Modeling and Analysis of Hydraulic Interchange of Surface and Ground Water

A dissertation submitted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy in Hydrology

by

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In memory of Dr. George B. Maxey-

His enthusiasm and encouragement will be long remembered
ABSTRACT

MODELING AND ANALYSIS OF HYDRAULIC INTERCHANGE OF SURFACE AND GROUND WATER

Alfred B. Cunningham

A technique for modeling the hydraulic interchange between surface and ground water in an unconfined aquifer-river system has been developed. Modeling procedure consists of simultaneous solution of the Saint Venant equations for one dimensional open channel flow and the Boussinesq equation descriptive of two dimensional transient ground water flow, coupled by an expression for flow through channel wetted perimeter. Gelerkin finite element technique was utilized in the numerical solution algorithm.

Evaluation of model predictive capability was accomplished in three interrelated phases which included 1) determination of relative sensitivity of various model parameters; 2) calibration of model to Truckee River system (northern Nevada) and evaluation of predictive uncertainty; and 3) estimation of uniqueness of solution. Parameter sensitivity investigations provided information regarding the accuracy to which individual system parameters must be measured in order to insure successful model operation. Model predictive uncertainty, which was evaluated utilizing data from Truckee River system, consisted of statistical comparison of model output with field observations. Results indicated generally good overall predictive capability and included fair to good prediction of extreme high and low river stage
and ground water table elevations. Predictive reliability was observed to decrease with distance away from river channel. A procedure for quantifying uniqueness of model solution was developed based on chi square goodness of fit test. Uniqueness of solution investigation provided insight into the reliability associated with extrapolation of model results beyond the limits of observed field measurements.

Subsequent to evaluation of predictive capability the model was utilized to delineate interrelationships among individual hydrologic system components as well as to determine the degree of influence of various parameters on system behavior. The particular hypothetical hydrologic system investigated consisted of a 25 mile channel reach bounded on both sides by an unconfined aquifer of limited extent. Investigations were based on system response to an inflow hydrograph (peak flow = 26,000 cfs, time to peak = 1.67 hours, baseflow = 18,000 cfs) and consisted of: 1) separation of hydrograph into baseflow and direct runoff components; 2) theoretical verification of analytical techniques for describing baseflow recession; 3) identification of influence of aquifer parameters on baseflow recession; and 4) investigation of system response under both natural and induced stress conditions.
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INTRODUCTION

BACKGROUND

With the evolution of hydrology to the full status of a science considerable effort has been directed toward gaining a physical understanding of the various processes and components which comprise hydrologic systems. This ever-increasing study of the hydrologic cycle can be separated into at least two distinct categories depending on the philosophy and interpretive interests of the individual investigator. In one case research activities from various branches of the physical sciences have been directed toward understanding phenomena related directly or indirectly to the hydrologic cycle. This area of endeavor, which is termed "physical hydrology", has as its basic objective the scientific study of physical phenomena while, quite often, little emphasis is given to the ultimate application of this knowledge to the solution of practical problems. The alternative approach to hydrologic investigation might be termed "applied hydrology" which consists of the analysis of hydrologic systems for the explicit purpose of establishing quantitative relationships for describing the behavior of various components of the hydrologic cycle. The basic motivation for the development of applied hydrology has largely been one of necessity. For example, manipulation of surface water supplies by means of impoundment, interbasin transfer, and subsequent distribution has tremendously altered the behavior of many hydrologic systems thereby creating need for a wide spectrum of
design, analysis and management procedures. Similarly, in regions
where conjunctive use of ground and surface water is practiced, with-
drawal, transfer, and consumptive use of ground water has considerably
transformed the original water table configuration both in the
vicinity of withdrawal as well as in the area of use and subsequent
recharge. Furthermore, if historical trends persist, man's level of
use (and reuse) of water resource systems will continue to increase
along with the degree to which human activity disrupts or alters the
natural behavior of hydrologic systems.

In response to these and other related problems considerable
effort has been directed by applied hydrologists toward the develop-
ment of techniques for studying behavior of hydrologic systems in
response to both natural and artificial stress conditions. These
techniques, more commonly referred to as hydrologic models, exhibit
considerable diversity in structure but, nevertheless, can be broadly
categorized as being either parametric or stochastic in nature.
According to Amoroch and Hart, 1964, the definition adopted by the
American Society of Civil Engineers (ASCE) for "parametric hydrology"
is:

the development of relationships among physical parameters
involved in hydrologic events and the use of these relation-
ships to generate or synthesize non-recorded hydrologic
sequences.

The corresponding ASCE definition of "stochastic hydrology" is:

use of statistical characteristics of hydrologic variables
to solve hydrologic problems.

Hydrologic modeling techniques have been developed by necessity
to suit a variety of purposes, as is clearly evident both from the
number and diversity of past investigations. It is further evident from review of contemporary literature that most previous modeling efforts have concentrated either on ground water or on surface water systems depending on the motivation for a particular study. It is seldom indeed that both of these components have been studied in detail simultaneously. This is due, at least in part, to factors such as mathematical complexity of relationships governing the behavior of the ground water - surface water exchange process as well as the difficulty and expense encountered in obtaining concurrent field observations from a hydrologic system composed of both ground water and surface water components. However the exchange of water between surface streams and rivers and ground-water reservoirs is nevertheless important, both from the standpoint of understanding the physical process involved as well as improving methods for analysis and management of conjunctive use systems. It is therefore clear that need exists for further development of methodology for conjunctive modeling of ground and surface water systems.

OBJECTIVES AND SCOPE

The primary objectives of this study are to 1) develop a modeling procedure based on the linkage between the physical equations governing the movement of ground and surface water (referred to herein as a linked ground water - surface water model), 2) calibrate and analyze model performance for real and hypothetical hydrologic systems, 3) evaluate model predictive capability in terms of parameter sensitivity, predictive uncertainty, and uniqueness of solution, and
4) determine interrelationships between various system parameters and components.

Objective 1 was accomplished by drawing heavily on results of previous investigations. The conceptual framework of the linked model closely follows that of Pinder and Sauer, 1971, with the basic difference being the implementation of a finite element scheme for solution of the equations of motion. Also of considerable significance to model development was work by Cooley, 1975 and Cooley and Mein, 1976 which is discussed subsequently. Objective 1, therefore, does not represent a completely original effort by the author. Rather the author's main contribution as well as the major thrust of this dissertation effort lies in fulfillment of objectives 2, 3, and 4.

The scope of this study deserves discussion particularly with respect to the aspects which have general versus site-specific applications. For example, model development and subsequent sensitivity analysis were performed independent of data from any particular study area and thus have more general application. Similarly, investigations into interrelationships between system components and parameters have been carried out in such a manner as to yield results which are more or less of a general nature. However, methods utilized to evaluate such aspects as model predictive capability along with uniqueness of solution required that the model first be calibrated for a specified hydrologic system, thereby limiting the generality of results. The hydrologic system chosen for this particular study phase consists of the Truckee Meadows reach of Truckee River in northern Nevada.
STUDY REGION

Location

Truckee River basin is situated on the eastern slopes of the Sierra Nevada Mountains in eastern California and western Nevada as shown in Figure 1. The Sierra Nevada is a major mountain range extending along most of eastern California between the Cascade Mountains in the north and the Tehachapi Mountains in the south. The orientation of the range is roughly north-northwest to south-southeast with a gradually rising western slope and, especially in the southern half, a more abrupt eastern slope. Numerous major rivers drain the western slope. The central Sierra comprises the watersheds of the Yuba, American, Mokelumne and Stanislaus Rivers which flow into the Great Valley of California, as well as the headwaters of the Truckee, Carson and Walker Rivers which flow eastward into Nevada. The largest body of water in the range, Lake Tahoe, is surrounded by area of relatively high relief. The lake itself, at an elevation of about 6200 feet, is drained by the Truckee River which descends to 4400 feet in Reno enroute to Pyramid Lake. Besides Pyramid Lake, other nearby flat areas include large valleys to the west of Reno which contain several reservoirs near 6000 feet elevation, and the valleys lying along the eastern slope of the Sierra near 4500 feet elevation containing Reno, Washoe Lake, and Carson City. Surrounding Lake Tahoe, the upper Truckee River, and the high meadows are relatively high mountains; the Sierra on the west with crests to 8000 and 9000 feet and the Carson Range on the east with higher crests of 9000 to 10,000 feet.
Climate

The weather in the area surrounding the Truckee River basin is described by Klieforth, 1974, as being influenced mainly by air masses from the Pacific. Cyclonic storms bring precipitation generally in the period October to June with maximum amounts in the winter months (December to March). The months July through September are usually dry with occasional periods of thunderstorms in moist, tropical air from the southeast. There is great variability in precipitation from year-to-year, often of the order of 50 percent, and even greater spatial variation as a result of orographic effects. The average annual precipitation varies from less than 7 inches in Reno and Carson City to more than 70 inches on the west slope of the Sierra. Snowfall often exceeds 400 inches per year on the higher elevations where it comprises 60 to 90 percent of the total precipitation. Because of heavy forest cover over much of the area, the snowpack is retained for a long period extending into summer, resulting in sustained streamflow. The average water equivalent of the snowpack on April 1 varies from 20 to 40 inches between 6000 and 7000 feet and 40 to 50 inches above 8000 feet.

Klieforth, 1974, also describes the air flow for Truckee River Region as being predominantly westerly. Wind speeds are light most of the time but approach gale force over ridges in big storms. Sunshine is abundant most of the year and average temperatures are moderate. The biggest temperature variations occur in the large valleys where the diurnal range is of the order of 20 to \(30^\circ C\) much of the year. In July, average minima are 2 to \(5^\circ C\) and maxima 27 to \(30^\circ C\),
while in January average minima are -13 to -9°C and maxima about 5°C; extremes in the Tahoe basin are 35°C in summer and -32°C in winter. Fog forms frequently during storms on the west slope of the Sierra and between storms in the valleys east of the Sierra crest. During most of the year, the air is exceptionally clear with visibility of the order of 100 miles.

Streamflow

The Truckee River originates at Tahoe City, California, where outflow from Lake Tahoe is regulated by a seventeen gate concrete dam. Initially the river follows a northerly course for about fifteen miles along which significant runoff is contributed by numerous natural and regulated tributaries. Beginning near Truckee, California, a northeasterly alignment is assumed and persists for about forty miles to Reno, Nevada. In this reach substantial flow is added from impoundment facilities which include Prosser, Boca, and Stampede Reservoirs. Very little additional runoff is contributed to the Truckee River below Reno where a northeasterly course assumed before the river system terminates at Pyramid Lake. A wide variation in channel slope is observed through the Truckee River System. In the upper reaches of Truckee River and streams discharging into Lake Tahoe, slopes of the order of 500 feet/mile are found while in lower reaches below Reno, slopes are comparatively flatter, generally less than ten feet/mile.

Runoff from the Truckee River basin is mainly the result of snow-melt occurring during late spring and early summer. As the Truckee River and other principal tributaries are largely regulated, flow in
the Truckee River is generally moderate except for snowmelt season. Annual runoff volumes at Farad range from a low value of 133,000 acre-feet in 1931 to a high of 1,432,000 acre-feet in 1907.

Major flood events on the Truckee River are usually the result of heavy, warm rains in December or January. These floods are characterized by a high peak and short duration, varying from three to six days. Lesser snowmelt floods occur more frequently due to rapid melting of snowpack during late spring and early summer. These floods are characterized by moderate peak flows and a prolonged duration of from one to four months. Total runoff volumes of snowmelt floods are usually substantially greater than rain floods.

A third type of flooding in the Sierra is the result of convective storm activity occurring primarily during summer months. Although high intensity thunderstorms frequently cause severe localized flooding on tributary streams they seldom cause significant flooding throughout the Truckee River system. Floods of this variety rarely have durations of more than a few hours, however, instantaneous peak flows of up to 430 cfs/sq. mi. have been observed on drainage basins of seven to ten square miles in area.

Truckee Meadows Study Area

Truckee Meadows is but one of a series of intermontane basins which lie along the eastern front of the Sierra. According to Thompson and White, 1964, these basins are formed by a combination of faulting, tilting, and warping with erosion and deposition playing a
major role in shaping present topography. Additional work by Thompson and Sandberg, 1958 (p. 1275) suggest that depth of alluvial sediments can range up to 2800 feet at certain points thereby giving rise to an extensive ground-water flow system. The Truckee Meadows is also the site of the Reno-Sparks urban area as well as extensive agricultural development. Local water demands are met by diversions from the Truckee River, which traverses the meadows, by direct runoff from surrounding mountainous drainage basins, and to lesser degree by wells. This combination of urbanization and agriculture serves to considerably alter the natural hydrologic system with respect to both temporal and spatial distribution of ground and surface water. Moreover lack of meaningful historic records for diversion, irrigation, urban runoff, and well pumpage virtually precludes the possibility of identifying individual cause and effect relationships at many locations in the meadows without considerable additional time and effort. It was therefore of necessity to locate an area within the Truckee Meadows relatively free from human influence in which to monitor the data necessary to model the ground water - surface water interchange process.

The actual study area selected, which is shown in Figure 2, consists of a relatively small section of the alluvial fan of the Truckee River. The ground-water system located immediately south of the river is virtually free from the effects of well pumpage, however, some influence from irrigation and subsequent recharge is present. In order to monitor water table fluctuations in the vicinity of the river, a total of five PVC cased piezometer holes were drilled to
Figure 2 Study reach of Truckee River
various depths at the locations indicated in Figure 2. The lithology pattern encountered in drilling the piezometer holes is described in Figure 3 and basically consists of assorted combinations of sand, silt, and gravel layers. The absence of extended clay lenses is notable here in that it suggests the presence of a basically unconfined aquifer system.

Water table elevations in the five piezometers were monitored on a weekly (and occasionally daily basis) for water year 1975 and roughly the first half of 1976. It was soon discovered, however, that piezometers F-4 and F-5 were influenced by irrigation practices to such a degree as to preclude their effective use in a modeling investigation such as this. Thus studies involving model calibration, verification, and analysis were carried out using only the data from piezometers F-1, F-2, and F-3. River stage elevations used in this investigation were obtained at the Boynton Lane Bridge concurrently with water table elevations in piezometers.
Figure 3  Piezometer data and lithology information
OVERVIEW OF HYDROLOGIC MODELING

The hydrologic modeling effort attempted herein is restricted to a specific type of hydrologic problem, namely the interchange between surface and ground water. As will be discussed subsequently, the basic approach is one of constructing a physically based model based on the partial differential equations which approximately govern the behavior of ground and surface water systems. To highlight the range of application of this particular model as well as to provide perspective, an overview is now presented outlining how this study relates in context to the general topic of hydrologic modeling. A review of related literature is also presented.

