An LP Embedded Simulation Model for Conjunctive Use Management Optimization

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Hydrology

by

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Equations for groundwater flow and surface water continuity are embedded in a linear programming hydraulic management model for the optimal conjunctive use of a groundwater and surface water system. This management model uses groundwater and surface water variables directly as decision variables. Problem reduction methods by reduction of decision variables and reduction of constraints, is of little utility. Time was incorporated in a lumped transient formulation in which all time periods are executed at once and in a step-wise procedure in which final heads from the current time step are used as initial heads in the next time step. The hydraulic management approach is valid for the groundwater model, but appears too simplistic for the conjunctive use model. The embedding method is useful for small problems over a limited number of time steps, but encountered numerical difficulties when applied to a large real-world problem.
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CHAPTER I.
INTRODUCTION

The Need for Management

Groundwater has an important role in water supply systems everywhere, but especially in arid or semi-arid regions where surface water supplies fluctuate. As more and more users are competing for limited quantities of water and a region's population and utility of resources are restricted by water supplies, water management is being recognized as a need planning agencies must address. Effective management can increase the efficient use of water by determining the optimal (or most desirable) set of guidelines for water utilization.

Where both groundwater and surface water resources are available, efficient management utilizes the concept of conjunctive use. Conjunctive use is the coordinated and planned operation of both surface and ground water supplies to meet water requirements while also conserving water (Todd, 1980). This may involve importing, storing, pumping, recharging and distributing water from several sources.

Conjunctive use can be especially beneficial in basins approaching full development of water resources, because the joint yields of surface water and groundwater will be larger and more economical than the separate yields. Hall and Dracup (1970) state that the greatest potential for conjunctive use management lies in alleviating the problems
associated with hydrologic uncertainty; utilizing the underground storage capacity of an aquifer for containing artificially recharged surface water during wet periods, to provide adequate water supplies during dry periods.

Conjunctive use has also been presented as a bridge between water use, control and conservation within a region, and all of the potential water sources in that region (Yevejevich, 1979). Many authors state that conjunctive use requires an impoundment of surface water, so that excesses can be transferred at an optimal rate to groundwater storage. While groundwater storage is retained primarily to meet water requirements in dry periods, surface water meets most of the daily water needs. Feasibility of such an approach is dependent upon operating an aquifer over a range of water levels (Todd, 1980). The aquifer must have space to store recharged water without detrimental effects, and must also have water available in storage when pumping is needed. This approach seems particularly useful for irrigation, where water is needed in large quantities and surface supplies cannot always meet demand.

Management is needed because of inadequate surface water supplies and groundwater overdrafts. There are localized problems of inadequate surface water supplies in many regions of the United States. The U.S. Water Resources Council (1978) projects that even more regions will have
serious problems of inadequate surface water supplies by the year 2000. Although nationally the United States has an ample supply of water when both surface and underground sources are considered, the uneven distribution of precipitation and location of demand centers can cause regional or local water shortages. Groundwater overdrafts are occurring in 38 of 106 subregions identified by the U.S. Water Resources Council. Eight of the subregions were described as having an extensive overdraft problem, and in some of those areas groundwater levels are declining from seven to ten feet per year. The most dramatic examples of groundwater overdraft occur in the High Plains from Nebraska to Texas, south-central Arizona and parts of California.

Optimal Management

In the past few decades, numerical simulation models have been developed and used as tools for evaluating water resources. Often, both groundwater and conjunctive use management models involve an optimization procedure to best use water resources. The system is mathematically modeled; however, accurate modeling requires proper parameter identification. The model is an abstract representation of the system's physical performance. The equations describing the system's state must be accurate. Dreizin and Haimes (1977) caution that using only one equation to describe a system may ignore areas which are really described by different
equations. They suggest dividing a system into areas governed by different equations and connecting the area models by boundary conditions.

Often the mathematical model is a simplification of the real system. Despite any approximations or simplifying assumptions, the model must be a valid representation of the problem in order that the relative effects of different decisions can be accurately predicted. Model prediction must have a high correlation with reality, otherwise the decisions derived will be meaningless (Hillier and Lieberman, 1980).

The procedure used to optimize groundwater and conjunctive use management is called systems analysis, which is a scientific approach to the search for an optimal management decision. The systems approach does not automatically take all water resource planning considerations into account, but it does offer the framework of a well organized strategy to provide management decisions based on a model of the water resources system (Beaver and Frankel, 1969).

**Objective, Scope and Limitations**

The purpose of this study is to extend an existing methodology combining a simulation model with an optimization model, to incorporate a surface water source, and to determine the applicability of this methodology to large
scale problems. To test this methodology, a management model is developed for the optimal conjunctive use of a groundwater and surface water system. The model provides optimal decisions at several points in time regarding pumping and recharge rates, well locations and quantities of surface water and groundwater withdrawn to satisfy demand. The model has a simulation submodel embedded within the optimization model, and makes decisions with regard to physically based objective functions.

A management model is first developed for a groundwater system alone, then the model is extended to incorporate a surface water source. The physical simulation of groundwater flow and continuity of surface water flow are incorporated into the model, and the optimal feasible solution which satisfies given management objectives is found using linear programming (LP). The optimal feasible solution is determined by using the governing equations of groundwater flow as constraints in the model. The resulting problem, consisting of a linear objective function subject to linear constraints, is solved by the primal simplex method.

The equation of state describing groundwater flow is written in finite difference form, and management constraints are also included. Both physical and management constraints are manipulated to utilize different objective functions, accommodate variations in boundary conditions,
time increments, dimensions of the finite difference grid and to change restrictions on pumpage and demand.

Any model is only as good as its inherent assumptions. A mathematical representation of any real system cannot completely describe the entire complex system. The solution derived from the mathematical model is, therefore, an approximation within the limitations of the simplifying assumptions. The assumptions made in developing this model are:

1) The aquifer is continually confined. Unconfined conditions introduce nonlinearity into the equation of state used for groundwater flow. The equation of state for a confined aquifer is linear, and is used as part of the constraint set. The management constraints and objective functions are also linear, and because of this linearity, the problem is solved by linear programming.

2) The boundary conditions are assumed constant throughout a given planning period.

3) There is no hydraulic connection between the surface water source and the confined aquifer. However, an artificial connection can be established between the aquifer and stream by artificial recharge and streamflow augmentation.
4) Groundwater flow in the confined aquifer can be adequately described in two dimensions, and transmissivity and storage coefficient values are considered as averaged vertically.

5) Water quality aspects are not considered. Groundwater and surface waters are assumed to be chemically compatible.
CHAPTER II.
LITERATURE REVIEW

General Statement

Management models are used to model responses of a particular system to different management alternatives. Use of numerical simulation models has helped hydrologists to understand complicated aquifer systems and to test hypotheses regarding the behavior of groundwater flow systems. Existing models can be categorized into groundwater models and models for the conjunctive use of groundwater and surface water. Both quantity and quality aspects of water management are considered in many of the existing models and these models are used to evaluate the impacts of water withdrawals, groundwater-surface water interactions, and contaminant movement.

Models may also be divided into prediction models, which simulate system behavior in response to stress, and resource management models, which integrate hydrologic prediction with explicit management decision procedures. An excellent summary of available models is given by Bachmat, et al. (1980). The value of both groundwater and conjunctive use management models is that they allow efficient assessment of alternative system designs in accordance with the physical, economic, social and political constraints on system development.
Prediction models solve equations for quantitative aspects of groundwater flow such as direction and rate of flow, hydraulic head, drawdown and stream-aquifer interactions. The response of the system to a particular impulse is simulated. Simulation analysis provides answers pertaining to specified system variables, and will continue to be a tool for evaluation of impacts upon aquifer systems. Simulation models are sometimes used to evaluate the most desirable management solution among a variety of alternatives. This is achieved by examining system response using numerous combinations of system variables; however, the best solution may not necessarily be discovered. Use of simulation analysis exclusively does not allow incorporation of management constraints and objectives.

Optimization models, however, provide decisions based on all of the possible feasible solutions to the physical problem, as well as on management constraints. Decisions are consistent with stated management objectives and constraints. Management models may employ both simulation and optimization techniques in deriving their solutions. In contrast to purely physically based prediction models, management models can explicitly consider the economic, technological, environmental and political aspects of the problem. A combined simulation-optimization model considers the system's physical behavior and determines the best decisions based on objectives and constraints placed on the
Simulation Models for Groundwater Management

Both digital and analog computer methods have been used for groundwater flow simulation. Simulation involves reproducing historical water level fluctuations resulting from stresses imposed on the system: artificial recharge, natural recharge, pumping and natural discharge. The mathematical equations governing groundwater flow are combined with some initial conditions, boundary conditions and aquifer parameters to provide the basis for simulation. The model imitates aquifer response to given stresses through time, deriving a solution showing the aquifer’s behavior resulting from a particular stress.

Computer simulations are performed using finite difference or finite element techniques. Rushton and Tomlinson (1971) critically examined and compared simulation models using discrete time steps (implicit, explicit and alternating direction implicit (ADI)) with the continuous time of an analog model. They studied the effects of time increments on the model solutions, concluding that larger time increments can cause inaccuracies in the solutions and, when using the alternating direction implicit method, can cause
problems in stability and convergence. They recommended using a small time increment during periods when heads may be changing rapidly, such as when pumps are turned on or off.

Pinder and Bredehoeft (1968) used the implicit finite difference method to simulate transient flow in an unconfined aquifer in Nova Scotia. The model was designed for a heterogeneous anisotropic aquifer with irregular boundaries and vertical leakage to the aquifer. They constructed an electric analog model to verify the simulation model results.

Later, Pinder (1970) used the alternating direction implicit method to solve transient, isotropic, heterogeneous, confined and unconfined groundwater flow problems. The source term in the flow equations was modified to account for pumpage, constant recharge, leakage and evapotranspiration. An updated version of Pinder's program by Trescott, et al. (1976) is widely used. It allows a choice between three equation solving schemes: ADI, line successive overrelaxation, and the strongly implicit procedure. This last solution method has been shown to require less computer time and have less problems with numerical instability than the other two equation solving schemes.

Prickett and Lonnquist (1971) developed another widely used groundwater simulation model. Their model simulates
two dimensional, transient, confined flow and also incorporates variable time and grid spacings, water table and leaky artesian conditions, and barrier and recharge boundary conditions. They used finite difference equations with the iterative alternating direction implicit (IADI) solution procedure.

Simulation models are often used to explore groundwater management alternatives. The model is repeatedly executed under various scenarios. Combinations of stresses may be evaluated to obtain different solutions, but combinations are numerous and evaluation of all possibilities is impractical and could be expensive. Potentially valuable management alternatives may never be considered in the combination process. When the best solution is needed, for example, the location and quantity of artificial recharge which maximizes hydraulic head, the simulation model must be incorporated into an optimization model, because the simulation model alone does not yield an immediate optimal answer.

Optimization Models for Groundwater Management

Groundwater management models integrate hydrologic prediction with explicit optimal management decision seeking techniques. Management models allow the assessment of alternative system designs in accordance with constraints on system development and determine the best decision among all of the alternatives.
An excellent summary of groundwater management models is given by Bachmat, et al. (1980). They discuss the additional elements of management models which make them conceptually more complex than simulation models. The four elements are: 1) a submodel for finding the most appropriate decisions for well locations and pumping rates; 2) a submodel for predicting the outcome of a decision such as water levels or contaminant concentrations; 3) a constraint set which governs admissible decisions and outcomes, that is, maximum pumpage rates, appropriated rights, limits on drawdowns and demand, limits on hydraulic gradients; 4) an objective function which evaluates the decisions, heads, pumpages, or concentrations.

The first element is included in the optimization procedure; the second is the simulation model embedded in the management model. The constraint set comprising the third element includes the upper and lower bounds placed on heads and pumpages, and the minimum demand specified. The fourth element, the objective function, is evaluated by the optimization procedure.

Bachmat, et al. (1980) stress the importance of correct parameter estimation and suggests that the reliability of models improves as parameter estimation improves. They also discuss the type of optimization submodel which suits different problems. When policy decisions are based upon
stochastic inputs, they recommend dynamic programming. Dynamic programming (DP) produces a decision at any time step which predicts the resulting state of the system as an input to the next time step. DP has no simulation capability; it provides solutions for lumped parameter models. DP can be used for nonlinear systems, however.

Lumped parameter models do not consider the spatial variation of system characteristics; they give a solution in terms of a state variable representing the average system condition. For example, in many conjunctive use models, the variable representing the state of the groundwater aquifer is an average drawdown value, which does not reflect areal fluctuations in drawdown over the aquifer.

When management models are deterministic, linear or nonlinear programming are recommended. Linear programming (LP) can incorporate a simulation submodel in the constraint formulation, thus ensuring an overall optimum. LP has the capability of simulation, thus LP models are classified as distributed models. Distributed parameter models apportion model results over all system variables, instead of using one state variable to represent the entire system.

Distributed and lumped parameter models can both consider temporal variation. Generally, LP models are distributed models, although a few LP lumped models do exist. DP models for groundwater management are always of the lumped
parameter type. In DP solutions there are up to three state variables for each time step, and these state variables are an overall measure of system variables, while LP solutions give values for each system variable for each time period.

Lumped parameter groundwater models have no simulation capabilities, and therefore have been used less for strictly groundwater management modeling than distributed parameter models in recent years. Lumped parameter models are used extensively for conjunctive use purposes, however, and will be discussed in the next section.

Another classification of groundwater management models has been presented by Gorelick (1983). He divides combined simulation-optimization models into two categories; hydraulic management models and policy evaluation and allocation models. The difference is that the first category is primarily concerned with groundwater hydraulics and managing stresses such as pumping and recharge, while the second category is composed of models used to evaluate complicated economic interactions or groundwater-surface water allocation problems. Both categories utilize an optimization technique. The latter is generally used for conjunctive use models.

Groundwater hydraulic management models are based on groundwater flow equations, have both spatial and temporal
components, and are distributed parameter models. Management decisions and aquifer simulation are obtained simultaneously as these models treat hydraulic heads and aquifer stresses directly as decision variables. The two types of hydraulic management models are described by Schwartz (1976), as influence and transformation models.

Influence models, called "response matrix models" by Gorelick (1982), incorporate the effects of time, space and boundaries into "influence coefficients" (also called "algebraic technological functions" (ATF) and "response functions") which are evaluated only for locations and times of interest. The equation of state is formulated in a linear "cause and effect" form; the influence function linearly relates system input to system output. The Theis equation is an example of an influence function in which drawdown is related to pumpage by a coefficient which is a function of aquifer parameters. System input, such as pumpage, is multiplied by a system unit impulse response function, and the result is the system output response, such as drawdown.

The influence coefficient can be calculated either analytically or externally by a numerical simulation model. The Theis and Cooper-Jacob equations are examples of analytical response functions. Numerically calculated response functions are derived from a simulation model in the following manner: a system response to one unit of input per one
unit of time is simulated and then extended for any multiplication of input and time units (Haimes, 1977). Each unit response describes the influence of a unit system stress upon hydraulic heads at locations of interest. A matrix of the influence coefficients is included in the management model.

The transformation model represents aquifer parameters by water level transformation equations for discrete cells. These are continuity equations which must be written for each cell for all time periods the model encompasses.

Schwartz (1976) concludes that the transformation model is preferable for a small number of cells over many time periods, because the matrix of simultaneous equations becomes too large when many cells are considered. He suggests that the influence model be used for a large number of cells because it is less restricted by the number of cells. As the influence coefficients are written only for pumpage and observation points of interest, the locations must be predetermined. Over time, the water level at a non-chosen point in the aquifer may go below some critical level, and model results will not show this. Therefore, Schwartz (1976) states that influence models are not as useful over many time periods as transformation models are.

Maddock (1972) used linear systems theory to develop a groundwater management model for a multiwell system
utilizing influence functions. His procedure is as follows: 

a) the domain is discretized; 
b) values of aquifer parameters are assigned to each cell; 
c) the boundary conditions are specified and put into the model; 
d) the well locations are designated; 
e) a pumpage of one unit is assigned to one well and zero units to the other wells, and drawdown at each node location is evaluated by simulation. This step is repeated for each well location considered; 
f) the drawdowns calculated from the unit pumpage impulses are the influence coefficients utilized in developing a groundwater management model.

Heidari (1982) found that the optimal groundwater management plan depends upon the extent of aquifer protection which could be up for future development. He has also shown that the allowable drawdown depends upon the current and potential configurations of the aquifer. The allowable drawdown is highly over-approximated, and some wells may have to cease operations in order to prevent damage to the aquifer from lowering of the water table beyond an allowable percent. Even with optimal pumpage patterns over the planning horizon of ten years, he determined that some well fields will not be able to meet demand. 

An influence model was formulated using Maddock's approach for the Pawnee Valley area in south-central Kansas to develop a groundwater management plan for agricultural use over a ten year period (Heidari, 1982). A two dimensional finite difference model was first used to identify aquifer parameters and for simulation of future aquifer conditions. The unconfined system was approximated by treatment as a confined system. The objective function which Heidari used in the model was maximization of water supply. The allowable drawdown was limited to a percentage of the total aquifer thickness, and wells were restricted to pumping, at most, their appropriated water rights. Total pumping was constrained to meet demand during each time period. He used linear programming for the optimization procedure, and ran the model twice for five time periods of one and two year
Heidari (1982) found that the optimal policy varied depending upon the percent of aquifer saturated thickness which could be used for water supply, and the well field configuration used. Using the model results, he concluded that groundwater resources in the valley are highly over appropriated, and that some well fields will have to cease operation in order to prevent damage to the aquifer from lowering of the water table beyond an allowable percent. Even with optimal pumpage patterns over the planning horizon of ten years, he determined that some well fields will not be able to pump their appropriated amount. Heidari's model is one of the few large scale groundwater optimization examples reported in the literature.

Gorelick (1982) used influence coefficients to optimally manage groundwater pollutant sources. The influence coefficients were generated as a unit source-concentration response matrix by a linear solute transport model. The response matrix was used in the constraint equations in a linear programming model to optimize waste disposal schedules while maintaining water quality at supply wells. They used a hypothetical aquifer to demonstrate maximizing waste disposal while keeping contaminant levels within specified limits.
A second management model developed by Gorelick and Remson (1982) deals with the evaluation of an aquifer in order to locate potential disposal sites. The management model is arranged so that the optimal values of the dual variables provide information about the usefulness of each disposal site under consideration. The interest in this model is in the "shadow prices" rather than the waste concentration distribution. "Shadow prices" show the influence of a unit change in the corresponding right-hand-side values of the constraints upon the value of the objective function. These "shadow prices" are the concentrations that would result at a specified supply well from unit sources of waste. The model located all potential waste disposal sites which would not violate contaminant concentration limits at supply wells.

