ K(\theta) = K_0 \exp \{ \beta(\theta - \theta_0) \} \]  

(4)

where \( \beta \) is a constant and \( K_0 \) and \( \theta_0 \) occur during steady-state infiltration.

Black et al. (1969) and Davidson et al. (1969) have shown that during redistribution the hydraulic gradient is approximately unity, i.e. \( \partial H / \partial Z = -1 \), and that water will drain equally from all depths in the profile. Substituting equation (3) and equation (4) into equation (2) we have

\[ -az \frac{\partial \theta}{\partial t} = K_0 \exp \{ \beta(\theta - \theta_0) \} \]  

(5)

where \( a \) and \( \theta \) are evaluated at \( z \).

Integration of equation (5) at a given depth \( z \) from the initial conditions \( t' = 0, \theta' = \theta_0 \) to \( t' = t, \theta' = \theta \) yields

\[ \theta_0 - \theta = \frac{1}{\beta} \ln \left( 1 + \beta K_0 t / az \right) \]  

(6)

Libardi et al. (1980) considered the effects of the addition of 1 within the natural logarithm to be negligible for \( t \) greater than three hours. Under this condition equation (6) becomes
\[ \theta_0 - \theta = \frac{1}{\beta} \ln (t) + \frac{1}{\beta} \ln \left( \frac{8K_o}{az} \right); \ t > 3 \text{ hrs.} \quad (7) \]

Equation (7) takes the form of a linear relationship between the amount of water drained after steady-state is achieved \((\theta_0 - \theta)\) and the natural log of time. Linear regression of \(\theta_0 - \theta\) on \(\ln(t)\) produces values for \(\beta\) and \(K_o\).

Data Collection

Sample Locations

Figure 5 is a diagram of the study plot showing the locations of 40 individual sampling sites (infiltration boxes) and shallow stream channels. It is a square area covering 1.8 hectares (4.45 acres) and each side is 134 meters (440 feet) long. The shortest distance between two sample sites is 7.5 meters.

Samples were taken in two sets of two transects each. Transects A and C comprise one set and are perpendicular to elevation contours. The set containing transects B and D are parallel to contours. This arrangement will allow statements to be made concerning the relationship between the spatial structure of measured properties and the depositional environment.

Transects A and B are similar respective of the spacing of sampling locations. Likewise, transects C and D are similar. Such spacing of sampling sites allows
Figure 5. Map of the study site showing locations of infiltration plots along four transects.
determination of variability as a function of separation distance.

**Infiltration Plots**

Figure 6 presents an illustration of an individual sampling site. The site is constructed in such a way that water may be ponded and the changes in moisture content below the pond may be monitored through time. Walls consisting of 1 x 12 inch wooden planks were set 6 inches into the ground to create an impoundment area of 4 square meters, each side being 2 meters long. In the center of each enclosure, a 2 meter-long neutron access tube was installed to a depth of approximately 110 cm.

Standard size tubes with 2 inch O.D. and 1.9 inch I.D. were used and each was closed at one end by a rubber stopper equipped with an expansion bolt. The tubes were installed by first drilling holes with a 3 inch diameter hand auger. Because of the lack of soil cohesiveness due to dry conditions, collapsing of the walls occurred frequently. Many times the diameter of the hole doubled in size. To combat this, water was poured down the hole and allowed to infiltrate which gave enough cohesiveness to the soil that a 3 inch diameter hole could be maintained. The access tube was then placed in the hole and soil was backfilled around the tube in the reverse order that it was removed. The soil was
Figure 6. Drawing of an infiltration plot.
moistened so that it could be better packed around the tube.

Infiltration water used in this study was obtained from groundwater wells located in the southern portion of Frenchman Flat and at Mercury, the main base camp for NTS. The water was stored in a tank on the study site until it was applied to the infiltration sites. Water quality was such that infiltration would not be affected by an adverse reaction, such as dispersion, between the soil and the water. Kautsky (1984) and Kearl (1981) showed electrical conductivities and sodium absorption ratios for soil profiles near the study site that were indicative of a normal, non-dispersive soil. An average EC value for the first meter of soil obtained from both efforts was 400 umhos/cm while an average SAR value was 3.8.

A constant head of water was maintained at a depth of 4 inches. Maintenance of the constant head was accomplished in two ways. One method utilized several 55 gallon drums as reservoirs. The reservoirs were hydraulically connected by syphons so that each one had the same value of head. A 1/2 inch I.D Tygon tube connected these reservoirs to the ponded water in the infiltration enclosure. The rate of water application was varied accordingly by a screw-type clamp, placed at the end of the hose, such that the level of ponded water would remain constant. The second method drew water, via a garden hose, directly from the 5500 gallon holding
tank located on the site. The rate of water application was varied by a valve at the end of the hose leading into the infiltration enclosure. The water level was again maintained by adjusting the discharge from the hose accordingly.

Moisture content measurements were taken periodically during infiltration by a Troxler Model 3220 depth moisture gage at 30 cm intervals below the ground surface. Calibration of the instrument to the study soil was done in the laboratory, the details of which are presented in Appendix B. When moisture contents at the three depth intervals remained unchanged, steady-state conditions were assumed to exist and drainage of the profile could commence.

Following termination of water application, the remaining water in the enclosure was allowed to infiltrate. The surface of the infiltration site was then covered by plastic followed by a layer of soil approximately 5 cm thick, thus preventing surface evaporation. When no standing water remained in a given enclosure, its initial time was set to zero and changes in moisture content were monitored at various times afterwards.

Results and Discussion

The application of water on the infiltration sites began in late July of 1984. Only three sites, namely 1a, 2a, and 3a, were completed before the two thousand gallon
fiberglass storage tank that was being used at the time ruptured while being filled. The infiltration experiments continued in October of the same year after the present storage tank was placed on the site. The remaining six sites in transect A were completed before the weather became too cold to continue the fieldwork. The analyses of the data obtained from those nine sites showed that some problems existed with the field methods.

The drainage graphs, i.e. water content versus time since drainage began, for all nine sites showed that the profiles drained quickly then abruptly tapered off to a more or less constant water content. Figure 7 shows the drainage plots for sites 7a and 9a. Sites such as 5a, 7a, and 9a, which were observed infiltrating water faster than other sites in the transect, tended to achieve a constant water content more rapidly. This effect was also more pronounced in the first 30 cm interval than in the deeper intervals. Equation 6 assumes an exponential decay of water content through time. The observed drainage phenomenon thus had a profound affect on the calculated values.

Figure 8 shows plots of decreased water content \( (0_\theta-9) \) versus \( \ln(t) \) for sites 7a and 9a. The data should plot as straight lines but as can be seen the trends in the data change approximately at \( \ln(t)=3.0 \) or 24 hours. Questionable results were obtained when linear regressions were performed on all data points collected from the first 30
Figure 7. Drainage graphs for sites 7a and 9a.
Figure 8. Decrease in volumetric moisture content from steady state, in cc H₂O/cc soil vs ln(t) for sites 7a and 9a.
cm intervals of each site. The first 30 cm intervals are used here merely as illustrations of what occurred for each depth. For example, $B=87.9$ and $K_O=1825.9$ cm/hr for site 7a while $B=75.2$ and $K_O=332.4$ cm/hr for site 9a. The correlation coefficients of these regressions are .984 and .968, respectively. These results gave sufficient reason to question the field methods as they were being performed at the time because they were too high for this type of soil.

The field methods used in this study were somewhat different than those originally described by Libardi et al. (1980). Infiltration boxes described in the study mentioned above were 6 m on each side while neutron access tubes extended 2 m into the ground and the soil undoubtedly held more water before infiltration began than did the desert soil of this study. Initial moisture contents observed at sites which had been in place for several months in this study were approximately 7% by volume during dry months and 10 to 12% during wetter months. Because of the difficulties involved in transporting, storing, and applying water in a remote area it was decided to modify the field methods so that the most information could be obtained from the least amount of available water. Smaller infiltration boxes and shorter penetration depths of the access tubes achieved this goal, however, not without some consequences.

Extensive lateral movement of water is believed to have occurred at each infiltration site due to large
potential gradients created between the dry, undisturbed soil and the near-saturated soil beneath the infiltration sites. This lateral movement is illustrated in Figure 9 which shows the moisture content changes observed at site la and at a point located 75 cm (2.5 ft) outside the infiltration box. Disturbed cores taken at the same distance from several other sites throughout the study area confirmed that lateral water movement was widespread.

The data were inconsistent with observations made during a study which used another modified form of the method of Libardi et al. (1980). Jones and Wagenet (1984) surrounded neutron access tubes with 37 cm diameter plastic irrigation pipe inserted 15 cm into the ground. Infiltration proceeded by ponding water within this pipe. They found little evidence that radial flow proceeded farther than 5 to 10 cm from the plastic pipe and they attributed this to the rather high sand content (>50%). Texturally the soil of their study and that of this study are similar, both being classified as sand or silty loams. The drainage graphs for their individual sites confirmed that moisture content decreased exponentially. The differences in observed lateral flow may be attributed to wetter initial conditions or the magnitude of the head maintained during infiltration in the Jones and Wagenet (1984) study. The magnitudes of horizontal potential gradients would be
Figure 9. Change in moisture content, in cubic centimeters of water per cubic centimeter of soil, with time for site 1a and a point 75 cm away.
less when wetter initial conditions exist and conduction of water would be greater in the z direction due to greater ponding depths.

It may be instructive at this point to mention the results of other experiments which utilized the methods of Libardi et al. (1980). One experiment (C. Elliott, University of Nevada, Reno, personal communication, 1985) was conducted in Clear Creek watershed on the eastern flank of the Sierra Nevada, approximately 7 miles west of Carson City, Nevada. The method was modified to use a smaller infiltration box (1.22 m x 1.22 m) and the soil is developed on granitic parent material, texturally classified between gravelly sand and gravelly sandy loam (Boone, 1983) with a porosity of approximately 45%. Initial moisture contents average 8% with depth. The drainage plot for this experiment is shown at the top of Figure 10. The data are consistent with the assumption of an exponential decrease of moisture content with time after drainage began. Little lateral movement of water may have occurred due to the coarser texture of the soil. The drainage plot shown at the bottom of Figure 10 was obtained from an experiment conducted in Area 5 of NTS, approximately 2.5 miles south of the present study area (W. Ross, Desert Research Institute, Reno, Nevada, personal communication, 1985). The soil types at both locations are essentially the same as are the porosities and ambient moisture conditions. The neutron
Figure 10. Decrease in moisture content from steady state ($\Phi - \Phi_o$) in cu H$_2$O/cc soil vs ln(t) for two similar infiltration experiments.
access tube was approximately 4 m long and placed at the center of a 4.78 m x 3.73 m x 0.31 m pit which was used to hold the ponding water. It appears that there may be some deviation from strict exponential drainage from the 30 cm depth, however, the $K_o$ and $b$ values obtained from the regression seem reasonable. The 61 cm and 91 cm depth intervals show good exponential behavior.

