VALUATION OF A GROUND-WATER SUPPLY
FOR MANAGEMENT AND DEVELOPMENT

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ABSTRACT

This paper is an attempt to analyze problems of ground-water management and development in terms of valuation of a resource or property that represents a source of future money receipts. An analytical expression is derived which gives both present worth of gains forthcoming from resource exploitation over a variable time period and the remaining worth of a ground-water supply after it has been partially depleted. With water-level position selected as the denominator common to both the system and its economic worth, a course of exploitation is charted so that (1) present worth of future returns is maximized, and (2) water rights are protected to the extent that water levels are not lowered below the economic limit of pumping. The results enable a conceptual valuation of (1) decision rules for efficiency in management, (2) optimal mining yield for specified conditions, and (3) the state of development of the resource at any time.
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SYMBOLS

The more important symbols that appear in the text are defined below.

\( V \), Present worth of future net receipts.
\( \gamma \), Rate of interest.
\( e^{-\gamma t} \), present worth factor.
\( t \), time.
\( p(t) \), Net profit per unit pumped
\( q(t) \), Rate of withdrawal from storage.
\( R(t) \), Rate of natural replenishment.
\( T \), Time after which pumping is no longer economical.
\( t' \), Time after which pumping is limited to recoverable natural replenishment.
\( T' \), Time after which pumping is limited to recoverable replenishment if present worth is to be maximized.
\( p(T') \), Net profit per unit pumped at time \( T' \).
\( q(T') \), Withdrawal from storage at time \( T' \).
\( R(T') \), Recoverable natural replenishment at time \( T' \).
\( h_0 \), Pumping level in wells when pumping is in equilibrium with recoverable natural replenishment.
\( h(t) \), Water-level decline as a function of time.
\( h(T') \), Total water-level decline at time \( T' \).
\( \epsilon \), Net profit per unit pumped, evaluated with respect to \( h_0 \).
\( S \), Volume of ground-water removed from storage.
K,  Water-level decline per acre-foot of storage withdrawals.

1/K, Acre-feet of withdrawals per foot of water-level decline.

\( a_m \),  Mining yield which maximizes present worth.

In addition to obvious difficulties of a valuation of this type, exploitation of a productive asset such as water generates a whole suite of interesting social problems. These are the extra-social costs, as they are commonly referred to, which are sometimes inflicted on individuals as a result of public or private decisions (Hirschleifer, et al., 1969, p. 38). As policies that improve conditions for everyone are not generally possible in the development of water resources, and ground water in particular, names for arbitrary use of these assets have received considerable attention in recent years. The heart of the problem is...
INTRODUCTION

Purpose

This paper presents a conceptual framework for efficiency in management of ground-water basins. The method proposed utilizes a dollar value for water produced as an economic measure of the system and decisions pertaining thereto. The approach taken is to determine the influence of specified input factors on the time distribution of net costs and returns from mining ground water in arid basins in the Great Basin, or similar environments. The objective is to formulate decision rules for a wide variety of conditions so that problems of management associated with full exploitation of a water-supply system can be reduced to problems of valuation of any property that represents a source of future money receipts.

In addition to obvious difficulties of a valuation of this type, exploitation of a productive asset such as water generates a whole suite of interesting social problems. These are the extra-social costs, as they are commonly referred to, which are sometimes inflicted on individuals as a result of public or private decisions (Hirshleifer, et al, 1960, p. 79). As policies that improve conditions for everyone are not generally possible in the development of water resources, and ground water in particular, means for eradication of these effects have received considerable attention in recent years. The heart of the problem is
generally conceded to be withdrawal interdependency, and methods proposed for its abolishment include use tax and assignment of quotas (Milliman, 1956, p. 426-437); modification of the appropriation doctrine (Hirshleifer, et al., 1960, p. 66); and compensation in kind (Trelease, et al., 1965, p. 235). Loss to one group as a result of gain to another is also treated as a problem in valuation.

Review of Current Objectives and Concepts in Ground-Water Management

This section presents a brief review of prevailing concepts and objectives in the practice of ground-water management. Material selected for review provides a background for sections to follow, and is logically divided into three categories: (1) physical objectives, including conventional concepts of safe and alternative yield; (2) socio-economic objectives, including the broad theme of welfare economics and its influence on water law, and; (3) operations research to achieve an optimal design of a water-resource system in which the ground-water reservoir serves as a regulating device for the total water supply.
CONVENTIONAL CONCEPT OF SAFE YIELD

Ground water is considered a renewable resource to the extent that the hydrologic cycle in which it partakes is renewable. Of the precipitation that falls on the land surface, a certain amount is available to augment natural ground-water flow. This natural replenishment, considered a flow resource, has made available a large volume of ground-water storage, a stock resource, the latter normally considered nonrenewable when exploited in common with the flow. Under equilibrium conditions, the amount of precipitation augmenting ground-water flow may: (1) leave the aquifer as natural discharge; (2) be captured by wells, or; (3) be distributed between natural and artificial discharge, the net effect of which allows the system to function independently of storage. Exploitation of the flow resource can be sustained perennially whereas exploitation of the nonrenewable resource, or part of it, is referred to as "mining", implying a one-time reserve.

Many decisions pertaining to developmental aspects of ground water are based purely on hydrologic, or physical, considerations. The criterion for making these decisions is generally the convention of "safe yield", a controversial concept based on the renewability of the hydrologic cycle, or alternative yield, which may or may not be sustained perennially (A.S.C.E., 1961, p. 53). Safe yield was first defined by Meinzer (1923, p. 55-56) as "the rate at which water can be withdrawn from an aquifer for human use without
depleting the supply to the extent that withdrawal at this rate is no longer economically feasible". This definition was interpreted by Conkling (1946, p. 283) as an annual extraction of ground water which does not: (1) exceed average annual recharge; (2) lower the water table so that the permissible cost of pumping is exceeded, or; (3) lower the water table as to permit intrusion of water of undesirable quality. Protection of existing rights is a fourth condition suggested by Banks (1953, p. 223).

The single-valued concept of safe yield defined above ambiguously encompasses hydrologic, economic, quality, and legal considerations. The controversial nature of the concept is clearly demonstrated in 43 pages of discussion of Conkling's pioneer paper (28 pages) by no less than 10 authorities. In an effort to remove some of the ambiguity, the Committee on Ground Water of the American Society of Civil Engineers introduced four concepts of yield from a ground-water basin (A.S.C.E., 1961, p. 52-69). These are defined as follows:

1) Maximum-sustained yield is the maximum rate at which water can be withdrawn perennially from a particular source.

2) Permissive-sustained yield is the maximum rate at which water can economically and legally be withdrawn perennially from a particular source for beneficial purposes, without bringing about
some undesired result.

3) Maximum-mining yield is the total volume of water in storage in a particular source that can be extracted and utilized.

4) Permissive-mining yield is the maximum volume of water in storage in a particular source that can economically and legally be extracted and utilized for beneficial purposes, without bringing about some undesired result.

The two categories of sustained yield are usually expressed as a volume per unit time, such as acre-feet per year, and can be maintained perennially. Maximum-sustained yield is a use rate that is determined by and limited to average input, or natural replenishment, and is a physical objective. Permissive-sustained yield is invariably less than natural input owing to physical or man-made limitations. A case in point may be a withdrawal rate less than annual replenishment designed to prevent salt-water encroachment, land subsidence, interference with existing rights, or some other undesirable result.

Mining yield is expressed as a volume of extractable nonrenewable water in a ground-water basin. In this sense it is an exhaustible resource of fixed supply, somewhat analogous to a mineral or petroleum deposit. It may be mined slowly, or rapidly, but the duration of extraction is definitely limited. Maximum-mining yield is that volume
of nonrenewable water that can be economically exploited whereas permissive-mining yield is that part of maximum-mining yield that can be exploited without bringing about some undesired result. Development of ground-water resources in some parts of New Mexico is reported to be accomplished on a maximum-mining yield basis (A.S.C.E., 1961, p. 68). A hypothetical application of the concept of permissible mining-yield might be the withdrawal of a volume of ground-water storage between a seaward salt water-fresh water interface and this same interface projected some tolerable distance inland.

The term "overdevelopment" is generally reserved for the condition where withdrawals exceed sustained yield. Such a condition has been reported in several places including Antelope Valley, California (Synder, 1955, p. 92-97) and Las Vegas Valley, Nevada (Malmberg, 1961, p. 20-21). Indeed, Bagley (1961, p. 146) estimates that over one-fourth of all ground-water withdrawals in the United States are mined, primarily in California, Arizona, New Mexico, and Texas.