CATEGORIES

Taken in the broadest sense methods of applied hydrologic systems investigations can be divided into two categories, namely "stochastic" and "parametric". Formal definitions for these terms have been given previously while a breakdown of each category is shown in Figure 4. Although not all of the methodologies listed here can be considered as modeling techniques mention of their existence complements the general discussion of hydrologic modeling.

Stochastic Methods

In recent times a number of stochastic models have been proposed which serve a variety of purposes (e.g., Hufschmidt and Fiering, 1966; Fiering and Jackson, 1971; U.S. Army Corps of Engineers, HEC 4, 1968).
PARAMETRIC AND STOCHASTIC HYDROLOGY

STOCHASTIC METHODS

Markov Chains
Monte Carlo Method

PARAMETRIC METHODS

Correlation Analysis
- regression analysis
- analysis of variance
- product-moment correlation

Partial System Synthesis with Linear Analysis
- unit hydrograph
- dimensionless hydrograph
- dimensionless snowmelt hydrograph

General Non-linear Analysis

General System Synthesis
- full synthesis
  - Stanford watershed model
- partial synthesis
  - surface water models
  - ground water models
  - linked models

Figure 4 Categories comprising parametric and stochastic hydrology
In general these and other similar methods can be classified as being either "Monte Carlo" or "Markovian" depending on the basic approach taken. Although theoretically different, both of these methods are generally used for the same purpose, namely the generation of artificial or synthetic data sequences which are statistically similar, to the observed data from which they are derived. When Monte Carlo methods are applied a recorded data sequence(s) is first analyzed to determine its intrinsic statistical properties. Next, an appropriate generation scheme is developed, quite often utilizing random number tables, which results in synthetic data sequences which reflect the same statistical properties as the original data. Monte Carlo methods are best suited for investigations in which the state of the hydrologic system at a given time is independent of previous system behavior. However, if the hydrologic system exhibits persistence (i.e., magnitude of events at a given time depend on previous events), then the system is considered to be Markovian in nature and can thus be treated by what is termed the theory of transition probability. A Markovian process can be described by the equation

\[ q_t = e q_{t-1} + \gamma r \]  \hspace{1cm} (1)

where \( q_t \) is the variable value at time "t", \( q_{t-1} \) is variable value from previous time step, \( e \) is a constant < 1.0, \( \gamma \) is a constant depending on the statistical properties of the recorded data, and \( r \) is a random variable. If \( e \) and \( \gamma \) are properly determined, this model can be used to generate statistically similar sequences of output by random sampling of \( r \).
Parametric Methods

Referring again to Figure 4 it is seen that a considerably greater breakdown of parametric methods is possible than in the case of stochastic methods. By way of overview, it should be stated that the fundamental purpose of parametric methods of modeling and analysis is the establishment of input-output relationships which approximate behavior of hydrologic systems. This task may be accomplished in a variety of ways. For example, correlation methods, which consist of such techniques as regression analysis, analysis of variance, or product moment correlation, can be used to develop statistical parametric models between hydrologic variables. Another approach is comprised of methods which employ the principal of superposition and proportionality to model various components of hydrologic systems (i.e., partial linear system synthesis). These techniques, which include, for example, the Unit Hydrograph (Sherman, 1932), Dimensionless Snowmelt Hydrograph (Cunningham, 1977), basically consist of a model prototype derived from observed data which operate linearly on a given set of input data to synthesize output. Although widely used linear models can only be expected to yield, at best, rough approximations since individual elements of real hydrologic systems rarely exhibit linear behavior. A third and possibly more powerful approach to developing input-output models consists of general non-linear analysis. Although application to hydrologic systems has occurred only recently, significant evidence of its potential for constructive use has been demonstrated by Amorocho, 1967, Prasad, 1967, and Jacoby, 1962.
General System Synthesis

The final category of parametric methods to be considered, namely general system synthesis, is by far the most fundamental to this investigation. It is here that the development of techniques for modeling the interchange between ground and surface water can be put in perspective with comparable techniques for modeling other aspects of hydrologic systems. As is seen in Figure 4, general system synthesis consists of two categories, i.e., full as well as partial system synthesis. Full system synthesis requires that the hydrologic model somehow encompasses all major components of the land phase of the hydrologic cycle. This does not imply, however, that in a full synthesis model all elements of hydrologic systems are necessarily treated to the same degree. Actually most such models have as their basic objective the synthesis of various aspects of direct surface runoff resulting from either assumed or historic precipitation input. Probably the best single example of a full synthesis investigation is the Stanford watershed model after Crawford and Linsley, 1960.

Although various versions and modifications have been published the fundamental concept underlying these models is that input precipitation is routed among individual processes in a hydrologic system in accordance with the set of system parameter values developed by the user. Once properly calibrated, the Stanford watershed model is capable of synthesizing the streamflow hydrograph resulting from an arbitrary precipitation input as well as other system variables such as evapotranspiration and ground-water storage. Although good results are possible (Crawford, 1964), depending on the adequacy of data
available for calibration, performance of this type of model is not always sufficiently reliable so as to place complete confidence in extended runoff histories. According to Amorocho, 1964 (p. 315) this lack of reliability is due to 1) errors in recorded input data, 2) effects of areal distribution of parameters, 3) imperfections of the model structure, and 4) nonuniqueness of the particular synthesis method. It should also be noted here that those four sources of error likewise apply to the following discussions concerning ground-water and surface-water models.

Partial System Synthesis

Partial system synthesis consists of structuring models which describe the behavior of a specific set of hydrologic system components in such a way as to preclude the necessity of full system synthesis. Three types of modeling approaches comprise this category, namely surface-water models, ground-water models and coupled or "linked" ground and surface-water models.

Surface-Water Models: The topic of surface-water modeling is generally concerned with simulating the disposition of transient surface waters in hydrologic systems. Although interaction between surface and ground water may not be entirely ignored in such investigations the subject is usually treated only to the degree necessary to include any significant influence on desired model output. For example, if the main objective in a modeling study is simulation of flood hydrographs resulting from high-intensity convective storms, net influx of moisture to the ground-water system (and subsequent
ground-water system response) may simply be represented as being an abstraction from the surface-water hydrograph.

Review of contemporary investigations discloses that numerous surface-water modeling techniques have been developed which serve a variety of purposes. The Flood Hydrograph Package (HEC-1, 1973) developed by U.S. Army Corps of Engineers, for example, has the capability to simulate flood hydrographs resulting from precipitation and runoff for a complex, multibasin, multichannel river system. Similarly, a generalized streamflow simulation system (Burnash, et al., 1973) developed by the joint Federal-State Flood Forecasting Center of California, has as its explicit purpose forecasting of regional flooding on a real-time basis. Of particular interest to the present investigation, however, are some recent modeling efforts which has as their main purpose the numerical simulation of unsteady flow in natural open channels. This modeling technique, examples of which include studies by Strelkoff, 1969; Strelkoff, 1970; Gupta and Moin, 1974; Amein and Pang, 1970, is explained in detail in a subsequent section.

Ground-Water Modeling and Linked Models: Approaches taken to simulate ground-water systems are equally if not more diversified than the surface-water modeling techniques previously discussed. A complete review of ground-water modeling is not attempted here. However, a review of the general approaches taken to simulate ground-water responses to variations in stage of a surface-water body is instructive. Approaches to the problem have ranged through nearly all of the
basic methodologies; Hele-Shaw models (e.g., Todd, 1955; Ibrahim and Brutsaert, 1965), analytical solutions to simplified flow equations (e.g., Cooper and Rorabaugh, 1963; Ferris, 1951; Hall and Moench, 1972; Moench and Kisiel, 1970; Pinder, Bredehoeft, and Cooper, 1969), electric analog models (e.g., Walton and Hills, 1967), numerical solutions to flow equations (e.g., Hornberger, Ebert, and Remson, 1970; Zucker, et al., 1973; Cooley and Westphal, 1974). In most of these studies, the interconnection with the surface-water body has been handled by two general methods: as a lateral known head boundary for one-dimensional flow (e.g., all of the analytical solutions cited) and as a recharge term analogous to that used in leaky aquifer theory (e.g., Cooley and Westphal, 1974; Walton and Hills, 1967). One of the more complete field studies which involves the interchange of ground and surface waters is that of Konikow and Bredehoeft, 1973. Their study involved a combined numerical model of mass transport and ground-water flow for the reach of the Arkansas River between La Junta and the Bent-Otero County Line, Colorado. The research showed that the numerical model was indeed capable of reproducing the ground-water levels and approximate TDS variations. However, the river stage fluctuations were regarded as known input, and the uniqueness of their solution was not investigated.

A very significant study involving the application of mathematical methods to stream-aquifer interactions is due to Pinder and Sauer, 1971. This study was primarily directed toward investigation of the modification of a flood hydrograph due to bank storage effects on a
hypothetical flood plain. Herein the dynamic equations describing one-dimensional open channel flow and an equation for two dimensional transient ground water flow were solved simultaneously, coupled by an expression for flow through the wetted perimeter of the channel. The hypothetical situation analyzed consisted of a straight, rectangular channel of constant slope bounded by a homogeneous, isotropic, unconfined aquifer. The following conclusions were reported: 1) Provided the appropriate physical parameters can be determined, the dynamics of the stream-aquifer system can be simulated by using available numerical techniques. 2) Bank storage attenuates a flood wave and this modification of the wave may be considerable in the lower segment of a long reach. 3) The extension of hydrograph base time by bank storage effects may generate a recession curve similar in appearance to one due to regional ground water flow. 4) The length of the channel reach and the hydraulic conductivity of the floodplain aquifer have a considerable influence on the modification of a flood wave by bank storage. 5) The response of a floodplain aquifer to the propagation of a flood wave along a hydraulically connected channel decreases rapidly with distance from the stream.

Another very enlightening investigation, at least in terms of relevance to this study is that of Cooley and Westphal, 1974. Their primary objective was to determine whether or not the available data and basic theory are adequate for describing the interrelations between ground and surface water in a real hydrologic system. The particular system chosen consisted of selected reaches of the Humboldt
River in northern Nevada. Preliminary to modeling the real hydrologic system a theoretical study was undertaken which compared the ability of four different models to simulate the interchange of ground and surface water. The four models involved were, in order of increasing simplification, variably saturated flow (the combination of flow in both the saturated and unsaturated regions) which allows the water table to form as the atmospheric pressure isobar, the classical free-surface theory in which the water table is treated as a moving constant pressure boundary, an approximation of the free-surface theory where the free-surface boundary condition is incorporated into the flow equation as a convolution integral and a generalized form of the Boussinesq equation. Incorporation of seepage to and from the river in the last two models was accomplished through use of a leakage term the magnitude of which is proportional to the difference between stream stage and mean head in the vertical in the sediments below the channel. The three cases which were solved with the four models include a river channel surrounded by homogeneous, isotropic sediments; and a river channel lined by a skin of sediments having low permeability which is surrounded by homogeneous, isotropic sediments. It was found that the results from using all four models to these test cases were very similar, thus favoring the use of the simplest technique (i.e., the Boussinesq equation) to model the real hydrologic system.

To simulate the interrelationships between the Humboldt River and ground water flow in contiguous sediments, the fluctuation of the river stage from its fall and early winter 1962 level was routed
empirically along the study reach using data from February 1962 through October 1962. Then, with the use of superposition theory, a linearized form of the Boussinesq equation together with an equation incorporating the transfer term were used to model fluctuations in ground water stage observed in test wells in the study area.

The principal conclusions from the final portion of the study which are of interest here are 1) The linearized Boussinesq approach for modeling fluctuations in ground water stage induced by changes in Humboldt River Stage reproduced most of the essential characteristics of the water level fluctuations in wells in the study area. 2) Calculated stage fluctuations at the well sites are highly insensitive to decreases in saturated thickness of the medial gravel unit away from the river, and fluctuations in ground water stage were modeled just as adequately using a constant saturated thickness as a variable one. 3) The evapotranspiration (ET) rate from ground water under the flood plain was significant and, based on a specific yield of 0.08, the component used in the model is about one-tenth of that obtained from lysimeter tanks located near Winnemucca. 4) Seepage losses vary with the hydrograph of river stage and, hence, may vary rapidly in time. Any attempt to measure seepage losses using an inflow-outflow method must involve simultaneous measurement at all stations if the river discharge is varying. 5) The highest seepage rates are associated with the early season hydrograph peak because the greatest head differential between stream stage and ground water stage (mean head in the vertical) beneath the stream occurs then. Subsequent stream hydrograph peaks produce lower seepage rates, even if the peaks are
higher than the early season peak. 6) Loading due to changes in river stage was apparently not an important factor in producing changes in ground water stage because the magnitude of the term to be added to the equation for ground water flow to account for loading was approximately three orders of magnitude smaller than the size of the term accounting for water transfer.

By far the most complete study in terms of structuring a coupled surface and ground water model is due to Freeze, 1972. His approach consisted of numerical solution to a set of equations representing three-dimensional, transient, saturated-unsaturated subsurface flow and one-dimensional, gradually varied, unsteady channel flow for the primary purpose of analyzing baseflow contributions to perennial flow in well defined channels. Subsequent to developing linkage and solution methodology Freeze analyzed the response of a hypothetical drainage basin to variations in rainfall and watershed parameters concluding, in part, that surface runoff in base flow dominant streams is influenced by rate and duration of rainfall, hydrogeologic configuration, saturated permeabilities, and unsaturated characteristics of soil types. These particular results are of importance in that they serve as guidelines in the present study for the investigation of the influence of aquifer parameters on baseflow recession.

The Freeze model is definitely more generalized than the approach chosen for the present study, primarily due to consideration of flow in the unsaturated zone along with the three-dimensional treatment of subsurface flow. Unsaturated flow was not considered in the present
study, and, because of computer storage limitations, subsurface flow modeling was reduced to two dimensions. Nevertheless, the present study does compliment Freeze's work to some degree in that it centers around system response to streamflow variations introduced at the upstream boundary. In all simulations reported by Freeze, the height of the stream remained below the seepage face exit point and thus reversals in the ground water gradient did not occur.
DEVELOPMENT AND SOLUTION OF FLOW EQUATIONS

INTRODUCTION

Based on results of the foregoing literature review the linearized version of the Boussinesq equation was selected for the ground water flow model for this investigation. This equation will subsequently be linked with the Saint Venant equations, which govern flow in natural open channels, in such a manner as to allow simultaneous numerical solution. However, as a prelude to discussion of linkage and solution procedures, a theoretical development of both Boussinesq and Saint Venant equations is first presented. Although such theoretical background is available in current literature, a brief outline of derivation procedures is necessary to identify the type and quantity of mathematical and physical assumptions inherent in the flow equations themselves. Such information lends perspective to subsequent analyses of model predictive capability.