The data from this study and those of the others presented show that the results obtained using this field method are indeed affected by the size of the ponding area, soil texture, and quite probably initial moisture conditions. Smaller ponding areas may be used in sandy, wetter soils or dry soils which are very coarse without causing excessive lateral movement that would affect how water drains from the profile. Large ponding areas, however, should be used in very dry soils to ensure that lateral movements will not affect the gravity drainage near the neutron access tubes. Small areas can be used in these latter environments if moisture changes are monitored frequently after drainage begins.

The remaining infiltration sites were completed during March and April of 1985. Because it had been shown that regression estimates of $K_o$ and $b$ were unreasonably high when data collected after 24 hours of drainage were included in the analyses and that this data was affected by lateral
movement of water, it was decided to collect moisture content data only between 3 and 24 hours after drainage began. After a profile had been draining for three hours, four to six moisture readings were taken at fifteen minute intervals. The measurement intervals increased to 30 minutes for a few hours then to longer intervals (at least 60 minutes but sometimes 5 to 6 hours depending on convenience) so that the first 24 hours of drainage could be well documented.

The drainage graphs for all 40 sites can be seen in Appendix C in addition to the results of the regression analyses which yield $K_0$ and $8$ values.

Figures 11 a, b, and c show the distributions of log $K_0$ values for 30 cm, 61 cm, and 91 cm, respectively. Logarithmic transformations were used because the $K_0$ values were so variable. Figures 12a, b, and c show the distributions of $\theta_0$ values for each depth. Figures 13a, b, and c show the distributions of log 8 values for each depth. Logarithms were used here for the same reasons as before. Measurements taken within the first 24 hours of drainage were retained for all infiltration sites in transect A except 3A, 6A, and 7A. Too few measurements were made within this crucial period so the experiments were repeated for these three in the spring of 1985.

As can be seen in Figures 11a through 13c, the soil hydraulic properties do vary both with depth and with
position over the study area. The goal now is to determine whether the populations of soil hydraulic properties are different for each depth. In addition, it is desirable to determine whether the spatial variations of the properties are caused by purely random processes or by underlying causes which may be connected to depositional processes operating in this fluvial environment.

It should be noted here that $K_0$ represents the steady-state hydraulic conductivity and should not be confused with saturated hydraulic conductivity, $K_s$. Entrapped air during infiltration did not permit complete saturation. This means also that $\theta_o$ must be considered as steady-state moisture content and should not be thought of as total porosity, $n$. The two are undoubtedly related but are not equivalent. It is unlikely that true saturation would ever be achieved under natural conditions so it is assumed that $K_0$ and $\theta_o$ represent the upper limits of hydraulic conductivity and moisture content.
Figure 11a. Distribution of $\log K_o$ values at 30 cm.
Figure 11b. Distribution of $\log K_o$ values at 61 cm.
Figure 11c. Distribution of log$K_o$ values at 91 cm.
Figure 12a. Distribution of $\theta_o$ values at 30 cm.
Figure 12b. Distribution of $\theta_o$ values at 61 cm.
Figure 12c. Distribution of $\theta_o$ values at 91 cm.
\[ \log \beta = 1.32 \]

\[ \log \beta = 1.70 \]

Figure 13a. Distribution of log\(\beta\) values at 30 cm.
Figure 13b. Distribution of $\log\beta$ values at 61 cm.
Figure 13c. Distribution of logβ values at 91 cm.
STATISTICAL THEORY AND APPLICATION

Introduction

This section explains the various statistical techniques used to describe the spatial variability of the hydraulic parameters that have been observed. For the most part, the techniques fall under the general heading of geostatistics. Matheron (1963) describes geostatistics as being a mathematical formalism concerned with the study of the spatial distribution of natural phenomena.

Various assumptions dealing with what is statistically unknown about the natural phenomena are made first. These are necessary to justify the quantification of spatial structure and subsequent interpolation of parameters. Following the stating of these assumptions is a description of how to characterize the parameter distributions in addition to calculating their means and variances. In some cases this may be all that is necessary to describe the observed parameter spatial distribution. Identification of any trends in the data follows. If there are no trends, spatial structure may be identified by calculating experimental semivariances. If spatial structure is apparent, accurate interpolation at unsampled locations within the study area can proceed.
Regionalized Variables and Random Functions

A regionalized variable (abbreviated ReV) is one that is distributed in space. More specifically, it is a function \( f(x,y,z) \) that has a value at every location \((x,y,z)\) coordinates) in a domain and is too irregular to be easily described mathematically (Journel and Huijbregts, 1978). The measured hydraulic parameters in this study are thus considered ReV's. A ReV characteristically exhibits two components. One is random and local in nature while the other is more general and structured (Matheron, 1963). To properly discern the spatial variability of the ReV, this apparent contradiction must be acknowledged. It can be if the concept of a random function is adopted.

A random variable is one which assumes a value according to some specific probability distribution. For example, the value of a hydraulic parameter obtained in this study at each discrete location \( x_1, x_2, x_3, ..., x_{40} \), where \( x_n \) designates a spatial coordinate \( x_n, y_n \), is a realization of a random variable \( Z(x_n) \) defined at that point. The set of 40 values constitutes one realization of a set of random variables having the study area as their domain. This set of random variables can now be considered a random function. According to Journel and Huijbregts (1978) this definition of a random function expresses the random and structured nature of a regionalized variable 1) at a point, \( x_n \), in that \( Z(x_n) \) is a random variable and 2) over an
area in that \( Z(x_n) \) and \( Z(x_n+h) \), where \( h \) is some distance, are correlated, indicating the structure in the regionalized variable.

To completely define the random function, the distribution functions of the random variables at every possible location within the domain would be needed. The number of realizations to achieve this is far too large for practical purposes. Certain assumptions dealing with various degrees of spatial homogeneity must be made so that adequate information about the random function may be obtained from fewer realizations. The statistical term for spatial homogeneity is stationarity.

Stationarity

There are essentially two definitions of stationarity that are commonly evoked in geostatistics. One is second-order stationarity which calls for the first two moments of the random function, namely the mean and variance, to remain unchanged from place to place. The other is known as the intrinsic hypothesis which is weaker than the first but is still valid when interpolation at unsampled locations is desired. Detailed descriptions of these definitions may be found in Vieira et al. (1983), Journel and Huijbregts (1978) and Delhomme (1978) and only a short summary will be presented here.
The random function $Z(x_i)$ is stationary of order 2 when the expected value is not dependent on position $x_i$. Mathematically this is stated

$$E[Z(x_i)] = m \text{ for all } x_i$$  \hspace{1cm} (8)

where $E$ denotes expected value. The expected value is the mean of an infinite number of realizations. The assumption of stationarity of order 2 also includes that for each pair of random variables, the covariance function, $C(h)$, exists and depends only on separation distance $h$. This is shown as

$$C(h) = E[Z(x_i) \cdot Z(x_i + h)] - m^2 \text{ for all } x_i.$$  \hspace{1cm} (9)

Stationarity of covariance implies stationarity of the variance. The variance of a random variable is the same as the covariance of the random variable with itself so that from (9)

$$\text{var} \{Z(x_i)\} = E[Z(x_i) \cdot Z(x_i + 0)] - m^2 = E[Z^2(x_i)] - m^2 = E[Z^2(x_i)] - m^2 = C(0)$$  \hspace{1cm} (10)

Stationarity of the covariance also implies stationarity of the variogram, $2\gamma(x_i, x_i+h)$. The
variogram is a measure of correlation between observations and is defined as

\[ 2Y(x_i, x_i+h) = \text{E} \{ Z(x_i) - Z(x_i+h) \}^2. \]  
(11)

This can be expanded into

\[ 2Y(x_i, x_i+h) = 2Y(h) = \text{E} \{ Z^2(x_i) - 2Z(x_i)Z(x_i+h) + Z^2(x_i+h) \} \]  
(12)

which is still equal to

\[ 2Y(h) = \text{E} \{ Z^2(x_i) - m^2 - 2Z(x_i)Z(x_i+h) + 2m^2 + Z^2(x_i+h) - m^2 \} \]  
(13)

after adding and subtracting \(2m^2\). Because of the linearity of the operator \(\text{E}\), (13) becomes

\[ 2Y(h) = \text{E} \{ Z^2(x_i) \} - m^2 - 2\text{E} \{ Z(x_i)Z(x_i+h) \} - m^2 + \text{E} \{ Z^2(x_i+h) \} - m^2 \]  
(14)

and because

\[ \text{E} \{ Z^2(x_i) \} - m^2 = \text{E} \{ Z(x_i)Z(x_i+h) \} - m^2 = C(0), \]

(14) now becomes

\[ 2Y(h) = C(0) - 2C(h) - C(0) \]  
(15)
\[ = 2C(0) - 2C(h) \]

or

\[ \gamma(h) = C(0) - C(h) \] (16)

The parameter \( \gamma(h) \) is known as the semivariance or semivariogram. Stationarity of order 2 implies that \( \text{var} \{ Z(x_i) \} = C(0) \) meaning that there is finite variance. This indirectly leads to \( \gamma(0) = 0 \) from equation (16).

Unfortunately this is not always the case. Commonly \( \gamma(0) \) will equal some non-zero, positive value. This is known as the nugget effect, a term which has its origins in the early days of geostatistics when that discipline dealt solely with estimating ore reserves. The presence of a nugget effect indicates that the parameter is microregionalized meaning that noticeable differences in the parameter may occur over very small distances (Delhomme, 1978). A nugget effect may also be caused by high measurement error (Delhomme, 1978).

In this situation, especially if data interpolation is desired, a weaker assumption is applicable.

The intrinsic hypothesis of stationarity states that only the increment \( [Z(x_i) - Z(x_i + h)] \) is required to have a finite variance that is independent of position \( x_i \) for all magnitudes and directions \( h \). Mathematically this is stated as
\[ \text{var} \{Z(x_i)-Z(x_i+h)\} = E\{[Z(x_i)-Z(x_i+h)]^2\} \quad (17) \]

which becomes, after recalling the definition of the variogram

\[ \gamma(h) = \frac{1}{2} E\{[Z(x_i)-Z(x_i+h)]^2\} \quad (18) \]

The semivariance can be estimated at specified distances \( h \) by

\[ \gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [Z(x_i)-Z(x_i+h)]^2 \quad (19) \]

where \( Z(x_i) \) and \( Z(x_i+h) \) are realizations, or measurements, of the random function of positions \( x_i \) and \( x_i+h \) and \( N(h) \) is the number of pairs of measurements separated by distance \( h \). A discussion of the experimental variograms calculated in this investigation is presented in a subsequent section.

Having stated these assumptions concerning the nature of a random function, the task is now to describe the random functions which govern the spatial distribution of the hydraulic parameters.
Population Distributions

A chi-square ($x^2$) analysis tests how well the distribution of observations conforms to a specified distribution. The specified distribution can be of any type, e.g. normal, lognormal, exponential, or some arbitrary distribution, however, in this study only normal and lognormal distributions were considered. The procedure used to test the distributions of the study data has been outlined by Davis (1973). The observations were first tested to determine whether they conformed to a normal distribution at a specified level of significance. If they did not conform, they were again tested against a lognormal distribution.

The null hypothesis for the first part of the analysis states that the observations were drawn from a normal population with unknown mean and variance. The alternative hypothesis would then be that the population is not normally distributed.