The question of whether ground water should be managed on a sustained or mining yield basis is not yet fully resolved, and is controlled more by local conditions and demands rather than premeditated policy decisions in advance of their absolute necessity. This is understandable in that there is likely to be little public sympathy for an
announced depletion policy whereas a policy of sustained resource use is more acceptable to the general public. Whatever the merits of alternative yield concepts, they are definitely ingrained in ground-water management, at least in the United States, and have value to the extent of providing a first estimate subject to revision in future years.

WELFARE ECONOMICS

Goals of income maximization for society in general and profit-maximizing behavior for consumers of resource outputs have been suggested as an alternative approach in formulating analysis of resource problems (Hartman, 1965, p.4). These ideals originate from a branch of economics to which the term "welfare economics" is commonly applied, and which decision-makers in the water-resource field are not yet fully prepared to accept.

The theme of welfare economics is relatively easy to understand. In the usual case, private costs and revenues are assumed to be, respectively, social costs and benefits. Thus, the producer who maximizes the difference between costs and revenues maximizes benefits society receives from use of its resources. Income maximization for consumers of natural resources and an overall increase in natural wealth means also an increase in economic welfare.

As pointed out by Scott (1955, p. 58), optimum benefits
cannot accrue to society when the social cost of resource use is in excess of private costs of producing it. The same has been said for the case where private benefits exceed social gains.

Recognizing that goals of welfare economics can never be fully realized in the real world, a rule has been formulated which states that any social, legal, economic, or institutional change is desirable which results in:

1. everyone being better off, or
2. someone being better off and no one being worse off than before the change (Buchanan, 1959, p. 124-125). This Pareto optimum rule is one of the cardinal points of welfare economics, and provides guide lines for courses of action that lead to desired changes. The part of the rule requiring no one be worse off as a result of a change is the equalizer required to mitigate the undesirable aspects related to unregulated exploitation of a common pool.

Review of Common-Pool Problems

Ground-water resources, by virtue of their migratory nature and possession through rules of capture, are one of a family of common-property resources. The most substantial physical property of a common pool is withdrawal interdependency; that is, individual withdrawals may reduce the quantity of water available to other users at a given cost level (Hirshleifer, et al, 1960, p. 59-66). Thus, with unregulated exploitation where water goes to those who
physically reduce it to possession, it is seldom in the interest of a single user to conserve water as what is not withdrawn this year is free to be withdrawn by others, and therefore not available at some future date (Gordon, 1958, p. 118-119). Financial incentives and uncertainties associated with this type of exploitation requires on the part of competitive pumpers a maximization of short-run values of their withdrawals, thereby consciously or unconsciously equating future values of water left in storage to zero (Renshaw, 1963, p. 285).

It is apparent from this brief critique that much thought has been given to the common-pool problem, the consensus of which is that some type of intervention is desirable when the onus of exploitation cost is nonspecifically distributed among members of the producing industry or society as a whole. Examples in areas other than water resources are common. Federal intervention in the petroleum industry prior to and following World War I to regulate output or otherwise influence production practices is classic in this respect. Regulatory control has since been relinquished to the Interstate Oil Compact Commission which was originally founded to circumvent federal interests. The Commission serves in an advisory capacity to the states on such matters as quota and production practices that are in the best interests of the industry; suggested policies are generally accepted and enforced under an authority
tantamount to police power. We observe further the legal institutions for the wildlife and fisheries industry where constraints on season, size, species, quota, territory and equipment efficiency, among others, are collectively interpreted as management. For a detailed account, the reader is referred to Gordon (1954, p. 124-142) for what is probably the best and most adequate contemporary treatment of the economic theory of a common-property resource and the fisheries industry in particular. Common usage of waterways for waste discharge is another serious problem where exploitation costs are seldom specific to any individual polluter, and large scale intervention imminent. Returning to the original problem, mitigation of the negative effects associated with unregulated exploitation of a ground-water resource has received considerable attention in recent years. Some advanced solutions are discussed below.

Proposed Solutions to Common-Pool Problems

Scott (1955, p. 119-121) has discussed policies designed to short circuit unregulated exploitation of a common pool. Some of these policies in one form or another are already incorporated in ground-water statutes of several states. The main theme of Scott's thesis is to reduce a physically nonspecific resource into one possessing "specificity" through simulation of a sole-ownership concept. The term "specificity" is used to classify resources in terms of
incidence of cost of output. If all costs arising from production are borne by producers of the resource, the resource is specific to its owners; the resource is non-specific if production costs are shared by the producing enterprise, the industry as a whole, or society in general.

Advantages of sole ownership are obvious in regard to the petroleum or fisheries industry, and may be equally advantageous in the case of ground-water development. For example, the rational sole producer will reduce user costs whenever possible, which effectively amounts to conservation. More explicitly, it is rational of the sole owner to achieve optimum well spacing, demand only a reasonable off-take from each well or well field, prevent waste, and, if water levels have to be lowered, to achieve lowering somewhat uniformly. It is also in the sole owner's interest to take steps to arrest salt-water intrusion or land subsidence and to artificially recharge the ground-water supply if the practice is economically and physically feasible. Indeed, an unsatisfactorily resolved problem with artificial recharge when many pumpers overlie a basin of large dimensions is the unequal distribution of benefits among those who pay; or, as may be the case, accumulation of benefits to those who do not pay. In short, sole ownership makes user costs and benefits specific to one manager.

For reasons cited above, it is not likely that competitive pumping will be established within the structure of a single enterprise, with the result that many unacceptable
characteristics of a common pool vanish. Three methods have been suggested for achieving this end: (1) pro-rating, or assignment of quotas; (2) unitization, or collectively treating divided interests as undivided interests; and (3) sole-ownership itself. In discussing these aspects as they apply to water resources, Hirshleifer and his colleagues (1960, p. 61-66) combine sole ownership and unitization under the heading "centralized decision-making", and suggest a use tax as another means to accomplish these ends.

Centralized decision-making is most easily managed through a water district or other entity charged with the responsibility of delivering water to a community. This management scheme allows the district to operate as a sole owner of the resource, thereby becoming the sole recipient of all costs of ground-water development and production. This form of management has been proposed for Las Vegas, Nevada, (Leeds, et al, 1961, p. 145-146), and Smith Valley, Nevada (Domenico, et al, 1966, p. 45). Assignment of quotas, on the other hand, is an unsatisfactory solution where private-property concepts prevail unless pumping quotas are designed to balance sustained yield.

Proponents devoted to equalizing social and private costs of development under the broad theme of welfare economics suggest compensation in kind as the classic solution to this problem. Water-law statutes for the state
of Utah contain such a clause (Hutchins, 1965, p. 106-107). Section 409 of the Model Water Use Act may also be interpreted as a compensation in kind clause (Trelease, et al, 1965, p. 235). This section contains doctrine that presumably can be used to replace existing statutes pertaining to water-level decline in the states of Nevada, Utah, Washington, Wyoming, and Kansas, in the interests of achieving water-law uniformity:

"Where application is made for permit and there is sufficient water available, but the use under the permit would interfere substantially and materially with a domestic use previously initiated, or with the water supply, water diversion facilities, or water power of a preserved use or a use made under a permit previously initiated, the commission may issue a permit subject to the condition that the permit holder furnish to the person whose use is interfered with a quantity of water or power equal to that lost by reason of interference."

The unit of measure of cost inflicted on the hypothetical injured party in the citation above is the deprived benefit, in this case water or power (or perhaps increased power requirements as a result of water-level decline).

Ground-Water Law-A State's Solution

Ground-water resources are appropriated in various manners according to laws and institutions in individual states. In general, prevailing doctrines ignore the use tax and sole ownership method and rely on the assignment of quotas within a larger framework of unitization or centralized state control. Under the appropriation doctrine, which is applied almost unequivocally in the most
arid states where water is relatively scarce and most uses consumptive (Thomas, 1951, 1951, p. 4), quotas or rights to use are based on priority in time of beneficial use. A "quota" in this sense implies that each user has a specific share. Centralized or state control comes into play in curtailing ground-water development once all assigned quotas effectively exhaust safe yield, as in Nevada and Utah; in dictating minimum well-spacing restrictions in areas of overdraft or in the vicinity of surface-water rights; in revoking rights in the event beneficial use criteria are not satisfied; in establishing priorities of use in the event water is scarce and additional water required; or in authorizing withdrawals that limit water-level decline to a specified amount per year.