The other type of distributed parameter model, the transformation model, has also been used for optimization of water levels and water supply. A variation of the transformation equation approach, called the embedding method, was first presented by Aguado and Remson (1974). The embedding method uses finite difference or finite element approximations of the governing partial differential equation for groundwater flow as constraints in an LP model. They used simple one and two dimensional examples to show that groundwater flow equations could be used as constraints in an optimization model. Finite difference approximations were
used in both transient and steady state examples, and the objective function used was maximization of hydraulic head. For the confined example, the governing equation is linear and was used directly in the LP formulation. For the unconfined example, the equation used was linear with respect to hydraulic head squared.

Alley, et al. (1976) used a transient finite difference equation for a confined aquifer to maximize hydraulic heads while maintaining a specified total flow from system wells. The transient approach they used was to optimize each time step separately, using final head conditions from the current time step as initial conditions for the next time step. The transient approach of Aguado and Remson (1974) was to treat all time steps in the LP model at once, resulting in a very large matrix.

Both Aguado and Remson (1974) and Alley et al. (1976) concluded that the embedding method is useful for studying aquifer behavior while achieving some management goal by optimizing a linear function of the groundwater variables. The advantage of this method is that both hydraulic head and sink/source terms are treated as decision variables simultaneously; thereby making the mathematical model self contained. They applied their models to small, hypothetical problems.
Willis and Newman (1976) presented a dynamic management model for the optimal development of the water supply resources of a regionally confined groundwater basin. They used finite elements to formulate the equation of state for groundwater flow. The solutions provided management decisions regarding well development sites and optimal pumping rates needed to meet a water demand in each planning period. The possible well locations were predetermined based on basin hydrogeology. The optimal pumping schedule was sought from these possible development sites. They state that the technique of embedding finite element simulation equations within an optimization model allows determination of policy alternatives as well as system states simultaneously. This is also true for finite difference simulation equations, and is a general property of transformation models.

In this research, the embedding method has been used, although this method results in a very large matrix. Gorelick (1983) states that the response matrix approach eliminates the very large matrices which occur when the embedding method is used. Both approaches have drawbacks. The embedding method requires one equation written for each node for each time period, resulting in a very large matrix, but the solution gives hydraulic heads at each node, which is valuable if the distribution of head over the aquifer is needed. The response matrix approach uses equations written only for nodes of interest, so the matrix is much smaller,
but the solution gives hydraulic heads only at the nodes for which equations were written. To obtain heads elsewhere in the aquifer requires use of a simulation model. The response matrix approach also requires the initial use of a simulation model to generate the matrix coefficients, whereas the embedding method does not require the use of another model.

The response approach requires predetermined well locations; the embedding method does not. In areas where new well locations are of interest, the embedding method with finite difference may be more valuable. In areas where only existing wells are to be modeled, the response matrix approach or the embedding method with finite elements may be more useful. All simulation-optimization approaches require considerable computer memory and execution time because of the memory needs of the LP solution procedure. The sparse, banded matrix of the embedding method develops computational instability problems for very large matrices. The density of the matrix for the response approach can also cause computational problems, because LP was developed for sparse matrices. Some researchers have suggested eliminating very small response coefficients to reduce the response matrix density. The embedding method, when used with LP, requires linearity of the equation discretized. As the response approach uses the principle of linear superposition, it too requires linearity.
Conjunctive Use Management Models

Simulation Models for Conjunctive Use Management

The model for conjunctive use is an extension of a groundwater management model to incorporate additional sources of water. Use of more than one water source to meet demand, importation of water and artificial recharge are common features of conjunctive use models. Simulation models for conjunctive use simulate groundwater and surface water as a complete system, and some models include combinations of surface and underground storage and transmission.

Young and Bredehoeft (1972) studied the management problems of conjunctive groundwater and surface water systems. They developed a simulation model which represented the physical response of the stream-aquifer system to changes in river flows, diversions and pumping, and included an economic indication of the water user's response to variations in supply and cost. Their model aids in determining the effects of unrestricted groundwater development on associated streamflows. Key decision variables in the Young and Bredehoeft model (1972) were the capacity and distribution of wells in the aquifer, while other variables in the model included streamflow diversions related to water rights, timing and location of groundwater withdrawals and recharge. The model included aquifer recharge from irrigation return.
waters and loss of water from the stream to the aquifer due to pumping. They used a finite difference approximation of the equation describing groundwater flow.

Trescott, et al. (1976) developed a digital simulation model for an alluvial aquifer hydraulically connected with a stream. They assumed that water table fluctuations were small compared with the aquifer thickness; therefore, they used the governing equation for a confined aquifer. They postulated wells in several locations to determine which locations resulted in the least drawdown when the wells were operating; however, their well location scenarios do not guarantee the optimal well placement for minimization of drawdown. Their model is an example of how a simulation model is used many times with postulated well locations in an attempt to optimize an objective.

Tyson and Weber (1964) developed a model to find the most economical plan for operation of groundwater basins in conjunction with surface storage and distribution facilities in Southern California. They studied the feasibility of importing water and artificially recharging depleted aquifers. As the groundwater basins were composed of eleven major aquifers, an "equivalent aquifer" was designed to represent the area's composite aquifers. The aquifer was divided into small polygonal zones and, using an analog circuit, the system's physical parameters were adjusted until
historical water level fluctuations were simulated satisfactorily.

The electric analog model was used to record the response from pumpage at each node; these responses were the influence coefficients used in their digital computer model. The study results were a set of injection and extraction flow rates at predetermined nodes. The authors chose scenarios involving differing amounts of imported, artificially recharged and pumped water. Although they were using influence coefficients, they were simulating predetermined decisions; there was no optimal seeking algorithm combined with their model.

Chun, et al. (1964) like Tyson and Weber (1964) used influence functions to superimpose solutions for varying conjunctive operation plans in a basin. The alternative plans combined a pattern of extractions with an extraction schedule, a spreading schedule for artificial recharge and methods for prevention of seawater intrusion. The management scenarios used were quite limited and the model did not utilize an optimization technique. They did, however, attempt to obtain the most economical combination of pumping and storage facilities.
Optimization Models for Conjunctive Use Management

Optimal conjunctive use models can often be categorized as policy evaluation and allocation models, where hydraulic management is not the only objective. Usually these models examine water allocation problems with economic objectives. Fowler (1964) presented guidelines for optimal basin management by conjunctive use. He states that both surface and subsurface storage capacities must be integrated to obtain the optimal amount of water conservation and the most economic utilization of storage resources. He suggests that surface pipeline distribution systems be integrated with groundwater basin transmission characteristics to provide the minimum cost distribution system. Fowler’s last suggestion is that one operating agency should have control of both surface and groundwater and all facilities connected with them.

Buras (1963) used stochastic dynamic programming with a lumped parameter model to optimize the operation of a dam in conjunction with an aquifer. His model attempted to solve three problems: 1) determination of design criteria for both the dam and groundwater recharge facilities; 2) determination of the areal extent the system could serve; and 3) development of an operating policy specifying drafts from the reservoir and pumpage from the aquifer. For each stage there were three state variables: 1) the amount of water
available in the aquifer; 2) the amount of water available in the surface reservoir; and 3) the amount of water in the recharge facility. Each stage also had three decision variables: 1) the amount of water released from the surface reservoir for irrigation; 2) the amount of water released for groundwater replenishment; and 3) the amount of water pumped from the aquifer. Buras used stages of one year, and found that after eight stages the system asymptotically approached a steady-state situation.

Another lumped parameter conjunctive use model utilizing dynamic programming was developed by Cochran (1968, 1973). His model was developed for the Las Vegas Valley aquifer in conjunction with Lake Mead. A model of the resource systems and institutional framework was coupled with projected future water requirements. Two management objectives were considered; providing water to consumers at least cost and providing water at maximum quality. Cochran found that the average water quality delivered to consumers could be enhanced over time by conjunctive use management. He also found that groundwater mining would economically benefit Las Vegas Valley; however, the amount of mining is dependent upon the storage in Lake Mead.

Parametric linear programming, a variation of linear programming in which several parameters are varied systematically, was applied to an optimal conjunctive use problem in
the San Gabriel Valley by Dracup (1966). He considered five water sources to be divided among three types of water users: agricultural, industrial and municipal use. His model allowed artificial recharge of the groundwater source. Planning decisions were optimized over a 30-year time span. System decisions relied on three separate hypotheses: 1) the mining of groundwater for the first five years with drafts thereafter restricted to the "safe yield"; 2) maximizing groundwater withdrawals; 3) holding groundwater levels constant for ten years, followed by 20 years of artificial recharge to restore water levels to their condition at the beginning of the study.

Study results included the optimal annual allocation of the five sources to each of the three users and the average cost of water to each user over the planning period. Some problems with Dracup's approach are: 1) while some of the processes are nonlinear, his approach requires linearity; 2) use of an annual average does not allow for seasonal and monthly variations which affect the sizing and operation of supply facilities; 3) as the model is of the lumped parameter type, the spatial distribution of pumpage is not accounted for.

Downing, et al. (1974) discuss the stages of optimal development for conjunctive use on a regional basis in the United Kingdom. Their primary objective was rational
exploitation of an initially undeveloped aquifer for purposes of maintaining adequate river flows. They followed four rules: 1) The river must be maintained at a prescribed flow, although water may be withdrawn to meet demand. Augmentation from the aquifer is allowed; 2) If the aquifer and stream are hydraulically connected, intake wells are placed near the stream to withdraw water from the stream, thereby deriving water purification through the aquifer; 3) Groundwater can be used directly as supply from wells, but additional groundwater may have to be pumped for streamflow augmentation; 4) When the river flow is in excess, the excess may be removed.

The final stage in their development plan is artificial recharge of the aquifer. They state that large scale over-development, without artificial recharge, should be for limited periods and is possible only if a regional distribution system exists which allows flexibility in the distribution of water from different sources. In their approach groundwater appears to have less importance than surface water. They seem unconcerned with maintaining hydraulic head levels, while they are concerned with maintaining minimum flow in the stream.

Chaudry, et al. (1974) developed a lumped parameter optimal conjunctive use model for the Indus Basin. They divided the basin into subsystems, optimized each of the
subsystems, recombined the subsystems, and satisfied the interconnective constraints. Water quality was ignored because it wasn’t a problem. Groundwater and surface water were both used for irrigation. As water was being lost to the sea during part of the year, the model incorporated the storage potential of the groundwater basin during wet periods. Their objective was to optimize surface storage facilities, channel capacities and pumping capacities while minimizing the design cost of each subsystem. The optimal operating procedure was determined for both dry and wet seasons. The optimization was carried out within each subsystem by selecting various design combinations, optimizing the combinations, and choosing the design which minimized cost. All functions were assumed linear or piecewise linear for the DP optimization procedure. They concluded that the aquifer could meet storage requirements, that they had found the least cost alternatives and that conjunctive use of groundwater and surface water would allow demand to be met for irrigation in that area.

Taylor (1970) used a variation of the influence function method in a LP model applied to a stream-aquifer system along the Arkansas river in Colorado. His simple example was designed to show how conjunctive use could meet irrigation water demand from two sources while minimizing stream depletion. The aquifer was divided into three cells parallel to the river. A set of response curves was developed
showing the effect of aquifer stresses upon streamflow. Constraints specified maximum pumping from each cell, and the water demand for each month.

Morel-Seytoux (1975) used a distributed parameter model to present a simple case of conjunctive use management in which an assumed homogeneous, isotropic, unconfined aquifer interacts with a stream. The equation of state for the unconfined aquifer was linearized, and one stream was assumed to traverse the aquifer. Influence functions were developed to relate the groundwater-stream interaction which occurred during pumping. LP was used to optimize three different pumping scheme problem formulations: maximization of beneficial use, minimization of pumpage, and minimization of surface storage.

Haimes (1977) describes the general approach for hierarchical management models for aquifer-stream systems in which the problem is designed at several sublevels. Yu and Haimes (1974) used the hierarchical approach to optimize a complicated regional water allocation problem. They state that the multilevel approach divides a regional water resources problem into sublevel problems, optimizes the sublevel problems, and then optimizes the overall problem. Yu and Haimes (1974) state that the successful optimization of the overall problem requires feedback between levels. Haimes (1977) describes the hierarchical approach in his
book. The output from each submodel is the pumping and artificial recharge plans, and the surface water requirement plan. The output is used as input to a higher level, and the model iterates, sending feedback to the first level. The second level is composed of two sets of functions: the drawdown in a given cell due to net pumping and the total amount of water induced from the stream into cells. The second level also considers operation of a surface reservoir with stochastic stream flow, reservoir storage limits and allocation of surface supplies subject to a penalty cost for streamflow depletion. Haimes used the following constraints in addition to the system equations required for an optimal solution: 1) minimum demand must be met; 2) drawdown must not exceed a maximum; 3) pumping capacities must follow physical limitations; 4) recharge wells must follow physical limitations; 5) surface water demand must not exceed supply; 6) infiltration rate must follow physical limitations.

In general, policy evaluation models are used for very complicated systems and are of the lumped parameter type, although the optimal decisions from a policy evaluation model can be put into a simulation model to obtain the spatial distribution of drawdown. Conjunctive use models considering economics calculate pumping costs based on average drawdowns over the aquifer. The assumption that an average drawdown can accurately reflect pumping lifts could underestimate pumping costs. However, distributed parameter models
are linear and cannot take nonlinear pumping costs into account. When any type of model is used to calculate pumping costs, caution must be used in interpreting the results because in-well drawdowns could be greater than predicted by the model.
CHAPTER III.
MODEL DEVELOPMENT

General Statement

The model developed in this study is a hydraulic management model for the optimal conjunctive use of a groundwater and surface water system. Constraints for the groundwater portion of the model are developed first. Conjunctive model constraints are then added. This study utilizes the embedding method in which finite difference approximations of the governing partial differential equation for confined groundwater flow are used as part of the constraint set. Equations for surface water continuity, limitations on artificial recharge and streamflow augmentation, and restrictions on pumping rates and drawdown are also incorporated. Both the groundwater model and the conjunctive use model developed in this study are distributed parameter models. Constraints are combined with a variety of objective functions, and the models are solved by linear programming.

Groundwater Management Model

Many simulation studies are undertaken with the ultimate aim of evaluating aquifer management alternatives. In such situations what is needed is not a simulation model alone, but a combined simulation and management model. The joint model should contain an accurate representation of the system under consideration and should enable optimal
The model developed for this study is a management model containing the essence of a simulation model. Such a model allows the use of groundwater variables directly as decision variables in the management model. The model considers a confined aquifer, and as the equation describing groundwater flow in a confined aquifer is linear, the resulting constraint set is linear. The partial differential equation for confined groundwater flow is approximated using implicit finite difference methods, resulting in a system of linear algebraic equations. These equations, along with management constraints and a linear objective function, constitute a linear programming formulation. Optimization of various objectives enables examination of different management alternatives. Optimal solutions are functions of the boundary conditions, initial conditions, hydraulic properties of the medium, discretization scheme, and objective function. The model provides the optimal decisions at several points in time regarding pumping rates, well locations, quantity of groundwater contributed to the total water supply, and spatial distribution of hydraulic heads.
Derivation of Finite Difference Equation

The partial differential equation describing two dimensional, transient groundwater flow in a heterogeneous, anisotropic, confined aquifer is (Bittinger et al., 1967; Remson et al., 1971):

\[
\frac{\partial}{\partial x} (T \frac{\partial H}{\partial x}) + \frac{\partial}{\partial y} (T \frac{\partial H}{\partial y}) = \frac{\partial H}{\partial t} + q \tag{3.1}
\]

where:

- \( T_x \) = aquifer transmissivity in x direction
- \( T_y \) = aquifer transmissivity in y direction
- \( H \) = hydraulic head
- \( t \) = time
- \( S \) = aquifer storage coefficient
- \( q \) = net groundwater withdrawal rate per unit area
- \( x, y \) = cartesian coordinates

A numerical solution for Eq. (3.1) can be obtained using the method of finite differences. In the finite differences method, derivatives are replaced by the ratio of the changes in variables over a discrete interval. If a grid is superimposed over an aquifer, Eq. (3.1) can be converted into a set of NP algebraic equations, involving NP unknowns, where NP is the number of nodal points (see Figure 1). If several time periods are used, an equation must be written for each node for each time period, yielding NP*NT equations, where NT is the number of time periods. The finite difference form for Eq. (3.1), referring to Figure 2,
Figure 1:

Finite difference grid showing index numbering convention.
Figure 2:
Coefficients of unknowns and definition of transmissivity with respect to the five point star used in deriving the difference equations. The time axis is vertical.
can be expressed as (Prickett and Lonnquist, 1971; Freeze and Cherry, 1980):

\[
A_{ij} * H_{ij,t} - B_{ij} * H_{i-1,j,t} - C_{ij} * H_{i+1,j,t} - \\
D_{ij} * H_{i,j-1,t} - E_{ij} * H_{i,j+1,t} - F_{ij} * H_{i,j,t-1} - \\
Q_{ij,t} = 0
\]

Equation (3.2) may be solved numerically using either an explicit or implicit approximation (Wang and Anderson, 1982). In the implicit approximation, or forward difference, the head at (i,j) at the (t+1) time level is expressed in terms of the values of heads at the nodes surrounding (i,j) at the previous time step. In the implicit scheme, or backward difference, the head at the four nodes surrounding (i,j) at the (t+1) time level are obtained from the solution of a system of linear equations. All of the heads for one time period must be solved for. Equation (3.2), adopted in this management model is the implicit formulation.