A standard normal distribution can be divided into a number of segments, the area of each equal to the probability that a random observation from a standard normal distribution will fall within that segment. The expected number of observations that would fall within a segment can be determined then compared with the number of sample observations that fall within that same interval. In this way a test statistic can be devised to determine whether the
number of sample observations that fall within a segment is significantly different from what is expected.

The test statistic, $\chi^2$, is calculated as

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

(Davis, 1973) where $k$ is the number of segments, $O_i$ is the number of observations within segment $i$, and $E_i$ is the expected number for that segment.

In this study the standard normal distribution was divided into eight segments. The boundaries for these were $-\infty$ to $-1.15$, $-1.15$ to $.67$, $.67$ to $.32$, $.32$ to $0.0$, $0.0$ to $.32$, $.32$ to $.67$, $.67$ to $1.15$, $1.15$ to $\infty$. These segments are all of equal area. Because there are 40 observations the expected number that would fall within each segment is 5. The sample observations were first standardized by

$$Z_i = \frac{X_i - \bar{X}}{s}$$

(21)

where $Z_i$ is the $i$th standardized observation and $X_i$ is the raw observation while $\bar{X}$ is the mean and $s$ is the standard deviation, both of which are estimated from the spatially distributed samples. The number of observations
which fell into each segment \((O_i)\) was recorded and the test statistic was calculated.

The number of degrees of freedom for this test equals 5 (number of segments \(-3\)). One degree of freedom is lost due to the act of placing the observations into the segments. Two additional degrees of freedom are lost because the population mean, \(\mu\), is estimated by the sample mean, \(\bar{x}\), while the population variance, \(\sigma^2\), is estimated by the sample variance, \(s^2\) in the standardization of the observations (Davis, 1973). The critical \(\chi^2\) value of this analysis is 12.83 for a significance level \((\alpha)\) of 2.5%. A significance level of 2.5% means that we are willing to reject the null hypothesis (that the sample observations are drawn from a normal population) one time out of forty when it is true.

Table 1 shows the results of this analysis. Means, variances, calculated \(\chi^2\) values and distribution types are presented for the hydraulic properties. Distributions which had calculated \(\chi^2\) values greater than 12.83 could not be considered normal, i.e. the null hypothesis had to be rejected. The distributions were then tested to determine if they had been drawn from lognormal populations. Logarithms of the raw data were standarized and assigned to appropriate segments. New \(\chi^2\) values were calculated and if they were less than 12.83, it could be concluded that there is no reason to believe that the logarithms of the parameters did
Table 1. Results of $x^2$ Analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>30 cm mean</th>
<th>30 cm variance</th>
<th>61 cm $x^2$ mean</th>
<th>61 cm variance</th>
<th>91 cm $x^2$ mean</th>
<th>91 cm variance</th>
<th>$x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_0$</td>
<td>13.7</td>
<td>537.7*</td>
<td>23.4</td>
<td>4.71</td>
<td>14.5*</td>
<td>20.8</td>
<td>6.56</td>
</tr>
<tr>
<td>log $K_0$</td>
<td>0.919</td>
<td>0.255</td>
<td>2.8</td>
<td>0.564</td>
<td>0.095</td>
<td>2.4</td>
<td>0.615</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>30.3</td>
<td>4.52</td>
<td>8.8</td>
<td>30.4</td>
<td>5.07</td>
<td>2.4</td>
<td>29.3</td>
</tr>
<tr>
<td>$\theta$</td>
<td>45.6</td>
<td>90.9</td>
<td>8.8</td>
<td>38.7</td>
<td>187.4</td>
<td>9.6</td>
<td>41.0</td>
</tr>
<tr>
<td>log $\theta$</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>1.59</td>
</tr>
</tbody>
</table>

* Variance of untransformed values found using the following equation by Haan (1977).

\[ S_x^2 = \bar{x}^2 (\exp(S_y^2) - 1) \]

where $Y = \log x$, $\bar{x}$ = mean of $x$, $S_y^2 = (\ln 10)^2 \cdot \text{var}(Y)$
not conform to a normal distribution. In other words, the raw parameters could be considered lognormally distributed.

Figures 14, 15, 16 and 17 show cumulative probability plots of log $K_0$, $\theta_0$, $\beta$ and log $\beta$, respectively for all three depths. Normal distributions should plot as straight lines. As can be seen in Figure 14, the distributions of log $K_0$ values at 61 and 91 cm are more similar than the distribution at 30 cm which may indicate that they were drawn from the same population. It is evident from the increase in curvature of the plotted populations of $\beta$ values in figure 16 that the variance of $\beta$ increases with depth.

The steady-state hydraulic conductivity populations from all three depths are shown to be lognormally distributed. This is in agreement with results presented by Jones and Wagenet (1984) who used the same field methods as this study to find $K_0$ values. Lognormal distributions of saturated $K$ values were also reported by Nielsen et al. (1973) and Russo (1983) though different methods were used to acquire them. Jones and Wagenet (1984) also reported that the distributions of $\beta$ values they observed could be considered either normal or lognormal which is the case for this study as well.
Figure 14. Population distributions of log$K_0$ values at 30, 61, and 91 cm plotted on normal probability paper.
Figure 15. Population distributions of $\theta_o$ values at 30, 61, and 91 cm plotted on normal probability paper.
Figure 16. Population distribution of $\beta$ values at 30, 61, and 91 cm plotted on normal probability paper.
Figure 17. Population distributions of log\(\beta\) values at 30, 61, and 91 cm plotted on normal probability paper.
Trend Surface Analysis

Trend surface analysis uses polynomial models which best describe a dependent variable as a function of its space co-ordinates. It is less sophisticated than other geostatistical techniques which attempt to describe spatial distributions of parameters because quantification of parameter spatial variability is not incorporated in the analysis. In order to adhere to assumptions concerning stationarity, any trends in spatially distributed data must be identified and removed before any statement can be made about spatial variability. Davis (1973) describes the procedure used in this study to fit trend surfaces to the data and only a summary will be presented here.

Trend surface analysis is an adaptation of another statistical technique known as multiple regression. Multiple regression analysis assumes that the relationship between one variable and several others can be modelled by

\[ Y = \sum_{j=0}^{m} \beta_j X_j + \epsilon \]  

(22)

where \( Y \) is the dependent variable, \( m \) is the number of independent variables, \( \beta_j \) is known as the model parameter associated with independent variable \( X_j \), and \( \epsilon \) is the increment by which any \( Y \) may deviate from the model. The variable \( X_0 \) in (22) is always equal to 1 to provide an
estimate of intercept $\beta_0$. Considering only first-order model at the present, a trend surface model takes a form of (22) such as

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$$

where $X_1$ and $X_2$ are the rectangular space coordinates $x$ and $y$, respectively.

The parameters $\beta_0$, $\beta_1$, $\beta_2$, and $\varepsilon$ are unknowns in equation (23). Because it changes for each observation, $Y$, $\varepsilon$ is difficult to determine (Draper and Smith, 1966). However, $\beta_0$, $\beta_1$, and $\beta_2$ remain unchanged and they must be estimated using the observations available. The predicted observation $Y$ can now be defined as

$$Y = b_0 + b_1 X_1 + b_2 X_2$$

where $b_0$, $b_1$, and $b_2$ are estimates of the corresponding $\beta$ parameters. The estimates are obtained by using the method of least squares.

Considering $n$ sets of observations, equation (24) can be written as

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i$$

(25)
and so the sum of squares due to deviation can be written as

\[ S(\beta_0, \beta_1, \beta_2) = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 X_{1i} - \beta_2 X_{2i})^2 \]  \hspace{1cm} (26)

The parameter estimates in equation (26) will be chosen such that when they are substituted for their corresponding \( \beta \) parameters, \( S(\beta_0, \beta_1, \beta_2) \) will take on its lowest possible value. In other words, the square of the deviations will be minimized.

The process of finding the least-squares estimates of the \( \beta \) parameters begins by taking the partial derivative of \( S \) with respect to each parameter and setting it equal to zero. Mathematically, this is

\[ \frac{\partial S}{\partial \beta_0} = -2 \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 X_{1i} - \beta_2 X_{2i}) = 0 \]  \hspace{1cm} (27a)

\[ \frac{\partial S}{\partial \beta_1} = -2 \sum_{i=1}^{n} X_{1i} (Y_i - \beta_0 - \beta_1 X_{1i} - \beta_2 X_{2i}) = 0 \]  \hspace{1cm} (27b)

\[ \frac{\partial S}{\partial \beta_2} = -2 \sum_{i=1}^{n} X_{2i} (Y_i - \beta_0 - \beta_1 X_{1i} - \beta_2 X_{2i}) = 0 \]  \hspace{1cm} (27c)

which, after substituting \( b_0, b_1, \) and \( b_2 \) for \( \beta_0, \beta_1, \) and \( \beta_2, \) yields the set of normal equations.
\[ nb_0 + b_1 \Sigma X_1 + b_2 \Sigma X_2 = \Sigma Y \quad (28a) \]
\[ b_0 \Sigma X_1 + b_1 \Sigma X_1^2 + b_2 \Sigma X_1 X_2 = \Sigma Y X_1 \quad (28b) \]
\[ b_0 \Sigma X_2 + b_1 \Sigma X_1 X_2 + b_2 \Sigma X_2^2 = \Sigma Y X_2 \quad (28c) \]

where the summation notation has been suppressed for convenience. Equations (28a-28c) may be written in matrix form as

\[
\begin{bmatrix}
  n & \Sigma X_1 & \Sigma X_2 \\
  \Sigma X_1 & \Sigma X_1^2 & \Sigma X_1 X_2 \\
  \Sigma X_2 & \Sigma X_1 X_2 & \Sigma X_2^2
\end{bmatrix} \begin{bmatrix}
  b_0 \\
  b_1 \\
  b_2
\end{bmatrix} = \begin{bmatrix}
  \Sigma Y \\
  \Sigma Y X_1 \\
  \Sigma Y X_2
\end{bmatrix} \quad (29)
\]

which makes the solutions for \( b_0 \), \( b_1 \), and \( b_2 \) much easier when using a computer since only one matrix inversion followed by a single matrix multiplication are needed.

The significance of the model can be tested by performing an analysis of variance. This process separates total variance into components having different sources. The total variance of the independent variable in trend surface analysis can be divided into that which is due to
regression and that which cannot be accounted for by the regression, or that which is due to deviations.

There are \((n-1)\) degrees of freedom associated with total variance where \(n\) is the total number of samples. The degrees of freedom associated with regression, \(m\), equal one less than the number of \(b\) coefficients which have been estimated. For a first-order surface, \(m = 2\). The degrees of freedom associated with the deviations are the difference between the first two.

An analysis of variance, or ANOVA, proceeds by first calculating the sums of squares associated with the sources of variances. The total sum of squares, \(SS_T\), can be calculated as

\[
SS_T = \sum_{i=1}^{n} Y_i^2 - \frac{\sum_{i=1}^{n} (\bar{Y})^2}{n} = \sum_{i=1}^{n} (Y_i - \bar{Y})^2. \tag{30}
\]

The sum of squares due to regression is defined as

\[
SS_R = \sum_{i=1}^{n} \hat{Y}_i^2 - \frac{\sum_{i=1}^{n} (\hat{Y})^2}{n} = \sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2 \tag{31}
\]

while the sum of squares due to deviations is the difference between the other two, namely
\[ SS_D = SS_T - SS_R = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^{n} e_i^2. \]  

(32)

A relative estimate of the amount of variation in \( Y \) which is due to the regression is known as the "goodness-of-fit", \( R^2 \). This is determined as \( R^2 = \frac{SS_R}{SS_T} \). The more variation which can be explained by the regression, the closer \( R^2 \) is to 1.