Under the riparian or land-ownership doctrine, which is a common policy in eastern and mid-western United States where water was at least thought to be abundant and most uses nonconsumptive, a "quota" in the broader sense of the term is a reasonable use with respect to requirements of other riparians. Such a loosely defined quantity constitutes a poor definition of a property right, and is only realistic if water is indeed abundant with respect to demands for its use. Under this policy, new rights may be established as long as new use is reasonable, priorities are not assigned on the basis of time, and centralized control is often reduced to a judicial matter in ascertaining the reasonableness of a particular use.
Several versions of this doctrine are applied in practice. The correlative rights doctrine of California, for example, recognizes all rights as being equal, and appropriate shares or quotas are assigned on the basis of historic use in the event of shortages. The absolute ownership version, on the other hand, places no limitations on withdrawals or their effect on neighboring riparians, and is a classic example of common ownership of a natural resource and exploitation under competitive pumping. Absolute ownership and reasonable use doctrines do not generally differentiate between a renewable and nonrenewable supply.

It is not germane to this study to give a detailed account of water law, as others have already done so (Milliman, 1959, p. 41-63; Bagley, 1961, p. 144-174; Trelease, 1965, p. 1-47). Further, it is beyond the scope of this work to investigate which of the prevailing doctrines results in the closest agreement between social and private costs and benefits, and whether more emphasis should be placed on a private-property concept, that is, definable quotas in terms of legal, physical, and tenure certainty, or on greater centralized control. Milliman (1959, p. 59-63) suggests state ownership of water resources has always been common in western United States, and current eastern trends also point in this direction. For our purposes, it is sufficient to recognize groundwater law as a control that attempts to minimize extra-
social costs by reducing a non-specific resource to a quasi-specific state through private-property concepts. Intervention of this kind may not always result in an optimum course of resource development.

OPERATIONS RESEARCH

A third approach to resource management entails optimization of an integrated ground-water and surface-water system or, less commonly, optimization of an aquifer as a separate unit independent of a regional water supply. In either case, this approach requires operating rules such that the system is operated in an optimal manner. These are generally determined through some economic or social objective associated with uses to which the water is put. Accordingly, the criterion of safe yield is abandoned in favor of the yield required to meet these objectives and still satisfy specified levels of constraining criteria (Bear and Levin, 1967, p. 403-404).

The optimal planning problem has been discussed in detail by Bear and Levin (1967, p. 401-412). Inputs to the system are identified in terms of replenishment, both natural and artificial, and outputs take the form of pumpage and spring flow. State variables describe the instantaneous level of operation, such as volume of water in storage at any given time. A change in state (storage) may depend on deterministic factors, such as artificial replenishment, factors associated with the system's state itself,
such as spring discharge or outflow, or factors subject to control, such as pumping rates. The latter are referred to as decision variables. A mathematical function of decision and state variables, termed an "objective function", indicates relative desirability of outputs from the system. Optimal planning is that combination of decision variables which best meet the objectives of the system. This sequence of decisions, generally expressed in terms of pumping and artificial recharge rates, constitutes a policy of operation. This approach has been applied to a carbonate aquifer in Israel with the objective of determining an operating policy which maximizes present value of future net benefits (Harpaz and Schwarz, 1967).

Another approach to optimal utilization of ground water is presented by Burt (1964, p. 80-93). The volume of water pumped at any time is determined by the storage available, thus constituting an operation policy. The objective criterion is maximization of present worth of net benefits. Other work in this area includes studies in California related to a least-cost combination of ground and surface-water components of the California Water Plan (Chun, et al, 1964, p. 79-95; Chun, et al, 1967, p. 426-434); optimization of conjunctive operation of aquifer and surface-water reservoirs (Buras, 1963, p. 111-131; 1963 a, p. 492-501); and optimal allocation of ground water between competing uses by linear programming (Castle and Lindeborg, 1961).
A necessary requirement for successful execution of most of the optimizing schemes cited above is sole ownership of the resource to be exploited. This requirement is fulfilled in the absence of private property in water, as in Israel, or with centralized control in the case of water districts in some parts of California. Otherwise, an inevitable conflict exists between the holders of right to pumpage and the operators of the basin for reservoir purposes. Thomas (1957, p. 425-427) suggests a compensation policy for removal of this conflict.

The Present Research

The main objective of the present research is to develop a conceptual guide to management and development that incorporates the more significant aspects of welfare criteria and operations research yet retains the physical criteria of management acceptable in most of the United States. The aquifer's development and operation are considered from the point of view that a fixed ground-water supply is available, the enterprise or community dependent upon the supply expands, and a greater use rate is demanded. Assuming economic criteria are important not only in formulating administrative regulations concerning development and use, but in providing decisions in the event of controversy, a decision is regarded as favorable if: (1) it is aimed at maximizing the welfare of a region, and; (2) it takes into account existing rights. These aspects are
examined with regard to ground-water mining.

ENVIRONMENT OF APPLICATION

The mining of a ground-water resource can lead to one or several unfavorable results including increased cost of production or diminishing profits owing to a declining water level; land subsidence; salt-water encroachment; and other forms of quality deterioration. In the absence of a common denominator that meaningfully describes the composite of these effects, attention will be focused on the physical environment of the Basin and Range Province or any other area characterized by: (1) considerable ground-water storage capacity owing to several hundreds or thousands of feet of saturated sediments; (2) little or no opportunity for water-quality deterioration as a result of full development, and; (3) limited recharge.

In view of these conditions, change in water level is used as the common denominator between the system and its economic evaluation since the depth to usable water is generally of primary concern to prospective users.

PRESENT WORTH CALCULATIONS

Decision-making in ground-water basins commonly entails choice between alternative courses of actions. The value of alternative decisions may be comparable on the basis of the profit (or loss) each will yield in the future, discounted back to the present. The valuation of prospective
future receipts or disbursements in this manner is termed a "present worth" calculation. According to Grant (1950, p. 112), present worth of a prospective series of future money receipts is the present investment that the future receipts would just repay with the interest. Similarly, present worth of future disbursements may be thought of as the sum which, if invested now at interest, would provide exactly the funds needed to make these disbursements.

In this analysis, it is convenient to follow the example of Hotelling (1931, p. 140) and allow the number of times the interest is compounded to approach infinity. The present worth factor $1/(1+y)^t$ for annual compounding is then replaced by $1/e^{yt}$, where $y$ is the rate of interest and $t$ is time. The present value of a unit of cost or profit to be obtained at time $t$ is then $e^{-yt}$. 

Solution of (1) for the case of ground-water production requires an appropriate expression for the net revenue function, or integral. Let $p(t)$ designate net profit per unit pumped as a function of time. Rate of pumping per unit time (for convenience taken on an annual basis) is assumed to be the sum of recoverable natural replenishment.
DEVELOPMENT OF THE THEORY

The Mathematical Model

In private planning, the optimum state of resource development is generally taken as the state in which future net revenues created by resource use are maximized over time. This criterion has been discussed at length by Ciriacy-Wantrup (1952, p. 65-223) and applied to a practical problem by Hotelling (1931, p. 137-175). Formulated in terms suitable to this analysis (Ciriacy-Wantrup, 1942, p. 82)

\[ V = \int_{0}^{t'} [(X(q,t) - C(q,t))e^{-\gamma t}dt \]  

(1)

where \( V \) is present worth of future net revenues, \( t' \) is the length of planning period, and \( X \) and \( C \) are gross revenue and cost functions, respectively, of rate of use \( (q) \) and time \( (t) \). The present worth calculation reduces future revenues in relation to their distance in time from the present. Inherent in this concept is the axiom that the market rate of interest be used by an entrepreneur in his calculations.