The coefficient matrix is tridiagonal and has five unknowns. The domain \( R \) has \( N \) internal nodes, as shown in Figure 3. Assume that the head is specified at all boundaries, so that

\[
A_{ij} = [T_x(i,j) + T_x(i-1,j)] \frac{\Delta x}{\Delta t} + [T_y(i,j) + T_y(i,j-1)] \frac{\Delta y}{\Delta t} + S(i,j) \frac{\Delta x \Delta y}{\Delta t}
\]

\[
B_{ij} = T_x(i-1,j) \frac{\Delta x}{\Delta t}
\]

\[
C_{ij} = T_x(i+1,j) \frac{\Delta x}{\Delta t}
\]

\[
D_{ij} = T_y(i,j-1) \frac{\Delta y}{\Delta t}
\]

\[
E_{ij} = T_y(i,j+1) \frac{\Delta y}{\Delta t}
\]

\[
F_{ij} = S(i,j) \frac{\Delta x \Delta y}{\Delta t}
\]
\[ Q_{ijt} = \text{pumping rate at node } (i,j) \text{ contributing to water supply during time } t \]

\[ QR_{ijt} = \text{recharge rate at node } (i,j) \text{ during time } t \]

The constant or known terms in Eq. (3.2) are A, B, C, D, E and F and the unknown terms are H, QP and QR. The equation (3.2) may be solved numerically using either an explicit or implicit approximation (Wang and Anderson, 1982). In the explicit approximation, or forward difference, head at \((i,j)\) at the \((t+1)\) time level is evaluated in terms of the known values of heads at the nodes surrounding \((i,j)\) at the previous time step. In the implicit scheme, or backward difference, the heads at the four nodes surrounding \((i,j)\) at the advanced time level \((t+1)\), in addition to the head at point \((i,j)\) are unknowns. The heads at the \((t+1)\) time level are obtained from the solution of a system of linear equations. All of the equations for one time period must be solved to provide knowns for the following time period. Equation (3.2) adopted in this management model is the implicit formulation.

The coefficient matrix is tridiagonal and has five unknowns in each equation (Remson et al., 1971). Suppose the domain R has NP internal nodes, as shown in Figure 3. Assume that the head is specified on all boundaries, so that
Figure 3: Finite difference grid over map of aquifer system having nine internal nodes.
these are Dirichlet boundary conditions. Writing Eq. (3.2) for each internal node results in NP finite difference equations (see Figure 4). For each additional time period, a new block of NP equations is generated. In matrix notation the system of simultaneous equations can be written:

\[ \mathbf{aH} = \mathbf{b} \]  

(3.3)

where \( \mathbf{a} \) is a matrix, \( \mathbf{H} \) is the vector of hydraulic heads, and \( \mathbf{b} \) is the vector of the right-hand-side values, defined by the system withdrawals, initial conditions and boundary conditions.

When Eq. (3.1) is solved numerically by a simulation model, the solution is a set of hydraulic head values for designated nodes in the aquifer system resulting from a given set of pumpage rates. When pumping rates and well locations are unknowns, in addition to heads being unknowns, Eq. (3.1) becomes indeterminate.

Well locations and pumping rates are unknowns in a management model because the model seeks pumping locations and rates which optimize a specified objective. The model must be free to choose where and how much to pump in order to meet the objective. This is the major difference between a simulation model and a distributed parameter optimization model; in the first, system stresses are specified, while in
Figure 4: Matrix form of the finite difference equations for nine internal nodes.
the second, stresses are part of the model solution. The indeterminate representation of Eq. (3.1), used as part of the constraint set in the management model, can be written in matrix form as:

$$\begin{bmatrix} a & H \end{bmatrix} + [\mathbf{I}] \mathbf{Q}_P = \mathbf{b}$$

(3.4)

where \( \mathbf{Q}_P \) is a vector of the unknown pumpages and \( [\mathbf{I}] \) is an identity matrix.

Constraints

System constraints may be physical, economic, institutional, operational or legal. Economic constraints will not be discussed as the model developed does not have an economic objective function. The physical model requires an equation of state which mathematically describes the transition of the system state from one stage to another. In this study the equation of state used is Eq. (3.2), which is the finite difference form of Eq. (3.1). Equation (3.2) describes the relationship between hydraulic head and pumpage for each node at each time period. This constraint ensures that the physical law of groundwater flow is satisfied.

Legal and institutional constraints in the model can appear as bounds on groundwater head and pumpages. There
must be a lower limit on the amount of water withdrawn from the aquifer, i.e. a specified water demand must be satisfied for each time period.

\[ \sum_{k=1}^{NP} Q_{Pkt} \geq D_t \text{ for all } t=1 \ldots NT \]  

where \( D_t \) is the water demand during time period \( t \), and \( k \) is an index corresponding to an \((i,j)\) location in the finite difference grid.

Pumpage from a well cannot exceed the appropriated right for that well, or exceed the well's capacity, whichever is smaller. Pumpage can be greater than some positive lower limit if the modeler wishes to force a particular well to pump. These bounds are a combination of legal and operational constraints.

\[ Q_{P^L_{kt}} \leq Q_{P_{kt}} \leq Q_{P^U_{kt}} \text{ for all } t=1 \ldots NT; k=1 \ldots NP \]

where \( Q_{P^L_{kt}} \) and \( Q_{P^U_{kt}} \) are the upper and lower bounds, respectively, on pumpage for well \( k \) during time period \( t \). To force a well to pump a specified amount, the upper and lower bounds may be set equal to the same specified value. In the case of a natural recharge node, both bounds may be set equal to the same negative value, forcing a recharge well at
that node.

Hydraulic head at any node may not drop below a specified value because of limits on drawdown, maintenance of confined conditions, restrictions on groundwater mining, creation of unwanted head gradients, or creation of excessive pumping lifts. Heads also may not exceed a certain value at any node. This is important in cases of upconing from artificial recharge. This constraint can be expressed as:

\[ H_{kt}^L \leq H_{kt} \leq H_{kt}^U \quad \text{for all } t=1\ldots NT; k=1\ldots NP \quad (3.7) \]

where \( H_{kt}^L \) and \( H_{kt}^U \) are the lower and upper bounds on heads at node \( k \) during time period \( t \).

All decision variables (H's and QP's) have upper and lower bounds. Ordinarily, bounds on the decision variables would introduce additional constraints which would enlarge the problem size. However, the linear programming subroutine package used in this study treats bounds on decision variables in a special way which does not increase the constraint matrix size. The matrix formulation for the groundwater management model is shown in Figure 5.
Figure 5: Matrix formulation for the groundwater management model.
Objective Functions

A management model optimizes some objective subject to a set of physical, economic and legal constraints. There are two basic types of management plans (Heidari, 1982): one in which the objective function is based on an economic measure such as cost, gross return or net return, and the other in which the objective function is based on the physical capability of the system. The model developed in this study uses the latter plan, and seeks to maximize hydraulic head or water supply subject to constraints on the system’s physical functioning. The possible objective functions can be expressed as follows:

(i) Maximization of total water supply:

\[
\text{Max } X_o = \sum_{k=1}^{NP} \sum_{t=1}^{NT} Q_P^{kt} \]

where \( X_o \) is the objective function variable, \( Q_P^{kt} \) is the pumpage for well \( k \) during time period \( t \), and each well location \( k \) corresponds to an internal grid point \((i,j)\) in the finite difference model.

(ii) Maximization of hydraulic head over the groundwater basin:

\[
\text{Max } X_o = \sum_{k=1}^{NP} \sum_{t=1}^{NT} H_{kt} \]

where \( H_{kt} \) represents the hydraulic head at well \( k \) at the end of time period \( t \).
where \( H_{kt} \) is the hydraulic head at well \( k \) at the end of time period \( t \).

Pumpage and head values are determined by the optimization procedure; these variables are called the decision variables in the linear programming (LP) model. There are problems with adopting each type of objective function. Equation (3.9) is useful when the main objective is obtaining the desired water yield while minimizing drawdown in the aquifer. The problem with this type of objective function is that the model tends to locate pumping wells as close to recharge areas as possible and to use only a few wells, thereby creating excessive pumping lifts. This problem can be avoided by setting proper lower bounds on hydraulic head and proper upper bounds on pumpage, which forces the model to use more wells. When Eq. (3.8) is used as the objective function, the model uses up all the allowable water because conservation is not included in the objective function.

**Incorporation of Time**

**Lumped Transient Approach**

This model generates a tridiagonal matrix for all nodes at all time periods. Interrelated decisions are made at several points in time, and the resulting constraint matrix consists of several blocks, one for each time period (see Figure 6). The matrix surrounding the blocks is filled with
The multi-period production planning problem is an example of a broad class of problems called multistage or “staircase” problems because the system states at the end of the previous time period are the system states at the beginning of the current time period. The states are transferred to the succeeding states by control variables.

In this model formulation, the state variables represent pumps and present pumpage and water levels.

The relationships between state variables and control variables for a multi-stage problem can be expressed mathematically as follows:

\[
\begin{align*}
\mathbf{x}_t &= \mathbf{a}_t \mathbf{x}_{t-1} + \mathbf{y}_t, \\
\mathbf{x}_0 &= \mathbf{x}_{-1}
\end{align*}
\]

where \( \mathbf{a}_t \) and \( \mathbf{y}_t \) are coefficient matrices, \( \mathbf{x}_t \) is the vector of the right-hand-side values, \( \mathbf{x}_{t-1} \) is the state variable vector for the \( (t-1) \)th period, \( \mathbf{x}_0 \) is the state variable vector for the \( t \)th period, and \( \mathbf{a}_t \) is the control variable vector for the \( t \)th period.

**Figure 6:** Staircase structure of the matrix for multi-time period problems.

Mathematically, Eq. (1.10) expresses that the states at the end of the previous time period, \( \mathbf{x}_{t-1} \), are transferred to the states at the end of the current period, \( \mathbf{x}_t \), by the \( t \)th-period control variables, \( \mathbf{a}_t \).
The multi-period groundwater management problem is an example of a broad class of problems called multi-stage or "staircase" problems (Dantzig, 1963). This type of problem is called a "staircase" problem because the system states at the end of the previous time period are the system states at the beginning of the current time period. Old states are transferred to new states by control variables. In this model formulation, the state variables represent heads and the control variables represent pumpage and/or recharge.

The relationship between state variables and control variables for a multi-stage problem can be expressed mathematically as follows:

\[ \mathbf{a}_t \mathbf{x}_{t-1} + \mathbf{b}_t \mathbf{x}_t + \mathbf{c}_t \mathbf{x}_t = \mathbf{b}_t \]  

(3.10)

where \( \mathbf{a}_t \), \( \mathbf{b}_t \) and \( \mathbf{c}_t \) are coefficient matrices, \( \mathbf{b}_t \) is the vector of the right-hand-side values, \( \mathbf{x}_{t-1} \) is the state variable vector for the \((t-1)\)th period, \( \mathbf{x}_t \) is the state variable vector for the \(t\)-th period and \( \mathbf{x}_t \) is the control variable vector for the \(t\)-th period.

Mathematically, Eq. (3.10) expresses that the states at the end of time period, \( \mathbf{x}_{t-1} \), are transferred to the states at the end of the period, \( \mathbf{x}_t \), by the \(t\)-th period control variables, \( \mathbf{x}_t \). The incorporation of the system's time
dependency into the model can be seen by expressing the relationship between several time periods as follows:

\[ c X_t - 2 + g X_t - 1 + y X_t = b_t - 1 \]  \hspace{1cm} (3.11)

\[ c X_t - 1 + g X_t + y X_t = b_t \]  \hspace{1cm} (3.12)

\[ c X_t + g X_t + y X_t = b_t + 1 \]  \hspace{1cm} (3.13)

Assuming that the aquifer's physical properties are time invariant, the resulting coefficient matrices \( c \), \( g \) and \( y \) are identical for all time periods. Notice that the state variables are the coupling variables and that their corresponding columns are the coupling columns. The coupling variables link the problem at different time periods. This interconnection between variables causes the staircase appearance of the coefficient matrix.

**Step-Wise Optimization Approach**

Numerical instability problems arose when the matrix became large, hampering the completion of many time periods, so an attempt was made to use step-wise optimization. Each time period was executed separately, adopting optimal final heads from the current time step as the initial heads in the next time step. This is the transient method used by Alley et al. (1976), and can be considered as a succession of
management models, one for each time period. Gorelick (1983) states that the deficiency in the step-wise method is that management decisions made over the short term may contradict long term management goals.

Gorelick's (1983) reasoning was valid for the objective function of maximization of water supply. By the last time step, when all time steps were lumped into one matrix, heads were drawn down to their lower limits. The number of time steps and total length of time the problem considered affected the optimal solution. When step-wise optimization was used, final heads were drawn down to their lower bounds, and when input as initial conditions to the next time step, demand could not be satisfied and the problem became infeasible. The short term goal of maximizing pumpage caused the long term goal of meeting water supply to become infeasible.

When the objective function was maximization of heads the step-wise approach was valid. Optimal solutions were obtained for the hypothetical example described in Chapter 4 and compared for the lumped transient formulation and the step-wise optimization procedure. The solution from the first five time steps using step-wise optimization was identical to the solution of the first five time steps from the lumped transient approach. Because the solutions from the two methods matched, the step-wise optimization
procedure was extended to twenty time periods, a problem size which could not execute as a lumped transient problem. Only when long term goals were considered was the step-wise transient procedure applicable. When both short term and long term goals were contained in the same management model, step-wise optimization favored short term goals, and caused long term goals to become infeasible.

**Conjunctive Use Management Model**

The model for conjunctive use is an extension of the groundwater management model to incorporate a surface water component, allow artificial recharge of the aquifer and streamflow augmentation (see Figure 7). This model, like the groundwater management model, is a combination of simulation and optimization submodels. The groundwater flow equation and equation for streamflow continuity are utilized in the simulation submodel, with management constraints added. Groundwater and surface water variables are used directly as decision variables in the management model.

An aquifer's response to stress is a function of the boundary and initial conditions, transmissivity, storage, and positioning of development within the system. Therefore, the conjunctive model developed in this study is a distributed parameter model. The advantages of using a distributed model can be illustrated by showing some of the
Figure 7: Conjunctive model diagram.

\begin{align*}
\text{Stream Inflow} & \quad \text{Water Supply from Stream (X)} \\
\text{Artificial Recharge (QR)} & \quad \text{Augmentation (QA)} \\
\text{Discretized Aquifer} & \quad \text{Pumpage (QP)} \\
\end{align*}
disadvantages of using a lumped model.

Bredehoeft, et al. (1982), discuss the common pitfalls in applying a lumped approach to a groundwater system. Typically, the lumped parameter model uses an input-output water budget analysis which treats the aquifer essentially as if it behaves as a surface water system. This approach ignores the fact that the time delays in a groundwater system differ from those in a surface water system and the response of most groundwater systems is not equally distributed. A distributed model enables the evaluation of system states temporally and spatially, while a lumped model evaluates states only temporally. For these reasons, the model developed in this study is a distributed parameter conjunctive use model, and this study attempts to determine the usefulness of such an approach.

Constraints

The constraint set from the finite difference formulation of the governing equation for groundwater flow, Eq. (3.4), is used as part of the constraint set in the conjunctive use model. Additional terms are added to account for artificial recharge and streamflow augmentation. For the conjunctive model the groundwater flow equation is:

\[aH + IQP - IQR + IQA = b \quad (3.14)\]
where \( QR \) is a vector of artificial recharge, \( QA \) is a vector of streamflow augmentation and other terms are as previously defined.

Other constraints are management and continuity constraints. Demand must be satisfied for each time period, but demand can now be met from two sources:

\[
\sum_{k=1}^{NP} Q_{Pt} + \sum_{l=1}^{LL} X_{lt} \geq D_t \quad \text{for all } t = 1, \ldots, NT \quad (3.15)
\]

in which \( X_{lt} \) is the water supply from stream reach 1 during time period \( t \) and \( LL \) is the total number of channel reaches considered.

Recharge to the aquifer must equal stream flow used for recharge:

\[
\sum_{k=1}^{NP} Q_{Rkt} - \sum_{l=1}^{LL} Y_{lt} = 0 \quad \text{for all } t = 1, \ldots, NT \quad (3.16)
\]

where \( Y_{lt} \) is the streamflow from reach 1 for recharging the aquifer in time period \( t \).

Groundwater pumped for streamflow augmentation must equal the amount augmented to the stream.
where $Z_{lt}$ is the groundwater augmented to stream reach $l$ during time period $t$.

Surface water continuity must also be satisfied. The amount of streamflow entering a specific reach, minus the amounts withdrawn for water supply and for recharge to the aquifer, plus the amount augmented from groundwater equals the streamflow entering the next reach.

$$Q_{St} - X_{lt} - Y_{lt} + Z_{lt} = Q_{Sl+1,t} \quad \text{for all } t = 1 \ldots NT; l = 1 \ldots LL$$

where $Q_{St}$ and $Q_{Sl+1,t}$ are the volume rate of streamflow entering reaches $l$ and $l+1$ at time $t$, respectively. Evaporation losses are neglected.

Upper and lower bounds are placed on streamflow in each reach, on recharge and on streamflow augmentation. Bounds are necessary to keep all of the streamflow from being used up, to comply with surface water rights, and to keep augmentation to within the stream's capacity.

$$Q^{L}_{St} \leq Q_{St} \leq Q^{U}_{St} \quad \text{for all } t = 1 \ldots NT$$
\[ Y^L_{lt} \leq Y_{lt} \leq Y^U_{lt} \quad \text{for all } t=1, \ldots, NT \quad (3.20) \]

\[ Z^L_{lt} \leq Z_{lt} \leq Z^U_{lt} \quad \text{for all } t=1, \ldots, NT \quad (3.21) \]

where the superscripts L and U denote the lower and upper bounds on each variable. Bounds are also placed on heads and pumpage as described in the groundwater model formulation section.

The number of unknowns in the conjunctive model includes the state variables of heads and streamflow, and the control variables of pumpage, recharge, augmentation and streamflow for water supply. The model solution chooses where and how much to pump for supply and which stream reaches water is withdrawn from. The conjunctive model has many more system stresses and many more decision variables than the groundwater model. An example conjunctive model matrix formulation for two time periods is shown in Figure 8.

Objective Functions

The objective functions for conjunctive use management, like the objective functions for groundwater management, are based on the physical capabilities of the system. The objective functions used in the conjunctive model formulation can be summarized as follows:
Figure 8: Matrix formulation for the conjunctive model.
(i) Maximization of hydraulic head over the groundwater basin:

\[
\text{Max } X_o = \sum_{t=1}^{NT} \sum_{k=1}^{NP} H_{kt} \tag{3.22}
\]

(ii) Maximization of total water supply, where water can be withdrawn from both the aquifer and the stream:

\[
\text{Max } X_o = \sum_{t=1}^{NT} \sum_{k=1}^{NP} \left( \frac{1}{2} Q_{Pkt} + \frac{1}{2} LL_{kt} \right) \tag{3.23}
\]

(iii) Minimization of augmentation and recharge to minimize water transfer between the two sources:

\[
\text{Min } X_o = \sum_{t=1}^{NT} \sum_{k=1}^{NP} \left( \frac{1}{2} QR_{kt} + \frac{1}{2} QA_{kt} \right) \tag{3.24}
\]

(iv) Minimization of total water supply:

\[
\text{Min } X_o = \sum_{t=1}^{NT} \sum_{k=1}^{NP} \left( \frac{1}{2} Q_{Pkt} + \frac{1}{2} LL_{kt} \right) \tag{3.25}
\]

(v) Maximization of supply while minimizing water transfer:

\[
\text{Max } X_o = \sum_{t=1}^{NT} \sum_{k=1}^{NP} \left( \frac{1}{2} Q_{Pkt} + \frac{1}{2} LL_{kt} - \frac{3}{2} Y_{lt} \right) \tag{3.26}
\]
The number of decision variables is greatly increased in the conjunctive model, compared to the groundwater model, but, the number of constraints is not greatly increased. Problem size is dependent upon the number of nodes in the finite difference grid, the number of time steps and the number of stream reaches. The number of decision variables is equal to \(NP \times NT \times 4 + LL \times NT \times 4\). The number of constraints is equal to \(NP \times NT + NT \times 3 + LL \times NT\).