DERIVATION OF BOUSSINESQ EQUATION

For ground water flow, the conservation-of-mass principle requires that the rate of increase or decrease of fluid mass in an element situated in the flow field be equal to the difference between the rates of influx and efflux; that is, there can be no gain or loss in mass. For the differential control volume of dimensions $dx_1$, $dx_2$, $dx_3$, (Figure 5), the mass influx rate through face ABCD is mass inflow rate to ABCD = $\rho_w V_{x_1} dx_2 dx_3$  \[ (2) \]
Figure 5  Differential control volume.
where \( V_{x_1} \) is the specific discharge normal to area \( dx_2 dx_3 \) and \( \rho_w = \) mass density of water. The outflow rate through face EFGH is

\[
\text{Mass outflow rate from EFGH} = (\rho_w V_{x_1} + \frac{\partial}{\partial x_1} \rho_w V_{x_1}) dx_2 dx_3 \tag{3}
\]

which allows for the possibility of nonuniform density, or specific discharge or both. Similar expressions follow for AEHD and BFGC.

Combining the net inward flux is the difference between mass inflow and outflow,

\[
\text{Net inward flux} = -(\frac{\partial}{\partial x_1} \rho_w V_{x_1} + \frac{\partial}{\partial x_3} \rho_w V_{x_3}) dx_1 dx_2 dx_3 \tag{4}
\]

By the principal of conservation of mass the net inward flux equals the rate at which water is accumulating in the differential element. According to Domenico, 1972 (p. 216-220), the later quantity is

\[
\frac{\partial (\Delta M_w)}{\partial t} = \rho_w (\alpha \rho_w g + n \beta \rho_w g) \Delta x_1 \Delta x_2 \Delta x_3 \frac{\partial h}{\partial t} \tag{5}
\]

where \( \frac{\partial (\Delta M_w)}{\partial t} \) is the time rate of change of fluid mass within the element in response to a change in fluid pressure \( \Delta P \), \( n \) is aquifer porosity, \( g \) is acceleration due to gravity, \( \alpha \) is vertical compressibility (defined as \( 1/E_\alpha \), the bulk modulus of aquifer), \( \beta \) is fluid compressibility (defined as \( 1/E_\beta \), the bulk modulus of the fluid), and \( h \) is hydraulic head.

Equating Equations 4 and 5 and canceling common terms yields

\[
-(\frac{\partial}{\partial x_1} \rho_w V_{x_1} + \frac{\partial}{\partial x_3} \rho_w V_{x_3}) = \rho_w (\alpha \rho_w g \\
+ n \beta \rho_w g) \frac{\partial h}{\partial t} \tag{6}
\]

The term in parenthesis on the right hand side of Equation 6 is called specific storage \( S_s \). As defined by Hantush, 1964, \( S_s \) is the volume of water that a unit volume of aquifer releases from storage because of
expansion of the water and compression of the grains under a unit decline in average head within the unit volume.

Assuming \( p_w \) to be constant and specifying \( W \) as a source-sink term Equation 6 can be written in Cartesian tensor notation as

\[
- \frac{\partial v_i}{\partial x_i} = \frac{\partial h}{\partial t} + W
\]

(7)

where \( v_i \) is the component of specific discharge in the \( x_i \) Cartesian coordinate direction, \( h \) is hydraulic head, \( t \) is time, \( S_s \) is specific storage, \( W \) is a source-sink term, positive for a source. All terms here have units of \( 1/t \), which is conceptually the same as discharge per unit volume.

Equation 7 applies at any point in a three-dimensional orthogonal coordinate system, and numerical solutions of it are, in theory, no more difficult than for the two-dimensional version. However, because of difficulties associated with basic data acquisition, computational time and effort and volume of computer program input, it is advisable to reduce the dimensionality of Equation 7 whenever possible. This may be accomplished through the process of integration over an interval \( x_3 \), the vertical coordinate direction.

Referring to Figure 6, if the interval of interest ranges from \( x_3 = z_L(x_1,x_2) \) to \( x_3 = z_u(x_1,x_2) \), then Equation 7 is integrated to yield

\[
\begin{align*}
\int_{z_L}^{z_u} \frac{\partial v_i}{\partial x_i} \, dx_3 &= \int_{z_L}^{z_u} \frac{\partial h}{\partial t} \, dx_3 - \int_{z_L}^{z_u} W \, dx_3 \\
\end{align*}
\]

(8)
Figure 6 Diagram illustrating flow continuity in the $X_2$ direction for a control volume of porous medium bounded on top and bottom by $z_u$ and $z_L$, respectively. View in $X_1$ direction would be analogous.
We now assume \( S_s \) to be an effective value such that

\[
\int S_h \frac{\partial h}{\partial t} \, dx_3 = S_s \int \frac{\partial h}{\partial t} \, dx_3
\]

(9)

By use of Darcy's law, \( v_i = -K_{ij} \frac{\partial h}{\partial x_i} \), and the concept that the conductivity tensor, \( K_{ij} \), is an effective value over the interval \( z \) to \( z' \), Cooley, 1974 has shown that Equation 8 can be integrated to yield

\[
\frac{\partial}{\partial x_i} \left\{ K_{ij} \left( \frac{\partial}{\partial x_j} \left( z_u - z_L \right) \right) - h_u \frac{\partial^2 h}{\partial x_j} + h_L \frac{\partial^2 h}{\partial x_j} \right\}
\]

(10)

where \( h = \int h \, dx_3/(z_u - z_L) \), \( h_u \) is \( h \) at \( z_u \), and \( h_L \) is \( h \) at \( z_L \). It is now assumed that

\[
\left| \frac{\partial}{\partial x_i} \left\{ K_{ij} \left( h - \bar{h} \right) \frac{\partial h}{\partial x_j} + \left( h - \bar{h} \right) \frac{\partial^2 h}{\partial x_j} \right\} \right| \ll \left| \frac{\partial}{\partial x_i} \left( K_{ij} (z_u - z_L) \frac{\partial \bar{h}}{\partial x_j} \right) \right|
\]

(11)

which, for nearly planar \( z_L \) and \( z_u \) of low slopes and (or) small vertical hydraulic gradients, is a good approximation. The equation then becomes

\[
\frac{\partial}{\partial x_i} \left( K_{ij} (z_u - z_L) \frac{\partial \bar{h}}{\partial x_j} \right) = \frac{V_{un}}{n_{u3}} + \frac{V_{ln}}{n_{L3}} = S_s (z_u - z_L) \frac{\partial \bar{h}}{\partial t} - v_s
\]

(12)

where \( v_s = \int w \, dx_3 \).
Note that neither $z_u$ nor $z_L$ can be the water table in Equation 12 because both were assumed to be independent of time. Thus Equation 12 approximates conditions for a confined aquifer system.

Dimensional analysis of Equation 12 shows that each term has dimensions of length/time. Conceptually this equation now represents flow conditions along a fully penetrating vertical line through a control volume as shown in Figure 7. The first term, \( \frac{\partial}{\partial x_i} \left( K_{ij} (z_u - z_L) \right) \), represents the total net lateral (two dimensional in $x_1$ and $x_2$ directions) specific discharge crossing the line, $-\nu_{un}/n_{13} + \nu_{Ln}/n_{L3}$ represents the net vertical specific discharge through the top and bottom of line (note, $n_{13}$ and $n_{L3}$ are direction cosine terms which transform $\nu_{un}$ into specific discharge in the $x_3$ direction), $S_s (z_u - z_L)$ is the summation of specific discharge released from compression storage at all points along the line, and finally $v_s$ the specific discharge moving into or out of control volume along the vertical line (i.e. line source or sink).

It is likewise possible to demonstrate how to modify Equation 12 to be valid for unconfined flow. By replacing $z_u$ by $H$, the water table elevation, the following equation is obtained (Cooley, 1974).

\[
\frac{\partial}{\partial x_i} \left( K_{ij} \left( (H-z_L) \frac{\partial H}{\partial x_j} - (H-H_L) \frac{\partial z_L}{\partial x_j} \right) \right) = \frac{\nu_{un}}{n_{13}} + \frac{\nu_{Ln}}{n_{L3}}
\]

\[
= S_s \left( (H-z_L) \frac{\partial H}{\partial t} - (H-H_L) \frac{\partial H_L}{\partial t} \right) - v_s
\]

(13)

Equation 13 was originally derived by Hantush, 1964, (p. 301).

Cooley, 1974 also demonstrates how it can be reduced to the form of
Figure 7  Generalized flow conditions along a fully penetrating line through control volume
the Boussinesq equation (Bear, 1972, p. 378) through use of the Dupuit-Forchheimer assumptions together with the assumptions that

\[ |\frac{(H-z_L)}{L}\frac{\partial H}{\partial t}| > |\frac{(H-h)}{L}\frac{\partial H}{\partial t}| \]  

(14a)

\[ \frac{\partial H}{\partial x_j} = \frac{\partial H}{\partial x_j} \]  

(14b)

and

\[ \frac{\partial H}{\partial t} = \frac{\partial H}{\partial t} \]  

(14c)

The resulting equation is

\[ \frac{\partial}{\partial x_j}\left( K_{ij} (H-z_L) \frac{\partial H}{\partial x_j}\right) - \frac{\nu_{un}}{n_{u3}} + \frac{\nu_{Ln}}{n_{L3}} = S_y (h-z_L) \frac{\partial H}{\partial t} - v_s \]  

(15)

The free surface boundary condition (e.g., Todsen, 1971, p. 209),

\[ - \nu_1 \frac{\partial H}{\partial x_1} - \nu_2 \frac{\partial H}{\partial x_2} + v_3 = S_y \frac{\partial H}{\partial t} - I = \frac{\nu_{un}}{n_{u3}} \]  

(16)

where \( S_y \) is specific yield and \( I \) is infiltration rate, is now substituted into Equation 15 to yield

\[ \frac{\partial}{\partial x_i}\left( K_{ij} (H-z_L) \frac{\partial H}{\partial x_i}\right) + \frac{\nu_{Ln}}{n_{L3}} = \left( S_y + S_s (H-z_L) \right) \frac{\partial H}{\partial t} - I - v_s \]  

(17)

Establishment of the final linearized form of Boussinesq equation is accomplished by incorporating assumptions somewhat less stringent than those leading to Equation 17. Here it is assumed that

\[ \frac{\partial H}{\partial x_i} = \frac{\partial H}{\partial x_i} \]  

(18a)

\[ \frac{\partial H}{\partial t} = \frac{\partial H}{\partial t} \]  

(18b)
and
\[ \frac{\partial}{\partial x_1} \left( K_{ij} (h - z_L) \frac{\partial h}{\partial x_j} - (h - h) \frac{\partial h}{\partial x_j} \right) > > \frac{\partial}{\partial x_1} \]

(18c)

With these assumptions Equation 13 becomes (Cooley, 1974, p. 8)
\[ \frac{\partial}{\partial x_j} \left( K_{ij} (h - z_L) \frac{\partial h}{\partial x_j} \right) + \frac{V_{ln}}{n_L^3} = \left( S_Y + S_g (h - z_L) \right) \frac{\partial h}{\partial t} - I - v_s \]  

(19)

DERIVATION OF SAINT VENANT EQUATIONS

The Saint Venant equations serve to approximately describe the one-dimensional unsteady, spatially varied flow of a homogeneous liquid in a rigid channel of arbitrary shape and alignment. This pair of equations, one based on conservation of mass (continuity), the other on conservation of either energy or momentum, pose a distinct advantage over more general equations in that they have only two unknown dependent variables, depth and average velocity (assuming channel geometry and lateral discharge are known. The Saint Venant equations thus provide a system which can ostensibly be solved simultaneously (although not analytically) once the appropriate initial and boundary conditions have been specified. The following derivation of both continuity and dynamic equations has been modified after Chow, 1959 (p. 525).

Continuity Equation

The continuity equation for unsteady, spatially varied flow in an open channel may be developed by applying the principle of
conservation of mass to a deforming, differential control in a region bounded by two arbitrary channel sections (Figure 8a). In unsteady flow, the discharge, \( Q \), changes with distance in direction of flow, \( x \), as \( \frac{3Q}{3x} \), and the depth, \( y \), changes with time at a rate \( \frac{3y}{3t} \). The change in discharge through space in the time \( dt \) is \( (\frac{3Q}{3x}) \ dx \ dt \). The corresponding change in channel storage in terms of the top width, \( T \), is \( T \ dx (\frac{3y}{3t}) \ dt \) which, assuming \( T \frac{3y}{3t} = \frac{dA}{3t} \), can also be written as \( \frac{dx (3A/3t)}{3t} \ dt \).