Solution of (1) for the case of ground-water production requires an appropriate expression for the net revenue function, or integrand. Let \( p(t) \) designate net profit per unit pumped as a function of time. Rate of pumping per unit time (for convenience taken on an annual basis) is assumed to be the sum of recoverable natural replenishment
Provided that there is withdrawal from storage, water levels will decline, and cost of extraction will increase. In the absence of any influence on the part of individuals to affect market price for the case where water is a factor in production (free competition), profit per unit pumped is a diminishing quantity, approaching zero at the economic limit of pumping. Designating the time when such pumping is no longer economical as "time of exhaustion" \( T \), present worth is formulated as an integral over this time period:

\[
V_E = \int_0^T [p(t)q(t) + p(t)R(t)]e^{-\gamma t}dt
\]  

(3)

If withdrawals are to be limited to recoverable replenishment at some time \( t' < T \), present worth must be restated to incorporate the value of perennial use of recoverable replenishment for all \( t > t' \). Two conditional assumptions are now applied. For \( t > t' \):

\[
p(t) = p(t,t')
\]
\[
R(t) = R(t')
\]

The first condition merely states that profit per unit pumped is a function of both \( t \) and \( t' \). This condition recognizes the fact that profit per unit pumped is a
diminishing quantity throughout the period of mining \( (t < t') \), and can vary thereafter \( (t > t') \) with price relevant to water use; that is, with enterprise selection. The second assumption requires a more detailed explanation.

An inherent assumption of the sustained-yield concepts discussed earlier is that safe yield or some part of it will eventually be recovered by pumping wells provided that annual pumping does not exceed natural replenishment. In other words, artificial withdrawals from the system will eventually eliminate natural discharge, the long-term net effect of which allows the system to function independently of storage. This point of view ignores the fact that recoverable natural replenishment is often a dynamic factor which depends on the state of storage development itself, the location and spacing of wells, and the external boundaries. In this analysis, natural replenishment is assumed to be recoverable only as long as operating levels in the reservoir are being lowered to make the recovery physically possible. Thus, the argument that \( R(t) = R(t') \) for \( t > t' \).

For the case where withdrawals are reduced to recoverable replenishment at some time \( t' \),

\[
V(t') = \int_0^{t'} p(t)q(t)e^{-\gamma t}dt + \int_0^{t'} p(t)R(t)e^{-\gamma t}dt + \int_{t'}^{\infty} p(t,t')R(t')e^{-\gamma t}dt
\]

(4)
The three terms in (4) give, respectively, present worth of net receipts forthcoming from use of ground-water storage, use of natural recharge over the period of mining, and natural recharge after mining ceases. All values are discounted back to the time mining began. The third term in (4) is an evaluation of the remaining worth of the ground-water basin after it has been partially depleted. As $t'$ approaches $T$, the value of this term approaches zero, and (4) reduces to (3).

Further clarification is required for the term "exhaustion". As used above and throughout this paper, the term exhaustion is used in an economic sense. In large areas characterized by thick sections of saturated materials, physical exhaustion may be a remote possibility. On the other hand, prolonged pumping of a water-supply system may not be possible when cost of capturing a given quantity exceeds revenue obtained from its use. Exhaustibility then involves a discontinuity in use resulting largely from an increase in production costs. This concept applies equally well to other exhaustible assets, such as mineral deposits, where exhaustion usually implies abandonment of low-grade ores that are currently (or comparatively) uneconomical to mine; to oil, gas, and steam deposits subject to large decreases in pressure; and to forests when only low-quality materials remain.
AN OPTIMAL COURSE

Intermediate between finite exhaustion and infinite preservation of storage is the mining period terminating at some $T'$ over which net revenue is to be maximized. The philosophy of such a policy suggests the existence of an optimal mining yield, or volume of extractable, nonrenewable storage which can be exploited in common with recoverable natural replenishment. The exhaustion of this quantity, which must come sooner or later, offers no threat to continued resource use and maximizes present worth of future net receipts. The execution of such a policy requires insight into how much water should be removed from storage, at what rate, and over how long a time period; and how much should be left in the reservoir, and at what level. These are surely quantitative questions which may at least be quantitatively formulated, and to which attention is now directed.

To examine the course of exploitation which would be "best" as defined above, inquiries are made as to the maximum value of (4) with respect to the time $T'$ mining should stop. Applying conditions for a maximum, there results

$$\frac{dV(T')}{dT'} = p(T') q(T') e^{-\gamma T'} + p(T') R(T') e^{-\gamma T'}$$

$$- p(T') R(T') e^{-\gamma T'} + \int_{T'}^{\infty} \frac{d}{dT} [p(t, T') R(t)] e^{-\gamma t} dt = 0$$

(5)
or
\[ p(T')q(T')e^{-\gamma T'} + \frac{dR(T')}{dT'} \int_{T'}^{\infty} p(t,T')e^{-\gamma t}dt + R(T') \int_{T'}^{\infty} \frac{dp(t,T')}{dT'} e^{-\gamma t}dt = 0 \quad (6) \]

The first term of (6) represents the net value of the ground water mined at time \( T' \), discounted back to the present. The second term is the present value (net) of natural replenishment recovered at time \( T' \). Once captured, this incremental volume is available in perpetuity; thus, its worth takes the form of a perpetual annuity starting at time \( T' \), as indicated by the limits of integration. The third term, which is negative as long as profit per unit pumped diminishes with decline of water levels, represents the present worth of the loss in value of \( R(T') \) owing to the increased lift incurred at time \( T' \). The disbursements required to pay for this loss are perpetual in that water-level decline is permanent. Thus, permanent benefits derived from recovery of natural recharge are accompanied by a perpetual added expense of lower operating levels in the reservoir required to make the recovery possible.

The conditions for a maximum described in (6) are an application of the economist's definition of best profit where marginal revenue equals marginal cost (Samuelson, 1964, p. 532-533). Interpreted literally, it is profitable
to continue ground-water mining as long as annual revenue
generated from use of ground-water storage, plus the
capitalized annual value of increases in recoverable re-
plenishment, exceeds the capitalized annual cost of loss
in value of recoverable replenishment. Decision rules
identical at least in principle to the one just stated
have been arrived at by application of dynamic programming
to storage control (Burt, 1964, p. 80-93) and by account-
ing procedures employed to ascertain the value of water

The result of three extreme conditions applied to
the general model are reviewed briefly. If prices relevant
to water use are increasing as fast as production
costs related to greater lift so that profit per unit
pumped is constant, (6) reduces to

\[ p(T')q(T') + \frac{dR(T')}{dT'} \frac{p(T')}{\gamma} = 0 \]  

(7)

In that neither term in (7) can be negative by definition,
the mining of a ground-water resource for this idealized
condition can profitably go on "indefinitely", that is,
until it is physically exhausted. On the other hand, if
profit per unit pumped is constant for all \( t > T' \), (6) be-
comes

\[ p(T')[q(T') + \frac{1}{\gamma} \frac{dR(T')}{dT'}] + \frac{R(T')}{\gamma} \frac{dp(T')}{dT'} = 0 \]  

(8)

This equation shows more clearly the perpetual annuities
and disbursements discussed earlier.

Equation (6) reduces to

\[ p(T')q(T') + \frac{R}{\gamma} \frac{dp(T')}{dT} = 0 \]  

(9)

if natural replenishment is constant, independent of the state of storage development, and completely recovered when equal to annual pumpage, and if profit per unit pumped is constant for all \( t > T' \). The decision rule now states that ground-water mining is profitable as long as annual net revenue generated from mining exceeds the capitalized annual loss in value of recharge. As in the general case, loss in value of recharge is interpreted in terms of greater lift required for its extraction. The conditions expressed in (6) through (9) are equally valid for the case where progressive deterioration of water quality continually diminishes net profit per unit of water applied in some productive process.

The mathematical models presented are fairly general in that output is dictated by several time-varying inputs with complex interactions. As with most abstract models, data required for their solution are equally complex or altogether unknown, and further abstraction is often required. Thus, an output may be sought for the few most causitive variable inputs, all other factors held constant. For example, it is possible to investigate for a given rate of withdrawal the influence of increased price relevant to
water use, or an increased rate of recovery of recharge for different well locations, all other factors held constant. Each of these examples is merely a special case of (6), and each requires prior knowledge of the time variability of the factor investigated. Whatever the intended purpose, further simplification is required for numerical evaluation of the decision rules.

**Simplifying Assumptions**

The unknown future behavior of several economically relevant variables affecting profit per unit pumped may necessitate assumptions about the future similar to those required in a study by Kelso (1961, p. 1114-1115); that is, no changes are anticipated in: (1) technology relevant to water use and development; (2) relationships among prices relevant to water use, and; (3) water-yielding capacity of aquifer systems with depth to usable water. Under these assumptions, the profit function is a projection from a current or postulated level of price and technology into the future, affected solely by water-level behavior.