Mean squares are estimates of variance and can be found by dividing sums of squares by their associated degrees of freedom. The variation in \( Y \), then can be estimated by

\[ MS_T = \frac{SS_T}{n-1}. \]  

(33)

The variance in \( Y \) due to regression is found by

\[ MS_R = \frac{SS_R}{m} \]  

(34)

while the variance unaccounted for by the regression, or that which is due to deviations, is calculated as

\[ MS_D = \frac{SS_D}{n-m-1}. \]  

(35)
The next task is to determine whether the total variance is sufficiently explained by the regression or whether it is more random in character. This is done by comparing the two component variances using an F probability distribution.

The F statistic is a ratio of two variances which have been determined by randomly sampling the same normal population (Davis, 1973). The shape of the distribution will change with changes in sample sizes and thus is dependent on the degrees of freedom associated with each variance in the ratio. The distribution describes the probability of obtaining a specified ratio of variances drawn from the same population. Because of this, it can be used to test equality of variances.

The F statistic for each trend surface analysis is found by

$$F = \frac{MS_R}{MS_D}. \quad (36)$$

The critical value of F for 2 and 37 degrees of freedom and a level of significance of 2.5% is 4.14.

The null hypothesis that is tested states that there is no difference between the variance due to regression and that due to deviation from regression, i.e. both sample
variances have been drawn from the same population. Formally, the null hypothesis and alternative are

$$H_0: \beta_1=\beta_2=\beta_3=...=\beta_m=0$$

$$H: \text{at least one } \beta_i \neq 0$$

which shows that what is actually tested is whether the partial regression coefficients are significantly different from zero.

A first-order trend surface analysis was performed using a computer program found in Davis (1973) and the results are presented in Table 2. It may be noted that the calculated F statistic is less than the critical F value in all cases. Although the null hypothesis is not rejected for each data set it may be advantageous to determine whether the data can be modeled by a higher order trend surface. For the purposes of this study only one higher order trend surface will be considered.

A second-order trend model takes the form of

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1^2 + \beta_4 X_2^2 + \beta_5 X_1 X_2 + \epsilon$$

where all variables have already been described. In this case, however, the sum of squares due to deviation is a
Table 2. Results of First-Order Trend Surface Analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>30 CM</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>61 CM</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>91 CM</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>b&lt;sub&gt;0&lt;/sub&gt;</td>
<td>b&lt;sub&gt;1&lt;/sub&gt;</td>
<td>b&lt;sub&gt;2&lt;/sub&gt;</td>
<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>F</td>
<td>b&lt;sub&gt;0&lt;/sub&gt;</td>
<td>b&lt;sub&gt;1&lt;/sub&gt;</td>
<td>b&lt;sub&gt;2&lt;/sub&gt;</td>
<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>F</td>
<td>b&lt;sub&gt;0&lt;/sub&gt;</td>
<td>b&lt;sub&gt;1&lt;/sub&gt;</td>
<td>b&lt;sub&gt;2&lt;/sub&gt;</td>
<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
</tr>
<tr>
<td>K&lt;sub&gt;0&lt;/sub&gt;</td>
<td>15.608</td>
<td>-0.456</td>
<td>0.2762</td>
<td>5.2%</td>
<td>1.02</td>
<td>7.457</td>
<td>-0.1657</td>
<td>-0.0617</td>
<td>6.8%</td>
<td>1.35</td>
<td>8.475</td>
<td>0.0801</td>
<td>-0.2270</td>
</tr>
<tr>
<td>logK&lt;sub&gt;0&lt;/sub&gt;</td>
<td>0.8606</td>
<td>-0.0218</td>
<td>0.0252</td>
<td>13.1%</td>
<td>2.79</td>
<td>0.7630</td>
<td>-0.0145</td>
<td>-0.0021</td>
<td>6.7%</td>
<td>1.34</td>
<td>0.7268</td>
<td>-0.0120</td>
<td>0.0025</td>
</tr>
<tr>
<td>θ&lt;sub&gt;0&lt;/sub&gt;</td>
<td>29.910</td>
<td>-0.0807</td>
<td>0.1078</td>
<td>12.1%</td>
<td>2.55</td>
<td>30.141</td>
<td>-0.0903</td>
<td>0.1082</td>
<td>11.8%</td>
<td>2.47</td>
<td>28.760</td>
<td>-0.0423</td>
<td>0.0846</td>
</tr>
<tr>
<td>β</td>
<td>49.883</td>
<td>-0.1354</td>
<td>-0.2107</td>
<td>2.1%</td>
<td>0.394</td>
<td>46.814</td>
<td>0.2633</td>
<td>-0.8945</td>
<td>14.1%</td>
<td>3.02</td>
<td>40.060</td>
<td>0.6242</td>
<td>-0.5154</td>
</tr>
<tr>
<td>logβ</td>
<td>1.705</td>
<td>-0.0017</td>
<td>-0.0028</td>
<td>4.0%</td>
<td>0.774</td>
<td>1.6501</td>
<td>0.0020</td>
<td>-0.0087</td>
<td>12.6%</td>
<td>2.66</td>
<td>1.5961</td>
<td>0.0047</td>
<td>-0.0048</td>
</tr>
</tbody>
</table>
function of six model parameters. Mathematically formulated this is

\[ \sum_{i=1}^{n} e_i^2 = S(\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5). \]  \hspace{1cm} (40)

The normal equations which produce the estimates of the model parameters are found in the same manner as for a first-order trend surface analysis. The significance of the fit is also tested in the same manner as for the first-order surface. However, because the degrees of freedom associated with the sum of squares due to regression increases to 5, the critical F value becomes 2.94 at the same level of significance. An additional test can be formulated to determine the significance of the second-order fit over the first-order fit.

The sum of squares due to the increase in order of the trend surface, \( SS_{RI} \), can be calculated by

\[ SS_{RI} = SS_{R2} - SS_{R1} \]  \hspace{1cm} (41)

where \( SS_{R2} \) is the sum of squares due to regression for the second-order surface and \( SS_{R1} \) is the sum of squares due to regression for the first-order surface. \( SS_{R2} \) has five degrees of freedom associated with it and because there are two degrees of freedom associated with \( SS_{R1} \),
SS_{RI} has three degrees of freedom. The mean squares due to increased regression from the higher order model is then $SS_{RI}/3$.

The $R^2$ is calculated in the same manner as for a first-order model. This measurement will always increase when higher order surfaces are fitted to the data. In fact, $R^2$ will be 1 when the number of model parameters becomes $(n-1)$ (Davis, 1973). The objective of the analysis is to determine whether the increase is significant.

An $F$ statistic can be calculated which compares the variance of $Y$ accounted for by an increase in model order to that variance which is unaccounted for by the second-order model. The $F$ statistic then is

$$F = \frac{MS_{R1}}{MS_{D2}}$$

where $MS_{D2}$ is the mean squares due to deviation which has 34 degrees of freedom associated with it. The critical value of $F$ for 3 and 34 degrees of freedom at 2.5% level significance is 3.55. A calculated $F$ statistic greater than this value causes the null hypothesis to be rejected.

Formally stated, the null hypothesis and alternative are

$$H_0: \beta_3 = \beta_4 = \beta_5 = 0$$
\[ H_1: \text{at least one } \neq 0 \]  \hspace{1cm} (44)

The test now is whether the additional model parameters from the higher order surface are significantly different from zero. In other words the test determines whether the higher order trend surface model accounts for more of the variability in \( Y \) than does the lower order model.

Table 3 shows the F statistics calculated from fitting second-order surfaces to each data set. This was also done using a computer program from Davis (1973). Again, these values are less than the critical F value so the hypothesis of no regression must be accepted. It is not necessary to compare the two models since neither are appropriate to explain the variation in \( Y \).

It can be concluded, after applying first- and second-order trend surface analyses, that each of the hydraulic parameters in each depth interval exhibits no trend, or drift, and thus partially fulfills the criteria for stationarity.

Identification of Variance Structure

Experimental Semivariogram

The first step in identifying the variance structure in the spatially distributed data of this study was to calculate the experimental semivariogram. The semivariance is estimated at discrete lag values by
Table 3. Results of Second-Order Trend Surface Analysis

<table>
<thead>
<tr>
<th>Depth (cm)</th>
<th>Parameter</th>
<th>$b_0$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$R^2$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 cm</td>
<td>$K_0$</td>
<td>-9.9358</td>
<td>1.1338</td>
<td>2.501</td>
<td>0.0289</td>
<td>-0.1055</td>
<td>-0.0042</td>
<td>12.5%</td>
<td>0.967</td>
</tr>
<tr>
<td></td>
<td>log $K_0$</td>
<td>0.0113</td>
<td>0.0253</td>
<td>0.1149</td>
<td>0.0008</td>
<td>-0.0053</td>
<td>-0.0010</td>
<td>10.0%</td>
<td>1.49</td>
</tr>
<tr>
<td></td>
<td>$\theta_0$</td>
<td>-0.3639</td>
<td>-0.0068</td>
<td>-0.0005</td>
<td>0.0244</td>
<td>-0.0074</td>
<td>16.7%</td>
<td>1.37</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>28.037</td>
<td>1.4463</td>
<td>1.4005</td>
<td>0.0385</td>
<td>-0.2060</td>
<td>0.0374</td>
<td>17.6%</td>
<td>1.45</td>
</tr>
<tr>
<td></td>
<td>log $\beta$</td>
<td>1.5291</td>
<td>0.0108</td>
<td>0.0097</td>
<td>0.0004</td>
<td>-0.0018</td>
<td>0.0004</td>
<td>19.1%</td>
<td>1.61</td>
</tr>
<tr>
<td>61 cm</td>
<td>$K_0$</td>
<td>12.521</td>
<td>-0.8068</td>
<td>-0.4097</td>
<td>0.0365</td>
<td>-0.0167</td>
<td>0.0221</td>
<td>21.4%</td>
<td>1.85</td>
</tr>
<tr>
<td></td>
<td>log $K_0$</td>
<td>1.3138</td>
<td>-0.0802</td>
<td>-0.0424</td>
<td>0.0025</td>
<td>0.0002</td>
<td>0.0015</td>
<td>15.9%</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>$\theta_0$</td>
<td>-0.3033</td>
<td>0.2504</td>
<td>-0.0124</td>
<td>0.0156</td>
<td>-0.0133</td>
<td>19.0%</td>
<td>1.63</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>-3.338</td>
<td>-0.052</td>
<td>0.1269</td>
<td>0.0284</td>
<td>0.1118</td>
<td>28.9%</td>
<td>2.76</td>
<td></td>
</tr>
<tr>
<td></td>
<td>log $\beta$</td>
<td>2.032</td>
<td>0.0352</td>
<td>0.0434</td>
<td>0.0013</td>
<td>0.0004</td>
<td>0.0012</td>
<td>28.1%</td>
<td>2.66</td>
</tr>
<tr>
<td>91 cm</td>
<td>$K_0$</td>
<td>-10.943</td>
<td>0.1105</td>
<td>0.9043</td>
<td>-0.0295</td>
<td>-0.104</td>
<td>0.0020</td>
<td>5.2%</td>
<td>0.375</td>
</tr>
<tr>
<td></td>
<td>log $K_0$</td>
<td>0.6899</td>
<td>0.0074</td>
<td>-0.0157</td>
<td>-0.0001</td>
<td>-0.0013</td>
<td>0.0014</td>
<td>5.2%</td>
<td>0.373</td>
</tr>
<tr>
<td></td>
<td>$\theta_0$</td>
<td>24.241</td>
<td>0.2324</td>
<td>0.6426</td>
<td>-0.0071</td>
<td>-0.0070</td>
<td>-0.0107</td>
<td>16.0%</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>-3.339</td>
<td>-3.339</td>
<td>-3.339</td>
<td>-0.0659</td>
<td>0.0014</td>
<td>0.1115</td>
<td>16.7%</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td>log $\beta$</td>
<td>1.8163</td>
<td>-0.0076</td>
<td>-0.0345</td>
<td>0.0005</td>
<td>0.0000</td>
<td>0.0012</td>
<td>19.1%</td>
<td>1.60</td>
</tr>
</tbody>
</table>
Each lag value is an integer multiple of the unit lag distance. In this study the unit lag distance is 7.5m.