When the second type of objective function is used, there is a tendency to use all of the water available for supply because the model does not incorporate conservation. This may be beneficial, in the case where as much water as possible is needed for irrigation over a short period of time. If there are not storage facilities available or no immediate need for the total supply, the model withdraws too much water. Supply can be maximized at a specific "demand center" of the aquifer by setting the upper bounds on pumping to zero at nodes outside of the demand center. When the first objective function is used, there is a tendency to artificially recharge to the upper bounds, so again the bounds must be carefully set.

**Solution Procedure**

An optimization model possesses an algorithm which searches feasible alternative solutions and selects the
"best" solution based upon the objective. The optimal solution can readily be found in multi-variable problems, and the solution is an explicit function of the decision variables. A limitation of optimization models is that they require simplification of the real system in order to preserve the tractability of the problem. Consequently, the optimal solution obtained is restricted to being the optimum for a simplified problem, and is optimal for the real problem only if the simplifying assumptions are not significant.

Generally, optimization models are first used as "screening" models to eliminate inferior decision sets unworthy of further analysis. The promising alternative policies, initially suggested by an optimization model, can then be examined in further detail by a simulation model.

Linear Programming (LP)

Linear programming is concerned with the efficient use or allocation of limited resources to meet desired objectives. LP problems are characterized by the large number of solutions satisfying basic conditions. Mathematically, LP problems deal with nonnegative solutions to underdetermined systems of linear equations. "Underdetermined" pertains to systems in which there are more unknowns than equations. Generally, an underdetermined system of equations has no solution or an infinite number of solutions. By combining linear constraints with a linear objective function, the
An optimization model contains two types of variables, decision variables and parameters. Decision variables are variables of unknown value initially, whose values are determined by the model. Parameters are known system values and are put into the model to tailor it to a particular system. Aquifer properties such as transmissivity and storage coefficient are parameters, while pumpage, heads and stream withdrawals are decision variables. Decision variables may be divided into control and state variables, where heads are state variables and pumpages are control variables. State variables describe the condition of the system, which is a function of the value of the control variables.

Decision variables are limited to their feasible values by constraints, which may be physical, economic, institutional, legal or operational. Constraints mathematically describe the limitations of the system. The objective function in an optimization model is a measure of the system effectiveness, and is an explicit mathematical function of the decision variables. The generalized form of an LP model with bounded variables, in which the value of the decision variables is restricted, can be expressed as (Hillier and Lieberman, 1972):

underdetermined system with many solutions can be transformed into a system with one optimal solution for a particular objective function.
Max $X_o = \sum_{j=1}^{J} C_j x_j$  \hspace{1cm} (3.27)

subject to:

$$\sum_{j=1}^{J} a_{ij} x_j \ (\leq, =, \geq) b_i \quad \text{for all } i = 1, \ldots, I$$ \hspace{1cm} (3.28)

$$x_j^L \leq x_j \leq x_j^U \quad \text{for all } j = 1, \ldots, J$$ \hspace{1cm} (3.29)

or, in matrix form:

$$\begin{align*}
\text{Max } X_o &= \mathbf{C} \mathbf{X} \\
\text{subject to:} &\quad \mathbf{aX} \ (\leq, =, \geq) \mathbf{b} \\
&\quad \mathbf{x}^L \leq \mathbf{X} \leq \mathbf{x}^U
\end{align*}$$ \hspace{1cm} (3.30)

subject to:

$$\begin{align*}
\mathbf{aX} \ (\leq, =, \geq) \mathbf{b} \\
\mathbf{x}^L \leq \mathbf{X} \leq \mathbf{x}^U
\end{align*}$$ \hspace{1cm} (3.31)

$$\begin{align*}
\mathbf{x}^L \leq \mathbf{X} \leq \mathbf{x}^U
\end{align*}$$ \hspace{1cm} (3.32)

where $X_o$ is the objective function variable, $\mathbf{C}$ is the vector of objective function coefficients, $\mathbf{X}$ represents the vector of decision variables, $\mathbf{a}$ is the matrix of constraint coefficients (the "$a$" matrix), $\mathbf{b}$ is the vector of the right-hand-side values, and $\mathbf{x}^L$ and $\mathbf{x}^U$ indicate the lower and upper
bounds on the decision variables, respectively.

The LP model takes the indeterminate set of simultaneous equations and bounds on decision variables, and solves the system of equations by an iterative procedure which optimizes the objective function. The solution technique used in this study is the primal simplex method (Dantzig, 1963), which proceeds from one basic feasible solution to another in such a way that the value of the objective function is continually improved. The simplex method has the property of solving a linear program in a finite number of iterations.

Constraints are formulated either as inequalities or equations. In order to define the constraint boundaries, each constraint of \( \leq, \geq, \) or \( = \) type is converted to an equation by introducing an additional variable, called a surplus variable for a \( \geq \) constraint, a slack variable for a \( \leq \) constraint and an artificial variable for a \( = \) constraint. These are added to form a starting basis. The equality constraints then define a flat geometrical shape in n-dimensional space called a hyperplane (Hillier and Lieberman, 1972). All feasible solutions fall on the boundaries of the hyperplanes with the optimal solution belonging to the set of feasible solutions.
Groundwater Management Model LP Format

By comparing the LP format Eqs. (3.30)-(3.32) to the groundwater management model Eqs. (3.4)-(3.9), the LP features can be matched to the groundwater management model features. $C$, the objective function coefficients, are 0, 1, or -1 depending upon the objective function used. The vector of decision variables, $X$, corresponds to both control variables and state variables, $QP$ and $H$. The "a" matrix, $a$, is the set of constraint coefficients which, when multiplied by the decision variables, forms the constraint equations. The constraint matrix includes the finite difference coefficients and the demand coefficients. The right-hand-side values, $b$, are derived from knowns in Eq. (3.2) and the water demand Eq. (3.5). Lower bounds on decision variables satisfy LP nonnegativity constraints.

Conjunctive Use Management Model LP Format

The equations used as constraints and objective functions for the conjunctive model are all linear so that the problem can be solved by linear programming. As is the case for the groundwater management model, the optimal solution is a function of the problem formulation, including any simplifications. Referring to Eqs. (3.30)-(3.32), which are the generalized form of a LP model, the optimal conjunctive use model can be compared to the LP format. The objective function coefficients, $C$, are always 1, 0 or -1 depending
upon the objective function used. The vector $X$ represents
the decision variables $H$, $QP$, $QR$, $QA$, $QS$, $X$, $Y$ and $Z$. The
right-hand-side coefficients, $b$, include elements from the
finite difference formulation, water demand, and streamflow
continuity. The constraint matrix, $A$, includes the tridiag­
onal matrix of finite difference equations and the elements
from the demand and continuity equations. Again, lower
bounds on decision variables satisfy nonnegativity require­
ments. As in the groundwater management model, the conjunc­
tive LP model is solved by the primal simplex method
(Dantzig, 1963).

Problem Reduction

Decomposition of the staircase problem has been studied
in detail by Fourer (1982, 1980), and according to him, an
efficient algorithm for solution of staircase problems has
not yet been devised. Groundwater management problems util­
izing the embedding approach result in a banded matrix with
a staircase structure. None of the current literature using
the embedding approach mentions any attempt at decomposing
the staircase problem structure, or computational problems
resulting from the staircase structure.

This research has attempted to reduce problem size by
two approaches: reduction of decision variables and reduc­
tion of constraints. The first is accomplished using the QP
As the number of basic variables (slack, surplus and artificial) added by the linear programming solution procedure is equal to the number of constraints, the number of decision variables increases by slightly more than 50 percent. For problems of considerable size, the addition of these extra variables increases the size of the problem considerably. The first $NP*NT$ artificial variables, corresponding to the constraints from the finite difference formulation, can be eliminated by noting that the first $NP*NT$ rows of the pumpage matrix form an identity matrix. In other words, the identity matrix portion of the QP matrix can be utilized directly as a starting basis. However, the last $NT$ rows of the QP matrix, corresponding to the demand constraints, prevent the entire QP matrix from being an identity matrix. These nonzeroes in the last $NT$ constraints can be eliminated by performing row operations.

Row operations are ways of manipulating a system of simultaneous equations without changing the solution. One of these row operations may be performed by adding any multiple of any row to any other row. The coefficients (1's) in the demand constraints can thus be changed to zeroes by
subtracting each of the finite difference constraints from the demand constraint belonging to the same time period. When this is accomplished, the QP matrix becomes an identity matrix, and, along with the surplus variables from the demand constraints, forms the starting basis. An illustration of row operations is shown in Figure 9. Each entire equation must be subtracted, causing coefficients to appear in the demand constraints under columns corresponding to the head array, and causing the right-hand-side values of the demand constraints to change. The new groundwater management model formulation is shown in Figure 10. The number of decision variables is thus reduced by NP*NT. The basis is then operated on as in the regular simplex method.

Reduction of Constraints

The second method used to improve program efficiency was generalized upper bounding (GUB). The method is a variation of the simplex method for equations having the property that each variable has at most one nonzero coefficient (Dantzig and Van Slyke, 1965). When variables can be grouped such that the sum of the variables in each group equals a constant, they can be called group constraints or GUB constraints. The algorithm uses a working basis of regular constraints for the simplex method (in this case the finite difference constraints). If Mreg is the number of regular constraints and Mgub is the number of generalized
Finite Difference Coefficients (amatrix)

\[
\begin{bmatrix}
-a_{11} & a_{12} & a_{13} \\
-a_{21} & a_{22} & a_{23} \\
-a_{31} & a_{32} & a_{33} \\
-a_{41} & a_{42} & a_{43}
\end{bmatrix}
\]

Pumpage Matrix

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & \geq D \\
1 & 1 & 1 & 1 & \geq D \\
1 & 1 & 1 & 1 & \geq D \\
1 & 1 & 1 & 1 & \geq D \\
0 & 0 & 0 & 0 & -D \\
0 & 0 & 0 & 0 & -\sum b_i
\end{bmatrix}
\]

Figure 9: Row operations to change the pumpage matrix into an identity matrix.
Figure 10: Matrix form with the pumpage matrix as the initial starting basis.
upper bounding constraints, then the working basis has the size of $M_{\text{reg}} \times M_{\text{reg}}$, which for large $M_{\text{gub}}$ is a substantial reduction in computation (Dantzig, 1963). The block angular structure of the demand constraints, shown in Figure 11, is suitable for GUB constraints (Hillier and Lieberman, 1972). Because the demand constraints are of the $\geq$ type, a surplus variable must be added to each demand constraint. The GUB method requires the coefficients in the last $M_{\text{gub}}$ equations to be $+1$, but with modifications will accept $-1$ coefficients. The procedure is described in detail by Dantzig and Van Slyke (1965) and by Lasdon (1970).

Computational Efficiency of Different Formulations

The memory requirements for all three formulations are similar (approximately 162K for four time periods and 36 nodes) because the LP model requires a significant amount of memory to execute regardless of the programming technique used. Execution time was reduced by the two problem reduction techniques used, particularly by the reduction in constraints. For the 36 node hypothetical example, with an objective function of head maximization, a plot of execution time versus time periods used for all three formulations is shown in Figure 12. The reduction in execution time, in light of the computational problems which developed when the matrix became very large, was not significant enough to warrant the effort expended in programming the two reduction
Figure 11:
Structure of constraint coefficients for generalized upper bounding (from Hillier and Lieberman, (1972), pg. 146).
Figure 12: Comparison of the three problem formulations.
techniques because the reduction is only significant when
the matrix is large. An interesting development of the two
reduction approaches is that when the problem data had a
symmetrical solution, the different programming techniques
would choose different symmetrical solutions as the optimum.

Verification of Model Development

Groundwater management model development was verified
using data from Alley et al. (1976) who utilized a
hypothetical aquifer and solved the problem of head maximiza-
tion over four time periods. Their program was developed
independent of this study, and agreement between the two
solutions verifies the correct development of the LP model
in this study. In order to assure that the optimization
procedure did not compromise the simulation submodel, Alley
et al. (1976) used the LP solution of pumpages as input to
Prickett and Lonnquist's (1971) numerical simulation model.
The resulting heads from the simulation model were the same
as the heads in the optimal solution from the LP model, as
found in this study. Alley et al. (1976) therefore con-
cluded that the LP procedure correctly predicts heads
resulting from the chosen pumpages.

Experimental Mathematical Programming Subroutine Package

The experimental mathematical program (XMP) is a set of
subroutines for linear programming developed by Marsten (1981). XMP is a hierarchically structured library of FORTRAN subroutines for large scale LP problems which is a subservient system and does not contain a main program. The driving program must be written by the user. The only practical limit on the size of problems solvable by XMP is the amount of main memory space available from the computing system used (Marsten, 1981). The model in this study was executed on a CDC Cyber 730 computer with 262K of available memory.

**Hypothetical Data Results**

Discussion of and results from use of one hypothetical data set, with 30 nodes, is presented here. There were enough nodes that the effects of adding known, fixed increments and space increments could be observed, yet there were few enough nodes that several other scenarios could be executed. The spatial distribution of the increments was
CHAPTER IV.
MODEL APPLICATIONS

General Statement

The model developed in this research was applied to five problems of varying sizes: 12, 36, 56, 156 and 384 nodes. The first three examples used hypothetical data sets, while the last two used data from the Las Vegas Valley aquifer with differing grid spacings. Utilization of different data sets allowed exploration of computational difficulties arising from increasing matrix size, comparison of the simulation-optimization procedure with problems of varying complexity, and exploration of a large scale application of the embedding approach. The model formulations were first applied to the hypothetical data sets, then to the large real-world problem. Some computational instability problems which arose in the hypothetical data when many time periods were used also arose in the real data when just a few time periods were used.

Hypothetical Data Example

Discussion of and results from use of one hypothetical data set, with 36 nodes, is presented here. There were enough nodes that the effect of varying bounds, time increments and space increments could be observed, yet there were few enough nodes that several time periods could be executed. The spatial distribution of the transmissivity (in
ft²/day) for the 36 internal node data set is shown in Figure 13. The transmissivity distribution was assumed isotropic and heterogeneous to avoid the possibility of symmetrical solutions (which probably do not exist in nature). The storage coefficient was .001 at each node. Bounds on pumpages were varied, and natural recharge was incorporated by forcing recharge along a boundary in one case. Initial head values were 100 feet at each node. Upper and lower bounds on heads were 200 feet and 0 feet, respectively, with 0 feet indicating the bottom of the confining layer. Zero flux boundaries were used around the entire aquifer area. When natural recharge was incorporated, it was forced along the western boundary of the aquifer. Demand for each time period was 1 acre-foot/year.

Groundwater Model Applications

The following examples illustrate how the model can be used. The purpose of testing the model with hypothetical data was to determine model utility, and the validity of various objective functions.

Case 1

The objective function was maximization of heads (Eq. 3.9), and the problem was executed for four time steps of one year each. Bounds on pumpage were the same for each node: 10,000 cubic feet per day (cf/d) for the upper bound
\[ T_x = T_y \text{ at each node} \]

Figure 13:
Transmissivity distribution for the 36 node data set.
and 0 cf/d for the lower bound. Resulting heads, well locations and pumping rates are shown in Figure 14 and Table 1. The model chose four well locations, three of which were pumped at their upper bounds. Wells were located near boundaries. This objective function was used in both the lumped transient approach and the step-wise optimization approach with successful and identical results.

Case 2

The objective function was maximization of total pumping (Eq. 3.8) with a restriction on pumping at each node. Upper bounds on pumping were 10,000 cf/d, and the problem was executed for four time periods of one year each. The optimal head distribution is shown in Figure 15. The model pumped all wells at their upper bounds i.e. 10,000 cf/d; the upper bounds were the limiting factor on the feasible space. Because upper bounds were specified on pumping rates, the model did not use up all of the available water for supply by the last time period, and heads were therefore not drawn down to their lower bounds. Feasible space existed for the next time period, and thus demand could be satisfied in the next time period. Final heads could be used as initial heads in the next time step, and a step-wise optimization procedure could be utilized. The step-wise approach was valid for this case because final heads were above their lower bounds leaving feasible space for water withdrawal in
Figure 14: Resulting optimal head distribution for case 1.
Table 1: Optimal pumpage pattern for case 1.
Figure 15: Resulting optimal head distribution for case 2.
The purpose for using this objective function with restrictions on pumping is to show that the limiting factor on the feasible space can be the upper bound on pumping. In case 3, the limiting factor on the feasible space is the lower bounds on heads.

Case 3

Again the objective function was maximization of pumpage (Eq. 3.8), but the bounds on pumpage were unrestricted. For this type of objective function with no upper bounds, the limiting factor on the feasible space was the lower bounds on heads. Two time periods were used to show that whatever the number of time periods used, heads went to their lower bounds by the end of the last time period. For two time periods, results are shown in Figure 16 and Table 2. Notice that by the end of the second and final time period heads were at their lower bounds. Whatever the number of time steps used, heads at the final time step were always drawn down to their lower bounds when pumpage was unrestricted. The optimal solution of well locations and pumping rates was directly dependent on the number of time steps used. The fact that heads were always drawn down to their lower bounds by the final time period precluded using this objective function with unrestricted pumpage in a
Figure 16: Resulting optimal head distribution for case 3.
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<td>301909.</td>
</tr>
<tr>
<td>(6,3)</td>
<td>313169.</td>
<td>(6,6)</td>
<td>621909.</td>
</tr>
</tbody>
</table>

Table 2: Optimal pumpage pattern for case 3.
step-wise optimization approach. The short term goal of pumpage maximization took precedence over the long term goal of meeting demand. When heads from the final time step (at their lower bounds) were used as initial heads for the next time step, demand could not be satisfied and the problem would be infeasible.