With these assumptions, it is possible to write general expressions for the profit function for the case where pumping is in equilibrium with recharge

\[ p = P_o - F_o - m.c. \times h_o \]  

and where equilibrium does not exist between pumpage and recharge
\[ p(t) = P_0 - F_0 - \text{m.c.} (h_0 + h(t)) \]  

(11)

where \( P_0 \) designates price received from some entrepreneurial activity per unit of water applied in the process; \( F_0 \) is fixed cost per unit of water delivered; m.c. is the long-run marginal cost of pumping, or cost of lifting one acre-foot of water one foot (Renshaw, 1963, p. 291), which is assumed constant; \( h_0 \) is the pumping level in wells previous to mining when pumping is in equilibrium with recharge; and \( h(t) \) is added lift as a function of time. For the condition where recoverable recharge is balanced by pumping, profit per unit pumped in (10) is designated "\( \epsilon \)" and (11) is expressed

\[ p(t) = \epsilon - \text{m.c.} \ h(t) \]  

(12)

Here, \( p(t) \) is interpreted as the net price received from some entrepreneurial activity per unit of water applied in the process.

Some results of a study of costs and returns from agricultural production in the upland desert valleys in Nevada by Rogers and Neely (1966) may be used to demonstrate the manner in which these equations are handled. In the Orovada area, a per-acre yield of 400 lbs. of certified alfalfa seed at $0.42 per lb. will generate about $168.00 per acre to pay for all expenses. For an annual application of two acre-feet of water per acre, \( P_0 \) is taken...
as $84.00. Fixed costs per acre-foot pumped amount to about $62.00. This includes all operating outlays exclusive of well-field operation, such as labor, harvest, hauling, etc; real estate tax; interest on real and non-real estate investment; and long-term amortization of one well per 240 acres, pumping apparatus, and required diversion works. This leaves a net of $22.00 per acre-foot pumped to pay for pumping water and to yield a profit. In 1965, operating costs averaged about $4.57 per acre-foot. Taking the long-run marginal cost of pumping at $0.05 per acre-foot per foot of lift and an average equilibrium lift of 90 feet, profit per unit pumped for this particular enterprise amounts to

\[ p = 84 - 62 - 0.05 \times 90 = 17.50/\text{acre-foot} \] (13)

From (11), \( p \) reduces to zero at a lift of 440 feet.

When price received \( P_0 \) as well as fixed costs \( F_0 \) cannot realistically be conceived as constant, profit per unit pumped will have a component of variation independent of water-level behavior. Further, the marginal cost of pumping \( m.c. \) may be subject to variation owing to increased lift or technological advance. To incorporate these conditions, the right hand side of (10) is designated "\( \epsilon(t) \)" and (12) becomes

\[ p(t) = \epsilon(t) - m.c.(t)h(t) \] (14)

which is a general expression for profit per unit pumped
as affected by price relevant to water use, variations in the marginal cost of pumping, and water-level decline.

Principal Factors Influencing Present Worth

The simplifying assumptions employed in the preceding section allow an idealized expression for net profit per unit pumped which varies with water-level decline and with price relevant to water use. Given any analytical relationship between water-level decline and storage withdrawal, it is possible to investigate on a partial basis the influence of each variable expressed in the equation for present worth; that is, a specific rate of withdrawal is arbitrarily selected and present worth calculated for a range of one variable, all other factors assumed constant. Although the results obtained relate to the particular assigned values, the overall relationships hold for any situation.

The analytical relationship sought is of the form

$$ h(t) = \int \frac{d}{dt} p(t) dt $$  \hspace{1cm} (13)

where $h$ is the volume of ground-water removed from storage, and $dh/dt$ is the rate of water-level change with respect to storage withdrawals. As we are concerned in this section with demonstrating the influence of certain factors on present worth rather than applications to real situations, $dh/dt$ is assumed constant for an ideal distribution of pumping. Equation (14) is then expressed.
APPLICATION OF THE THEORY

Principal Factors Influencing Present Worth

The simplifying assumptions employed in the preceding section allow an idealized expression for net profit per unit pumped which varies with water-level decline and with price relevant to water use. Given now an analytical relationship between water-level decline and storage withdrawal, it is possible to investigate on a partial basis the influence of each variable expressed in the equation for present worth; that is, a specific rate of withdrawal is arbitrarily selected and present worth calculated for a range of one variable, all other factors assumed constant. Although the results obtained relate to the particular assigned values, the overall relationships hold for any situation.

The analytical relationship sought is of the form

\[ h(t) = \int_0^t \frac{dh}{dS} q(\tau) d\tau \]  

(15)

Where \( S \) is the volume of ground water removed from storage, and \( dh/dS \) is the rate of water-level change with respect to storage withdrawals. As we are concerned in this section with demonstrating the influence of certain factors on present worth rather than applications to real situations, \( dh/dS \) is assumed constant for an ideal distribution of pumping. Equation (14) is then expressed
\[ p(t) = \epsilon(t) - m.c.(t)K \int_0^t q(\tau) d\tau \]

where \( K \) is water-level decline per acre-foot of storage withdrawals in an area of pumping. The reciprocal of \( K \) (acre-feet of withdrawals per foot of water-level decline) is another idealized index of reservoir behavior. For other than an idealized distribution of pumping, use of a constant value for \( dh/dS \) may not be realistic from a hydrologic point of view.

A linear rate of mining \([q(t) = bt]\) is assumed for a period of pumping designed to economically exhaust a groundwater supply. Substitution of (16) into (3) and performing the required integration gives

\[ V_I = [e^{-\gamma T} - 1] \left[ \frac{3(m.c.)Kb^2}{\gamma^4} - \frac{eb}{\gamma^2} \right] \]

\[ + [Te^{-\gamma T}] \left[ \frac{3(m.c.)Kb^2}{\gamma^3} - \frac{eb}{\gamma} \right] \]

\[ + [T^2e^{-\gamma T}] \left[ \frac{3(m.c.)Kb^2}{2\gamma^2} \right] \]

(17)

for the first part of the integral, or returns from mining, and:

\[ V_{II} = [e^{-\gamma T} - 1] \left[ \frac{(m.c.)KrR}{\gamma^3} - \frac{eR}{\gamma} \right] \]

\[ + [Te^{-\gamma T}] \left[ \frac{(m.c.)KrR}{\gamma^2} \right] \]

\[ + [T^2e^{-\gamma T}] \left[ \frac{(m.c.)KrR}{2\gamma} \right] \]

(18)
for the second part of the integral, or returns from annual recharge during the period of mining.

Time of exhaustion is arrived at from (16) by setting \( p(t) \) equal to zero and assuming all other variables are constant.

\[
T = \left[ \frac{-2e}{(m.c.)(Kb)} \right]^{1/2}
\]  

(19)

The synthetic data chosen for this example are:

\( 1/K \) equals 3500 acre-feet/foot of head decline; \( b \) equals 2000 acre-feet/year\(^2\); \( \gamma \) equals 0.05; \( \epsilon \) equals $17.50/acre-foot; \( R \) is assumed constant and equal to 20,000 acre-feet/year, and; \( m.c. \) equal to $0.05/acre-foot\(^2\). For convenience, the net revenues from mining and pumping annual recharge are designated \( V_I \) and \( V_{II} \), respectively. Their sum, which is \( V_E \), is also shown in the results.

Figure 1A shows values of present worth for a range of \( 1/K \), all other factors assumed constant. Incremental additions to present worth are small for incremental additions to \( 1/K \) beyond 10,000 acre-feet/foot. This comes about because of the influence of \( 1/K \) on exhaustion time (19) and the time-discount calculation of (17) and (18). For an unconfined system, or for a confined system during its initial stages of dewatering, \( 1/K \) may be several times larger than for a completely confined system. If aquifer boundaries intrude, \( 1/K \) may rapidly become smaller or larger with time, depending on whether the boundaries are
barrier or recharge of water. If pumping from an extensive ground-water system is confined to a relatively small area, the rate of water-level decline may decrease because of depression expanding in radius. Thus, \(1/K\) will depend not only on the geometry and hydraulic properties of a ground-water basin, but also on the local distribution of wells, seasonal pumping requirements, and the manner in which withdrawals increase with time.