Other studies such as Russo (1983), Vieira et al. (1981), and Russo and Bressler (1980) have shown that the distances at which certain hydraulic properties, similar to $K_0$ measured in this study, are no longer correlated range between about 40 and 80m. Specifically, Russo (1983) measured saturated hydraulic conductivity, $K_s$, using a method similar to the one utilized in this study but modified to include suction gradients obtained from tensiometer measurements. Vieira et al. (1981) measured infiltration rates with single-ring infiltrometers while Russo and Bressler (1980) determined $K_s$ values using a modified air-entry permeameter. Because it was possible that this study area would exhibit similar variance structure of a similar hydraulic parameter, it was decided to construct a sampling scheme that would yield more detailed information about the variance structure at distances less than 40m. In addition, the chosen scheme would cause most semivariances to be estimated from numerous pairs of data. The greater the number of pairs, the more reliable the estimate. Semivariances could be estimated at six lag distances less than 40m in length.
Originally, semivariance estimations were limited to integer multiples of the unit lag and to two directions only. Differences in directional semivariograms would indicate anisotropic properties. However, semivariograms calculated using this criteria showed much scatter which made identification of structure and anisotropy difficult. It was decided that structure may be more easily seen if more pairs of data points could be included in the estimation of semivariance for a particular lag. Therefore, the restrictions concerning lag and direction had to be eased.

More data points may be included in the estimation of a particular semivariance by considering a range of directions and separation distances. If zero degrees defines the direction of a vector parallel to local elevation contours and if the number of degrees increases counterclockwise, then all vectors having directions between 45° and 135° would be grouped into one class. All semivariances estimated for this class are considered having directions perpendicular to local elevation contours. Likewise, all vectors having directions between 135° and 225° would be grouped into another class with a generalized direction parallel to contours. Vectors having magnitudes of $h \pm 0.5$ were defined as having the same magnitude, $h$. In this way, more reliable semivariances could be obtained by including a greater number of pairs of data in each estimation. Table 4 shows the
Table 4. Increase in Number of Estimation Pairs for Unrestricted Method.

<table>
<thead>
<tr>
<th>h</th>
<th>Restricted Parallel to Contours N(h)</th>
<th>Restricted Perpendicular to Contours N(h)</th>
<th>Unrestricted Parallel to Contours N(h)</th>
<th>Unrestricted Perpendicular to Contours N(h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>9</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>10</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>12</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>8</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>10</td>
<td>31</td>
<td>30</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>6</td>
<td>15</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>15</td>
<td>15</td>
<td>39</td>
<td>39</td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>6</td>
<td>38</td>
<td>44</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>6</td>
<td>21</td>
<td>24</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>4</td>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td>6</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
<td>4</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
<td>4</td>
<td>25</td>
<td>24</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>4</td>
<td>18</td>
<td>19</td>
</tr>
</tbody>
</table>
number of pairs of data included in the estimation of semivariances for sixteen lag distances, two directions, and for both restricted and unrestricted considerations.

Figures 18a through 22c illustrate the experimental semivariograms for $K_0$, $\log K_0$, $\theta_0$, $\beta$ and $\log \beta$ for each depth. The logarithms of $K_0$ and $\beta$ were included because Delhomme (1978) suggested that variance structures of lognormally distributed parameters may be more easily identified if the parameters are logarithmically transformed first.

There are three semivariograms for each hydraulic parameter. One has been determined using only those vectors having directions defined as being perpendicular to contours. Another was determined using only those vectors with directions defined as being parallel to contours. The third is the total, nondirectional semivariogram which is the average of the first two.

There appears to be little significant differences between the directional semivariograms for $K_0$ values at all depths (see Figures 18a, 18b, and 18c). In addition, each directional semivariogram shows much scatter. The total semivariograms are less scattered because they are averages of the directional semivariograms.

The total semivariogram for the $K_0$ values at 30 cm shows an anomalously low semivariance at one lag. This may
Figure 18a. Directional and total semivariograms for $K_o$ at 30 cm.
Figure 18b. Directional and total semivariograms for \( K_\theta \) at 61 cm.
Figure 18c. Directional and total semivariograms for K\textsubscript{o} at 91 cm.
signify increased correlation at this distance or it may simply be an artifact of relatively few (18) observation pairs used to estimate the semivariance. It would be easier to determine whether the low semivariance is real if the semivariogram exhibited a silled structure. Different types of semivariogram models are discussed in the next section.

The total semivariogram for $K_0$ at 61 cm shows a linear structure in combination with a nugget effect. The total semivariogram for $K_0$ values at 91 cm shows no structure, or a pure nugget effect, which indicates that the parameter is randomly distributed in space.

The semivariograms of the logarithmic transformation of $K_0$ values, shown in Figures 19a through 19c generally show less scatter.

The directional semivariograms of log $K_0$ values for 30 cm are now different. Values of log $K_0$ perpendicular to contours may be correlated at distances less than two or three lag distances (15 or 22.5 meters). The semivariance beyond this distance remains relatively constant. This is indicative of a silled model. The semivariogram for directions parallel to the contours, however, reveals regularly oscillating semivariances which repeat themselves every seven or eight lags. The combination of the two results in a linear structure similar to that found in the total semivariogram of $K_0$ values at 61 cm.
Figure 19a. Directional and total semivariograms for logK at 30 cm. Best fit theoretical model is shown.
Figure 19b. Directional and total semivariograms for log K at 61 cm. Best fit theoretical model is shown.
Figure 19c. Directional and total semivariograms for logK₀ at 91 cm.
The logarithmic transformations decreased the scatter in the directional semivariograms of log $K_o$ at 61 cm but the linear structure is maintained for the total semivariogram.

Scatter is decreased drastically in the semivariograms at 91 cm. The total semivariogram of log $K_o$ now reveals a classic pure nugget effect.

The semivariograms of $\theta_o$ values shown in Figures 20a, b, and c for directions perpendicular to contours show distinct linear trends. The trends are greater at 30 cm and 91 cm than at 61 cm. The semivariograms for directions parallel to contours show much more scatter and are structureless. There is an anomalously high semivariance at a lag of 14 for each depth which is also noticeable in the total semivariograms. The total semivariogram for $\theta_o$ values at 30 cm shows a slight linear trend while the total semivariograms for 61 cm and 91 cm exhibit no structure. The high semivariances at lag 14 are quite evident in these semivariograms. As shown in Figure 21a, there does not appear to be any structure in any of the three semivariograms for $\beta$ at 30 cm. The semivariograms of $\beta$ for 61 cm and 91 cm are similar (see Figures 21b and 21c). The semivariograms for directions perpendicular to contours at both depths exhibit linear structures. Both semivariograms determined parallel to contours show more scatter than their counterparts in addition to an anomalously high semivariance at lag 14.
Figure 20a. Directional and total semivariograms for $\theta_0$ at 30 cm. Best fit theoretical model is shown.
Figure 20b. Directional and total semivariograms for $\Theta_0$ at 61 cm.
Figure 20c. Directional and total semivariograms for \( \theta_0 \) at 91 cm.
Figure 21a. Directional and total semivariograms for B at 30 cm.
Figure 21b. Directional and total semivariograms for $\beta$ at 61 cm.
Figure 21c. Directional and total semivariograms for $\beta$ at 91 cm.
The total semivariograms are similar in that both show pure nugget effects and sharp increases in semivariances at lag 14.

The semivariograms of the logarithmic transformations of $A$, seen in Figures 22a through 22c, do not reveal any significant differences in structure.

### Theoretical Model Descriptions

The variance structure within a study area may be satisfactorily described by fitting one, or a combination, of several predetermined mathematical functions to the calculated semivariances. This is essential if accurate interpolation at unsampled locations is desired. Descriptions of these functions are presented here as summaries of more detailed explanations given by Vieira et al. (1983), Journel and Huijbregts (1978) and Delhomme (1978).

There are two types of theoretical models in common usage: those with sills and those without.

Models with sills, or transition models, are characterized by regularly increasing semivariances up to some constant value, the sill, which represents the total variance in the study area. The distance at which the sill is achieved is known as the range. Observations are correlated at distances less than the range while no correlation exists at greater distances. When the range is smaller than the
Directional and total semivariograms for logβ at 30 cm.

Figure 22a.
Figure 22b. Directional and total semivariograms for logβ at 61 cm.
Figure 22c. Directional and total semivariograms for logβ at 91 cm.
shortest sampling interval, a pure nugget effect exists. This indicates that there is no variance structure and that therefore the parameter in question is randomly distributed throughout the study area.

There are essentially four classifications of transition models: 1) linear; 2) spherical; 3) Gaussian; and 4) exponential. In the following descriptions, \( C_0 \) will denote a nugget effect, which may or may not exist, \( C \) will denote the sill and \( a \) is the range. Visual representations of the models are shown in Figure 23.

1) linear

\[
Y(h) = C_0 + Ah \quad \text{for } 0 < h < a
\]
and \( Y(h) = C \) \quad \text{for } h > a

where \( A \) is the slope between 0 and \( a \).

2) spherical

\[
Y(h) = C_0 + C\left(e^{-\frac{3h}{2a}} - \left(\frac{h}{a}\right)^3\right) \quad \text{for } 0 < h < a
\]
and \( Y(h) = C \) \quad \text{for } h > a

After \( C_0 \) and \( C \) have been determined, \( a \) is found by first extending a straight line from the \( y \)-axis at \( C_0 \), tangential to the points closest to the origin, up to the sill. The distance at which this line intersects the sill should be \( 2/3a \). The model should behave linearly up to \( 1/3a \).
Figure 23. Four types of transition models: a) linear, b) spherical, c) exponential, d) Gaussian (Delhomme, 1978).
3) Gaussian

\[ Y(h) = C_0 + C[1 - \exp(-h^2/a_0^2)] \] for all \( h \) \hfill (48)

where \( a_0 = a/\sqrt{3} \). Because the sill is approached asymptotically the range is normally considered the distance at which \( Y(h) = 0.95C \).