Case 4

The objective function was minimization of pumpage, subject to the same demand as the previous examples. This objective function has the same form as Eq. 3.8, but the objective function coefficients are multiplied by -1. Results for four time periods of one year each are shown in Figure 17 and Table 3. Notice that the demand was satisfied as in case 1, but with different pumping locations. Well locations chosen were not unique for this objective function, because the only pumpage restrictions were that demand must be satisfied and pumping rates could not exceed their upper bounds. The solution is not unique. Comparing Figures 14 and 17, the total head at the end of each time period for case 1 is higher than for case 4. The solution chosen was the first solution in the feasible space.
Figure 17: Resulting optimal head distribution for case 4.
### Table 3: Optimal pumpage pattern for case 4.

<table>
<thead>
<tr>
<th>Node</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</thead>
<tbody>
<tr>
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<td>0.0</td>
<td>5803.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(3,2)</td>
<td>0.0</td>
<td>0.0</td>
<td>10000.0</td>
<td>0.0</td>
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<tr>
<td>(1,3)</td>
<td>10000.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(4,3)</td>
<td>10000.0</td>
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<td>0.0</td>
<td>0.0</td>
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<td>10000.0</td>
<td>10000.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(1,5)</td>
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<td>10000.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(2,5)</td>
<td>0.0</td>
<td>10000.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(3,6)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>5803.0</td>
</tr>
<tr>
<td>(5,6)</td>
<td>0.0</td>
<td>0.0</td>
<td>10000.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(6,6)</td>
<td>0.0</td>
<td>0.0</td>
<td>10000.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Case 5

This case was considered to show that well locations can be predetermined. The objective function was maximization of heads (Eq. 3.9), but pumping was allowed only at four selected wells. Pumping rates at these wells and resulting heads are shown in Figure 18 and Table 4. Wells can be forced at specific locations by setting the upper bounds on pumpage at nodes other than the selected wells to zero. This is useful when wells already exist in an area, and the optimal pumping pattern for existing wells is needed.

Case 6

The objective function was maximization of heads and constraints were the same as for case 1, but natural recharge of one-third acre-ft/year exists along the western boundary of Figure 13. Natural recharge can be considered by setting both the upper and lower bounds on pumping at recharge nodes to the quantity of recharge desired. Recharge nodes are negative pumping nodes. Results are shown in Figure 19 and Table 5. The model chose the same pumping locations as case 1.
Figure 18: Resulting optimal head distribution for case 5.
<table>
<thead>
<tr>
<th>Node</th>
<th>Time Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>(3,2)</td>
<td>5803</td>
</tr>
<tr>
<td>(4,3)</td>
<td>10000</td>
</tr>
<tr>
<td>(2,4)</td>
<td>10000</td>
</tr>
<tr>
<td>(5,5)</td>
<td>10000</td>
</tr>
</tbody>
</table>

**Table 4:** Optimal pumpage pattern for case 5.
Figure 19: Resulting optimal head distribution for case 6.
Table 5: Optimal pumpage pattern for case 6.
Case 7

Case 1 was used again, but artificial recharge was allowed. Artificial recharge can be incorporated by adding a negative identity matrix multiplied by a vector of artificial recharge, and setting a limit on the total amount of artificial recharge allowed during each time period. Incorporating this matrix adds to the problem size. Therefore, only three time periods were considered. The maximum amount of artificial recharge allowed during each time period was 5,000 cf/d. The upper bound on recharge at each node was also 5,000 cf/d, and the lower bound on artificial recharge at each well was 0 cf/d. The optimal solution is shown in Figure 20 and Table 6. Well locations and pumping rates chosen were the same as the solution for case 1. One artificial recharge well was chosen, and it was located as far from the pumping center as possible. The model tends to draw down heads in one area and build up heads in another area in order to maintain the sum of the heads over the aquifer at the highest level.

Conjunctive Model Applications

The 36 node data set was extended to incorporate a surface water source (see Figure 21). The stream was initially divided into three sections. However, as the total quantity of surface water withdrawn for supply could come from any of the sections, and as no attempt was made to distinguish
Figure 20: Resulting optimal head distribution for case 7.
Table 6: Optimal pumpage and recharge pattern for case 7.
Upper bound on streamflow = 25,000cf/d
Lower bound on streamflow = 15,000cf/d

Figure 21: Conjunctive model diagram for the 36 node hypothetical data set.
between the sections, the stream was treated as one unit. This reduced a few of the decision variables and constraints. If surface water withdrawals were made they could be made at any point along the stream with the same reduction in flow. Therefore, the stream section division was arbitrary without either an economic or a water quality reason for the division. So the stream was specified as one section.

As in the groundwater management model, the conjunctive use model only considers the system’s physical aspects. Water treatment costs, pumping costs and transportation costs are neglected.

**Case 8**

The objective function used was maximization of heads, as in the groundwater management model. When this objective function was used an extreme transfer of water took place as the model alternately depleted and recharged the aquifer. This occurred because no costs were included for water transfer between the two sources. To avoid unnecessary water transfer, this objective function for the conjunctive model was modified as follows:

\[
\text{Max } X_0 = \sum_{t=1}^{T} \sum_{k=1}^{N} \left( H_{kt} - \frac{1}{2} \right) - \sum_{t=1}^{T} \sum_{l=1}^{N} \left( Y_{lt} \right) \tag{5.1}
\]
This modified objective function penalizes augmentation and thus prevents excessive water transfer. The two terms in this modified objective function are noncommensurate; therefore, the problem is a multi-objective formulation. However, it is used as a device to avoid mass transfer of water. Results are shown in Figure 22 and Table 7 for two time periods. The model chose to satisfy demand totally from the aquifer, and to use available surface water for recharging the aquifer. Apparently, using available surface water for recharging the aquifer instead of for meeting demand maintains heads at a higher level than simply reducing the quantity of groundwater pumped would.

Case 9

The objective function is to maximize supply (Eq. 3.23) from both sources for two time periods of one year each. Upper bounds on pumpage were 10,000 cf/d. Results are shown in Figure 23 and Table 8. The model pumped all wells at their upper bounds during both time periods, and augmented the stream up to its upper bound of 25,000 cf/d during both time periods. The optimal solution includes both removal of surface water for supply (although some of the surface water removed was from augmentation), and groundwater withdrawals for supply. Some sort of "penalty" for unnecessary water transfer during the same time period should be imposed.
Figure 22: Resulting optimal head distribution for case 8.
Table 7: Optimal pumpage, recharge and augmentation pattern and stream water supply for case 3.
Figure 23: Resulting optimal head distribution for case 9.
### Table 8: Optimal pumpage, recharge and augmentation pattern and stream water supply for case 9.

<table>
<thead>
<tr>
<th>Node</th>
<th>Time Period</th>
<th>Node</th>
<th>Time Period</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>10000</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
</tr>
<tr>
<td>All</td>
<td>0</td>
<td>All</td>
<td>0</td>
</tr>
</tbody>
</table>

#### Water Supply from Stream (cf/d) Augmentation (cf/d)

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Node</th>
<th>Time Period</th>
<th>Node</th>
<th>Time Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>18000</td>
<td>1</td>
<td>1</td>
<td>10000</td>
</tr>
<tr>
<td>2</td>
<td>16000</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(2,3)</td>
<td>(6,2)</td>
<td></td>
<td>0</td>
</tr>
<tr>
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<td>10000</td>
<td></td>
<td>10000</td>
</tr>
</tbody>
</table>

#### Stream Flow (cf/d)

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Node</th>
<th>Time Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
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<td>2</td>
</tr>
<tr>
<td>2</td>
<td>21000</td>
<td>2</td>
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<tr>
<td>Beginning</td>
<td>23000</td>
<td>21000</td>
</tr>
<tr>
<td>Ending</td>
<td>15000</td>
<td>15000</td>
</tr>
</tbody>
</table>
Case 10

The objective function was minimization of water transfer between the groundwater and the surface water sources (Eq. 3.24). This objective function chose the same pumping locations as the objective function of minimization of total supply. Demand was satisfied from the aquifer, but the solution of well locations chosen was not unique (see Figure 24 and Table 9). No recharge or augmentation took place and no water was withdrawn from the stream. The chosen well locations were not unique because the model was not trying to optimize an objective incorporating hydraulic heads, it was only trying to satisfy demand while staying within the restrictions placed on pumping rates.

Case 11

The objective function was minimization of total supply (Eq. 3.25). Demand was met completely by the aquifer but pumping locations were non-unique. Augmentation and artificial recharge took place in both time periods (see Figure 25 and Table 10). The model augmented the stream up to its upper bounds in both time periods, but withdrew available surface water for recharge. As in case 9, augmentation and recharge took place during the same time period because there was no penalty placed on water transfer. Recharge and augmentation locations also seem non-unique.
Figure 24: Resulting optimal head distribution for case 1D.
### Optimal Pumpage, Recharge and Augmentation Pattern

<table>
<thead>
<tr>
<th>Node</th>
<th>Time Period</th>
<th>Pumpage (cf/d)</th>
<th>Artificial Recharge (cf/d)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>All 0.</td>
</tr>
<tr>
<td>(3,4)</td>
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<td>10000.</td>
<td>All 0.</td>
</tr>
<tr>
<td>(1,5)</td>
<td>1</td>
<td>10000.</td>
<td>All 0.</td>
</tr>
<tr>
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<td>1</td>
<td>10000.</td>
<td>All 0.</td>
</tr>
<tr>
<td>(3,6)</td>
<td>0</td>
<td>0.</td>
<td>10000.</td>
</tr>
<tr>
<td>(4,6)</td>
<td>0</td>
<td>0.</td>
<td>10000.</td>
</tr>
<tr>
<td>(5,6)</td>
<td>0</td>
<td>0.</td>
<td>10000.</td>
</tr>
<tr>
<td>(6,6)</td>
<td>0</td>
<td>0.</td>
<td>10000.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Node</th>
<th>Time Period</th>
<th>Water Supply from Stream (cf/d)</th>
<th>Augmentation (cf/d)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2</td>
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<td>0.</td>
</tr>
<tr>
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<td>2</td>
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<tr>
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### Stream Flow (cf/d)

<table>
<thead>
<tr>
<th>Time Period</th>
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<th>23000.</th>
<th>21000.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ending</td>
<td>23000.</td>
<td>21000.</td>
<td></td>
</tr>
</tbody>
</table>

**Table 9:** Optimal pumpage, recharge and augmentation pattern and stream water supply for case 10.
Figure 25: Resulting optimal head distribution for case 11.
### Pumpage (cf/d) vs. Artificial Recharge (cf/d)

<table>
<thead>
<tr>
<th>Node</th>
<th>Time Period</th>
<th>Node</th>
<th>Time Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
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<tr>
<td>(2,5)</td>
<td>10000.</td>
<td>(3,6)</td>
<td>0.</td>
</tr>
<tr>
<td>(3,6)</td>
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<td>(4,6)</td>
<td>0.</td>
</tr>
<tr>
<td>(4,6)</td>
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<td>(5,6)</td>
<td>0.</td>
</tr>
<tr>
<td>(5,6)</td>
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<td>(6,6)</td>
<td>0.</td>
</tr>
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### Water Supply from Stream (cf/d) vs. Augmentation (cf/d)

<table>
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<th>Node</th>
<th>Time Period</th>
</tr>
</thead>
<tbody>
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<tr>
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### Stream Flow (cf/d)

<table>
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<th>Time Period</th>
</tr>
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</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>Beginning</td>
</tr>
<tr>
<td>Ending</td>
</tr>
</tbody>
</table>

Table 10: Optimal pumpage, recharge and augmentation pattern and stream water supply for case 11.
Case 12

The objective function was to maximize water supply while minimizing water transfer (Eq. 3.26). Heads were drawn down to their lower bounds when no restriction was placed on pumping. For the case where the upper bounds on pumpage were 10,000 cf/d, the model withdrew all available water from the aquifer and from the stream (see Figure 26 and Table 11). No water transfer took place because there was a penalty for augmentation. When the initial value of streamflow used in the model was below the lower bound specified for streamflow, the model augmented the stream up to its lower bounds to maintain a minimum flow, even though there was a penalty.

Las Vegas Valley Data Example

The Las Vegas Valley area was modeled because it is a large, very heterogeneous area and suitable for examining the groundwater management model in a real situation. A location map of the Las Vegas Valley area is shown in Figure 27. The "Las Vegas Valley Aquifer" is a conglomerate of aquifers, which previous modelers have divided into a near surface aquifer, and shallow, middle and deep zone confined aquifers. The shallow, middle and upper portion of the deep confined aquifers have been grouped into the "principal aquifer zone", which is the source for 95 percent of the
Figure 26: Resulting optimal head distribution for case 12.
<table>
<thead>
<tr>
<th>Pumpage (cf/d)</th>
<th>Artificial Recharge (cf/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Node</strong></td>
<td><strong>Time Period</strong></td>
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<tr>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Water Supply from Stream (cf/d)</th>
<th>Augmentation (cf/d)</th>
</tr>
</thead>
<tbody>
<tr>
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<td><strong>Node</strong></td>
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</table>

<table>
<thead>
<tr>
<th>Stream Flow (cf/d)</th>
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<tbody>
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</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>Beginning</td>
</tr>
<tr>
<td>23000.</td>
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<tr>
<td>21000.</td>
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<tr>
<td>Ending</td>
</tr>
<tr>
<td>15000.</td>
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<tr>
<td>15000.</td>
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</tbody>
</table>

Table 11: Optimal pumpage, recharge and augmentation pattern and stream water supply for case 12.
Figure 27: Location map for Las Vegas Valley.
groundwater produced in the valley (Harrill, 1976). The principal aquifer zone has been modeled as one aquifer by many researchers (Cochran, 1968, 1973; Harrill, 1976; Broadbent, 1980), and is the aquifer zone modeled in this study. This principal aquifer zone is modeled as a single confined aquifer with a two dimensional model.

The Las Vegas Valley principal aquifer zone is highly heterogeneous, with transmissivities varying from 1,000 gallons per day per foot (gpd/ft) to 300,000 gpd/ft (Harrill, 1976; Broadbent, 1980). Transmissivity and storage coefficient data used in this study were from Broadbent's (1980) calibration of historical water level fluctuations from aquifer stresses. Natural recharge used in this model was 30,000 acre-ft/yr, and was input along the northern, western and southern model boundaries as forced recharge wells (Harrill, 1976).

Originally, a 1 mile by 1 mile grid spacing was used over most of the aquifer, resulting in 384 nodes. This size problem obtained the computer memory needed to execute, but numerical difficulties developed and a solution could not be achieved. The problem matrix was very large: 385 constraints and 768 decision variables. The solution was blowing up because XMP shut itself off after iterating at length. XMP performs numerical checks on itself, and if the solution becomes numerically unstable, shuts itself off.
In order to decrease the number of constraints and decision variables, a 2 mile by 2 mile grid spacing was used over the entire aquifer, resulting in 156 nodes. Each potential well location represented 4 square miles. An attempt was made to optimize the sum of the heads subject to actual total principal aquifer zone withdrawals for the period 1955-1974. The purpose of this example was to see if optimal well locations and pumping rates would produce an aquifer potentiometric surface very different from the actual 1974 potentiometric surface. The question was whether optimal development from the start would have made a difference in the present day water level configuration.

Actual water levels for 1955 and groundwater demands for the valley during 1955-1974 were obtained from Harrill (1976) and used with the transmissivity data, storage coefficient distribution and natural recharge as input to the optimization model. Initial head values are shown in Figure 28. The model was run in a step-wise formulation with an objective function of head maximization, because it could not run for more than two time periods as a lumped transient formulation. Each time period was optimized separately with a time increment of one year, for a total time of twenty years.

All twenty time periods executed, and demand was satisfied in each time period. Pumping was located near recharge
Figure 28:
Initial (1955) heads for Las Vegas Valley
(from Harrill, 1976)
areas in the optimal solution. Resulting heads from the optimization procedure are very different from actual 1974 heads, which maintained a head gradient across the valley. Initial heads (1955), graded from approximately 2600 feet on the western side of the valley to 1600 feet on the eastern side of the valley. The resulting head distribution after 20 years of optimal pumping ranged from 1300 feet to 1800 feet (see Figure 29). After the first few time steps, heads slowly started flattening out, and did not remain higher in the recharge areas. A graph of initial heads versus final heads across the valley is shown in Figure 30.

The sum of the 1974 heads was greater than the sum of the 1955 heads. The model located wells near recharge areas, making use of recharge capture and drawing down heads in areas where they were initially high. The head gradient from west to east in the valley disappeared, and the resulting head distribution is almost flat. The probable cause of this flattening out of the head gradient is the objective function of head maximization. This objective function does not account for topography, and thus does not account for pumping lifts. The assumption was made in the development of this model that head maximization corresponded to drawdown minimization. This assumption is not valid in areas where topography varies. A better objective function in this case would be minimization of lift, where topography would be considered.
Figure 29: Optimal 1974 heads for Las Vegas Valley after 20 years of pumping using the step-wise approach.
Figure 30:
Optimal 1974 heads and initial 1955 heads from West to East across Las Vegas Valley.
Factors Affecting Optimal Solutions

Effect of Time and Space Increments

Using an objective function of maximization of head, time and space discretizations were varied to determine if a point would be reached where computational instability occurred based on the size of the matrix elements. Although the space increment was varied between 1000 feet and two miles, and the time increment was varied between four days and one year, no difficulties were encountered. The problem executed for all grid sizes and time increments tried.

Decreasing the grid size for the same total area increased the number of nodes, thus increasing the number of possible well locations. Optimal well locations chosen were in the same vicinity regardless of the space discretization used. Of more concern was whether changing the time increment would affect either the optimal well locations or optimal pumping rates in the solution. In a real application, the space increment would be chosen based on available data and the number of nodes desired. The space increment would more likely be fixed than the time increment, especially since widely scattered aquifer data would not justify a very small grid spacing. Available data would dictate the space discretization used.