Although large values of \(1/K\) tend to postpone economic exhaustion of a given rate of production, too slow a rate of production may postpone profitable return into the future than warranted by the rate of interest. From Figure 1B the larger the interest rate for a given time-distribution of future revenues from the present, set in for a given rate of withdrawal, the lower the present value of such revenues. It follows that greater time-distance for a given interest rate may increase present worth, which may favor economic depression in the present at the expense of future earnings. The inverse is the rate of interest and the size of the enterprise (0.05 to 0.10) results in small changes in present worth.

All other things being equal, the net value per unit of water with respect to pumping should remain constant if demonstrated to be the major parameter influencing present worth (Figures 1A, 1B, 1C, 1D).

**Fig. 1.** Present worth versus increasing values of (A) \(1/K\); (B) \(\gamma\); (C) \(\varepsilon\); (D) \(m.c.\)
barrier or recharge in nature. If pumpage from an extensive ground-water system is confined to a relatively small area, the rate of water-level decline may decrease as cones of depression expand, and 1/K will increase. Thus, 1/K will depend not only on the geometry and hydraulic properties of a ground-water basin, but on the spatial distribution of wells, seasonal pumping requirements, and the manner in which withdrawals increase with time.

Although large values of 1/K tend to postpone economic exhaustion for a given rate of production, too slow a rate of production may postpone profits further into the future than warranted by the rate of interest. From Figure 1B the larger the interest rate for a given time-distribution of future revenues from the present, that is, for a given rate of withdrawal, the lower the present value of such revenues. It follows that a shorter time-distance for a given interest rate will increase present worth, which may favor early depletion on the part of pumpers tending to maximize future receipts. Large changes in the rate of interest over the normal range expected for private enterprise (0.05 to 0.10) results in small changes in present worth.

All other factors as above, the net value per unit of water with respect to the equilibrium pumping level is demonstrated to be the most significant parameter influencing present worth (Figure 1C)
A fourth item of some significance is variation in what has been termed "marginal cost of pumping", or cost of lifting one unit of water one foot. Although assumed constant, this cost can increase in the case where depth to usable water is increased; or decrease in the case of technological innovation, as progressively going from diesel, to electrical power, to natural gas, or technological advance in the case of improved pump efficiency. For the lower range of marginal cost, small changes noticeably influence present worth. For a high marginal cost of pumping, shorter exhaustion times reduce both total revenue and its time-span from the present, the effects of which tend toward neutralization for the particular data of the problem.

The remainder of this study deals with two special cases. The first case, which is the simplest conceivable, demonstrates the incidence of cost and benefit in the exploitation of a ground-water resource, the conceptual compatibility of goals of income maximization and compensation in kind, and the existence of an optimal mining yield somewhat analogous to the concept of permissive-mining yield. The second case is merely a problem-solving exercise for an agriculturally based area of ground-water mining in Nevada. The most important aspect of this problem is that ground water has value only to the extent to which it can be used as a factor in agricultural production, and recoverable replenishment is dictated by the location of wells.
and their influence on natural discharge.

A Simple Model of Storage Inventory

As mentioned previously, the general model (6) reduces to

\[ p(T')q(T') + \frac{R}{\gamma} \frac{dp(T')}{dT'} = 0 \]  

(20)

for the following assumptions: (1) natural replenishment is constant, independent of the state of storage development, and completely recovered when equal to annual pumpage, and; (2) profit per unit pumped is constant for all time beyond \( T' \). The problem-solving approach applied to this simple model, as outlined in this section, is equally valid for more complex cases.

OPTIMUM MINING YIELD

The argument for moving storage in a ground-water basin to some level \( h_0 + h(T') \) is the net revenue generated from use of the one-time reserve between \( h_0 \) and \( h(T') \). More specifically, exhaustion of an optimal one-time reserve and perennial use of natural replenishment thereafter is desirable if maximization of future net revenue is a management objective. This course of exploitation is not only the most profitable, but protects water rights in flow to the extent that water levels are not lowered below the economic limit of pumping. A first approximation of the optimal mining yield is determined as follows.
Profit per unit pumped at time $T'$ is expressed as
\[ p(T') = \varepsilon - m \cdot c \cdot h(T') \]
where $h(T')$ is the added lift imposed on the basin at the termination of the mining period. It follows that
\[ \frac{dp(T')}{dT'} = -m \cdot c \cdot \frac{dh(T')}{dT'} \]
where $dh(T')/dT'$ is the water-level change during the last year of mining. The change in storage during any years pumping is expressed
\[ \frac{dS(t)}{dt} = q(t) \]
and, for the last years mining,
\[ \frac{dS(T')}{dT'} = q(T') \]
Substituting these relationships into (20)
\[ [\varepsilon - m \cdot c \cdot h(T')] \frac{dS(T')}{dT'} - R[m \cdot c \cdot \frac{dh(T')}{dT}] = 0 \]
This equation is easily solved for $h(T')$
\[ h(T') = \frac{\varepsilon}{m \cdot c} - \frac{R}{\gamma} \frac{dh(T')}{dS(T')} \]
Equation (26) gives the permissible added lift that can be imposed on a basin, after which withdrawals should be limited to recoverable replenishment. As (26) is valid only at $T'$, which is not yet known, no direct solution is available. For an idealized distribution of pumping,
The volume of ground-water storage \( a_m \) utilized over the most profitable mining period dictated by (20) is

\[
a_m = \int_0^{T'} \frac{dh(t)}{dh(t)} \frac{dS(t)}{dS(t)}
\]

(28)

For the idealized yield-response assumption cited above

\[
a_m = \frac{\varepsilon}{m.c.K} - \frac{R}{\gamma}
\]

(29)

The quantity \( a_m \) represents the optimal mining yield, or volume of extractable, nonrenewable storage that can be exploited in common with annual recharge. It is analogous to permissive-mining yield in that it is independent of the rate of mining and represents a part of the mining yield which would eventually exhaust (economically) the supply; that is, a part of \( \varepsilon/m.c.K \), or the maximum-mining yield.

The final form of \( a_m \) as given by (29) is interesting in that all the parameters of the problem are included and
arranged as one would intuitively expect. For example, as natural recharge becomes small, or the rate of interest becomes large, optimal mining yield approaches the maximum-mining yield. That is, a large rate of interest or a low rate of flow favors exploitation of a one-time reserve at the expense of a flow. Similarly, for a given R and \( Y \), any factor that increases maximum-mining yield increases the optimal amount which can be withdrawn. Optimal mining yield is thus conceptually viewed as the difference between the total volume of usable storage and the "capitalized" annual volume of natural replenishment.

DETERMINING THE LENGTH OF THE MINING PERIOD

To help carry the conceptual analysis, a specific rate of mining will be assumed and certain calculations carried out. In order that the results can be compared to facts already known, a linear rate of mining and the synthetic data employed in construction of Figure 1 will be used here. The results obtained are thus useful only from the point of view of interpretation for more realistic situations.

Profit from mining for any year \( t' \) is the product of the quantity of storage mined that year and its net unit value

\[
p(t') q(t') = [e - m.c.K_0 \int_{t'}^{t} q(\tau) d\tau] q(t')
\]  

(30)

Capitalized annual loss in value of recharge is then
easily obtained

\[ \frac{R \ dp(t')}{\gamma \ dt'} = - \frac{R}{\gamma} m \ c \ K \ q(t') \]  

(31)

The decision rule of (20) states that mining should continue until (30) and (31) are numerically equal. A graphical presentation of these equations is given in Figure 2 for a linear rate of mining and the data of Figure 1.

Figure 2 is interpreted as follows. The height of the curve \(p(t')q(t')\) for any year represents the net receipts from mining for that year. The height of \(\frac{R \ dp(t')}{\gamma \ d(t')}\) for any year represents the capitalized annual loss in value of recharge. Viewed in another way, \(\frac{R \ dp(t')}{\gamma \ d(t')}\) represents a part of annual gains from mining which, if invested at \(\gamma\) at their time of occurrence, would repay for all time the reduction in value of recharge attributable to mining operations for that year. For any year's operation, this sum represents a hypothetical return to the mine required to make all future operating costs identical to what they were previous to mining. The "net" is obviously the area between the curves, which is yet to be discounted. Incremental returns from development beyond \(T'\), if invested at \(\gamma\), are not sufficient to pay for incremental losses by an amount equal to the vertical distance between the curves. This is another way of stating the necessary conditions for which \(V(T')\) is a maximum, and demonstrates the conceptual compatibility of ideals of income maximization.
and compensation.