4) exponential

\[ Y(h) = C_0 + C[1 - \exp(-h/a_0)] \] for all \( h \) \hfill (49)

where \( a_0 = a/3 \). A tangent extending from the \( y \)-axis at \( C_0 \) reaches the sill at \( a_0 \). Again, because the sill is approached asymptotically the range is considered that distance where \( Y(h) = 0.95C \).

Models without sills indicate phenomena for which variances and covariances cannot be defined. The intrinsic hypothesis, i.e. stationarity of increments only, must be embraced for these models to have meaning. These models all assume the form

\[ Y(h) = C_0 + Ah^B \quad 0 < B < 2; \] for all \( h \) \hfill (50)

and examples of three different values of \( B \) are shown in Figure 24. Variance structures modeled in this way indicate phenomena which are irregular yet correlated regardless of separation distance throughout the study area.
Figure 24. Graphic displays of $C + \text{Ah}^B$ for three values of $B$ (Delhomme, 1978).
Fitting the Theoretical Model

The choice of a theoretical semivariogram model is critical in a study such as this because the accuracy of interpolated points depends on how appropriate the model is. Appropriate models have been fitted directly to experimental semivariograms based on minimum mean square deviation (Russo, 1983) and on techniques which weight semivariances according to the number of pairs of observation used in estimating them (Yost et al., 1982a). Other procedures produce theoretical models directly from the spatially distributed data. Theoretical semivariogram models for the data in this study were found by analyzing the data using a computer package known as BLUEPACK (Delfiner et al., 1976). A brief description of the automatic structure identification portion of this package is presented.

After visual inspection of the total experimental semivariograms, only three data sets indicated possible structure. From 30 cm these were log Ko and Go, and from 61 cm the set was log Ko. The remaining data sets were found to exhibit pure nugget effects at distances which will be considered in data interpolation.

Automatic structure identification procedures were applied to the three data sets. This is an option of BLUEPACK (Delfiner et al., 1976) which is a package of kriging-related programs. The procedure fits polynominal generalized covariance functions which take into consideration
the presence of any trends in the data but neglect the possibility of anisotropy. When there is no trend, which is the case for this study, the generalized covariance function is related to the semivariogram by $K(h) = -\gamma(h)$. The procedure begins by determining the order $k$ of the trend. The order is 0 when there is no trend, 1 when the trend is linear and 2 when it is quadratic. Coefficients of the generalized covariance functions are then estimated using weighted least squares theory. There may be more than one function that satisfies the theory's criteria. The validity of a particular function is checked by using it to interpolate values at a number of sample locations using neighboring observations. The function which can best reproduce the known values is retained.

The results of this procedure reveal that the variance structures of the parameters may all be represented by a linear model of the form

$$\gamma(h) = C_0 + Ah$$

(51)

Table 5 shows the values of $C_0$ and $A$ for each parameter while Figures 19a, 19b, and 20a show the functions drawn on the total experimental semivariograms.
Table 5. Theoretical Model Coefficients of Three Total Semivariograms.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Depth</th>
<th>$C_0$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log K_0$</td>
<td>30cm</td>
<td>0.176</td>
<td>0.00653</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>30cm</td>
<td>3.6</td>
<td>0.0591</td>
</tr>
<tr>
<td>$\log K_0$</td>
<td>61cm</td>
<td>0.0675</td>
<td>0.00374</td>
</tr>
</tbody>
</table>

Kriging

Once the spatial variabilities of the regionalized variables, i.e. the hydraulic parameters, have been defined it is desirable to produce accurate displays showing their spatial distributions. This can be accomplished using an interpolating technique known as kriging. This method incorporates information about variance structure so only those parameters that have been shown to have structure will be mapped. The following theoretical description of the technique is a summary of more detailed information presented in Vieira et al. (1983), Vieira et al. (1981), and Burgess and Webster (1980). The actual computations were performed by another option of BLUEPACK (Delfiner et al., 1976).

The technique estimates the value of some parameter, $z$, at a chosen location, $x_0$. Let $x_i = (x_i, y_i)$ represent spatial coordinates. To do this, a weighted moving average of the form

$$z^*(x_0) = \sum_{i=1}^{n} \lambda_i z(x_i)$$

(52)
is used where \( z^*(x_0) \) is the estimated value at \( x_0 \), \( n \) is the number of observations in the "neighborhood" that are weighted, \( \lambda_i \) is the weight associated with observation \( z(x_i) \). The number of observations in the neighborhood ranges from 8 to 16 and is fixed by BLUEPACK depending on the drift. Recalling that \( z(x_i) \) is a realization of the random function, \( Z(x_i) \), and that \( Z(x_i) \) is subject to the assumptions of the intrinsic hypothesis, (46) can be written as

\[
Z^*(x_0) = \sum_{i=1}^{n} \lambda_i Z(x_i)
\]  

(53)

The weights, \( \lambda_i \), will provide the best linear estimator if conditions concerning unbiasedness and minimum variance of estimation are satisfied. The estimation will be unbiased when

\[
E[Z^*(x_0) - Z(x_0)] = 0
\]  

(54)

and the estimation variance will be minimized when

\[
E[(Z^*(x_0) - Z(x_0))^2] = \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \gamma(x_i, x_j) + 2 \sum_{j=1}^{n} \lambda_j \gamma(x_0, x_j)
\]  

(55)

is minimized. The term \( \gamma(x_i, x_j) \) is the semivariance of a vector with its origin at \( x_i \) and extending out to \( x_j \).
Substituting equation (52) into (53) and applying the linearity of $E$ we have

$$E[Z^*(x_o)-Z(x_o)]=E[\sum_{i=1}^{n} \lambda_i Z(x_i)-Z(x_i)]$$

$$=E[\sum_{i=1}^{n} \lambda_i Z(x_i)]-E[Z(x_i)]$$

$$=\sum_{i=1}^{n} \lambda_i E[Z(x_i)]-E[Z(x_i)] = 0 \quad (56)$$

By assuming that there is no trend, i.e. stationarity exists, we can say

$$E[Z(x_i)] = m$$

and so (50) now becomes

$$m(\sum_{i=1}^{n} \lambda_i -1) = 0 \quad (57)$$

and therefore the estimation will be unbiased when

$$\sum_{i=1}^{n} \lambda_i = 1 \quad (58)$$
Equation (55) can be minimized with respect to the weights, \( \lambda_i \), and subject to (58) by using Lagrangian techniques which take the n partial derivatives of (55) and setting them all to zero. Mathematically this is

\[
\frac{\partial}{\partial \lambda_i} \left[ \mathbb{E} \left[ \left\{ Z(x_o) - Z(x_o) \right\}^2 \right] - 2u \sum_{i=1}^{n} \lambda_i \right] \tag{59}
\]

where \( u \) is the lagrange multiplier. This results in a set of normal equations such as

\[
\sum_{j=1}^{n} \lambda_j y(x_i, x_j) + u = y(x_i, x_o), \quad i=1, 2, \ldots, n \tag{60}
\]

where

\[
\sum_{j=1}^{n} \lambda_j = 1
\]

These can be expressed in matrix form as

\[
\begin{bmatrix} \lambda \\ u \end{bmatrix} = A^{-1}b \tag{61}
\]

where
\[
A = \begin{bmatrix}
\gamma(x_1, x_1) & \gamma(x_2, x_1) & \ldots & \gamma(x_n, x_1) & 1 \\
\gamma(x_1, x_2) & \gamma(x_2, x_2) & \ldots & \gamma(x_n, x_2) & 1 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\gamma(x_1, x_n) & \gamma(x_2, x_n) & \ldots & \gamma(x_n, x_n) & 1 \\
1 & 1 & \ldots & 1 & 0
\end{bmatrix}, \quad (62)
\]

\[
b = \begin{bmatrix}
\gamma(x_1, x_0) \\
\gamma(x_2, x_0) \\
\vdots \\
\gamma(x_n, x_0) \\
1
\end{bmatrix}, \quad (63)
\]

\[
\lambda = \begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\vdots \\
\lambda_n \\
\mu
\end{bmatrix}, \quad (64)
\]

The minimum estimation variance or kriging variance, \( \sigma_k^2 \) is given by

\[
\sigma_k^2 = b^T \begin{bmatrix}
\lambda \\
\mu
\end{bmatrix} \quad (65)
\]

where \( b^T \) is the transpose of \( b \).

Figures 25 through 27 show the interpolated contour maps of the three hydraulic parameters in addition to contour maps of standard deviations resulting from the BLUEPACK analyses. The standard deviations are the square roots of the kriging variances.
Figure 25. Kriged contour map (top) and associated standard deviations for $\log K$ at 30cm.
Figure 26. Kriged contour map (top) and associated standard deviations for $\theta_o$ at 30 cm.
Figure 27. Kriged contour map (top) and associated standard deviations for $\log K_o$ at 51 cm.
The spatial distributions of these parameters do not reveal any overall relationships to depositional processes which operate on this area of the alluvial fan. In other words, there are no distinct zones of low hydraulic conductivity which correspond to the channels shown in Figure 5. Likewise, there are no similarities in the spatial distribution of log $K_0$ values at 30 cm to that shown of log $K_Q$ values at 61 cm. There does, however, appear to be a relationship between high $\Theta_0$ values and high log $K_0$ values in the first 30 cm of the study area.

The distributions of standard deviations depend only on the semivariogram and the position of the interpolated point. This is evident in Figures 25, 26, and 27 where the lowest standard deviations i.e., the smallest estimation errors, occur at the intersections of transect C with transects B and D. The interpolated points near these areas have been estimated using more observations than points interpolated at other locations. Displays of standard deviation distributions may show where additional sampling can increase the accuracy of the corresponding interpolated contour maps.
SUMMARY AND CONCLUSIONS

Hydraulic properties were measured at three depth intervals at 40 locations distributed throughout an area of soil developed on an alluvial fan. The method used to measure the parameters used drainage data obtained from infiltration experiments. Because of excessive lateral flow, only observations made during the first 24 hours of drainage were considered.

Using $x^2$ analyses, steady-state hydraulic conductivity, $K_0$, was found to be lognormally distributed at all three depths. Steady-state moisture content, $\theta_o$, was found to be normally distributed, at a level of significance of 2.5%, at all three depths. The dimensionless parameter $\beta$ was found to be normally distributed at the same level of significance for 30 cm and 61 cm and lognormally distributed at 91 cm.

A comparison of sample means and variances and cumulative probability plots shows that the population of log $K_0$ values for 30 cm is quite different from those observed at 61 and 91 cm. The populations of $\theta_o$ values for each depth are very similar. Although the means of $\beta$ values for 30 and 61 cm are similar the variance increases with depth. The population of $\beta$ values at 91 cm becomes quite different as
it becomes lognormal. It is assumed that these populations are different because of horizonation present in the soil profile.

No first- or second-order trends were observed for any parameter at any depth. This showed that assumptions concerning stationarity were correct and that variance structure identification could proceed.