Since water is assumed incompressible, the net change in discharge plus the change in storage should be zero; that is,

\[
\left( \frac{3Q}{3x} \right) dx dt + T dx \left( \frac{3y}{3t} \right) dt = \left( \frac{3Q}{3x} \right) dx dt + dx \left( \frac{3A}{3t} \right) dt = 0 \tag{20}
\]

Simplifying, Equation 20 becomes

\[
\frac{3Q}{3x} + T \frac{3y}{3t} = 0 \text{ (21a)}
\]

or

\[
\frac{3Q}{3x} + \frac{3A}{3t} = 0 \text{ (21b)}
\]

At a given section, \( Q = VA \); thus Equation 21b becomes

\[
\frac{3(VA)}{3x} + T \frac{3y}{3t} = 0 \tag{22}
\]

**Dynamic Equation**

Again considering the fluid element in Figure 8a, the energy balance across the element can be written as,
Figure 8a Continuity of unsteady spatially varied flow
\[ Z + y + \frac{\alpha v^2}{2g} = z + dz + y + dy + \frac{\alpha v^2}{2g} + d \left( \frac{\alpha v^2}{2g} \right) + h_{L}^{x+dx} \] (23)

where \( Z \) = elevation of channel bottom above an arbitrary horizontal datum, \( y \) = depth of flow, \( \alpha \) = energy coefficient, \( V \) = average velocity, \( g \) = gravitational acceleration, \( h_{L}^{x+dx} \) = total energy dissipated in the channel between \( x \) and \( x + dx \). Simplifying, Equation 23 leads to

\[ d(z + y + \frac{\alpha v^2}{2g}) = h_{L}^{x+dx} \] (24)

In the case of unsteady flow the energy loss occurring in open channel flow is composed of two parts: 1) the loss due to friction, \( h_{f} \), and 2) an additional independent loss due to acceleration, \( h_{a} \). An expression for \( h_{a} \) and \( h_{f} \) can be developed with the aid of Figure 8b which describes the energy balance for unsteady flow through a differential element. Based on Newton's second law the force due to acceleration \( 3V/3t \) acting on a unit weight \( Y \) of fluid is equal to \( (\gamma/g) 3V/3t \). If it is assumed that the channel slope, \( S_{c} \), is small then the acceleration is assumed to be in the \( x \) direction and its vertical component is considered negligible. Thus, the work done by this force through a distance \( dx \) across the differential element in Figure 8b is \( (\gamma/g) (3V/3t)dx \). This amount of work is equal to the energy loss due to acceleration. Dividing by \( \gamma \) the acceleration loss becomes

\[ h_{a} = \frac{1}{g} (3V/3t)dx \] (25)

The frictional energy loss component, \( h_{f} \), can be expressed in terms of the friction slope, \( S_{f} \), as
Figure 8b: Simplified representation of energy in unsteady flow.
\[ h_f = S_f \, dx \]  \hspace{1cm} (26)

Here \( S_f \) can be expressed in terms of Manning's \( n \) and hydraulic radius, \( R \), (Chow, 1959, p. 263) as

\[ S_f = \frac{n^2 \frac{V^2}{2g}}{2.208 R^{4/3}} \]  \hspace{1cm} (27)

Substituting Equations 25 and 26 into Equation 24 results in

\[ d (z + Y + \frac{V^2}{2g}) = -S_f \, dx - \frac{1}{g} \frac{\partial V}{\partial t} \, dx \]  \hspace{1cm} (28)

Dividing through by \( dx \) in Equation 28 and utilizing partial differential.

\[ \frac{3(z+Y)}{3x} + S_f + \frac{3}{3x} \left( \frac{\partial V^2}{2g} \right) + \frac{1}{g} \frac{\partial V}{\partial t} = 0 \]  \hspace{1cm} (29)

or

\[ \frac{\partial V}{\partial x} + \frac{\partial V}{\partial x} + \frac{1}{g} \frac{\partial V}{\partial t} + \frac{3z}{3x} + S_f = 0 \]  \hspace{1cm} (30)

This is the general dynamic equation for gradually varied unsteady flow assuming no lateral inflow to channel.

In the case of lateral inflow, \( q \), to the channel another term must be added to Equation 30 to account for the additional energy added. Strelkoff, 1969, defines this term to be \((V-u_x)q/gA\) where \( u_x \) is the component of inflow velocity in the \( x \) direction. This additional energy term, together with the assumptions that \( -\frac{3z}{3x} = S_o \) and \( \alpha = 1.0 \). Allow Equation 30 to be written as

\[ \frac{1}{g} \frac{\partial V}{\partial t} + \frac{V}{g} \frac{\partial V}{\partial x} + \frac{\partial V}{\partial x} - S_o + S_p + \frac{(V-u_x)q}{gA} = 0 \]  \hspace{1cm} (31)
Equation 31 together with Equation 22 which becomes
\[ T \frac{\partial y}{\partial x} + \frac{\partial (VA)}{\partial x} - q = 0 \] (32)

Comprise the Saint Venant equations which approximately describe gradually varied, unsteady, spatially varied, open channel flow.

**Assumptions and Approximations**

As was evident from the foregoing derivation procedures both physical and mathematical assumptions and approximations abound in the development of Boussinesq and Saint Venant equations. The nature of these assumptions is indeed diversified ranging from more or less trivial, such as the assumption of constant \( \rho_w \), to rather severe approximations of physical phenomena, such as homogeneity and isotropy. At this point it is difficult to assess the true impact of assumption and approximation in terms of their ultimate effect on model predictive accuracy. Nor is it really possible to completely isolate this effect due to additional error encountered in approximate numerical solution techniques as well as errors in acquisition of field measurements. However an organized review of the foregoing assumptions and approximations is useful from the standpoint of retaining a proper perspective.

In development of Boussinesq equation by far the most important assumptions made were those of homogeneity and isotropy of storative and transmissive properties of porous media. Specifically these are manifested in derivation procedures by the assumption of effective values of \( K_{ij} \) (i.e., \( K \)), \( S_y \) and \( S_s \) throughout the entire aquifer
system. However despite their gross distortion of reality these assumptions do not constitute as great a threat to model predictive accuracy as might be initially envisioned. Reasons include: 1) model application to a specific system does not usually require output to be generated at all possible locations, therefore system parameters such as $K$, $S_y$, and $S_z$ can be calibrated with respect to only those points where output is obtained; 2) the finite element solution technique employed here does in fact allow system parameters to vary for each element; and 3) standard techniques for estimating field values of system parameters are generally not intended to detect spatial and directional variations, therefore effective use of mathematical relationships developed for non-homogeneous, anisotropic conditions is often precluded.

Other noteworthy assumptions in Boussinesq development include application of Dupuit-Forchheimer condition, (Bear, 1971; p. 361-366) together with the assumption that Darcy's law applies throughout the aquifer system. Although not as restrictive as the previously discussed approximations of homogeneity and isotropy, these assumptions nevertheless lose considerable validity at locations where drawdown is large such as, for example, in the vicinity of pumped wells. The assumption of constant fluid density completes the summary of physical approximations used in Boussinesq development while notable mathematical approximations are given by Equations 11, 14a, 14b, 14c, 18a, 18b, and 18c.

A similar review can be given of the derivation of Saint Venant
equations. As was stated in the derivation, these equations apply only under conditions of gradually varied open channel flow. This limitation restricts the use of Saint Venant to simulate unsteady flow in natural channels in that certain physical conditions must be avoided if meaningful results are to be achieved. For example, solution of Saint Venant equations by standard numerical techniques requires that the channel be subdivided into a number of finite sections over which hydraulic characteristics are assumed uniform. However it is entirely possible in this procedure to specify channel section characteristics which violate or at least stress the condition of gradually varied flow. Specific situations include sudden expansion or contraction of channel width between connecting sections, spedivication of section geometry and slope so as to create a hydraulic jump or bore, or slope changes which require flow regime to change from subcritical to supercritical. Such conditions, depending on their degree, could generate results of intolerable inaccuracy or prohibit solution altogether.

Another restriction of importance is the assumption that bed slope, $S_o$, be sufficiently small so that vertical acceleration is negligible - a condition which can conceivably be violated in both natural and man-made channels. Other somewhat less important approximations include: 1) that channel top width is given by $dA/dy$, 2) that kinetic energy coefficient, $a$, is unity, and 3) that slope of energy grade line, $S_f$, is given by Equation 27.
SOLUTION PROCEDURE

The linearized Boussinesq equation given by Equation 19 approximately describes unsteady, two dimensional flow throughout an unconfined aquifer with the exception of the region directly beneath the channel bottom. For flow in this region the storage-yield term in Equation 19 i.e., \( S_y + S_s (\frac{h}{z_L}) \) is replaced by the storage coefficient, \( S_c \), along with the addition of the exchange term \( \frac{q_g}{b} + 2y \). Thus with these changes, the equation for ground-water flow below the channel becomes

\[
\frac{\partial}{\partial x_1} \left( K_d \frac{\partial h}{\partial x_1} \right) + \frac{V_{ln}}{n_{L3}} = S_c \frac{\partial h}{\partial t} + \frac{q_g}{b + 2y} - 1 - v_s
\]

where \( K_d \) is aquifer transmissivity (\( d \) is saturated thickness below channel), \( S_c \) is storage coefficient, \( q_g \) is lateral ground-water inflow per unit channel length, \( b \) is channel bottom width, \( y \) is channel depth of flow.

Rewriting Saint Venant Equations using compatible notation yields

\[
\frac{1}{q} \frac{\partial V}{\partial t} + \frac{V}{q} \frac{\partial V}{\partial l} + \frac{\partial u_x (q_1 + q_g)}{gby} - S_O + S_f = 0 \quad (34)
\]

\[
T \frac{\partial V}{\partial t} + \frac{\partial (Vby)}{1} - (q_1 + q_g) = 0 \quad (35)
\]

where \( l \) is length along the channel flow path, \( q_1 \) is surface lateral inflow, \( q_g \) is ground water later inflow.

Finite Element Method

Numerical solution of Equations 19, 33, 34, and 35 was accomplished with a finite element scheme based on the Gelerkin method (see
Zienkiewicz, 1971, or Norrie and de Vries, 1973, for general explanation). Operationally this technique involves: 1) specification of approximating functions for \( V \) (or \( Q \)), \( y \), and \( h \) in terms of \( x_1 \) which involve unknowns to be determined for each element; 2) multiplication of Equations 19, 33, 34 and 35 by weighting functions derived from the approximating functions; 3) integration of the resulting equations over each element, which leads to a set of ordinary differential equations in time involving the unknowns; and 4) integration of the ordinary differential equations over time.

Application of Gelerkin finite element procedure reduces the first order, non-linear partial differential equations of Saint Venant to a set of approximate non-linear algebraic equations which are amenable to solution by standard matrix algebra techniques. As an illustration Cooley and Moin, 1976, (p. 766) show that the final finite element algebraic form of Saint Venant equations for node "i" other than boundary nodes can be written as:

\[
\begin{align*}
    a_{1i} \bar{v}_{i-1} + a_{2i} \bar{q}_{i-1} + b_{1i} \bar{v}_i + b_{2i} \bar{q}_i + c_{1i} \bar{v}_{i+1} + c_{2i} \bar{q}_{i+1} &= d_{1i} \\
    a_{3i} \bar{v}_{i-1} + a_{4i} \bar{q}_{i-1} + b_{3i} \bar{v}_i + b_{4i} \bar{q}_i + c_{3i} \bar{v}_{i+2} + c_{4i} \bar{q}_{i+2} &= d_{2i}
\end{align*}
\]

(36)

where \( a_{1i}, a_{2i}, \) etc. are coefficients, \( d_{1i}, d_{2i}, \) etc. are vectors involving all known terms at the current time step. A similar though somewhat more complicated result is obtained for finite element approximation of Boussinesq equation. Complete documentation of finite
element derivations for both Saint Venant and Boussinesq equations is given by Sinclair, 1977 and therefore not repeated here.

**Linkage of Ground and Surface Water Equations**

In order to solve Equations 18, 33, 34 and 35 simultaneously another expression describing the movement of water between ground and surface systems must be defined. According to Pinder and Sauer, 1971 (p. 65), this expression can be developed from Darcy's law as

\[
\frac{q_g}{b+2y} = \frac{-K_c (y_O + y - h)}{\Delta y}
\]

where \( K_c \) is channel bottom hydraulic conductivity, \( y_O \) is datum to channel to channel bottom, \( y \) is depth to flow, \( h \) is hydraulic head in aquifer relative to datum immediately outside of channel bottom sediments, \( \Delta y \) is thickness of channel bottom sediments. Equation 37 after rearrangement replaces the second term of the right hand side of Equation 33.

Equations 19 or 33, 34 and 35 represent three equations in three unknowns, namely \( V \) (or \( Q \)), \( y \) and \( h \), which can be solved simultaneously using Equation 37 and following the procedure outlined in the flow chart in Figure 9. After defining the necessary physical parameters it is necessary to first solve the channel and aquifer system equations for steady flow conditions. This involves: 1) Assuming a steady state channel flow and solving Equations 34 and 35 (with time derivatives set = 0) assuming \( q_g \) is initially 0. This results in initial estimates of \( y \) (and also \( Q \)) at each channel node which serves as input to Equation 33 by way of Equation 37. 2) Ground water
Figure 9  Flow chart for iterative solution of river-aquifer system (modified after Pinder and Sauer, 1971)
equations (Equations 19 and 33) are then solved for steady flow conditions (again time derivatives set = 0). This results in a new h value which, when used with y from channel solution in Equation 37, allows a new \( q_y \) to be computed. 3) The open channel flow equations must now be resolved using the modified \( q_y \) value. This cycle is repeated (i.e., solving Equations 34 and 35 then 19 and 33) until the change in the ground water inflow term, \( q_y \), between successive iterations is reduced to within a predetermined error tolerance. This steady state solution to the aquifer-stream system forms the basis for study of system response to a transient flow sequence introduced at the upstream boundary.

To proceed with the solution of the transient problem the procedure indicated above is repeated for each time interval \( \Delta t \). The equations for open channel flow and ground water flow are solved repeatedly for the same point in time until the change in leakage is less than the error tolerance. When this occurs the simulation proceeds to the new time \( t + \Delta t \).

The computer algorithm for solution of the linked ground water-surface water equations was developed by Sinclair and Cunningham, 1976 for the CDC 6400 computer and appears in Appendix I together with supplemental user documentation. This program was modified from a previously developed algorithm for numerical solution of transient ground water flow (Cooley, 1975) together with the finite element algorithm for solution of Saint Venant equations by Cooley and Moin, 1976.
OVERVIEW OF ANALYSIS PROCEDURES

Representation of the physical processes comprising a hydrologic system by means of any modeling procedure is subject to uncertainty from several sources. First of all, model input can never be exactly the same as for the hydrologic system because of the error inherent in actually measuring time and space variation of system input. This is further complicated by the uncertainty associated with the particular methods used to simulate individual system processes as such methods inevitably fall short of achieving perfect simulation. Furthermore, comparison of model and system output only partially defines model uncertainty because, as is the case for system input, system output also can not be measured exactly. Thus the best that can be hoped for is to construct a model which reduces this uncertainty to within tolerable limits.

An in-depth discussion of uncertainty associated with hydrologic modeling is given by Amorocho, 1967, (p. 866) in which the requirements for equivalence between a prototype system and its model are given. Strictly speaking, a system and its model are "equivalent" if they are both capable of transforming the same inputs into identical outputs. This definition, however, is far too restrictive for application to modeling of hydrologic systems. Instead, a more suitable concept is that of "approximate equivalence" (Amorocho, 1967, p. 864). A hydrologic system and its model are considered to be
approximately equivalent if the model output does not differ from the measured system output by more than some prescribed error tolerance. For the purposes of this study approximate equivalence is formally expressed as follows:

Let M be a model of a system S such that

\[ I(X_1, X_2, X_3, t) = \text{input to } S \]

\[ I'(X_1, X_2, X_3, t) = \text{input to } M \text{ with } I' \text{ being some approximation of } I \]

\[ O(X_1, X_2, X_3, t) = \text{output from } S \text{ (as measured and subject to measurement error)} \]

\[ O'(X_1, X_2, X_3, t) = \text{model output} \]

Then M and S are considered to be approximately equivalent over some range of inputs if

\[ O(X_1, X_2, X_3, t) - O'(X_1, X_2, X_3, t) = \phi(X_1, X_2, X_3, t) \]

and \( \phi_1 \leq \epsilon \)

where \( \epsilon \) is the pre-assigned error tolerance. The overall relationship between a hydrologic system and its approximately equivalent model is illustrated in Figure 10.