Changing the time increment did not change the optimal
well locations and per day pumping rates. Regardless of the
time discretization used, optimal pumping locations and
rates chosen were always the same for given conditions.
Another concern was how much the time increment affected
resulting hydraulic head values. When one well was forced
to pump 1 acre-foot/year near the center of the aquifer at
node (3,3) (see Figure 31), the resulting heads varied. Six
time increments were tried: 365/20, 365/12, 365/8, 365/4,
365/2 and 365 days. These were chosen so that the number of
days elapsed would periodically correspond. Intuitively it
appeared that the smaller time increments gave better esti­
mates of drawdown, because smaller time increments more
closely approximate the analytic solution. However, as
shown in Figure 31, the drawdown estimates from different
time increments appear to reverse themselves at later times.
What actually happened is not that at later times the larger
time increments gave larger drawdown estimates, but that the
smaller time increments required many time steps to reach a
total time of 365 days. After 8 or 10 time periods, the
drawdown curves flattened out due to boundary effects.
Heads after the 8th or 10th time step did not change.

The amount of variability in the drawdowns predicted
from different time increments was several feet, but in
light of the errors in estimation of input parameters which
would occur in a real example, absolutely accurate drawdown
estimates could not be expected. Even though drawdowns are
Figure 31: Drawdowns from use of varying time increments.
somewhat underestimated at early times with a larger time increment, the larger time increment is justified in order to run the program over a longer total time period and to limit the number of iterations.

**Effect of Different Objective Functions**

A variety of objective functions was used to determine for which situations each would be applicable. Four objective functions were used in the groundwater management model: 1) maximize the sum of the heads for all time periods; 2) maximize the sum of the heads for the last time period only; 3) maximize water supply; and 4) minimize water supply. Minimize water supply is the negative of maximize water supply. The first two objective functions gave the same results for each time period; pumping locations, pumping rates and hydraulic heads were identical. To maintain heads at some later time, heads should be maintained through earlier times. An optimal solution at some future date requires optimal decisions through time leading up to the future. This type of objective function maintains heads over the aquifer through time while meeting demand and obeying restrictions on pumping rates and lower bounds on heads.

Using maximization of pumpage as the objective function either caused pumping which drew the piezometric surface down to the lower bounds on head or pumped at the upper bounds on pumpage, whichever was the limiting factor on the
feasible space. For the objective functions which maximize heads and maximize pumpage, problems had a clear goal to satisfy in choosing pumping locations and rates. For the objective function which minimizes pumpage, problems had no clear goal other than meeting demand and the problem solution was not unique. There were alternative optimal solutions, and the optimal solution chosen was the first which fit the feasible space. The objective of pumpage minimization did not correspond to the objective of head maximization, although in each case the same demand was met.

Five objective functions were used in the conjunctive model: 1) maximization of heads; 2) maximization of total water supply; 3) minimization of water supply; 4) minimization of water transfer in the form of artificial recharge and streamflow augmentation; and 5) maximization of water supply while minimizing water transfer. The first objective function was originally formulated just as in the groundwater management model. This resulted in an extreme transfer of water between the aquifer and the stream as the model alternately recharged and depleted the aquifer. This extreme transfer of water occurred because there were no costs for water transfer. This objective function was then modified to maximize head while minimizing streamflow augmentation. This modification gave better results.

When the objective was maximization of water supply,
both sources were used and drawn down to their lower bounds. Some water transfer occurred to augment stream supply. When the objective function was modified to maximize water supply while minimizing water transfer, no water transfer occurred. It was expected that water would be stored in the aquifer from artificial recharge during early time periods to be used during later time periods for supply or augmentation. Instead, the model withdrew all the water possible during each time period. This is because the model was not developed to place a future value on stored water.

The model was also executed with an objective function of minimizing water transfer between the aquifer and the stream. No transfer took place and supply was satisfied. The well locations and pumping rates chosen were identical to those chosen with the objective function which minimized water supply. Demand was met by pumpage only and the solution was non-unique in both these cases. The solutions chosen were the first which fit the feasible space.

**Factors Affecting Program Execution**

In general, execution time was greatly affected by factors other than program formulation. As the number of time periods increased the execution time increased exponentially. An increase in the number of nodes also causes an increase in execution time. For an equal number of
constraints, the problem with more time periods took longer to execute than the problem with more nodes. As a problem's feasible space became increasingly limited, i.e. bounds on pumpage or heads restricted, or demand increased, more wells were selected in the solution, and the problem took longer to complete.

For a given problem, the objective function chosen affected the computation time. When the objective was to minimize pumpage, the LP procedure chose wells arbitrarily and found a solution quickly. This was because the solution was not unique. When the objective was to maximize pumpage with upper bounds specified, the program reached a solution quickly, but, with upper bounds on pumpage unrestricted, the feasible space became very large and sometimes computational problems kept the LP procedure from reaching a solution. Maximization of hydraulic head was the objective function which required the most execution time.

Slight increases in computational time used were noted for either longer time or increased space increment. A larger discretization scheme increased the size of the individual matrix elements. Time increments used were from a few days up to one year, while space increments used were from 1000 feet up to two miles. Longer time or larger space increments did not cause any noticeable computational problems.
The memory and execution time requirements, for a given data set, were much larger for a combined simulation-optimization model than for a simulation model alone. Originally, it was expected that the upper limit on the number of nodes and number of time periods which could execute would be limited by the computer used, but, for large data sets, some computational instability difficulties arose proving to be the limiting factor.

**Computational Instability**

Problems of different sizes and complexity were tested to determine the model's potential for numerical instability. It was found that an extremely large data set (384 nodes) would not execute at all, a large data set (156 nodes) would execute for up to 1 time period, a medium sized data set (36 to 56 nodes) would execute for up to 5 time periods and a small data set (12 nodes) would execute for up to 10 time periods. This lack of execution occurred because of computational instability which arose during the LP solution procedure. Fortunately, the LP code used checks itself during computations, reports any problems, and terminates execution if problems arise. Infrequently occurring problems were: 1) pivot rejections because of the pivot element being too small; 2) development of a singular matrix; 3) the objective function not changing; and 4) development of a column with no nonzero elements. The latter two problems
occurred only once each, and both occurred for the case where the objective function was to maximize pumpage with no upper bounds.

By far the most frequently occurring problem was that for large matrices the program would iterate at length, some matrix elements would become too small, and to prevent the solution from blowing up the linear programming code would shut itself off. Elango and Rouve (1980) used LP with the finite element method and hypothesized that for a large number of equality constraints problems may develop. Gorelick (1983) states that commercial LP codes use lower-upper triangular decomposition of the base matrix, and that there are numerical difficulties with lower-upper basis factorization when LP is used with banded matrices. Current literature and this study show that for small problems the embedding method works well. However, for large problems, this study found that numerical difficulties arise.

Discussion of Results

Originally it was thought that the size of the individual matrix elements would have an effect on the problem's stability, so hypothetical data were manipulated to force matrix elements to vary by two orders of magnitude. Larger matrix elements caused no additional instability problems beyond those mentioned in the computational instability section. Large matrix elements were caused in part by coarse
time and space increments, but the size of the increments did not affect the program's ability to reach a solution. Coarse discretizations were needed in order to execute the model over a longer amount of total time and to restrict the number of nodes so the program could run. Large time increments resulted in smaller estimates of drawdown, but this was justified in light of using optimization models as screening models and recommending that optimization results not be used without further analysis.

This model could be used with varying objective functions for different situations provided that the area to which the model is applied is small and the number of time steps is limited. For an undeveloped aquifer, where optimal well locations are desired, the objective of maximization of heads maintains heads over the aquifer while choosing the best well locations and pumping rates to meet projected demands. However, a relatively undeveloped aquifer may not have enough data about the transmissivity and storage coefficient distribution to justify use of a distributed parameter model. In this case a coarse space discretization controlled by the amount of data available is recommended. Otherwise, the model would be optimizing a hypothetical rather than actual distribution of aquifer parameters. For an undeveloped area, the objective of maximization of supply would decide what the maximum withdrawals could be given an allowable level of depletion.
In a developed area, data on aquifer parameters would be more extensive than in an undeveloped area. Space increments should be small enough to represent all wells or well fields. The objective of head maximization could either be used to determine pumping rates for existing well locations, or to propose new well locations. Maximization of head could also be used to propose artificial recharge locations based on already existing well locations. The objective of maximization of supply could be used to determine how much water could be withdrawn using existing facilities given constraints on demand. This might be useful in determining if future appropriations could be met.

Applying this type of a distributed parameter model for groundwater management to small problems appears valid for the objective function of head maximization and pumpage maximization. This type of distributed parameter model does not appear very useful for the conjunctive formulation though. The division of stream sections is artificial. In order to choose stream sections some criteria, such as cost or water quality, must be applied to allow the model to distinguish between sections. Otherwise, the division of sections is an arbitrary one.

Ideally, a conjunctive model should be executed for a long period of time to obtain optimal decisions about when to recharge and which water source should be used to satisfy
The size of this distributed parameter conjunctive model precludes using many time steps. A conjunctive model should have some distinction, other than physical characteristics, between the groundwater source and the surface water source, such as cost or water quality. This type of model assumes that the two sources are of equal desirability, which may or may not be true, and also assumes that economics would not make a difference in choosing a source. Costs, such as pumping costs or water treatment costs, are nonlinear, and could not be used in an LP model because of the requirement of linearity. The hydraulic management approach produces more worthwhile results for a groundwater model than for a conjunctive model. Hydraulic management, by itself, is apparently too simplistic an approach for conjunctive use management.

Perhaps using a combination of a resource allocation and policy evaluation model with a hydraulic management model would be the best approach for conjunctive use management. A conjunctive model could be run as a lumped parameter model initially, taking pumping and water treatment costs into consideration. Then the optimal quantity of groundwater to be pumped or recharged could be input into a distributed parameter groundwater management model for determination of optimal well locations and pumping rates. The spatial distribution of drawdown from the hydraulic management model could be compared to the average drawdown from the
lumped parameter model used to calculate pumping costs, and pumping cost estimates could be revised based on the areal distribution of drawdown. In-well drawdown estimated either a simulation model or an optimization model might underestimate actual drawdowns.

A combined simulation-optimization model was developed by embedding the finite difference equation of the partial differential equation for groundwater flow in a confined aquifer into a linear programming optimization model. The resulting system of linear simultaneous equations was solved by the primal simplex method. The system of equations formed a banded matrix, and all time periods under consideration were incorporated into the matrix, forming a large number of constraints when many planning periods were considered. The system of equations was also solved in a step-wise approach, where each time period was optimized separately.

Programs were developed for a groundwater aquifer alone, for an aquifer with allowable artificial recharge, and for an aquifer in conjunction with a stream where both artificial recharge and streamflow augmentation were allowed. Two attempts at reducing the problem size to improve computer computational efficiency and memory requirements were unsatisfactory. Memory requirements remained approximately the same for all three approaches, while the amount of reduced computational time did not warrant the effort expended in modifying the original model formulation.
CHAPTER V.
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

A combined simulation-optimization model was developed by embedding the finite difference approximation of the partial differential equation for groundwater flow in a confined aquifer into a linear programming optimization model. The resulting system of linear simultaneous equations was solved by the primal simplex method. The system of equations formed a banded matrix, and all time periods under consideration were incorporated into the matrix, forming a large number of constraints when many planning periods were considered. The system of equations was also solved in a step-wise approach, where each time period was optimized separately.

Programs were developed for a groundwater aquifer alone, for an aquifer with allowable artificial recharge, and for an aquifer in conjunction with a stream where both artificial recharge and streamflow augmentation were allowed. Two attempts at reducing the problem size to improve computer computational efficiency and memory requirements were unsatisfactory. Memory requirements remained approximately the same for all three approaches, while the amount of reduced computational time did not warrant the effort expended in modifying the original model formulation.
Different objective functions were used in the model to test the validity and utility of each. Some objective functions in the conjunctive formulation needed modification after they proved not useful in their original form. The effect of different time and space increments on the solution was explored because of the restriction on the number of constraints the model could handle. Use of larger space and time increments allowed the execution of larger problems (in terms of area covered, and number of planning periods which could be executed). Smaller time periods gave larger estimates of drawdown, but the difference in drawdown predicted using a larger time increment compared to drawdown predicted using a smaller time increment was small enough to justify use of a larger time increment, especially since the number of time steps must be limited. Some numerical problems with the lumped transient approach were encountered and described. Current literature on the subject of the embedding method does not discuss computational difficulties in detail.

A step-wise optimization approach gave results equivalent to lumped transient approach results for the objective functions of maximization of heads and maximization of pumpage when pumpage bounds were restricted. The step-wise approach was then used with both the hypothetical data and the Las Vegas Valley aquifer data to execute the model over 20 time periods.
Conclusions and Recommendations

The embedding methodology combines optimization and numerical simulation methods to formulate and solve groundwater management problems. Physical and management constraints on the system are solved simultaneously. A physical representation of the aquifer system should be included in a management model because a lumped parameter approach does not adequately describe the spatial characteristics of the groundwater system.

Although embedding a simulation model inside an optimization model is possible, and simple conceptually, in practice this method works well only for small scale problems over a limited number of planning periods. The method, in its current state of development, does not seem beneficial for large scale, complicated, real world problems. Current literature describes only small scale examples of the embedding approach, while this research utilized a large scale problem over many time periods. Numerical difficulties encountered restrict the utility of this method for real world problems. Another limiting factor of the embedding approach is that distributed parameter models have large data requirements. Use of a distributed parameter model is justified only when data are available. Otherwise, the model is optimizing hypothetical data, not the real system, and results may have little correlation with reality.
Because of simplifying assumptions and the coarseness of the space and time increments used in the combined simulation-optimization model, the use of post-optimization simulation with finer time and space increments is recommended. Optimization models are good for plan formulation; they can generate new management schemes. Simulation models can assess the performance of the schemes and refine plans suggested by the optimization model. Only an optimization model can generate new plans for stated objectives, but because of their numerical complexity, optimization models cannot be executed with the space and time detail of simulation models. Therefore, optimization models should be considered as screening models which offer plans for consideration in further detail. The results from an optimization model should not be used without detailed analysis and hydrologic judgement.

The methodology described in this research needs to be better tailored to the banded matrix type of problems which apply to groundwater flow systems. A more efficient solution algorithm needs to be developed for solving such matrix formulations; the development of such a solution algorithm, however, belongs in the field of operations research and numerical analysis, not in the field of hydrology. Unless the embedding method can become computationally efficient and stable, it should be bypassed in favor of the response matrix approach, as large problems utilizing the latter have
been reported as successful in current literature (Heidari, 1982; Gorelick, 1982). The large response matrix problems reported used only a few time steps, and no detailed analysis of numerical difficulties in the response matrix approach has been reported.

Incorporation of the groundwater system’s physical functioning into a management model is a worthwhile approach. The extension of simulation-optimization methods to unconfined, nonlinear systems, and to aquifers hydraulically connected to streams needs to be addressed, but not with LP since the assumptions made to linearize the equation for groundwater flow in an unconfined aquifer might not always be valid, and might not produce valid results.

As management of water resources moves closer to a more realistic description of each system’s functioning, incorporation of stochastic behavior and uncertainty into system equations needs to be addressed. When a real world system is utilized in a management model for other than academic purposes, sensitivity analysis of the results should be conducted to determine which hydrologic parameters the model is most sensitive to, and how errors in parameter estimation affect the optimal solution. Sensitivity analysis could also indicate in which areas data collection could be improved. Then the management model could be executed again with improved parameter estimation suggested by the
sensitivity analysis.


REFERENCES CITED


Maddock, T. III., 1972, Algebraic Technological Function from a Simulation Model; Water Resources Research, v.8, no.1.


Pinder, G., 1970, A Digital Model for Aquifer Evaluation; Techniques of Water Resources Investigation, Book 7, Chapter C1, USGS, Washington, D.C.


APPENDIX A: USER'S MANUAL FOR PROGRAMS GWUSER AND CONJUN

CARD 1:

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>FORMAT</th>
<th>CARD COLUMN</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>IOBJ</td>
<td>I8</td>
<td>1-8</td>
<td>Objective Function type</td>
</tr>
<tr>
<td>LSECT</td>
<td>I8</td>
<td>9-16</td>
<td>Number of stream</td>
</tr>
<tr>
<td>NT</td>
<td>I8</td>
<td>17-24</td>
<td>Number of time periods</td>
</tr>
<tr>
<td>NR</td>
<td>I8</td>
<td>25-32</td>
<td>Number of elements in each row</td>
</tr>
<tr>
<td>MC</td>
<td>I8</td>
<td>33-40</td>
<td>Number of elements in each column</td>
</tr>
<tr>
<td>DT</td>
<td>F8.2</td>
<td>41-48</td>
<td>Time increment in days</td>
</tr>
</tbody>
</table>

CARD 2:

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>FORMAT</th>
<th>CARD COLUMN</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>DX</td>
<td>F8.0</td>
<td>1-8</td>
<td>X dimension of grid</td>
</tr>
<tr>
<td>DY</td>
<td>F8.0</td>
<td>9-16</td>
<td>Y dimension of grid</td>
</tr>
</tbody>
</table>

CARD 3:

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>FORMAT</th>
<th>CARD COLUMN</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRANS</td>
<td>I8</td>
<td>1-8</td>
<td>1 if isotropic 2 if anisotropic</td>
</tr>
<tr>
<td>GHBBOUND</td>
<td>I8</td>
<td>9-16</td>
<td>1 if bounds on heads are the same at each node; 2 if bounds on heads vary</td>
</tr>
</tbody>
</table>
GQBOUND

I8

17-24

1 if bounds on pumpages are the same at each node; 2 if bounds on pumpages vary

IU(3)

I8

25-32

1 if input data is in gallons per day; 0 otherwise

CARD 4:

VARIABLE NAME

BCHDR1(J)

FORMAT

10F8.4

CARD COLUMN

1-80

DESCRIPTION

Heads on boundary, first element in each row, starting with node (1,2)

CARD 5:

VARIABLE NAME

BCHDRN(J)

FORMAT

10F8.4

CARD COLUMN

1-80

DESCRIPTION

Heads on boundary, last element in each row starting with node (MC,2)

CARD 6:

VARIABLE NAME

BCHDC1(I)

FORMAT

10F8.4

CARD COLUMN

1-80

DESCRIPTION

Heads on boundary, first element in each column, starting with node (2,1)

CARD 7:

VARIABLE NAME

BCHDCM(I)

FORMAT

10F8.4

CARD COLUMN

1-80

DESCRIPTION

Heads on boundary, last element in each column, starting with node (2, NR)
The next cards contain input data about the aquifer parameters.