The optimal length of the mining period shown in Figure 2 should have been obtained by solving (30) for the appropriate rate of mining and substituting the result in (26). In this case

\[ T^* = \frac{1}{p_0^2} \left( \frac{n R}{p_0^2} \right)^{1/2} \]

or, from (19)

\[ T^* = \frac{T - \frac{2\pi}{\sqrt{yD}}}{\pi} \]

where \( T^* \) is time of exhaustion, from what has already been demonstrated in Figure 1, and from the expression for optimal mining yield (19), a factor that increases \( T \) for a given rate of production lengthens the optimal period of mining, increases optimal mining yield, and increases relative profitability. On the other hand, for decreasing \( A \) or increasing \( N \), both the mining period and the optimal yield are increased but relative profitability is attenuated.

Figure 3, which shows present value of various mining periods of length one year through \( T \), clearly demonstrates the relationships between \( V \), \( T \), \( A \), and \( N \). A more realistic water-level response to pumping would give a more pronounced maximum.

Fig. 2. Plot of net annual receipts from mining and capitalized annual loss in value of recharge.

As data requirements for a graph similar to Figure 2 are rather minimal, the practical aspects of this relationship
and compensation.

The optimal length of the mining period shown in Figure 2 could have been obtained by solving (30) for the appropriate rate of mining and substituting the result in (20). In this case

\[ T' = \pm \frac{2e}{m \cdot c \cdot KB} \cdot \frac{2R}{\gamma_b}^{1/2} \]  

or, from (19)

\[ T' = \pm \frac{T^2 - \frac{2R}{\gamma_b}}{\gamma_b}^{1/2} \]  

where \( T \) is time of exhaustion. From what has already been demonstrated on Figure 1, and from the expression for optimal mining yield (29), any factor that increases \( T \) for a given rate of production lengthens the optimal period of mining, increases optimal mining yield, and increases relative profitability. On the other hand, for decreasing \( R \) or increasing \( \gamma \), both the mining period and the optimal yield are increased, but relative profitability is attenuated.

Figure 3, which shows present worth for various mining periods of length one year through \( T \), clearly demonstrates the relationships between \( V_E, T', R, \) and \( \gamma \). A more realistic water-level response to pumping would give a more pronounced maximum.

The final form of \( T' \) as given by (32) or (33) is open to the same reasoning applied to optimum mining yield.

As data requirements for a graph similar to Figure 2 are rather minimal, the practical aspects of this relationship
Fig. 3. Length of the optimal mining period as influenced by (A) annual recharge; (B) rate of interest.
may be quite useful to management. For example, as long as annual gains from mining exceed the capitalized annual losses in value of recharge, optimal mining yield has not been exceeded. Thus, the relative magnitude of loss and gain versus time provides considerable insight into the state of development of the resource. Indeed, the concept of overdevelopment generally associated with exceeding safe yield is given other than purely physical significance when used in this context.

SOLVING FOR PRESENT WORTH

For the simplifying assumptions of this section, the general model (4) reduces to

\[ V(T') = \int_0^{T'} p(t)q(t)e^{-\gamma t}dt + \int_0^{T'} p(t)Re^{-\gamma t}dt \]

\[ + \frac{RP(T')}{\gamma} e^{-\gamma T'} \]  

(34)

In that there is practically no limit to the range and forms of variables that can be employed in (34), the question as to the most expedient treatment of this equation is answered by re-examining the linear example and Figure 2 in particular.

For a period of mining of length \( T' \), present worth described in (34) can be found by discounting the "net" between the two curves of Figure 2, and adding this sum to

\[ V = \int_0^{\infty} Re^{-\gamma t}dt \]
or $\varepsilon R/\gamma$. Equation (35) gives present worth of future net receipts associated with sustained yield of the resource. For the assumptions concerning the simple storage inventory model, the expression for present worth given in (4) reduces to (35) if ground-water withdrawals are limited to natural recharge. If the mining period extends beyond $T'$, the discounted net is calculated with respect to $T'$ but corrected for the discounted loss (the annual difference between the curves beyond $T'$) and then added to (35).

It follows, therefore, that rigorous solutions such as (17) and (18) are not required for analysis of present worth. It is only necessary to reduce historic or predicted gains and losses to a graph similar to Figure 2. The procedure described above is then followed to obtain numerical values of present worth.

THE RATE OF MINING

The demonstrations thus far have ignored the magnitude of $q(t)$ whereas a most profitable rate of production is the factor sought in many optimizing problems. In that optimal mining yield is independent of the rate of production, the ground-water supply is considered fixed from the point of view of current technology, enterprise selection, natural phenomenon, and rate of interest. Thus, there exists as many theoretical "maximums" as there are production rates designed to exhaust the optimal storage, suggesting the
rate giving the largest return may be impractically high. This follows from the fact that any \( q(t) \) that decreases the time span between future revenues and the present will favorably influence present worth. On the other hand, a lower "maximum" exists for a slow rate of production that may postpone profits further into the future than warranted by the rate of interest (Figure 1B). It follows that the conditions for a maximum described above as well as for the general model are necessary but not sufficient.

An Empirical Application

This application is worked out for Kings River Valley in northern Nevada. Hydrologic data required for the study have been abstracted from a report by Malmberg and Worts (1966). Enterprise budgets have been developed from a report on upland desert valleys which adjoin Kings River Valley, and which are similar in most respects (Rogers and Neely, 1966).

BACKGROUND INFORMATION

Prior to 1956, Kings River Valley was largely a sheep and cattle ranching area with a few domestic and stock wells. Over the period 1956 to 1963, private interests acquired or filed entries for agricultural development of public lands, and over 100 irrigation wells have been drilled. As most of the rights to pump water have
not yet been exercised, 44 wells produced most of the water pumped for irrigation farming in 1963. The cultivated area is entirely dependent on the ground-water supply for irrigation.

The valley has been divided into two subareas. Virtually all of the past pumpage and unexercised rights to pump additional water are concentrated in the Rio King subarea. The net pumping draft in 1963 was about 15,000 acre-feet, or 3000 acre-feet in excess of the estimated perennial yield of the subarea. The so-called "excess" of draft over yield is academic in that practically all of the water pumped over the period 1957 to 1963 came from storage. In other words, the concentration of pumpage in the Rio King subarea has not yet affected natural discharge from the system, a typical characteristic of the early phases of ground-water development in the Basin and Range Province.

Rights to pump 60,000 acre-feet of water per year have been granted in the Rio King subarea. In 1962, the State Engineer invoked a temporary moratorium which prevents further well drilling. If all permitted rights to pump were exercised, the net pumping draft would be about 40,000 acre-feet per year. This estimate is based on the assumption that one-third of the gross pumpage will return to the reservoir by deep percolation. Assuming further that this amount of pumpage will allow complete recovery
of natural discharge, the excess of pumpage over yield may eventually be about 28,000 acre-feet per year.

**DESIGN OF THE PROBLEM**

In the Rio King subarea, ground water has value only to the extent to which it can be used as a factor in agricultural production, and recoverable replenishment is dictated by the influence of pumping wells on natural discharge. The mathematical model described by (8) appears most appropriate for these conditions

\[
p(T')[q(T') + \frac{1}{\gamma} \frac{dR(T')}{d(T')} + \frac{R(T')}{\gamma} \frac{dp(T')}{d(T')} = 0
\]  

(36)

Recognizing that

\[
p(T') = e^{-m.c. h(T')}
\]  

(37)

the termination level is easily derived

\[
h(T') = \frac{e}{m.c.} - \frac{R(T')}{\gamma \frac{dh(T')}{dT'}}
\]  

(38)

Optimal mining yield will be determined from the condition expressed in (36).

The data of this problem have been suggested by Malmberg and Worts (1966, p. 48). Withdrawals from storage increased from 2000 acre-feet per year to 15,000 acre-feet over the period 1957-1963. The total permitted rights to pump 60,000 acre-feet have been assumed to be exercised over the period 1963 to 1973 in a reasonably uniform manner.
Recoverable replenishment would increase over this period, again uniformly, and be more or less fully recovered by 1973. These assumed relationships are shown in Figure 4A.