Since the linked groundwater - surface-water model used in this investigation was constructed from the differential equations which approximately govern both open channel and groundwater flow it is considered to be a "physically based" model. An attractive feature of such a model is its potential, when properly calibrated, to simulate hydrologic events beyond the range of historic or measured data (this is in contrast to empirical and statistical models which
Figure 10  Relationship between a hydrologic system and its approximately equivalent model (modified after Amoroch, 1967)
are generally considered valid for use only over the range of data used in their construction). However, trade-offs do exist in taking this physically based modeling approach, particularly in terms of complexity of both mathematical derivation and development of a solution technique. This complexity is obvious from the theoretical development presented previously where statements regarding both assumptions and mathematical compromise are made in such profusion so as to cast some doubt, at least at this point, as to the reliability of the final product. It is therefore paramount in such a modeling procedure as this to undertake a systematic and detailed analysis in order to assess the models' true capabilities and limitations, for its complexity certainly precludes blind application to field problems. This requires evaluation of $\phi_1$ which is an index illustrative of model predictive capability.

Model analysis was carried out in three phases: 1) determination of the relative sensitivity among the respective model parameters 2) calibration of the model to the designated study area (lower Truckee River System), and based on comparison of observed and predicted values of system output, evaluation of predictive uncertainty, and 3) estimation of uniqueness of solution.

Results obtained from these three types of analysis allow conclusions to be drawn regarding several aspects of model operation. One such aspect is determination of the relative importance of individual parameter values to successful model operation. Stated another way, the question being considered is which parameters are most
sensitive in terms of achieving acceptable modeling accuracy? Answers to this question provide valuable information regarding the degree of precision with which individual parameters need to be measured in the field.

Another desired result is determination of the degree of uncertainty associated with model predictions. This has been addressed herein by way of statistical analysis of comparisons between observed field data sequences for the Lower Truckee River System and corresponding predicted system outputs.

The final topic of interest concerns investigating the uniqueness of model solution. The concept of uniqueness is important because, if it can be quantified in some way, it serves as a measure of the reliability of model results which extend beyond the limits of measured field data. Estimates of model uniqueness developed herein are based on the model results obtained for the Lower Truckee River System.

PARAMETER SENSITIVITY

The basic purpose of conducting a sensitivity analysis on any hydrologic model is to establish relative sensitivities among the respective parameters. This information is useful particularly from the standpoint of determining the accuracy to which individual parameters must be known in order to insure successful model operation. For purposes of this study the term "sensitivity" is formally defined as:
a measure of how completely variations in a particular model input parameter are transmitted through and reflected as corresponding variations in model output.

However, in the case of the linked ground-water - surface-water model, application of this definition to the evaluation of parameter sensitivity is by no means straightforward. The problem in this case has to do with properly defining the terms input and output. As has been demonstrated in the theoretical development of the model, multiple rather than single hydrologic variables can be designated as being input or output. Consider, for example, the case where the linked model is applied to a system which includes a river reach located in an agricultural area and bounded on both sides by an unconfined ground water table. In this hypothetical case, input variable to the linked model might include river inflows to the system, well pumpage rates, irrigation diversions and returns, and evapotranspiration, precipitation and infiltration within the study area. Corresponding output variables could be river flows within the study reach, net seepage rate between river and aquifer, and fluctuations in ground-water table surface. However, if alternative field circumstances were to prevail, the model may need to be operated with different combinations of the above variables serving as model inputs and outputs. The main point here is that no general statement can be made regarding which variables are to serve as inputs and those which constitute model outputs without first specifying the particular hydrologic system to be modeled. This more or less precludes any general evaluation of parameter sensitivity for the linked model. Instead the analysis has been carried out relative to the particular set of input and output
variables which are best suited to the particular hydrologic system being studied.

In this study parameter sensitivity is evaluated by defining a sensitivity index, $S$, as the average of the absolute percentage differences between the predicted and observed output values. Mathematically $S$ is given by:

$$S = \frac{\sum_{i=1}^{n} \left| \frac{O_i - P_i}{O_i} \right| \times 100}{n}$$

where $O_i =$ system output value for time step "i"

$P_i =$ model output prediction for time step "i"

$n =$ number of time steps considered

Thus by systematically varying individual parameters while keeping model input constant, the relative sensitivity of the model to changes in each of its parameters can be examined by comparison of $S$ values. Note here also that $S$ is computed with respect to only one output variable.

The hydrologic system which has been modeled for the purpose of determining parameter sensitivity is essentially that used by Pinder and Sauer, 1971. The system consists of a flood plain aquifer which extends 130,000 feet along the length of the channel and is 1400 feet across the valley; it is surrounded by impermeable material on all sides. The hydraulic conductivity of the aquifer is 864 ft/day and the initial saturated thickness ranges from 220 feet at the upstream boundary to 90 feet at the downstream boundary. The stream flows along the axis of the valley through a straight rectangular channel.
with constant cross section and a slope of 0.001. The ratio of hydraulic conductivity of the stream bed to its thickness along the wetted perimeter of the stream is 4,000 ft/day and, therefore, is not a limiting factor in the amount of water entering the aquifer. The channel is 100 feet wide and the initial depth of flow is 20 feet.

The rectangular finite element grid representative of the hydrologic system under investigation is shown in Figure 11. Note that the vertical scale has been exaggerated to provide resolution necessary for visual analysis of water surface elevation contours (i.e., node points are 10,000 feet apart horizontally and 500 feet apart vertically). The dots in this illustration depict element nodes and therefore represent the locations at which the model provides solution values for the variables of interest. The dashed lines bordering the system represent "no-flow boundaries" and thus define the entire ground-water system. There are a total of thirteen river elements for this system and the interchange flow values the net seepage rate either into or out of the river over each of these elements.

This hypothetical hydrologic system was studied in terms of its response to the passage of a flood wave superimposed on a base flow of 18,000 cfs as shown in Figure 12. The flood wave has the following characteristics: initial flow = 18,000 cfs, peak flow = 26,000 cfs, time to peak = 1.67 hours, hydrograph time base = 12 hours.

In carrying out the model sensitivity analysis an initial model solution was obtained based on the parameter values listed above. Parameter sensitivities were then computed with respect to various
Figure 11  Rectangular finite element grid: nodes are 500 feet apart vertically and 10,000 feet apart horizontally
Figure 12  Inflow hydrograph used for analysis of hypothetical hydrologic system
model output variables by raising each parameter by first 10% and then by 100% while keeping all other parameters at their original values. Results of this analysis are displayed in Tables 1 and 2. As can be seen, output variables include river discharge, stage, and ground water - surface water interchange together with ground water table elevations at nodes 95, 96, and 97. Similarly ground water - surface water interchange has been studied over river element 9 which is the element bounded on the upstream boundary by node 94.

Analysis of Tables 1 and 2 provides insight as to the relative sensitivity of the different model outputs to changes in magnitude of the various parameters. Here it is seen that the model is most sensitive to changes in Manning's n and channel slope regardless of the type of output considered. At the 10% level of parameter change river stage and discharge along with ground water - surface water interchange are most sensitive to Manning's n, while water table elevations are most influenced by channel slope and secondly by Manning's n. These results differ only slightly at the 100% level of change wherein the aquifer hydraulic conductivity becomes the most sensitive variable in terms of the computation of ground water - surface water interchange.

The overall results obtained here are somewhat surprising due to the dominance of river parameters as the most sensitive. For example, it is seen by comparison of sensitivities that Manning's n is about 50 to 100 times more sensitive than aquifer parameters of specific yield or hydraulic conductivity in terms of modeling fluctuations in
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Water Table Elevation Node 95</th>
<th>Water Table Elevation Node 96</th>
<th>Water Table Elevation Node 97</th>
<th>River Discharge Node 94</th>
<th>River Stage Node 94</th>
<th>Ground Water - Surface Water Interchange Element 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Aquifer Hydraulic Conductivity (K)</td>
<td>.010</td>
<td>.016</td>
<td>.0212</td>
<td>.010</td>
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<td>16.550</td>
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<td>2. Aquifer Specific Yield (S_y)</td>
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<td>.026</td>
<td>.035</td>
<td>.022</td>
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<td>3. River Bottom conductivity (K_s)</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>4. Manning Roughness (n)</td>
<td>1.007</td>
<td>1.004</td>
<td>1.003</td>
<td>.639</td>
<td>6.695</td>
<td>32.366</td>
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<tr>
<td>5. Stage-Area Coefficient (AC)</td>
<td>.017</td>
<td>.019</td>
<td>.016</td>
<td>.002</td>
<td>.124</td>
<td>.180</td>
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<tr>
<td>6. Stage-Area Exponent (PA)</td>
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<td>.024</td>
<td>.022</td>
<td>.003</td>
<td>.163</td>
<td>.255</td>
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<td>Parameter</td>
<td>Piezometric Surface Node 95</td>
<td>Piezometric Surface Node 96</td>
<td>Piezometric Surface Node 97</td>
<td>River Discharge Node 94</td>
<td>River Stage Node 94</td>
<td>Ground Water-Surface Water Interchange Element 12</td>
</tr>
<tr>
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<td>-----------------------------</td>
<td>-------------------------</td>
<td>---------------------</td>
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<tr>
<td>1. Aquifer Hydraulic Conductivity (K)</td>
<td>0.052</td>
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<td>0.125</td>
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<td>2. Aquifer Specific Yield (S_y)</td>
<td>0.108</td>
<td>0.166</td>
<td>0.206</td>
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<td>0.210</td>
<td>122.250</td>
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<td>3. River Bottom Conductivity (K_s)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>5. Stage-Area Coefficient (AC)</td>
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<td>0.169</td>
<td>0.168</td>
<td>0.024</td>
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<td>6. Stage-Area Exponent (PA)</td>
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<td>2.532</td>
<td>2.528</td>
<td>0.571</td>
<td>16.809</td>
<td>45.161</td>
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<td>7. River Channel Slope (S_0)</td>
<td>30.227</td>
<td>30.222</td>
<td>30.217</td>
<td>1.296</td>
<td>12.805</td>
<td>111.73</td>
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</tbody>
</table>
ground water table elevations. This result is indeed welcomed, however, since river parameters are, by comparison, more easily and accurately measured than aquifer parameters.

CALIBRATION OF MODEL TO LOWER TRUCKEE RIVER SYSTEM

A discussion of the hydrologic and lithologic character of the lower Truckee Meadows study area has been given previously. However, more information and physical data are needed in order to properly calibrate the linked ground water - surface water model. Much of this information, which includes both aquifer and river parameters, is given in Table 3, together with the source or method by which individual parameter values were obtained.

Comparison of Field Data

Examples of the correspondence between observed river stage and ground water levels in nearby wells are given in Figures 13a, 13b, and 13c. Here agreement between fluctuations in river stage and ground water levels is seen to be quite good, at least for piezometers F-1 and F-2. There is, however, one particular aspect of well hydrograph behavior, namely the elevation difference between well levels and river stages, which is worthy of mention. It is apparent in Figure 13a, for example, that the elevation difference between observed river stage and piezometer F-1 water level is about 1.5 feet for the first three observations after which it reduces to about .9 feet. The departure then continues to remain stable at about .9 feet until sometime near the end of September at which time a gradual increase in departure begins and continues through March of the following year. A possible
Figure 13a Comparison of observed Truckee River stages and corresponding hydrograph for piezometer F-1, 1975
Figure 13b  Comparison of observed Truckee River stages and corresponding hydrograph for piezometer F-2, 1975
Figure 13c  Comparison of observed Truckee River stages and corresponding hydrograph for piezometer F-4, 1975
### Table 3  Final Parameter Values used in Simulation of Lower Truckee River System

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Method of Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aquifer Hydraulic Conductivity (K)</td>
<td>34.56</td>
<td>ft/day</td>
<td>Analysis of Pump test data</td>
</tr>
<tr>
<td>Aquifer Specific Yield ($S_y$)</td>
<td>0.001</td>
<td>dimensionless</td>
<td>Analysis of Pump test data</td>
</tr>
<tr>
<td>River Bottom Skin Hydraulic conductivity (KS)</td>
<td>25-35</td>
<td>ft/day</td>
<td>Analysis of river stage and well hydrograph data</td>
</tr>
<tr>
<td>Mannings Roughness Coefficient (n)</td>
<td>0.0242</td>
<td>dimensionless</td>
<td>Hydraulic survey of lower Truckee River</td>
</tr>
<tr>
<td>River channel Slope ($S_o$)</td>
<td>0.00329</td>
<td>dimensionless</td>
<td>Survey of lower Truckee River</td>
</tr>
</tbody>
</table>
explanation of this behavior could be that the hydraulic conductivity of the river bottom sediments does not remain constant during the entire year. Instead it is postulated that, at least for this particular reach of the lower Truckee River, the hydraulic conductivity of river bottom sediments is increased due to scouring during high flow periods and subsequently undergoes a gradual decrease during periods of relatively low flow. Referring again to Figure 13a, these assumptions appear to be substantiated in that the reduction of the river stage - water table departure from 1.5 to .9 feet begins early during the high flow period and is followed by a gradual increase in departure during low flow. Furthermore it is reasonable that the departure during high flow should remain roughly constant at its minimum value as it does here at about .9 feet. This is because at some time during the high flow period, scouring occurs to the point where river bottom hydraulic conductivity becomes as high as the aquifer conductivity. Under these conditions aquifer conductivity, which likely remains constant, governs the elevation differential between river stage and the surrounding ground-water table. However, as the river flow decreases deposition of progressively finer sediment occurs and, with time, the river bottom skin is reestablished. If this process continues without interruption, conceivably the river bottom conductivity would continue to decrease thereby causing a continual increase in the head differential between the river stage and surrounding water table.

Model Calibration

Values given for model parameters in Table 3 constitute, at best,
only rough approximations estimated in many cases by relatively crude field measurement techniques. For some parameters, such as Manning's n, published information is available (Chow, 1959) which allows limits to be set regarding the uncertainty of the parameter estimate. Such is not the case for other parameters such as, for example, river bottom conductivity for which only the judgement of the analyst is responsible for determining if the value used in the model is reasonable.

Because of the nature of parameter uncertainty calibration of the model for the study area was not simply a matter of programming the model with the observed parameter values. Rather calibration consisted of repeated trials using alternative combinations of parameter values (taking care to ensure that no value exceeded reasonable limits) and comparing model outputs against corresponding system outputs. "Outputs" is used here in the plural sense because, as was discussed earlier, the linked model produces more than one output depending on the problem specified. The model was considered calibrated when no significant increase in accuracy (i.e., comparison and observed output values) could be achieved by further variation in parameter values.