If the medium is isotropic (TRANS = 1), then the parameters are input, for each row, as follows:

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>FORMAT</th>
<th>CARD COLUMN</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRANX</td>
<td>10F8.0</td>
<td>1-80</td>
<td>Transmissivity in X direction across row</td>
</tr>
<tr>
<td>STOR</td>
<td>10F8.4</td>
<td>1-80</td>
<td>Storage coefficient across row</td>
</tr>
<tr>
<td>HEAD0</td>
<td>10F8.0</td>
<td>1-80</td>
<td>Initial heads across row</td>
</tr>
</tbody>
</table>

If the medium is anisotropic (TRANS = 2), then the parameters are input, for each row, as follows:

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>FORMAT</th>
<th>CARD COLUMN</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRANX</td>
<td>10F8.0</td>
<td>1-80</td>
<td>Transmissivity in Y direction across row</td>
</tr>
<tr>
<td>TRANY</td>
<td>10F8.0</td>
<td>1-80</td>
<td></td>
</tr>
<tr>
<td>STOR</td>
<td>10F8.4</td>
<td>1-80</td>
<td>Upper bound on pumpage</td>
</tr>
<tr>
<td>HEAD0</td>
<td>10F8.0</td>
<td>1-80</td>
<td>Low bound on pumpage</td>
</tr>
</tbody>
</table>

The next cards read in bounds on heads. If GHBOUND = 1, then the head bounds are input, as follows:

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>FORMAT</th>
<th>CARD COLUMN</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>UBOUND</td>
<td>F8.0</td>
<td>1-8</td>
<td>Default upper bound on head</td>
</tr>
<tr>
<td>LBOUND</td>
<td>F8.0</td>
<td>9-16</td>
<td>Default lower bound on head</td>
</tr>
</tbody>
</table>
If $G_{BOUND} = 2$, then the head bounds are input for each row, node by node for internal nodes only, as follows:

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>FORMAT</th>
<th>CARD COLUMN</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{HEAD}(I,J)$</td>
<td>8F10.2</td>
<td>1-80</td>
<td>Upper bounds on heads</td>
</tr>
<tr>
<td>$L_{HEAD}(I,J)$</td>
<td>8F10.2</td>
<td>1-80</td>
<td>Lower bounds on heads</td>
</tr>
</tbody>
</table>

The next cards read in bounds on pumpages. If $G_{QBOUND} = 1$, then the pumpage bounds are input as follows:

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>FORMAT</th>
<th>CARD COLUMN</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{QBOUND}$</td>
<td>F10.2</td>
<td>1-10</td>
<td>Default upper bound on pumpage</td>
</tr>
<tr>
<td>$L_{QBOUND}$</td>
<td>F10.2</td>
<td>11-20</td>
<td>Default lower bound on pumpage</td>
</tr>
</tbody>
</table>

If $G_{QBOUND} = 2$, then pumpage bounds are specified separately for each node, row by row, for internal nodes only.

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>FORMAT</th>
<th>CARD COLUMN</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{PUMP}(I,J)$</td>
<td>8F10.2</td>
<td>1-80</td>
<td>Upper bound on pumpage</td>
</tr>
<tr>
<td>$L_{PUMP}(I,J)$</td>
<td>8F10.2</td>
<td>1-80</td>
<td>Lower bound on pumpage</td>
</tr>
</tbody>
</table>

The next card reads in the demand for each time period.

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>FORMAT</th>
<th>CARD COLUMN</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEMAND(t)</td>
<td>4F15.8</td>
<td>1-60</td>
<td>Water demand for each time period</td>
</tr>
</tbody>
</table>

This is the end of the data requirements for program GWUSER. If program CONJUN is used, after the bounds on pumpage are input, the additional data are given as follows:
Input upper and lower bounds on streamflow in each section. The number of sections is LSECT.

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>FORMAT</th>
<th>CARD COLUMN</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>UQS</td>
<td>10F8.0</td>
<td>1-80</td>
<td>Upper bound on streamflow in each section</td>
</tr>
<tr>
<td>LQS</td>
<td>10F8.0</td>
<td>1-80</td>
<td>Lower bound on streamflow in each section</td>
</tr>
</tbody>
</table>

Input the flow rate at the upstream end of the system in cubic feet per day, for each time period.

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>FORMAT</th>
<th>CARD COLUMN</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>QSU</td>
<td>10F8.0</td>
<td>1-80</td>
<td>Flow rate at upstream end of system</td>
</tr>
<tr>
<td>DEMAND</td>
<td>4F15.8</td>
<td>1-60</td>
<td>Water demand to be met from both sources during each time period</td>
</tr>
</tbody>
</table>

The main program calls GWATER (for GWUSER) or XSETUP (for CONJUN) to generate the coefficient matrices, bounds on variables and right-hand-side values. The main program generates the objective function coefficients based on the objective function type, and feeds the columns of the coefficient matrix into XMP one by one.

XMP variables used in the main program and their dimension requirements are as follows:

Real Arrays

B: the right-hand-side array (MAXM)

BASCB: array containing the objective function coefficients of the basic variables (MAXM)

BASCB: array containing the lower bounds on the basic variables (MAXM)

BASUB: the array containing the upper bounds on the basic variables (MAXM)
CANDA: a table containing the non-zero coefficients of the columns that belong to the candidate list (COLMAX,P)

CANDCJ: a list containing the objective coefficient for each column that belongs to the candidate list (P)

COLA: used to hold the non-zero coefficients of a matrix column (COLMAX)

MEMORY: the main storage array; it contains all of the arrays for the problem data and the basis inverse representation (LENMY); memory can not be dimensioned larger than the value set for LENMY

UZERO: the array containing the value of the dual variables (MAXM)

YQ: used to hold the unpacked column that is about to enter the basis (MAXM)

**Integer Arrays**

BASIS: the list of basic variables (MAXM)

CAND: the candidate list; it contains the non-basic variables that are eligible to enter the basis during a series of minor iterations (P)

CANDI: a table containing the row numbers corresponding to the non-zeros of the column that belongs to the candidate list (COLMAX,P)

CANDL: a list containing the number of non-zeros in each column that belongs to the candidate list (P)

CJ: array of objective function coefficients in each column (COLMAX)

COLI: used to hold the row numbers corresponding to the non-zeros of a matrix column (COLMAX)

LJ: lower bound for decision variables

MAPA: a map of the data structure for the original problem data, consisting of pointers into the memory array (20)

MAPI: a map of the data structure for the basis inverse representation, consisting of pointers into the memory array (20)
ROWTYP: the array of row types; (MAXM)
+1 means less than or equal to
0 means equation
-1 means greater than or equal to
-2 means a free row (functional)

STATUS: an indicator for each variable: (MAXN)
0 means the variable is out at its lower bound
K means that this is the K-th basic variable
-1 means that this variable is out at its upper bound
-2 means that this is a free variable; once in the basis, it never leaves
-3 means that this is an artificial variable; once it leaves the basis, it never re-enters
-4 means that the variable is locked out of the basis at its lower bound
-5 means the variable is locked out of the basis at its upper bound

UJ: upper bound for decision variables

Variables local to the main program. Some of these are used by GWATER and XSETUP. Dimensions are for GWATER and XSETUP respectively.

CJX: array of all the objective function coefficients, (NPT2); (NPT*4+LST*4)
COLMAX: maximum number of non-zeros in each column
KOLA: value of the non-zeros in coefficient matrix, (NPT2,8); (NPT*4+LST*4,10)
KOLI: row number used to locate non-zeros, (NPT2,8); (NPT*4+LST*4,10)
KOLLEN: number of non-zeros in each column, (NPT2,8)
LH: array of lower bounds on decision variables (NPT2)
NCOLSA: number of structural variables (columns of the coefficient matrix)
LST: LSECT*NT
NR: number of elements in each column of the finite difference grid
MC: number of elements in each row of the finite difference grid
NP: \((NR-2)^*(MC-2)\), number of internal nodes in finite difference grid

NT: number of time periods

NPT: NP*NT

NPT 2: \((NP*NT)^2\)

MAXA: the maximum number of non-zeros that will be encountered during the current run, usually equal to MA

MAXM: the maximum number of constraints that will be encountered during the current run, usually equal to MA

MAXN: the maximum number of variables that will be encountered during the current run

UH: array of upper bounds on decision variables, \((NPT^2)\)

The objective functions are specified by the value of IOBJ, as follows:

For GWUSER

<table>
<thead>
<tr>
<th>IOBJ</th>
<th>TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>maximize head</td>
</tr>
<tr>
<td>2</td>
<td>maximize pumpage</td>
</tr>
<tr>
<td>3</td>
<td>minimize pumpage</td>
</tr>
</tbody>
</table>

For CONJUN

<table>
<thead>
<tr>
<th>IOBJ</th>
<th>TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>maximize head</td>
</tr>
<tr>
<td>2</td>
<td>maximize supply</td>
</tr>
<tr>
<td>3</td>
<td>minimize water transfer</td>
</tr>
<tr>
<td>4</td>
<td>minimize supply</td>
</tr>
<tr>
<td>5</td>
<td>maximize supply while minimizing water transfer</td>
</tr>
</tbody>
</table>
APPENDIX B: Flowchart for Main Programs GWUSER/CONJUN

START

DIMENSION ARRAYS

For GWUSER call Subroutine GWATER

For CONJUN call Subroutine XSETUP

Subroutine GWATER or XSETUP
(Reads in data, generates constraint matrix and bounds)

Call RHSHEAD (computes RHS values)

Call AMATRIX (Computes finite difference matrix)

Compute values of variables used for adjustable dimensions in XMP routines

Subroutine XMAPS (determines if enough memory has been dimensioned)

Set ROWTYP, equation types ≥, = or ≤?

A
Generate the objective function coefficients, CJX, based on value of IOBJ

Read in the columns one at a time and pass them to XMP through CJ, LJ, UJ, COLA, COLI, COLLEN

Subroutine XADDAJ (sets up constraint columns)

Subroutine XADDAJ (stores bounds on variables)

Start all decision variables at their lower bounds (STATUS = ∅)

Subroutine XSLACK (sets up slack, surplus and artificial variables as the starting basis according to the equation types)

Subroutine XPRIML (performs primal simplex method)
Subroutine XPRINT (prints out LP solution)

Subroutine XFORMAT (prints out solution in grid form)

STOP
APPENDIX C:

Program Listing for GWUSER/CONJUN
F is the maximum number of attractive non-basic variables to be placed in the candidate list.

LEN is the number of columns to scan in setting up the candidate list.

FACTOR is the re-factorization frequency, F=6.

LENH = 200

FACTOR = 20

LENH is the length of the memory array. The memory array contains the data structures for the
original problem data and the basis inverse representation.

LENH = 30000

PRINT specifies the level of output desired:
0 means ERROR MESSAGES only
1 means TERMINATION MESSAGES (OPTIMAL, UNBOUNDED, INFEASIBLE)
2 means a LOG LINE for every ITERATION.

PRINT = 1

N is the number of constraints. 
NCOLA is local to this program.
NCOLA is the number of structural variables (columns of the coefficient matrix).

N=1000

NCOLA=NF2

TYPE is the BOUND TYPE:
1 means LOWER BOUND-0, UPPER BOUND-INFINITY
2 means LOWER BOUND-0, UPPER BOUND-BOUND
3 means LOWER BOUND-0 for every non-free variable
4 means both bounds are general.

TYPE=1

NCOLA=NF2

MAY BE THE BOUND TYPE:
1 means LOWER BOUND-0, UPPER BOUND-INFINITY
2 means LOWER BOUND-0, UPPER BOUND-BOUND
3 means LOWER BOUND-0 for every non-free variable
4 means both bounds are general.

MAY BE THE BOUND TYPE:
1 means LOWER BOUND-0, UPPER BOUND-INFINITY
2 means LOWER BOUND-0, UPPER BOUND-BOUND
3 means LOWER BOUND-0 for every non-free variable
4 means both bounds are general.

MAY BE THE BOUND TYPE:
1 means LOWER BOUND-0, UPPER BOUND-INFINITY
2 means LOWER BOUND-0, UPPER BOUND-BOUND
3 means LOWER BOUND-0 for every non-free variable
4 means both bounds are general.

MAY BE THE BOUND TYPE:
1 means LOWER BOUND-0, UPPER BOUND-INFINITY
2 means LOWER BOUND-0, UPPER BOUND-BOUND
3 means LOWER BOUND-0 for every non-free variable
4 means both bounds are general.

MAY BE THE BOUND TYPE:
1 means LOWER BOUND-0, UPPER BOUND-INFINITY
2 means LOWER BOUND-0, UPPER BOUND-BOUND
3 means LOWER BOUND-0 for every non-free variable
4 means both bounds are general.

MAY BE THE BOUND TYPE:
1 means LOWER BOUND-0, UPPER BOUND-INFINITY
2 means LOWER BOUND-0, UPPER BOUND-BOUND
3 means LOWER BOUND-0 for every non-free variable
4 means both bounds are general.