Directional semivariograms were calculated using ranges of directions and separation distances. This resulted in one semivariogram for directions perpendicular to elevation contours and one for directions parallel to contours. Any differences in these two semivariograms would indicate an anisotropic parameter.

Only the directional semivariograms for log $K_0$ values at 30 cm revealed significant differences that may be attributed to anisotropy. The anisotropy would be due to highly directional processes, such as stream deposition of detritus, which affect the fabric of the alluvial deposit and thus, the soil which develops upon it. Hydraulic parameters are affected by this fabric. The periodicity of the semivariogram for directions parallel to elevation contours may reflect the periodic nature of the braided channels in this direction. If these channels are responsible for giving a particular fabric to the deposit-soil at any particular time, they may also be responsible for giving variance structure to hydraulic properties.
The automatic structure identification procedure, which did not account for anisotropy, showed that only log $K_0$ and $\theta_0$ values from 30 cm and log $K_0$ values from 61 cm had variance structure. The structure was defined by a linear model with no sill.

Jones and Wagenet (1984) observed no structure in $K_0$ and $\theta$ values which they obtained using a slightly different form of the method used here. They proposed that this was the case either because there was actually no variance structure present or because of insensitivity of the method. They suggested that insensitivity may have been caused by the simplifying assumptions made during the derivation of the drainage equation (7) or by experimental techniques such as the use of the neutron probe to measure moisture content. Perhaps semivariograms of logarithmic transforms of their $K_0$ and $\theta$ values would have shown better structure.

Semivariograms of $\theta_0$ and $\theta$ values for 61 cm and 91 cm showed anomalously high semivariances at fourteen lag positions. This is a separation distance of 105m. This indicates that the parameters are more dissimilar when they are observed at two locations 105m away from each other than when they are observed at any separation distance less than this. This may also indicate some overall structure but it was difficult to be certain without more information concerning correlations at greater separation distances.
Accurate contour maps of the three parameters shown to have variance structures were constructed using Kriging techniques. No relationships were observed between surface morphology, i.e. locations of stream channels, and the hydraulic properties.

The contour maps were included in this investigation merely to display the structural spatial variability that was observed. The utility of this interpolating technique in planning and modeling procedures in agriculture and hydrology should not be overlooked.

In conclusion it can be said that the hydraulic properties measured by the infiltration method used in this study do, in fact, exhibit variation with depth and a structured variation in the horizontal plane.

It is a reasonable assertion to say that variation with depth is caused by differences in textural properties due to horizonation and/or depositional features. However, this cannot truly be supported unless hydraulic properties are measured in a well defined horizon and statistically compared to the textural characteristics. This was beyond the scope of this study.

The fact that hydraulic property variation is structured in the horizontal plane implies two things. First, it is reasonable to suggest that the depositional environment may to some extent affect how hydraulic properties are dis-
tributed in space. Again, comparisons between textural and hydraulic properties would be needed to verify this. Regardless, the results of this study show that sampling conducted in areas where depositional processes are known should be structured to account for the directional aspects of the processes. More specifically, samples should be taken parallel to the dominant direction of deposition and perpendicular to it. In this way possible anisotropy may be distinguished. The second implication is that the sampling method used in this study was sensitive enough to permit structured variation of the hydraulic properties to be observed. Because of the problems brought about by lateral water movement and the fact that each infiltration site required a great deal of water (approximately 900 gallons per site) before steady-state conditions were achieved, this method should not be used in similar environments where many sites must be sampled.
RECOMMENDATIONS FOR FUTURE WORK

As was alluded to in Jones and Wagenet (1984), it may be that the results of an investigation such as this are unique to the method employed in measuring the hydraulic parameters. To test this, it is recommended that an additional investigation be performed at the study site using a different method to measure comparable hydraulic parameters. The spatial variabilities of the two sets of hydraulic parameters could then be compared. Significant differences in conclusions about spatial variability would indicate differences in sensitivities of the methods; the more sensitive method would produce values which retain more information about spatial dependence.

It is also recommended that further research be performed to determine the source of variation in the hydraulic parameters. It has been suggested that the major source of variation in this study is the textural differences in the soil brought about by the depositional processes acting in the area. However, as has been determined by Bresler et al. (1984) and Wagenet et al. (1984), soil chemical parameters may also affect the variability of hydraulic parameters. Multiple regression analysis would determine how much variation in $K_0$ is attributed to variation in such textural properties as percent sand, silt, and clay, mean grain dia-
meter, and sorting and such chemical properties as EC and SAR. A relationship between hydraulic parameters and soil characteristics may be useful in reconnaissance studies in that textural and/or chemical properties may be more easily obtained.
REFERENCES


Romney, E.M., V.Q. Hale, A. Wallace, O.R. Lunt, J.D. Childress, H. Kaaz, G.V. Alexander, J.E. Kinnear and T.L. Ackerman, 1973, Some characteristics of soil and perennial vegetation in northern Mojave Desert areas of the Nevada Test Site, UCLA No. 12-916, UC-48 Biomedical and Environmental Research, TID-4500, University of California, Laboratory of Nuclear Medicine and Radiation Biology.


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<tr>
<th>Horizon</th>
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<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2</td>
<td>17cm</td>
<td>Pale brown (10YR 6/1) sand, brown to dark brown (10YR 4/1) soils; very weak fine subangular blocky structure; soft, very friable, non-sticky; violently effervescent; no roots; pH 4.2; clear smooth boundary.</td>
</tr>
<tr>
<td>A3</td>
<td>79cm</td>
<td>Very pale brown (10YR 7/1) loamy sand; yellowish brown (10YR 5/4) moist; weak fine subangular blocky structure; soft, friable, non-sticky; violently effervescent; few medium to micro roots; pH 4.5; clear very boundary.</td>
</tr>
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<td>C1</td>
<td>34cm</td>
<td>Very pale brown (10YR 7/1) sand; yellowish brown (10YR 5/4) moist; very weak fine subangular blocky structure; soft, loose, non-sticky; violently effervescent; few medium to micro roots; pH 4.5; gradual boundary.</td>
</tr>
<tr>
<td>C2</td>
<td>344cm</td>
<td>Light gray (10YR 7/1) coarse sand, light yellowish brown (10YR 6/4) moist; massive structure; soft, loose, non-sticky; violently effervescent; few fine to micro roots; pH 8.1; boundary not seen.</td>
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</tbody>
</table>
## Appendix A

### Soil Profile Descriptions, Northern Frenchman Flat, Nevada Test Site

**Site R42 (from Romney et al. 1973)**

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<th>Description</th>
</tr>
</thead>
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<td>12cm</td>
<td>Pale brown (10YR6/3) sand, brown to dark brown (10YR4/3) moist; very weak fine subangular blocky structure; soft, very friable, non-sticky; violently effervescent; no roots; pH 8.2; clear smooth boundary.</td>
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<tr>
<td>A3</td>
<td>29cm</td>
<td>Very pale brown (10YR7/3) loamy sand, yellowish brown (10YR5/4) moist; weak fine subangular block structure; soft, friable, non-sticky; violently effervescent; few medium to micro roots; pH 8.7; clear wavy boundary.</td>
</tr>
<tr>
<td>C1</td>
<td>54cm</td>
<td>Very pale brown (10YR7/3) sand, yellowish brown (10YR5/4) moist; very weak fine subangular blocky structure; soft, loose, non-sticky; violently effervescent; few medium to micro roots; pH 8.3; gradual boundary.</td>
</tr>
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<td>Light gray (10YR7/2) coarse sand, light yellowish brown (10YR6/4) moist; massive structure; soft, loose, non-sticky; violently effervescent; few fine to micro roots; pH 8.1; boundary not seen.</td>
</tr>
<tr>
<td>Horizon</td>
<td>Depth</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
<td>-------------</td>
</tr>
<tr>
<td>Al</td>
<td>2cm</td>
<td>Brown (10YR5/3) fine sand, brown to dark brown (10YR4/3) moist; weak fine subangular blocky structure breaking to single grain; soft, friable, non-sticky; violently effervescent; few micro roots; pH 8.3; clear wavy boundary.</td>
</tr>
<tr>
<td>A2</td>
<td>12cm</td>
<td>Light gray (10YR7/2) loamy sand, yellowish brown (10YR5/4) moist; moderate medium subangular blocky structure; slightly hard, friable, non-sticky; violently effervescent; few medium fine and micro roots; pH 8.5; abrupt wavy boundary.</td>
</tr>
<tr>
<td>C1</td>
<td>32cm</td>
<td>Very pale brown (10YR7/3) sand, yellowish brown (10YR5/4) moist; weak fine subangular blocky structure breaking to single grain; soft friable, non-sticky; violently effervescent; abundant medium and fine roots; pH 8.5; clear smooth boundary.</td>
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<td>72cm</td>
<td>Very pale brown (10YR7/3) sand, brown to dark brown (10YR4/3) moist; single grain; soft friable, non-sticky; violently effervescent; abundant fine and micro roots; pH 8.3; clear smooth boundary.</td>
</tr>
<tr>
<td>C3</td>
<td>128cm</td>
<td>Very pale brown (10YR7/3) sand, brown to dark brown (10YR4/3) moist; single grain; loose friable, non-sticky; violently effervescent; few fine and micro roots; pH 8.1.</td>
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Site D1 (from Case et al. (1984))

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<tr>
<td>Alv 7cm</td>
<td>Light gray (10YR7/30) loamy sand; strong medium platy structure; weakly coherant when dry, non-sticky and non-plastic when wet; mild reaction with HCl from 0 to 4cm, violent reaction below (through-out profile); abundant 0.5mm vesicular pores; abrupt smooth boundary.</td>
<td></td>
</tr>
<tr>
<td>Al 21cm</td>
<td>Pale brown (10YR6/3 dry, 10YR6/4 moist) very gravelly sand; moderate very coarse prismatic breaking to weak thin platy structure; weakly coherant when dry, slightly sticky and non-plastic when wet; clear smooth boundary.</td>
<td></td>
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<tr>
<td>Clca 56cm</td>
<td>Light brown (7.5YR6/3 dry) with gravel (white carbonate coatings); single grained; loose when dry, non-sticky and non-plastic when wet; well bedded with depositional laminae 1 to 5cm thick, individual laminae are well sorted and pebbles are aligned; very abrupt wavy boundary.</td>
<td></td>
</tr>
<tr>
<td>II Alb 74cm</td>
<td>Pinkish gray (7.5YR7/3 dry) very gravelly sand; weak medium prismatic structure; weakly coherant when dry, non-sticky and non-plastic when wet; clear smooth boundary.</td>
<td></td>
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<tr>
<td>II C2 98cm</td>
<td>Light gray (10YR7/3 dry) sandy gravel; single grained; loose when dry, non-sticky and non-plastic when wet; lower boundary is an unconformity and truncates a soil profile; abrupt broken boundary.</td>
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Site D1 (Case et al., 1984) cont.