Results of the calibration exercise are expressed here in terms of comparisons of observed and simulated river stages and well hydrographs for the 1975 spring runoff season. In this problem the model input consisted of the observed 1975 daily hydrograph at the Reno Gage. Outputs consisted of river stage, ground water - surface water interchange and water table elevations in the immediate vicinity of the Boynton Lane Bridge. Comparison of final model results with observed
river stages and water table elevations is given by Figures 14a and 14b.

PREDICTIVE UNCERTAINTY

Quantification of predictive uncertainty is essential to evaluation of overall model performance. As stated previously, an analysis of this type is meaningful only in relation to the particular field problem tested and thus a generalized statement as to model predictive uncertainty in all possible situations is precluded. In this study, predictive uncertainty has been evaluated based on comparison of observed and simulated river flows and ground water levels for the Truckee River system.

The initial step in evaluating predictive uncertainty consisted of a comparison of basic statistical properties computed from thirty-two sets of observed and predicted hydrologic data for the 1975 runoff season. Results given in Table 4 indicate excellent agreement between predicted and observed mean values for all three variables, a result which serves as a good indication that the model has been calibrated adequately. Standard deviation values are also seen to be in close agreement while skew values diverge slightly.

An interesting observation can be made concerning the magnitude of the first and second order serial correlation coefficients in Table 4. Basically serial correlation is an indicator of the degree of carry-over or persistence present in a particular phenomenon from one time step to the next. Not only do the observed and predicted serial correlation coefficients compare favorably but their magnitudes (.63
Figure 14a  Comparison of observed and simulated water table fluctuations, piezometer F-1
Figure 14b  Comparison of observed and simulated water table fluctuations, piezometer F-2
Table 4: Statistical Comparison of Observed and Predicted Hydrologic Data
Statistics for Observed Data in Parenthesis

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Skew</th>
<th>First Order Serial Correlation Coefficient</th>
<th>Second Order Serial Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well F-1</td>
<td>4386.39 (4386.39)</td>
<td>.62 (.72)</td>
<td>.86 (.99)</td>
<td>.84 (.84)</td>
<td>.68 (.68)</td>
</tr>
<tr>
<td>(feet above M.S.L.)</td>
<td>(feet)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Well F-2</td>
<td>4386.39 (4386.39)</td>
<td>.63 (.69)</td>
<td>.86 (.56)</td>
<td>.84 (.82)</td>
<td>.68 (.63)</td>
</tr>
<tr>
<td>(feet above M.S.L.)</td>
<td>(feet)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Truckee River of Reno, 1963 Flood</td>
<td>6442 (6816)</td>
<td>5103 (5091)</td>
<td>.69 (.91)</td>
<td>.95 (.93)</td>
<td>.88 (.84)</td>
</tr>
<tr>
<td>(Cubic feet per second)</td>
<td>(Cubic feet per second)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
up to .95) suggest the possibility that synthetic data sequences could be generated successfully by techniques such as the sequential generation procedure proposed by Fiering and Jackson, 1971. Sequential generation of synthetic data is an attractive tool basically because of its simplicity and the fact that only modest quantities of field data are required. Once obtained, synthetic data sequences could be useful as a means of studying extreme variations in both river flow and ground water elevations which have not been observed in recorded field data.

Comparison of statistical properties provides a general overview of model performance. However, by far the most straightforward and meaningful method for evaluating predictive uncertainty is that of direct graphical comparison of model predictions with corresponding values of observed field data. Graphs of this type readily facilitate the construction of confidence bands which in turn allow probabilistic inference to be made regarding the reliability of model predictions. If confidence bands are chosen for the 95% level, as is customarily the case for this type of analysis, then the probabilistic inference would be that the true value of an output variable corresponding to any predicted point on the straight line as a 95% probability of lying between the confidence bands at that point.

Results of this graphical analysis for the lower Truckee River system concerning the uncertainty associated with model predictions of water table levels in piezometers F-1 and F-2 are given in Figures 15a and 15b while corresponding results for predictions of river flows
Figure 15a Observed and simulated water table elevations, piezometer F-1
Figure 15b  Observed and simulated water table elevations, piezometer F-2
at the Reno Gage are shown in Figure 15c. Of primary interest to this
study are the results of modeling water table fluctuations for this
procedure, in order to be successful, requires accurate simulation of
the exchange between ground and surface water. The most notable result
apparent here is that predictive accuracy decreases markedly with
distance away from the river. Comparison of observations and predict-
ions for well F-1 shows the individual points to result in a 95% confi-
dence band of about .35 feet while the more distant F-2 well results
exhibit a 95% confidence band of nearly .70 feet. Thus predictive
accuracy has decreased noticeably over a distance of about 300 feet.

Several other features of Figures 15a and 15b are noteworthy.
First of all, it is seen in both cases that high water table elevations
are predicted with about the same accuracy as low elevations. The
only real exception to this statement are several of the low level
points for well F-2 which are the result of the previously discussed
early season bias. A second feature of interest is the virtual absence
of bias in the predicted water table elevations, a fact evidenced by
the more or less even scatter of points both above and below the
straight lines.

Results of riverflow simulation given in Figure 15c are somewhat
less general in that they are not influenced by the exchange of ground
and surface water to the same degree as the prediction of water table
elevations. The points plotted are the result of modeling an observed
flood event (in this case the flood of February, 1963) over the 22.75
mile reach of the Truckee River from Farad to the Reno Gage. Clearly
Figure 15c Observed and simulated Truckee River discharge, Reno gage
this type of procedure does not require the ground water - surface water interchange to be modelled with any great degree of precision, simply because of the tremendous dominance of river flows compared with interchange flows. However results of this comparison are significant in that they serve to quantify the degree of accuracy with which river flows can be numerically simulated over long channel reaches. In general a very high degree of predictive accuracy was observed for river flow simulation with both high and low flows predicted quite accurately. This, together with the unbiased point scatter and narrow 95% confidence bands seen in Figure 15c, generally indicates that predictive uncertainty has been minimized in this example.

To complete the analysis of model predictive uncertainty another type of statistical test was performed on the model output. This consisted of assessing the degree to which residual values (i.e., the difference between observations and predictions) were normally distributed. Ideally, the calibrated model successfully predicts the deterministic component of the observed data, residual values should be normally distributed about a mean value of zero. Figures 16a, 16b, and 16c indicate the degree of conformity with a normal distribution for the three test cases. Generally good agreement is apparent for the water table elevation simulation for piezometers F-1 and F-2. In both cases the theoretical mean values of the residuals (i.e., the residual value corresponding to a probability of nonexceedence of 50%) are seen to be very nearly zero while a generally good fit of the points defining the observed distributions by the theoretical distribution lines is also observed. In the case of piezometer F-2 the lack
Figure 16a  Normal distribution fit of residual water table elevations, piezometer F-1
Figure 16b  Normal distribution fit of residual water table elevations, piezometer F-2
Figure 16c  Normal distribution fit of residual Truckee River discharges, Reno gage
of fit is due primarily to underprediction of the first three points on the rising limb of the well hydrograph. Residuals obtained from the simulation of the February 1963 flood hydrograph show a somewhat lesser but nevertheless tolerable degree of fit between the observed distribution and theoretical normal distribution.

UNIQUENESS OF SOLUTION

It has been stated previously that one advantage of physically based models is their potential for at least limited extrapolation beyond the limits of the recorded data base from which they were calibrated. However, reliable extrapolation is in fact possible only if the parameter values determined in the calibration process are unique values and if their use results in accurate replication of the observed phenomena. Thus evaluation of uniqueness of model results is fundamental to the comprehensive assessment of overall model performance and reliability.

As has been previously demonstrated predictive uncertainty of the linked model, which has been calibrated using measured and estimated parameter values, for the lower Truckee River system, appears to lie within tolerable limits. However no statement can be made at this point as to the possibility that other combinations of different but compensating magnitudes may in fact yield an equally good (or even better) comparison between model predictions and observed field data. In fact it is easily conceivable that such a range of parameter values does exist owing, if for no other reason, to the many assumptions made in the derivation of the physical equations of motion governing the
movement of ground and surface water. The problem at hand is therefore one of identification of the range over which parameters may vary and still yield statistically similar model results. Answers to this question will provide a quantitative index to model uniqueness and will serve as a basis for making judgements as to the reliability of extrapolated model results.

The procedure by which model uniqueness of solution is quantified is to a large degree arbitrary. Basically the problem is one of identifying some type of index of comparison between model predictions and corresponding observations and then recording the changes in this index in response to systematic variation in model parameter values. In this study an attempt has been made to adopt a procedure which conforms to a large degree with established statistical methods. The methodology adopted consists of three parts: 1) variation of model parameters in a systematic manner and recording of resulting model predictions, 2) comparison of model predictions with corresponding observations by application of the chi square test for goodness of fit, and 3) theoretical statistical analysis to determine the range of parameter values for which the model produces statistically similar and hence nonunique results.

**Systematic Parameter Variation**

To avoid an excessive number of computer runs using random combinations of parameter values, a systematic approach to parameter variation was adopted utilizing results obtained from the parameter sensitivity analysis. Beginning with the initial calibrated model for
the lower Truckee River system, parameter values were subsequently changed by designated percentages (i.e., 1\%, 2\%, 5\%, etc.) in such a manner as to collectively provide the maximum increase in the magnitude of model results. The decision as to whether a particular parameter was to be raised or lowered was made based on information obtained from studying parameter sensitivity which is summarized in Table 5. Here is can be seen, for example, that to achieve the maximum increase in predicted water table elevations the magnitude of hydraulic conductivity and Manning's n should be increased while specific yield, stage-area exponent and coefficient, and channel slope should be decreased. This procedure was repeated and model results obtained for variable of 1\%, 2\%, 5\%, 10\%, 20\%, and 40\% from their original values.

Computation of Chi Square

The chi square test is a standard statistical technique for determining probabilistically whether or not a predicted or estimated set of data values is statistically indistinguishable from their corresponding observations. The basic premise for this procedure is that the chi square statistic, $\chi^2_c$, which is defined by,

$$
\chi^2_c = \sum_{i=1}^{i=n} \frac{(O_i - P_i)^2}{P_i}
$$

where $O_i$ = observation for time period "i"

$P_i$ = prediction for time period "i"

$n$ = numbers of observations

obeys the theoretical chi square probability distribution as given by Benjamin and Cornell, 1970, P. 295).
Table 5 Reaction of Output Variables Corresponding to an Increase in Parameter Value for Theoretical Stream-Aquifer System

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Water Table Elevation Node 95</th>
<th>Water Table Elevation Node 96</th>
<th>Water Table Elevation Node 97</th>
<th>River Discharge Node 94</th>
<th>River Stage Node 94</th>
<th>Ground Water-Surface Water Interchange Element 126</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Aquifer Hydraulic Conductivity ($K_a$)</td>
<td>Increased</td>
<td>Increased</td>
<td>Increased</td>
<td><strong>$T_p$ = Unchanged</strong></td>
<td><strong>$Q_p$ = Increased</strong></td>
<td>Increased Decreased River-to-aquifer leakage</td>
</tr>
<tr>
<td>2. Aquifer Specific Yield ($S_y$)</td>
<td>Decreased</td>
<td>Decreased</td>
<td>Decreased</td>
<td><strong>$T_p$ = Unchanged</strong></td>
<td>$Q_p$ = Decreased</td>
<td>Decreased Increased River-to-aquifer Leakage</td>
</tr>
<tr>
<td>3. River Bottom Skin Hydraulic Conductivity ($K_s$)</td>
<td>Unchanged</td>
<td>Unchanged</td>
<td>Unchanged</td>
<td>Unchanged</td>
<td>Unchanged</td>
<td>Unchanged</td>
</tr>
<tr>
<td>4. Manning Roughness (n)</td>
<td>Increased</td>
<td>Increased</td>
<td>Increased</td>
<td>$T_p$ = Increased</td>
<td>$Q_p$ = Decreased</td>
<td>Increased Decreased River-to-aquifer leakage</td>
</tr>
<tr>
<td>5. Stage-Area Coefficient (AC)</td>
<td>Decreased</td>
<td>Decreased</td>
<td>Decreased</td>
<td><strong>$T_p$ = Unchanged</strong></td>
<td>$Q_p$ = Increased</td>
<td>Decreased Decreased River-to-aquifer leakage</td>
</tr>
<tr>
<td>6. Stage-Area Exponent (PA)</td>
<td>Decreased</td>
<td>Decreased</td>
<td>Decreased</td>
<td><strong>$T_p$ = Unchanged</strong></td>
<td>$Q_p$ = Increased</td>
<td>Decreased Decreased River-to-aquifer leakage</td>
</tr>
<tr>
<td>7. River Channel Slope ($S_q$)</td>
<td>Decreased</td>
<td>Decreased</td>
<td>Decreased</td>
<td><strong>$T_p$ = Unchanged</strong></td>
<td>$Q_p$ = Increased</td>
<td>Decreased Decreased River-to-aquifer leakage</td>
</tr>
</tbody>
</table>

* $T_p$ = Time to peak  
** $Q_p$ = Peak Flow
The chi square procedure involves testing the so-called "null hypothesis", at some predetermined probability level. In this analysis the null hypothesis, $H_0$, which was tested is

$$H_0: \chi^2_\text{obs} \text{ is equal (i.e., statistically indistinguishable from) to the theoretical value, } \chi^2_\text{th}, \text{ at some designated probability level}$$

In order to test the null hypothesis for significance it is first necessary to compute the chi square statistic values for model predictions obtained for the various levels of parameter variation. These statistics were computed via Equation 40 and appear in Figure 17 where they have been plotted against percent parameter change in piezometers F-1 and F-2.

**Statistical Comparison**

The hypothesis test procedure described above was carried out using theoretical chi square values for the 1%, 5%, 95%, and 99% significance levels. Results are shown in Figure 17 where the theoretical chi square values for the four significance levels are shown as horizontal lines superimposed on the computed chi square curves. Intersections between these horizontal lines and the curves are of fundamental importance because it is these points which allow quantitative inference to be made regarding the uniqueness of model results. For example, the 1% chi square line is seen to intersect at points corresponding to 7.4% parameter change for well F-1 and 8.7% change for well F-2. The corresponding interpretation is that the parameters
**Figure 17** Chi square non-uniqueness test
for the linked model can be changed in any combination up to about ±8% from their original best fit values and still produce model results which are statistically indistinguishable, and thus non-unique, at the 1% level. Thus, in this case, the percent parameter change range of 8% constitutes the "region of non-uniqueness" at the 1% significance level. In the case of the intersection of the 99% theoretical chi square line the interpretation would be to the effect that, if the models parameters are not estimated to within about ±28% of their true field values, then there is at least a99% probability that the predicted water table elevations will be statistically significantly different from actual observations. Similar statements would apply in this case for percent parameter changes corresponding to the 5% and 95% theoretical chi square lines intersections.