MAY BE THE BOUND TYPE:
1 means LOWER BOUND-0, UPPER BOUND-INFINITY
2 means LOWER BOUND-0, UPPER BOUND-BOUND
3 means LOWER BOUND-0 for every non-free variable
4 means both bounds are general.
C SPECIFY THE ROW TYPES.
C RowTyP = 1 is a LESS THAN CONSTRAINT
C RowTyP = 2 is a GREATER THAN CONSTRAINT
C RowTyP = 0 is an Equality CONSTRAINT
DO 902 J=1,NRT
C
902 CONTINUE
J=NRT
DO 903 J=J+1
C
903 CONTINUE
C
C N is the current number of variables.
N=0
C
C PLAN IN THE COLUMNS ONE AT A TIME AND PASS THEN
C TO XNF.
C COLLEN is the number of non-zeros.
C COLA is the list of non-zeros, and
C COLT is the corresponding list of row numbers.
C CJ is the objective coefficient (MAX ASSUMED).
C LJ is the upper bound (Must be specified if RowTyP=3 or 4)
C LJ is the lower bound (Must be specified if RowTyP=1)
C JX is a sequence number (the columns are numbered
C sequentially as they are entered).
C
C INITIALIZE OBJECTIVE FUNCTION COEFFICIENTS TO ZERO
DO 102 J=1,NRT
102 CJ(J)=0.
IF(CX(J).GE.0.2) GO TO 105
IF(CX(J).LE.0.4) GO TO 106
IF(CX(J).LE.0.1) GO TO 100
C MAXIMIZATION OF HEADS
DO 103 J=1,NRT
J=NRT
IF(CX(J).GT.0.) THEN
CJ(J)=1.0
ENDIF
103 CONTINUE
GO TO 100
C
C MAXIMIZATION OF PUNISHES
DO 105 J=1,NRT
J=NRT
IF(CX(J).GT.0.) THEN
CJ(J)=1.0
ENDIF
105 CONTINUE
GO TO 100
C
C MINIMIZATION OF PUNISHES
C
107 DO 104 *(NRT)=1,NRT
IF((H(J),BT,0.) . THEN
CJ(J)=1.0
ENDIF
104 CONTINUE
GO TO 100
C
C MAXIMIZATION OF HEAD FOR THE LAST PERIOD ONLY
C
100 NRT=1-NRT
J=NRT
IF((H(J),LTE,0.) GO TO 104
CJ(J)=1.0
104 CONTINUE
GO TO 100
C
C CONTINUE
C
108 CONTINUE
DO 120 J=1,NCOLS
IF((H(J),LTE,0.) THEN
CJ(C(J))
LI=H(J)
ELSE
CJ(C(J))
LI=ABS(H(J))
LI=
ENDIF
COLLEN=COLLEN(J)
DO 110 K=1,COLLEN
CDE(K)=COLLEN(J)
CDE(K)=H(J)
ELSE
CDE(K)=H(J)
ENDIF
110 CONTINUE
GO TO 120
C
C ENTER EACH COLUMN OF THE MATRIX INTO XNF
CALL XADDAJ,CJ,COLL, COLLEN, COLMAX,
X IDEMX,J,LEM9C,CMAP, STATUS
IF((H,J),LTE,2160) GO TO 120
IF((H,J),GE,3) J=0.0
ENDIF
C
C STORE BOUNDS ON EACH VARIABLE
CALL XADDAJ,HNDY+,IDEX,J,LEM9C,CMAP, STATUS,J)
120 CONTINUE
C
C START ALL OF THE STRUCTURAL VARIABLES AT THEIR
C LOWER BOUNDS.
DO 170 J=1,N
170 CONTINUE
C
C CONVERT EACH EQUATION INTO AN EQUALITY CONSTRAINT
C
I  ORMT(5I8F8.0)
READ(5,12) I'XfUY
WRITE(6,13)(X' Pu ,PY
11 FORMAT(4I8)
12 FORMAT(2F8.0)
13 FORMAT(5XrF0.2/r)
WRITE(6,14) '5Xr 16HDELTA X EQUALS rF0.2r/r
* 5Xr 16IIPELTA Y EQUALS r18. 2»//r)
READ(6,15) nXrliY
READ(5, 11 ) TRANSrOIIPOUNP»OQPOUNP r  I  UNIT
NRM1 -Mf<-1
NRM2-NR-2
MCM1=MC-1
MCM2=MC-2
NPpNRI12*MCM2
M l *  l=Nr*NT
Nri2^MPT*2
C
C
C REFINE HEAPS ON BOUNDARIES
REAP(5,20) <  BCIIDR1  <  J )  r  J-1  r  HCM2)
RE A r t  (Sr 20) (  BCHPRN (  J )  r  J-1  r  MCM2 >
Rt AIK Sr 20) !BCIIDC1<1),I«1 »NRM2)
REAP(5,20) (DCHDCM!I),I=1,NRM2)
20 FORMAT!1  OF8.0)
C
C INPUT OROUNDWATER AQUIFER PROFERTlRESr
C INITIAL CONDITIONS
C
C IF TRANS2 1
  IF THEN I  RANX IRANY
C IF GI|POUNP=lr THEN OFFER ANP LOWER HOUNDS ON HEAPS ARE
C I I I E  SAME AT EACH NODE
C IF TRANS=*2r THEN TRANX AND TRANY ARE INPUT SEPARATELY
C IF GIIP00NP=2r THEN All HEAP POUNDS ARE INPUT SEPARATELY
C
IFURANS.EU.DTIIEN
  DO 2t JC=1
  HCFAD<5
30) (TRANX(  IR,.IC) r  IR=1
NR)
REAP (  S
30) <  STOR <  IR
.I C )
IR=1
NR )
RE AD (5, 30) (HE ADO (  IRr.JC)
IR=1 rNR)
DO 19 IR=1,NR
TRANY(IR,JC)=1RANX(IRrJO)
19 CONTINUE
21 CONTINUE
ELSE
  DO 23 IO I
  MC
READ(S,30) (TRANX<IR,JC) 1R=1.NR)
READ(S,30) (TRANY<IR,JC)
I R M
NR)
READ (Sr 30) (STOR< IR,JC )
NR = - 1
NR)
READ (Sr 30) (IIEADO< IR,JC )
NR*1,
23 CONTINUE
21 CONTINUE
END IF
C
C ♦♦♦REAP IN PFFAIJl 1  UPPER AND I  ODER POUNDS ON HEADS
C
IF(OHPOUNP.EO.I>THEN
  READ(S,30) I I I I D O I I N P
L  HDOUND
UR I  IE <  A
30 )  UIIDOUND
I  HDOUND
DO 25 JC»2*HCM1
DO 26 TR-2
NRM1
IHPOUNP
26 CONTINUE
25 CONTINUE
ELSE
  DO 22 JC=2
  MCM1
READ!5,31) (IIEADO< IR, JC )
IR=2
MCM1)
READ(5,31) (LHFAP(IR,JC)
IR=2
NRM1)
WRITE(6,31)(LHEAD(IR,JC)
IR=2
NRM1)
WRITE(6,31) (LHEAD(IR,JC)
IR=2
NRM1)
22 CONTINUE
END IF
C
C ♦♦♦REAP IN UPPER ANP IOUER POUNDS ON PUMPAGE
C
IF(OOPOUND. EO.2)THEN
  DO 27 JC=2, MCM1
  READ!5,
31) (UPIJMP(IR,  JC)  ,  IR=2
NRM1)
READ!5,31) ILPUMP!IR,JC)rIR=2,NRM1)
27 CONTINUE
ELSE
  REAP!5,31) UOPOUND
LPOUND
DO 29 JC=1,MC
DO 28 IR=1,NR
UPIJMP (IR,JC)
LPOUMP!IR,JC)
LPUMP!IR,JC)LPOUND
29 CONTINUE
END IF
C
C READ IN THE WATER DEMAND FOR EACH PERIOD
READ!5,32) !DEMAND!IT)rIT=l,MT)
WR ITF!6r*> !DEMAND!IT),IT=1,NT)
C
C ECHO PRINT THE INPUT PARAMETERS
C
C ♦♦♦URIIE OUT HEAPO
TRANX
TRANX
STOR
94 FORMAT !5Xr 13IIINITIAL HEAPS,//)
95 FORMAT !//, 1  T i l l  RAN Y  ( OPP/I T  )  ,//)
96 FORMAT!//, 5X.  4 I I S I  OR,//)
97 FORMAT !5Xr 13IIINITIAL HEAPS,//)
33 FORMAT(1X,0//)
34 FORMAT(1X,0//)
35 FORMAT(1X,0//)
36 FORMAT(1X,0//)
C #######WRITE OUT BOUNDS ON SURFACE
37 WRITE(3490)
39 FORMAT(1X,0//)
40 FORMAT(1X,0//)
41 WRITE(3490)
42 FORMAT(1X,0//)
43 DO 393 JC=2-MC1
44 WRITE(3490)
45 FORMAT(1X,0//)
46 CONTINUE
47 IF ITMAT=1, THE
49 CONTINUE
50 DO 24 IJ=1-MH
51 CONTINUE
52 CONTINUE
53 CONTINUE
54 CONTINUE
55 CONTINUE
56 CONTINUE
57 CONTINUE
58 CONTINUE
59 CONTINUE
60 CONTINUE
61 CONTINUE
62 CONTINUE
63 CONTINUE
64 CONTINUE
65 CONTINUE
66 CONTINUE
67 CONTINUE
68 CONTINUE
69 CONTINUE
70 CONTINUE
71 CONTINUE
72 CONTINUE
73 CONTINUE
74 CONTINUE
75 CONTINUE
76 CONTINUE
77 CONTINUE
78 CONTINUE
79 CONTINUE
80 CONTINUE
81 CONTINUE
82 CONTINUE
83 CONTINUE
84 CONTINUE
85 CONTINUE
86 CONTINUE
87 CONTINUE
88 CONTINUE
89 CONTINUE
90 CONTINUE
91 CONTINUE
92 CONTINUE
93 CONTINUE
94 CONTINUE
95 CONTINUE
96 CONTINUE
97 CONTINUE
98 CONTINUE
99 CONTINUE
100 CONTINUE
101 CONTINUE
102 CONTINUE
103 CONTINUE
104 CONTINUE
105 CONTINUE
106 CONTINUE
107 CONTINUE
108 CONTINUE
109 CONTINUE
110 CONTINUE
MAXA IS THE MAXIMUM NUMBER OF NON-ZEROS ALLOWED
IN THE COEFFICIENT MATRIX.
MAXB IS THE MAXIMUM NUMBER OF CONSTRAINTS ALLOWED.
MAXA IS THE MAXIMUM NUMBER OF VARIABLES ALLOWED.
MAXA IS THE MAXIMUM NUMBER OF NON-ZEROS ALLOWED
IN ANY ONE MATRIX COLUMN.
MAXJ=MAXK=MAXL=MAXM=MAXN=
10000
F IS THE MAXIMUM NUMBER OF ATTRACTIVE NON-BASIC
VARIABLES TO BE PLACED IN THE CANDIDATE LIST.
MAXA IS THE NUMBER OF COLUMNS TO SCAN IN SETTING UP THE
CANDIDATE LIST.
FACTOR IS THE RE-FACTORIZATION FREQUENCY.
PRNSP=0.24
FACTOR=50
LENJ IS THE LENGTH OF THE MEMORY ARRAY. THE
MEMORY ARRAY CONTAINS THE DATA STRUCTURES FOR THE
ORIGINAL PROBLEM DATA AND THE BASIS INVERSE
REPRESENTATION.
LENJ=25000
PRINT SPECIFIES THE LEVEL OF OUTPUT DESIRED:
0 MEANS ERROR MESSAGES ONLY
1 MEANS TERMINATION MESSAGES ONLY, UNBOUNDED, INFEASIBLE)
2 MEANS A LEC LINE FOR EVERY ITERATION.
PRNSP=1
N IS THE NUMBER OF CONSTRAINTS.
M = IS LOCAL IN THIS PROGRAM.
M = IS THE NUMBER OF STRUCTURAL VARIABLES (COLUMNS
OF THE COEFFICIENT MATRIX).
MAXA=
MAXJ=MAXK=MAXL=MAXM=MAXN=
10000
PRNSP=0
BROTP IS THE BOUND TYPE:
1 MEANS LOWER BOUND=0, UPPER BOUND=INFINITY
2 MEANS LOWER BOUND=0, UPPER BOUND=BOUND
3 MEANS LOWER BOUND=0 FOR EVERY NON-FREE VARIABLE.
4 MEANS BOTH BOUNDS ARE GENERAL.
HYTE2 IS HYTE2=2, THEN EACH VARIABLE
1,...,HYTE2 MUST EITHER HAVE THE COMMON UPP.
ER OR ELSE HAVE A FREE VARIABLE IF THE REMAINING
VARIABLES HYTE2,...,N MUST EITHER HAVE LOWER
BOUND ZERO AND UPPER BOUND INFINITY OR ELSE BE A
FREE VARIABLE.
ROUND IS THE COMMON UPPER BOUND IF BROTP=2.
IF BROTP IS NOT 2, THEN HYTE2 AND ROUND ARE IGNORED.
BROTP=4
HYTE2=0
BROTP=0.
Determine if Enough Memory Has Been Dimensioned
CALL KPA (SP; IDER: LEMM; LEMM; LEMM;)
X MAX=MAX+1; MAXA; MAXJ; MAXK; MAXL; MAXM; MAXN; MAXN;
N IS THE CURRENT NUMBER OF VARIABLES, N=0
READ IN THE COLUMNS ONE AT A TIME AND PASS THEM
TO XMAX.
COLL IS THE NUMBER OF NON-ZEROS.
COLL IS THE LIST OF NON-ZEROS AND
COLL IS THE CONSISTENT LIST OF NON-ZEROS.
CJ IS THE OBJECTIVE COEFFICIENT (MAX ABS.
CU) IS THE UPPER BOUND (MUST BE SPECIFIED IF BROTP=3 OR 4)
CJ IS THE LOWER BOUND MUST BE SPECIFIED IF BROTP=4)
CU IS A SEQUENCE NUMBER THE COLUMNS ARE NUMBERED
SEQUENTIAL AS THEY ARE ENTERED.
CU=10
DO 110 CJ IN 1, NCOLL
110 IF (CJ.EQ.2) 100 TO 125
IF (CJ.EQ.3) 100 TO 145
IF (CJ.EQ.4) 100 TO 145
IF (CJ.EQ.5) 100 TO 155
C
C MAXIMIZATION OF HEAPS WHILE MINIMIZING WATER TRANSFER
DO 120 CJ IN 1, NMAX
120 C(J)=1.0
N1,M1=J, M1=J, N1=M1
DO 125 J=1, N1+M1
125 C(J)=1.0
DO 150 C(J)=250
MAXIMIZE SUM OF WITHDRAWALS FROM BOTH SOURCES

\[ \begin{align*}
&\text{MAX} \sum (\text{WITHDRAWAL}) \\
&\text{subject to} \quad \text{constraints}
\end{align*} \]

\[ \begin{align*}
&\text{for each variable}
\end{align*} \]

MINIMIZE WATER TRANSFER

\[ \begin{align*}
&\text{MIN} \sum (\text{TRANSFER}) \\
&\text{subject to} \quad \text{constraints}
\end{align*} \]

MINIMIZE WATER WITHDRAWAL FOR SUPPLY

\[ \begin{align*}
&\text{MIN} \sum (\text{WITHDRAWAL}) \\
&\text{subject to} \quad \text{constraints}
\end{align*} \]

MAXIMIZE WATER SUPPLY YET MINIMIZE WATER TRANSFER

\[ \begin{align*}
&\text{MAX} \sum (\text{SUPPLY}) \\
&\text{subject to} \quad \text{constraints}
\end{align*} \]

ENTER EACH COLUMN OF THE COEFFICIENT MATRIX INTO XMP

\[ \begin{align*}
&\text{CALL } \text{XMPD}(\text{matrix})\text{ into XMP}
\end{align*} \]

CONVERT EACH EQUATION INTO AN EQUALITY CONSTRAINT

\[ \begin{align*}
&\text{CALL } \text{XMPD}(\text{matrix})\text{ into constraints}
\end{align*} \]

INPUT PRIMAL SIMPLEX METHOD

\[ \begin{align*}
&\text{INPUT } \text{PRIMAL SIMPLEX}\text{ method}
\end{align*} \]

PRINT OUT VALUES OF DECISION VARIABLES

\[ \begin{align*}
&\text{CALL } \text{XPRD}(\text{matrix})\text{ into output}
\end{align*} \]

WRITE (I0L0Gr260) TERMINATION CODE

\[ \begin{align*}
&\text{WRITE } (\text{I0L0Gr260}) \text{Termination code}
\end{align*} \]

EXECUTE BUT VALUES OF DECISION VARIABLES

\[ \begin{align*}
&\text{CALL } \text{XPRD}(\text{matrix})\text{ into output}
\end{align*} \]

FINAL OUTPUT OF HEADS AND PUMPAGES

\[ \begin{align*}
&\text{FINAL } \text{OUTPUT}\text{ of heads and pumpages}
\end{align*} \]
WRITE(4,96)
96 FORMAT(//X,HEADER,/)  
DO 79 JC=1,JC  
33 FORMAT(11H0)  
34 FORMAT(1IF8.5)  
35 FORMAT(1IF8.5)  
WRITE(6,36)
36 FORMAT(11H0)  
C **READ IN UPPER AND LOWER BOUND ON FLOW**
IF(BOUND(IVER,1).EQ.1) THEN
  READ5(30) UBOUND(IVER)
  DO 29 JC=1,JC  
    UBOUND(JC)=UBOUND(IVER)
  LBOUND(JC)=LBOUND(IVER)
  CONTINUE
  ELSE
    DO 29 JC=1,JC  
      LBOUND5(JC)=(LBOUND(IVER)+1,2+NBND)
    CONTINUE
  DO 50 LBOUND(JC)=LBOUND(IVER)
  CONTINUE
C INPUT UPPER AND LOWER BOUNDS OF FLOW RATES
C FOR EACH STREAM SECTION AND DIFFERENT TIME
C PERIODS.
C UBD - UPPER BOUND OF STREAM FLOW (CFD)
C LBD - LOWER BOUND OF STREAM FLOW (CFD)
C READ5(30) UBOUND(JC)+1,LBOUND(JC))
C READ5(30) LBOUND(JC)+1,LBOUND(JC))
C DO 42 IT=1,IT  
      LBOUND(JC)=LBOUND(JC)+1
    CONTINUE
C Continue
C READ5(30) LBOUND(JC)+1,LBOUND(JC))
C DO 43 IT=1,IT  
      LBOUND(JC)=LBOUND(JC)+1
    CONTINUE
C Continue
C INPUT FLOW RATE AT UPSTREAM END OF CHANNEL
C SYSTEM FOR VARIOUS TIME PERIODS
C OSD - STREAM FLOW AT UPSTREAM END OF SYSTEM (CFD)
C READ5(30) (OSD(JC)+1,OSD(JC))
C READ(JC)+1,READ(JC))
C DO 44 IT=1,IT  
      READ(JC)=READ(JC)+1
    CONTINUE
C Continue
C INPUT THE WATER DEMAND FOR EACH TIME PERIOD
C DEMAND - TOTAL WATER DEMAND (CFD)
C READ5(32) (DEMAND(JC)+1,DEMAND(JC))
C FORMAT(11H0)  
32 FORMAT(11H0)  
C COMPUTE SPACE AND TIME INCREMENT CONSTANTS
C TX=9/7  
C TX=9/7  
X IT=3/7
X IT=3/7
Y IT=3/7
Y IT=3/7
C Set bounds for readings and pumping rates
C into arrays U and L.U
**COMPUTE THE RIGHT-HAND-SIDE OF THE FINITE DIFFERENCE CONSTRAINTS**

```c
XP=0
DO 59 II=1,NI
DO 57 JC=1,NJI
DO 58 IF=1,NIF
IM=1-1
IF1=IF1
JC1=JC1
59 CONTINUE
57 CONTINUE
58 CONTINUE
```

**CALL ENSURE(IT,IR,JC,IF,IM,IF1,JC1,IF,RH00,RH100,RH110,RH01,RH02,RH101,RF,FR,FRH00,FRH100,FRH110,FRH01,FRH02,FRH101,FRH111,X)
```

**SET UP THE R.H.S. CORRESPONDING TO WATER FLOW**

```c
LS=KFT
DO 60 IT=1,NT
LS(4+5*(IT))
60 CONTINUE
```

**SET UP R.H.S. FOR WATER BALANCE EQUATION**

```c
LS(KFT11)
DO 61 IT=1,NT
LS(4+6*(IT))=LS(4+5*(IT))
61 CONTINUE
```

**FILL IN THE CONSTRAINT MATRIX**

```c
XP=0
DO 62 IF=1,NIF
DO 63 IR=1,NIR
IM=1-1
IM1=IM-1
JCH1=JC1-1
JF1=JC1
62 CONTINUE
63 CONTINUE
60 CONTINUE
```

**CALL AGRIDICT(IF,JCH1,IF1,JF1,IF,RH01,RH02,RH101,RF,FR,FRH01,FRH02,FRH101,FRH111,X)
```

**SET UP CONSTRAINT COLUMN UNDER SURFACE FROM DRAINAGE ADDER**

```c
KP=KFT
DO 71 IT=1,NT
DO 70 IR=1,NIR
IF=KFT
IF(RH01)=2
COL1(1,1)=IF
COL1(1,1)=0.0
COL1(1,2)=0.0
COL1(NT,2)=0.0
70 CONTINUE
71 CONTINUE
```

**SET UP CONSTRAINT COLUMNS UNDER RECHARGE TO DRAINAGE ADDER**

```c
KP=KFT2
DO 90 IT=1,NT
DO 89 IR=1,NIR
IF(KF)=1
IF(RH01)=2
COL1(1,1)=IF
COL1(1,1)=0.0
COL1(1,2)=0.0
COL1(2,2)=0.0
90 CONTINUE
```

**SET UP CONSTRAINT COLUMN UNDER AUGMENTED FLOW FROM DRAINAGE TO STREAM**

```c
KP=KFT3
DO 110 IT=1,NT
DO 109 IR=1,NIR
 IF=KFT1
 IF(RH01)=2
 IF(RH01)=0
 IF(RH01)=KFT2
 COL1(1,1)=IF
 COL1(1,1)=0.0
 COL(1,2)=0.0
 COL(NT,2)=0.0
110 CONTINUE
100 CONTINUE
```

**SET UP CONSTRAINT COLUMN UNDER STREAM FLOW OS**

```c
RHS(KFT4)
RHS(KFT4)
DO 130 IT=1,NT
DO 129 IR=1,NIR
RHS(KFT4)
RHS(KFT4)
129 CONTINUE
130 CONTINUE
```

**SET UP CONSTRAINT COLUMN UNDER STREAM FLOW OS**

```c
IF(OL.FS.CT) GO TO 115
COL1(1,1)=IF
COL1(1,1)=1.0
COL1(1,2)=1.0
COL1(NT,2)=1.0
115 CONTINUE
```
CO4(4K05-2)=-1.0
GO TO 110
115 COLB4(4K05)-1
CO4(4K05+1)=ROW
116 COL4(4K05+1)=1.0
120 CONTINUE
130 CONTINUE

C SET UP CONSTRAINT COLUMN UNDER WATER
C SUPPLY FROM STREAM FLOW, X.
C
C KY=4FT41LIST
KY=4FT41
DO 130 I=1,10HT
DO 130 I=1,10HT
KY=4FT4
KY=4FT4
130 CONTINUE

C SET UP CONSTRAINT COLUMN UNDER STREAM
C FLOW FOR RETAINING, T.
C
C KY=4FT41LIST
KY=4FT41
DO 130 I=1,10HT
DO 130 I=1,10HT
KY=4FT4
KY=4FT4
130 CONTINUE

C SET UP CONSTRAINT COLUMN UNDER AUGMENTED
C STREAM FLOW FROM AUGMENT, Z.
C
C KZ=4FT41LIST
KZ=4FT41
DO 130 I=1,10HT
DO 130 I=1,10HT
KZ=4FT4
KZ=4FT4
130 CONTINUE

140 CONTINUE
150 CONTINUE
160 CONTINUE
170 CONTINUE
180 CONTINUE
190 CONTINUE
RETURN
END