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<td>Pink (7.5YR-7/4 dry) very gravelly sandy loam; moderate medium subangular blocky structure weakly cemented; carbonate coatings to 2 cm thick on pebble and cobble bottoms, irregular brittle masses of carbonate to 5 cm in diameter; clear broken boundary.</td>
</tr>
<tr>
<td>III C4</td>
<td>150</td>
<td>Light gray (10YR7/3 dry) very gravelly loamy sand; massive; weakly coherant when dry, non-sticky and non-plastic when wet; 20% cobbles and 50% pebbles, no preferred orientation; boundary not seen.</td>
</tr>
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**Site D2 (from Case et al. (1984))**

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</thead>
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<td>B2</td>
<td>25cm</td>
<td>Brown (7.5YR7/4 dry, 5/4 moist) sandy loam; strong coarse prismatic structure; slightly hard when dry, firm when moist, slightly sticky and non-plastic when wet; a few MnO2 coatings on red faces; clear smooth boundary.</td>
</tr>
<tr>
<td>B31ca</td>
<td>43cm</td>
<td>Brown (7.5YR7/4 dry, 5/4 moist) loamy sand; weak medium prismatic breaking to moderate medium subangular blocky structure; weakly coherant when dry, non-sticky and non-plastic when wet; weakly cemented in places; common soft carbonate coatings and filaments; 20% pebbles less than 5mm in diameter; clear smooth boundary.</td>
</tr>
<tr>
<td>B32</td>
<td>54cm</td>
<td>Pinkish gray (7.5YR7/3 dry, 6/3 moist) loamy sand; massive; weakly coherant when dry, non-sticky and non-plastic when wet; 20% pebbles less than 5mm in diameter; clear smooth boundary.</td>
</tr>
<tr>
<td>B33ca</td>
<td>90cm</td>
<td>Pinkish gray (7.5YR8/2 dry, 7/3 moist) loamy sand; weak medium subangular blocky structure; weakly coherant when dry, non-sticky and non-plastic when wet; 50% weakly cemented; abundant brittle carbonate pebble coatings, filaments and masses; 20% pebbles less than 5mm in diameter, becoming coarser downwards; clear smooth boundary.</td>
</tr>
<tr>
<td>Cl</td>
<td>135cm</td>
<td>Pinkish gray (7.5YR7/3 dry, 7/3 moist) very gravelly loamy sand; massive; loose when dry, non-sticky and non-plastic when wet; 50% cobbles and 15% pebbles; boundary not seen.</td>
</tr>
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Appendix B
FIELD SOIL MOISTURE DETERMINATIONS

Theory of Operation

The neutron soil moisture gauge method of determining the amount of water in a porous medium is based on the thermalization or slowing-down of fast neutrons emitted by a radioactive source.

The instrument used in this study was the Troxler Model 3222 Depth Moisture Gauge manufactured by Troxler Electronic Laboratories, Inc. of North Carolina. It essentially consists of a radioactive source-detection tube assembly connected to electronics that control the measurement and display of counted neutrons. The detector is a He-3 filled tube which effectively counts incident neutrons when they react with the gas and create electrical pulses. The radioactive source is a combination of 10mCi of Americium-241, which is an alpha emitter, and Beryllium. The alpha particles strike the Beryllium nuclei resulting in the creation of a "fast" neutron and a carbon-12 nucleus. The energies of the fast neutrons can range from 0 to 11 million electron volts (MeV) (Dickey, 1982).

Neutrons lose much of their kinetic energy and are slowed by collisions with hydrogen atoms in the surrounding soil. The depth moisture gauge measures the amount of hydrogen in the vicinity of the detector-source probe and the flux of slow neutrons that is detected is directly proportional to the amount of hydrogen present.
Fortunately, most of the hydrogen in the soil is present in the form of water.

Fast neutrons react in various ways with the material they travel through which eventually converts them to slow neutrons. The process by which a neutron loses all or part of its energy without being absorbed is known as scattering. Inelastic scattering occurs when the energy of a neutron is transferred to a nucleus and its direction of travel is changed. Collisions with comparatively larger nuclei change the neutron's direction of travel but do little to alter its energy. A neutron is absorbed when it enters a nucleus and a new nucleus is formed. The new nucleus is unstable and decays quickly by emitting gamma radiation or a proton or alpha particle if the energy of the incoming neutron was great enough. This does not allow the neutron to get back to the detection tube. A neutron may be captured by a nucleus and the probability of this happening is different for different elements.

Soil elements that dominate neutron capture (absorption) are, in order of decreasing effect, xenon, cadmium, boron, chlorine, iron, and potassium (Dickey, 1982). After a neutron is thermalized it must diffuse through the soil material, much like a gas, back to the detector to be counted. The presence of significant amounts of these absorbing elements may cause an underestimation of the number of slow
neutrons. Depth moisture gauges must be calibrated to each soil to account for differences in soil chemistry that might effect the measurement of slow neutrons.

Calibration of the Depth Moisture Gauge

The depth moisture gauge manufactured by Troxler Electronic Laboratories, Inc. is calibrated at the factory before shipment to the purchaser. This calibration procedure consists of recording the count ratio (measured count/standard count) for each of six standards. The standards are made of varying amounts of laminated sheets of plastic and a non-absorber of thermal neutrons. The moisture equivalent of the standards are determined by comparing measurements taken in the plastic to measurements made in pure water. The calibration of the standards are verified against measurements made in saturated and drained standards of clean sand.

The count ratio data is subjected to a regression analysis which performs a least-squares fit of a third order polynomial which is given as

\[ M = O_f + S (A_0 + A_1X + A_2X^2 + A_3X^3) \]

where \( M \) is moisture content (cm³/cm³), \( O_f \) is offset (cm³/cm³; normally zero), \( S \) is the slope (cm³/cm³; normally one), \( A_0, A_1, A_2, \) and \( A_3 \) are dimensionless coefficients.
and $X$ is the count ratio (Troxler Electronic Laboratories, Inc., 1980). This calibration curve is contained within the instrument's computer memory which allows for quick conversion from count ratio to moisture content.

As previously mentioned variations in soil chemistry and thus variations in amounts of neutron absorbing elements need to be accounted for in a new calibration of the instrument. Because more accurate measurements of moisture content were required for this study it was necessary to calibrate the instrument to the study soil.

Soil was collected from the DRI infiltration site (Kearl, 1981) located in Area 5 of NTS. A pit with the approximate dimensions of 6ft. x 6ft. x 1ft. was excavated and the soil was placed in two 55 gallon drums which were covered by the accompanying drum lids. The soil was then transported to the DRI office in Reno where the calibrations took place.

The soil was sieved through a 0.25 inch sieve to remove large gravel and cobbles. This would allow undisturbed soil cores to be more easily taken so that bulk densities and volumetric moisture contents could be determined. The soil was thoroughly mixed in large tubs before it was packed back into the drums. The bulk densities that were produced by this action were similar to what would be expected for the field soil. Different moisture contents were achieved by adding water during the mixing process between measurements.
After the soil was packed into a drum to a depth of approximately 26 inches, a 3-inch diameter hole was augered to within 7 inches of the bottom of the drum. A standard size (2.0 inch O.D., 1.9 inch I.D.) aluminum access tube, inches long and sealed at the one end with a 40 rubber expanding stopper, was placed in the hole. Soil that was removed by the auger was repacked into the space surrounding the tube. The tube was located in the center of the barrel which left approximately 11 inches of soil between the tube and the barrel sides. Under the driest conditions encountered, namely 7.8 percent by volume moisture content, the radius of influence of the instrument should have been slightly more than 10 inches (Troxler Electronic Laboratories, Inc., 1980). This distance decreases linearly with increasing moisture content.

A calibration run for a particular moisture content was initiated by taking three to four standard counts while the instrument was perched on the top of the tube. The resulting mean standard count was used in determining the count ratio. Sets of five measured counts were taken when the top of the source-detector probe was positioned at three, five, and seven inches below the soil surface. Thus there were fifteen measured counts obtained for each calibration run.
Four undisturbed soil cores were taken for each calibration run. Two were taken at six inches below the soil surface and two at twelve inches. Each core was located approximately six inches from the access tube and the opposite side of the tube from its pair. After pounding in the sampler, the cores were removed by carefully digging away the surrounding soil. Each core was placed in its own air-tight sample tin which was then weighed. With lids off, the tins were placed in a vacuum oven for 24 hours at a temperature of 110°C. The vacuum was then increased to 0.86 bar and maintained for another 24 hours. The tins were weighed again and volumetric moisture contents were determined by multiplying gravimetric moisture contents by bulk densities. The average of the four volumetric moisture contents were then calibrated to the average of the fifteen count ratios.

Five calibration points were obtained in this manner and a linear regression was performed. The data are presented in Figure B1. Figure B1 also illustrates the calibration curve for the instrument determined at the factory. A data point was obtained each time a measured count was determined and converted into moisture content in the machine. The correlation coefficients for each fitted line show that the relationship between count ratio
and volumetric moisture content is indeed linear for each case. It appears that only the slope of the new calibration curve is significantly different from the factory calibration curve which is in agreement with the findings of Dickey (1982). Count ratios obtained from the field were converted to volumetric moisture contents using the derived calibration curve.
Figure B1. Comparison of factory calibration curve to calibration curve determined for NTS soil.
Appendix C
Results of Regression Analyses

Tables C1 through C3 show the results of regression analyses performed on the raw data to obtain values for $K_0$ and $\beta$.

The units for $K_0$ are centimeters of water drained per hour (cm/hr) while those for $\theta_0$ are cubic centimeters of water contained in 100 cubic centimeters of soil multiplied by 100. These are volumetric percentages of moisture content. The units for volumetric moisture content are commonly expressed as cubic centimeters per cubic centimeters or cc/cc. The proportionality coefficient, $\beta$, is dimensionless.

Values of $n$ shown in each table represent the number of raw data points used in each regression analysis to obtain values of $K_0$ and $\beta$. These values also represent the number of raw data points used in finding $a$ and $b$ by regressing equation (3).

Correlation coefficients resulting from regressing equation (7) to obtain $K_0$ and $\beta$ are denoted as $r_1$. The correlation coefficients labeled as $r_2$ are those resulting from regressing equation (3).

Figures C1 through C10 show raw drainage data and the best fit regression line for each depth at each of the forty infiltration sites. The decrease in moisture content from steady-state conditions, $\theta_0 - \theta$, is expressed as cubic centimeters of water per one cubic centimeter of soil. The units are thus cc/cc but are not percentages.
Table C1. Hydraulic Parameters at 30 CM

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<tr>
<th>Site</th>
<th>Ko (cm/hr)</th>
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Figure C1. Regression plots for sites 1a, 2a, 3a, and 4a.
Figure C2. Regression plots for sites 5a, 6a, 7a, and 8a.
Figure C3. Regression plots for sites 9a, 1b, 2b, and 3b.
Figure C4. Regression plots for sites 4b, 5b, 6b, and 7b.
Figure C5. Regression plots for sites 8b, 1c, 2c, and 3c.
Figure C6. Regression plots for sites 4c, 5c, 6c, and 7c.
Figure C7. Regression plots for sites 8c, 9c, 10c, and 11c.
Figure C8. Regression plots for sites 12c, 1d, 2d, and 3d.
Figure C9. Regression plots for sites 4d, 5d, 6d, and 7d.
Figure C10. Regression plots for sites 8d, 9d, 10d, and 1ld.