Although the preceding analysis is reasonably straightforward, it is by no means complete in terms of total quantification of model uniqueness of solution. Nor is this really feasible to do simply because of the possibility that an infinite number of combinations of unrealistic but compensating parameter values may exist which produce acceptable results when comparison is made between model predictions and observations. However the results obtained from this study of model uniqueness are of considerable value in determining how, and to what extent, field estimation of model parameters should be carried out. Results herein demonstrate that, in the case of the lower Truckee River system, there is little to be gained by expending excessive effort to measure a particular parameter value to within less than about 10%. Similarly there is justification for attempting to obtain
parameter estimates which are within at least 28% of true field values. These findings, together with results obtained from the analysis of parameter sensitivity, serve as useful guidelines for the calibration of the linked ground water - surface water model to other hydrologic systems.
SUMMARY

Evaluation of the predictive capability of the linked ground water surface water model was carried out in three separate but interrelated phases, these being 1) determination of relative sensitivity of various model parameters, 2) calibration of model to lower Truckee River system and evaluation of predictive uncertainty and, 3) estimation of uniqueness of solution.

To evaluate parameter sensitivity a systematic methodology was developed which involved computation and comparison of a "sensitivity index" for each model parameter. It was found that the model was most sensitive by far to changes of ±10% in Manning roughness and channel slope regardless of the type of output considered. Similar results were obtained at the 100% level of parameter change.

Calibration of the model to lower Truckee River study area required trial and error adjustment of parameter values until an acceptable fit was obtained between model output and observed field measurements. There was, however, one aspect of system behavior which was unaccounted for by model calibration, namely the tendency for observed water table elevations to decline relative to river stage during prolonged periods of low flow. The postulated reason for this behavior is that the hydraulic conductivity of river bottom sediments, at least for the study area of the lower Truckee River, does not remain constant but instead increases during periods of high flow (and hence increased scour) then decreases gradually during periods of low flow due to re-deposition of sediment.
Model predictive uncertainty, which was evaluated by several methods, was found to lie within tolerable limits. Comparison of statistical properties for model output and corresponding field measurements showed good agreement between mean and standard deviation values along with fair agreement between skew coefficients. First order serial coefficients also agreed favorably and, because of their magnitude, suggested the possibility for sequential generation of realistic synthetic series of system data. Predictive uncertainty was further investigated by direct graphical comparison between model predictions and corresponding field observations. This analysis revealed the ability of the model to predict high and low water table elevations with about the same degree of precision. In addition, model accuracy in terms of predicting ground water elevations was observed to decrease with distance from the Truckee River.

Uniqueness of solution was investigated to determine, at least in part, the degree of reliability of model extrapolation beyond the limits of observed field measurements. Although somewhat arbitrary in nature the procedure for expressing uniqueness in quantitative terms was developed so as to conform as much as possible with established statistical methods. The three part methodology developed was based to a large degree on the standard chi square goodness of fit test. Significant results from the ensuing analysis were: 1) a "region of non-uniqueness" of model results exists at the 1% significance level for a range of parameter changes of ±8%. Put another way, the model parameter values can be varied in any combination by as much as ±8%
from their original "best fit" values and still produce model outputs which are statistically indistinguishable at the 1% significance level.

2) if model parameters are not estimated to within ±28% of their true field values, then there is at least a 99% chance that the predicted water table elevations will be statistically different from actual field observations.
ANALYSIS OF SYSTEM COMPONENTS AND PARAMETERS

INTRODUCTION

Up to this point application of the linked ground water - surface water model has been directed toward evaluation of overall model capabilities and performance. In this endeavor both real and theoretical hydrologic systems have been studied to gain insight into parameter sensitivity, predictive uncertainty, and uniqueness of solution. While these studies are necessary in order to understand the limitations of this modeling procedure they are, in fact, of a preliminary nature compared with other types of model applications which provide more generalized results. For example, once model capabilities and limitations have been assessed, it is possible to analyze the interrelationship among the individual components of certain hydrologic systems as well as to determine the influence of various parameters on system behavior. Efforts henceforth are directed toward this end and are composed of four closely related parts: 1) separation of hydrographs into baseflow and direct runoff components, 2) verification of analytical techniques for describing hydrograph recession, 3) influence of aquifer parameters on baseflow recession, and 4) analysis of interchange of ground and surface water. Here as before the hypothetical hydrologic system of Pinder and Sauer was utilized to carry out these analysis.

HYDROGRAPH SEPARATION

General Analysis

Division of a hydrograph into its components of direct runoff and
baseflow is known as hydrograph separation or hydrograph analysis. While various methodologies exist by which separation may be attempted (see Linsley, et al., 1975, p. 230) they are generally quite arbitrary and to a large degree unrealistic in that they do not include aquifer characteristics as input. The problem is further complicated by the difficulty encountered in attempting to obtain actual field measurement of the behavior of the ground water component during the period when direct runoff is active. It is possible, however, to utilize the linked ground water - surface water model to study this process as it occurs in a system consisting of a river channel reach bounded by an unconfined aquifer. The specific methodology followed here includes determining the initial steady state solution for the entire system, routing a hydrograph through the channel reach, and recording the movement of ground water into and out of bank storage. Specifically the ground water or base flow component was determined by routing the inflow hydrograph first assuming an impervious channel bottom, then again with the impervious restriction removed. The base flow hydrograph was then considered to be the difference of these two river hydrographs.

Baseflow separation was accomplished using the same theoretical hydrologic system and inflow hydrograph as were used in the study of parameter uncertainty (see Figure 12). Results, which appear in Figures 18a and 18b, allow some interesting observations to be made. It can be noted, for example, that the baseflow component first declines relative to its initial value then increases to a peak value after which a prolonged recession begins. This behavior appears
Figure 18a  Baseflow hydrographs with hydraulic conductivity varied; negative sign indicated flow into aquifer.
River Discharge (cfs x 100)

\[ K = 605 \text{ ft/day} \]
\[ S_y = .01 \]

Figure 18b  Baseflow hydrographs with specific yield varied; negative sign indicates flow into aquifer
reasonable since as the river rises there is flow from the stream into
the banks thereby decreasing the amount of baseflow. Furthermore,
baseflow should continue to decrease until stages in the river begins
to drop and bank storage returns to the channel. From this point on
baseflow into the channel would increase to some maximum value then
recede as bank storage is depleted.

Effects of Varying $K$ and $S_y$

Referring again to Figure 18a, it is possible to delineate the
effects of varying aquifer hydraulic conductivity, $K$, and specific
yield, $S_y$, on the profile of baseflow hydrographs. For example, as $K$
is lowered from 605 ft/day to 60.5 ft/day it is observed that minimum
baseflow value increases from -196 cfs to -88 cfs, timing of occurrence
of minimum baseflow is increased slightly, maximum baseflow value is
reduced from 116 cfs to 25 cfs, timing of maximum baseflow increases
approximately 1 hour, and rate of hydrograph recession decreases
markedly. Similarly when specific yield is decreased from .01 to .001,
(see Figure 18b) minimum baseflow increases from -715 cfs to -196 cfs
while timing of occurrence decreases slightly, maximum baseflow de-
creases from 200 cfs to 120 cfs along with decrease in the time of
occurrence, and recession rate decreases significantly.

Based on the foregoing results the linked model appears to provide
a satisfactory analytical tool for separation of the baseflow component
of hydrographs particularly during the period when the direct runoff
process is active. This technique is particularly attractive in that
aquifer characteristics are incorporated directly into solution pro-
procedure. However since this modeling approach does not include infiltration to ground water table during the direct runoff period these effects must by necessity be assumed negligible in order for the results to be valid. Also, by its very nature, a modeling approach such as this is probably too time consuming and expensive to justify using it solely for baseflow separation on an individual hydrograph basis. Instead the true utility of the linked model in terms of baseflow separation lies in the ability to generate a wide variety of theoretical baseflow hydrographs resulting from various combinations of aquifer and streamflow conditions. Availability of information of this type would then serve as a guideline to investigators concerned with making rapid approximations of the baseflow hydrograph component on a one-time basis.

BASEFLOW RECESSION ANALYSIS

Barnes (1940, p. 106) suggests that a hydrograph recession limb can be composed of runoff originating from as many as three separate sources these being surface runoff, interflow, and baseflow. In addition, Barnes demonstrates how these individual components of recession can be approximated by straight lines on a semilogarithmic plot of flow versus time, which are described mathematically by

\[ q_t = q_o e^{-K_r t} \]  

(41)

Here \( q_t \) is the streamflow occurring "t" time units after \( q_o \), and \( K_r \) is the recession constant representative of either surface runoff, interflow, or baseflow conditions. The degree to which a particular hydrograph recession conforms with Equation 41 is governed by many factors,
not the least of which is the characteristics of the watershed itself. Indeed analysis of actual hydrograph data (Linsley, 1975, p. 228 and Cunningham, 1976) often fails to reveal the presence of three distinct recession components.

Of the three recession components baseflow is one which is of primary interest to the present study. This is because baseflow is influenced to a much greater extent than either surface runoff or interflow by the interchange between ground and surface water. For this reason the linked model has application to the study of baseflow recession both in terms of providing theoretical justification for the relationship given by Equation 41, as well as identifying the degree of influence of system parameters on baseflow recession behavior.

Partial theoretical justification for Equation 41 is given by Singh (1968, p. 985-999) in which ideal baseflow curves were determined by numerical solution of the Boussinesq equation for both full and partial stream penetration. For a shallow aquifer and a fully penetrating stream, Singh found that the ideal baseflow curve failed to plot as a straight line on a semi-log paper but rather the recession rate continuously decreased with time. For a deep aquifer and a shallow-entrenched stream, however, it was found that the ideal baseflow did generally decay exponentially in accordance with Equation 41. The application of the linked ground water - surface water model to the theoretical study of baseflow recession does in fact constitute an extension of Singh’s study in that the combined effects of ground water seepage plus subsequent channel storage and routing are considered.
Singh's analysis did not include channel effects.

The theoretical hydrologic system on which the present study is based consists of essentially the same system and input as that used to study hydrograph separation, the only significant modification having to do with the inflow hydrograph. To adequately study baseflow it was deemed necessary here to use an inflow hydrograph which began at an initial steady state flow (18,000 cfs in this case) and was then subsequently reduced to 10,000 cfs where it remained throughout the analysis. By modeling the system response under these particular conditions it was possible to isolate and study the behavior of baseflow recession at any desired nodal location. Results given in Figure 19, which are for an arbitrary channel node located near the downstream boundary of the system, are typical of those obtained using a variety of system parameter values. Here it is clearly demonstrated that the theoretical baseflow recession, which includes both groundwater seepage and channel routing effects, does indeed conform to the exponential decay relationship given by Equation 41.

INFLUENCE OF AQUIFER PARAMETERS ON BASE FLOW RECESSION

Effects of Varying $K$ and $S_v$

The foregoing study of baseflow recession is significant in that it provides some theoretical justification, at least for the type of hydrologic system tested, for assuming that baseflow recession does indeed conform to a decaying exponential relationship. However the benefits obtainable from modeling do not end here. In fact, with only
Figure 19  Baseflow recession caused by seepage from ground-water reservoir
minor modification and extension, the linked model analysis can be made to yield some fundamental insights into how the baseflow recession process is controlled by aquifer parameters such as specific yield and hydraulic conductivity. This particular study was carried out using exactly the same hydrologic system and hydrograph as in the previous analysis of baseflow recession. An initial set of results was obtained by holding specific yield constant at 0.001 and varying hydraulic conductivity between 432 ft/day and 0.26 ft/day, than in a like manner, hydraulic conductivity was held constant at 60.5 ft/day and specific yield was varied systematically. The hydrograph recessions resulting from this procedure are shown in Figures 20a and 20b.

It is interesting to note in Figure 20a that, regardless of the value of hydraulic conductivity, the hydrograph recession ultimately conforms to a decaying exponential relationship. However, it is also obvious that the flow value, as well as the point in time at which conformance begins, is a function of hydraulic conductivity. To understand the nature of this relationship it is necessary to recall the results of the hydrograph separation study where it was demonstrated that the timing, magnitude, and duration of the baseflow hydrograph are related to aquifer hydraulic conductivity. Specifically it was found that baseflow hydrographs for high hydraulic conductivity values have a shorter duration, higher peak, and steeper recession than hydrographs obtained using low hydraulic conductivities. It is this same phenomenon which causes the variation of the baseflow recession curves in Figure 20a. For example, for a hydraulic conductivity of 432 ft/day, the baseflow recession is dissipated almost as rapidly as the channel
Figure 20a  Baseflow recession curves resulting from variations in hydraulic conductivity
Figure 20b  Baseflow recession curves resulting from variations in aquifer specific yield

K = 60.5 ft./day

S_y = 0.03

S_y = 0.02

S_y = 0.01

S_y = 0.005

River Discharge (cfs)

Time (hours)

5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135

S_y = 0.001

Figure 20b  Baseflow recession curves resulting from variations in aquifer specific yield
storage (it was determined here that channel storage effects persist for about three hours). However, if hydraulic conductivity is reduced to 25.9 ft/day, the baseflow hydrograph is seen to reach its peak and begin to recede after about seven hours.

Results given in Figure 20b provide similar insight into the relationship between baseflow recession and aquifer specific yield. Here it is seen that high specific yield results in an increased recession duration, a larger volume of recession flow, and correspondingly higher flow values than recession generated with lower specific yield values.

Relation of \( K \) and \( S_y \) to \( K_r \)

The foregoing results provide useful qualitative insight into the relationship between aquifer parameter values and baseflow recession. However, it is likewise possible, to develop analytic expressions for relating \( K \) and \( S_y \) to the recession constant \( K_r \). On a semi-logarithmic plot of hydrograph recession \( K_r \) is determined from the slope of the straight line segment in accordance with Equation 41 as

\[
K_r = \frac{-(\ln q_t - \ln q_o)}{t}
\]  

(42)

Thus, once \( K_r \) values are determined for each straight line segment in Figures 20a, and 20b, approximate functional relationships between \( K_r \) and \( K \) as well as \( K_r \) and \( S_y \) can be delineated graphically. The specific nature of these relationships is seen in Figures 21a and 21b which suggest equations of the form
Graphical relationship between $K$ and $K_r$

Figure 21a  Graphical relationship between $K$ and $K_r$
Figure 21b  Graphical relationship between $S_y$ and $K_r$
\[ K = aK_r \]  

(43)

and

\[ S_y = bK_r^x \]  

(44)

Thus a general equation takes the form

\[ K_r = cK(S_y)^y \]  

(45)

where \(a, b, c, x\) and \(y\) are constants.