Malmberg and Worts (1966, p. 48-49) estimate that the pumpage shown in Figure 4A would result in the equivalent of a subarea-wide decline of water levels of about 20 feet over the period 1963 to 1973. In terms of real behavior, they state further that water levels in the centers of pumpage may decline more than 100 feet. This latter estimate is based on historic response to pumping over the available period of record. In the absence of a cause-and-effect device such as an analog or mathematical model, identical assumptions are incorporated here to obtain rough estimates of water-level behavior in the centers of pumping (Figure 4B). Estimates of water-level decline shown in Figure 4B are probably higher than would actually occur in response to this pumping, and the calculations to follow are conservative.

Condition One - Storage Augmented by Return Flow

The results of this study are shown in Figure 5 for the condition that one-third of the pumpage is available to augment ground-water storage. Figure 5A is interpreted in the same manner as Figure 2. The height of the curve \( p(t)[q(t) + \frac{1}{Y} \frac{dR(t)}{dt}] \) for any year represents the net receipts from mining that year plus the capitalized annual
Fig. 4. Estimate of (A) pumpage and natural recharge; (B) water-level response to pumpage, Kings River Valley.
value of increases in recoverable replenishment. The
height of the curve at time $t$ for any year represents the
capitalized annual gains in value of recoverable replenish-
ment. This latter amount is the annual investment re-
quired to repay for all time the deduction in value of re-
charge. Conditions for a mine to be satisfied are satisfied at $T'$, or
about 24 years, as development parts (1944).

Figure (A) shows cumulative storage depletion and
water-level drop versus time. Optimum mining yield is
about 480,000 acre-feet, and is exhausted when water levels
drop about 192 feet. The concept of mining yield
focuses attention on the perspective net benefit from storage;
the assumption is made that 480,000 acre-feet of storage can be put to use. As the
average pumping lift in 1937 was about 6 feet, total lift
at time $T'$ is calculated to be about 238 feet. The added
lift at $T'$ can also be determined from (39).

Figure (B) shows present worth for various pumping pe-
riods of length one year through $T$. For this particular en-
terprise and for the special use of
annual recharge after $T'$ is simply more profitable
than...
value of increases in recoverable replenishment. The height of the curve \( \frac{R(t)}{v} \frac{dp(t)}{dt} \) for any year represents the capitalized annual loss in value of recoverable replenishment. This latter amount is the annual investment required to repay for all time the reduction in value of recharge. Conditions for a maximum are satisfied at \( T' \), or about 24 years after development starts (1980).

Figure 5B shows cumulative storage depletion and water-level decline versus time. Optimum mining yield is about 460,000 acre-feet, and is exhausted when water levels decline about 153 feet. As the concept of mining yield focuses attention on the permissive net draft from storage, the assumption concerning return flow suggests that 690,000 acre-feet of storage can profitably be put to use. As the average pumping lift in 1957 was about 85 feet, total lift at time \( T' \) is calculated to be about 238 feet. The added lift at \( T' \) can also be determined from (38)

\[
h(T') = \frac{10}{0.05} - \frac{12,000 \times 9.1}{48,000} = 154.5 \text{ feet} \tag{39}
\]

Figure 6 shows present worth for various mining periods of length one year through \( T \). For this particular enterprise and for the assumptions stated, perennial use of annual recharge after \( T' \) is only slightly more profitable than depletion of the remaining storage. A more realistic cause-and-effect relationship would eliminate this apparent indifference.
Condition Two - No Augmentation of Storage

Figure 7 shows the results of this study if return flows are not available to augment storage. Conditions for a maximum are satisfied about 17 years after development starts (1973). Optimal mining yield is about 441,000 acre-feet, and is exhausted when water levels decline about 147 feet. Annual gains from mining are considerably less than shown in Figure 6 owing to the assumed lack of return flow and, therefore, the small amount of storage that can practically be put to use.

The residual lift at $T'$ can also be determined from (38)

$$h(T') = \frac{12,000 \times 15.6}{0.08} - \frac{200}{40,000 + \frac{1200}{0.05}} = 48 \text{ feet}$$

(Figure 6 shows present worth for various mining periods of length one year through $T$.)

**EFFECT OF DEVIATIONS FROM PROBLEM ASSUMPTIONS**

The rate at which £0.05 were exercised in future years has no effect on optimal mining yield or the calculated termination level provided that (1) natural replenishment is fully recovered prior to achieving a terminal operating point; (2) rights to previously exercised [q(T') = 48,000], and; (3) the water level response to 28,000 acre-feet of storage withdrawals (total pumped...
Condition Two - No Augmentation of Storage

Figure 7 shows the results of this study if return flows are not available to augment storage. Conditions for a maximum are satisfied about 17 years after development starts (1973). Optimal mining yield is about 441,000 acre-feet, and is exhausted when water levels decline about 147 feet. Annual gains from mining are considerably less than shown in Figure 5 owing to the assumed lack of return flow and, therefore, the smaller amount of storage that can profitably be put to use.

The added lift at \( T' \) can also be determined from (38)

\[
h(T') = \frac{10}{0.05} - \frac{12,000 \times 15.6}{0.05} = \frac{48,000 + \frac{1200}{0.05}}{148 \text{ feet}}
\]

Figure 8 shows present worth for various mining periods of length one year through \( T \).

EFFECT OF DEVIATIONS FROM PROBLEM ASSUMPTIONS

The rate at which rights to pump are exercised in future years has no effect on optimal mining yield or the calculated termination level provided that (1) natural replenishment is fully recovered prior to achieving a terminal operating level in the reservoir \( [dR(T')/dT' = 0] \); (2) rights to pump 60,000 acre-feet are eventually exercised \( [q(T') = 48,000] \), and; (3) the water level response to 28,000 acre-feet of storage withdrawals (total pumpage
Fig. 7. Graphical presentation of (A) gains and losses associated with ground-water mining; (B) storage depletion and water-level response, Kings River Valley (no return flow).
Fig. 8. Present worth for various mining periods of length one year through \( T \), Kings River Valley (no return flow).
minus natural replenishment and return flows) will be about 9 feet, or about \(1/3000\) feet per acre-foot of withdrawals. The ground-water supply is thus considered fixed, and the production rate merely dictates relative profitability for uses upon which the worth of the reservoir is predicated. Too slow a rate of production may postpone profits further into the future than warranted by the interest rate whereas a rapid rate of production may result in untimely exhaustion of the storage reserve.

As a fixed ground-water supply is realistic only for very short planning periods, the figures given here are merely a first estimate subject to revision in future years. Optimal mining yield can increase with (1) a wider distribution of pumpage or pumpage from areas utilized to a limited extent; (2) technological advance or renovations affecting pumping costs; (3) favorable price variation and enterprise selection, and; (4) inflationary or other trends tending to increase the going rate of interest.
CONCLUDING STATEMENT

The optimum conditions for a transition from groundwater storage to natural replenishment have been investigated. If maximization of present worth is a management objective, the extreme cases of perennial use of natural replenishment (no mining) and depletion of the entire reserve (exhaustion) emerge as special cases of the general model. This is reflected through the expression for optimum mining yield, which may take on values ranging from zero to maximum-mining yield. A few observations regarding the significance and limitations of the analysis are in order.

First, it is recognized that the mathematical models constitute an arrangement of economic and physical facts which serve as a substitute for rule of thumb judgement. They have merit to the extent that they provide a conceptual guide to resource development that is definitely superior to unregulated exploitation, on one hand, and unguided restrictive intervention on the other. As a predicting tool, a forecaster, or a hand-book approach to management, they are severely limited by the assumptions required to analyze the complexities of a management science.

Consider the following difficulties of application. First of all, the net revenue function may be an insurmountable barrier when water is put to uses not easily
evaluated through the market place. Even where water is a factor in agricultural production, there still remains the problem of selecting its representative value when several crops are grown and several pumping levels exist.

A second and perhaps most important limitation is the general inaccessibility of some of the required data. Included here would be the rate at which natural discharge is recovered by wells, long-term changes in the yield-response characteristics of a basin, and long-term changes in the use to which water is put.

If the assumptions and limitations of the analysis are acceptable, or can be made acceptable by collecting and utilizing hydrologic and economic data not normally considered worthy of the required effort, the concepts presented may be usefully applied to management and planning. The ultimate objective is to translate physical data, such as water-level response and pumping, into meaningful economic terms. Useful to these objectives are the numerical evaluation of the decision rules presented and derivation of an optimal mining yield for specified conditions. In addition, there appear to be practical applications for criteria and methodology for ascertaining the state of development of the resource. In that a deterministic cause and effect relationship is presupposed, analysis of this type is logically aided by a physical model of the ground-water system.
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