Equations 43, 44, and 45 are significant in that they provide an analytical link between properties of streamflow recession and the aquifer parameters defining specific yield and conductivity. The primary implication here is that, with further refinement and calibration, a procedure could be developed whereby aquifer specific yield and hydraulic conductivity could be estimated directly from hydrograph recession data. Development and refinement of this methodology in a manner suitable for general application is, however, beyond the scope of the present study.

GROUND WATER - SURFACE WATER INTERCHANGE

An application of the linked ground water - surface water model which particularly lends itself to solution of practical problems is analysis of the exchange between ground and surface water under various system conditions. Generally speaking the linked model is capable of providing quantitative insight into any problem which is concerned with one or more of the following questions: 1) at a given time what is the water table configuration, river stage, and direction
and magnitude of ground water - river water interchange at selected points throughout an unconfined aquifer - river system? 2) How are these system components affected by spatial and temporal variations in system parameters? 3) How are these system components affected by artificial system stresses such as pumping, recharge, irrigation, diversion, impoundment and redistribution?

As an example consider the passage of an unregulated flood wave through a river reach which is hydraulically connected to an unconfined ground water system. Possible questions of interest in this situation include; 1) How does the water table surface change over time at a given point with the passage of the flood wave? 2) How far does the disturbance to the ground water table propagate laterally from the river channel? 3) What is the spatial distribution of temporary bank storage and how rapidly is it returned to the river channel? To carry the example a step further, consider that the behavior of the system is complicated by the influence of ground water withdrawal at one or more locations near the river channel. In this case it may be desirable to study, in addition to the questions already posed, the degree to which pumping at a given rate affects the interchange of ground and surface water. For example an interesting study from the standpoint of water rights might be to determine the maximum ground water withdrawal rate which can occur without altering the interchange pattern and increasing the net seepage rate into the aquifer.

To exemplify an approach to the solution of these and other similar problems the linked model was applied to the previously described
river-aquifer system of Pinder and Sauer. To illustrate the variety of results obtainable the system was analyzed under several types of conditions, namely with ground and surface water in steady state equilibrium, at selected intervals during the passage of the input flood wave shown in Figure 11, and again with ground water withdrawal occurring at a constant rate at a point near the river channel.

**Steady State System**

To analyze system behavior under steady state conditions water surface elevations (i.e., equipotential lines based on the Dupuit-Forchheimer assumptions) have been plotted (see Figure 22a) for ten foot intervals along with interchange flows for each river element in cfs. The steady state system river inflow is 18,000 cfs. Analysis of the water surface elevation contours reveals a flow system which begins with major withdrawal of river water into the aquifer over the first river element, is followed by minor withdrawals and returns over interior river elements, and terminates with a major return to the river over the final element. It is notable that most of the water surface elevation contours exhibit very little curvature even in the vicinity of the largest withdrawals and returns. For example, the 210 foot contour curves only slightly even though it forms the boundary between a large withdrawal element (i.e., -9.07 cfs from river element 1) and a significant return element (i.e., 250 cfs over river element 2). The only contours which do in fact show significant curvature are the 220 foot contour at the upstream boundary and the 90 foot contour at the lower boundary. The curvature of the 220 foot contour suggests
Figure 22a  Steady state system for river inflow of 18,000 cfs. Interchange flows are in cfs, water surface elevation contours are in feet. Dashed lines represent no-flow boundaries.
that a substantial water surface elevation gradient exists outward from
the river channel along the upstream boundary while, because of the
opposite curvature of the 90 foot contour, the gradient appears to be
toward the river. This behavior thus implies that a significant part
of the initial withdrawal of river water into the aquifer occurs along
or near the upstream boundary and, likewise, a significant return
occurs near the downstream boundary. Flow through interior elements
in a system of this type would, by necessity, be nearly parallel to the
river, an assumption substantiated here by the fact that the water
surface contour lines are nearly perpendicular to the direction of
flow (both in channel and aquifer) and further by the relatively small
magnitude of the interchange flow values for river nodes 2-12.

In analyzing these and other similar model results, it is well to
recall that they are best approximations of true system behavior and
thus should never be accepted blindly. As an example consider the
estimated interchange flows for elements 2 and 3 in Figure 22a.
Element 2 is seen to return a relatively large flow (2.50 cfs) to the
river while, for element 3, the interchange flow is seen to reverse
direction and flow back into the aquifer. These results, together
with the withdrawal of -.53 cfs from element 12 are very difficult to
justify from a physical standpoint. Instead it is more sensible to
assume that they are inconsistencies resulting from the numerous
approximations inherent in the derivation and subsequent numerical
solution of the flow equations. Thus while the overall steady state
flow configuration given by the model appears reasonable (i.e., large
Figure 22b  System configuration 1.39 hours after inception of hydrograph during rising limb.
withdrawal at top system boundary followed by minor returns over intermediate elements and a major return flow at lower system boundary) there is some reservation as to the exact magnitude and in some cases even direction of individual element interchange flows obtained from the model.

Rising Limb

As can be seen in Figure 22b introduction of a unimodal flood wave (see Figure 12) into the river channel caused a considerable change in the hydrologic system compared to its steady state condition. At a time 1.39 hours after the beginning of the hydrograph the interchange flow pattern is almost entirely outward from the river with river element 13 being the only exception. Furthermore, the withdrawals into the aquifer are largest over the initial river elements and then decrease moving downstream, as is reflected by the curvature of the water surface elevation contours. Here, as in the case of the steady state system, the curvature of the 90 foot contour indicates that the return flow to the river is occurring mainly along the lower system boundary. Also at this time the major component of transient aquifer storage is located near the river channel in the upstream elements.

Crest Segment

Figure 22c shows the condition of the system at a time where the hydrograph peak is located about midway through the river channel. Here the interchange flow pattern has again been altered considerably with return flow to the river now beginning over the first two river
Figure 22c  System configuration 2.78 hours after inception of hydrograph during crest segment. Hydrograph peak located about midway in system.
elements. Elements 3 through 13 show a definite withdrawal pattern with the majority of the withdrawal occurring from elements 5 through 12. It is also evident here that interchange flow values for river elements 3 and 4 are somewhat anomalous by virtue of the -72.1 cfs value for element 3 followed by the -28.3 cfs value for element 4. Indeed there is little if any physical justification for the element 4 outflow being greater than that of element 3 if in fact the hydrograph peak is located downstream. The only explanation for this behavior which seems at all reasonable is that it is the result of the same type of numerical instability as appeared in the steady state solution.

Recession Limb

Flow system configuration at two times during the hydrograph recession are demonstrated by Figures 22d and 22e. In Figure 22b, which reflects conditions 5 hours after hydrograph began, the return of transient aquifer storage back into the river has begun for all but the bottom two river elements. Compared with the cases examined previously the volume of aquifer storage is greater, as is evidenced by the general downstream shift in the water surface elevation contours. Also this storage is predominately situated nearer to the side boundaries than was previously the case. Turning now to conditions after 6.11 hours, it is seen that the system is rapidly returning to its initial steady state configuration. All river elements are returning flow to the channel with the magnitudes of these interchange flows greatly reduced from those in Figure 22d. Transient aquifer
Figure 22d  System configuration during recession limb 5.0 hours after hydrograph inception.
Figure 22e  System configuration during recession limb 6.11 hours after hydrograph inception.

\[ Q_{in} = 18,006 \text{ cfs} \]

\[ Q_{out} = 29,255 \text{ cfs} \]
storage has likewise been reduced as is seen by the upstream shift in the water surface elevation contours. The majority of the remaining transient aquifer storage is now located near the river in the lower part of the system. Further investigation revealed that the system returned to virtually a steady state condition after about 24 hours.

Ground Water Withdrawal

The final application of the linked model involved modeling system response due to artificial stress. The particular problem analyzed consisted of obtaining the steady state solution with withdrawal of ground water at a constant rate (and with no subsequent recharge) from nodes located near the lower system boundary. The river inflow to the system was again assumed to be 18,000 cfs thus facilitating comparison of results with previous steady state solution. Figure 22f illustrates system configuration resulting from a 5 cfs withdrawal from node 126 and, when comparison is made with Figure 22a, several observations are evident. For example it is notable that the 5 cfs demand is supplied virtually entirely from river elements 11 and 12 which now show river interchange flows into the aquifer of -2.56 cfs and -3.58 cfs respectively (compared with .56 cfs and -.53 cfs in the original steady state solution). Comparison of remaining interchange flow values reveals a virtually identical pattern as the original steady state solution for elements 1 through 10 and also element 13. Also, with the exception of the 110 foot contour in vicinity of the withdrawal node, the water surface elevation contour configuration is seen to be unaffected by the addition of the ground water withdrawal.
Figure 22f Steady state system with -5 cfs withdrawal from node 126.
From the foregoing results it is evident that for this particular problem the 5 cfs ground water sink does not disturb the overall system flow configuration but rather has only a local affect. In an attempt to develop contrasting results the problem was re-analyzed with the ground water sink location changed to node 124 which lies 1,000 feet farther away from the river channel. With only a few very minor differences the resulting steady state solution yielded virtually identical water surface elevation contours and interchange flow values thereby indicating that lateral distance from the river has little to do with the system configuration caused by the ground water sink in this problem.

The final analysis conducted consisted of raising the ground water withdrawal rate to 50 cfs with sink located again at node 124. As can be seen in Figure 22g, state solution of this problem resulted in considerable distortion of contour lines in the vicinity of the sink node when compared with the previous solutions. Basically the contour lines now form the configuration of the cone of depression from the withdrawal node superimposed on the original "unpumped" steady state water surface gradient. However, it should also be noted that the upstream extent of these withdrawal effects is still limited as contour lines and interchange flows for river elements 1 through 9 are still unchanged from previous solutions. Also obvious from Figure 22g, is that the effects of the 50 cfs ground water withdrawal do not extend across the river. However considering the large hydraulic conductivity value used (605 ft/day) together with the 18,000 cfs river flow this result does appear reasonable.
Figure 22g  Steady state system with -50 cfs withdrawal from node 124.
SUMMARY

In this phase of the study the linked ground water - surface water model was applied to a hypothetical hydrologic system in order to delineate interrelationships among individual system components as well as to determine the nature of the influence of various parameters on system behavior. The particular hydrologic system studied consisted of a 24.62 mile channel reach bounded on both sides by an unconfined aquifer of limited extent. Investigations were based on the response of the system to an inflow hydrograph (peak flow of 26,000 cfs, time to peak = 1.67 hours) and consisted of: 1) separation of hydrograph into baseflow and direct runoff components, 2) theoretical verification of analytical techniques for describing baseflow recession, and 3) influence of aquifer parameters on baseflow recession.

Results of the hydrograph separation analysis indicate that, as the hydrograph proceeded downstream, the baseflow component (which was originally effluent into channel) initially declined, then rose to a unimodal peak value, then receded. It was further demonstrated for the particular test problem analyzed that baseflow reversed from effluent to influent for a period of time during the passage of the hydrograph. In addition it was found that, by systematically varying \( K \) and \( S_y \), a decrease in \( K \) resulted in an increase in both magnitude and timing of minimum baseflow value along with a reduction of maximum baseflow peak and an increase in its time of occurrence. Also rate of hydrograph recession decreased markedly. Similarly a decrease in specific yield caused an increase in magnitude of minimum baseflow and
a decrease in both magnitude and timing of maximum baseflow and a
decreased hydrograph recession rate. Generally speaking the linked
model appears to provide a satisfactory analytical tool of separation
of the baseflow component of hydrographs. However, due to the degree
of time and effort necessary for calibration, a modeling approach such
as this is probably not justified in studies where baseflow separation
is not a primary objective. Instead the linked model appears to be
more useful as a means for describing baseflow behavior resulting from
a variety of combinations of aquifer and streamflow conditions. Such
results would allow at least qualitative guidelines to be developed
for making rapid approximations of the baseflow hydrograph component
on a one-time basis.

Analytical methods of hydrograph analysis available in current
literature are based on the assumption that baseflow recession can be
approximated by the decaying exponential relationship given in Equation
41. Results obtained from analysis of baseflow recession in this study
provide some degree of theoretical confirmation to this assumption in
that system outflow hydrographs, which were generated by simultaneous
numerical solution of the physical equations which approximately govern
a linked ground water - surface water system, did in fact produce
baseflow recessions which conformed to the decaying exponential rela-
tionship.

Influence of aquifer parameter values on baseflow recession was
also investigated by recording system response to systematic variation
of $K$ and $S_y$. Significant results include: 1) increased $K$ causes a
lower initial baseflow value and a steeper recession while, 2) increased $S_y$ resulted in higher initial baseflow and milder recession. In addition analytical relationships were developed which relate $K$ to $K_r$ and $S_y$ to $K_r$. It is envisioned that these relationships, if properly calibrated by means of further investigation, could allow the aquifer parameters $K$ and $S_y$ to be estimated directly from hydrograph recession data.

The process of exchange between ground and surface water together with various aspects of flow system configuration were investigated by modeling a hypothetical hydrologic river-aquifer system bounded on all sides by no-flow boundaries. Hydraulic conductivity and specific yield values of 605 ft/day and .01 respectively were assumed throughout the aquifer. The system was modeled in response to several types of conditions which included: 1) ground and surface water in steady state equilibrium, 2) at selected intervals during the passage of a flood wave, and 3) with ground water withdrawal at a constant rate from a single node near the river channel.

Results which were obtained for a steady state river inflow of 18,000 cfs, generally appeared reasonable however, several of the smaller magnitude interchange flow values were considered doubtful. The overall flow system configuration identified from modeling consisted of immediate withdrawal from the river into the aquifer along the upstream system boundary, followed by a sequence of minor withdrawals and returns to the river over interior nodes, and terminated by a substantial return to the river along the downstream system boundary.
Model analysis of unsteady conditions in the system during the passage of a flood wave enabled the temporal and spatial distribution of transient aquifer storage to be monitored along with variations in interchange flow pattern. General trends revealed in this particular problem were: 1) a build-up in aquifer storage close to the river in the upstream segment of system during hydrograph rising limb, 2) continued movement of ground water down gradient and away from river during crest segment, and 3) development of return flow pattern to river during hydrograph recession.

Subsequent model investigations involved analysis of system response to artificial stresses in the form of ground water withdrawals from selected nodes near the lower boundary. Withdrawal rates of -5 cfs and -50 cfs were studied for nodes positioned 500 feet and 1500 feet away from river channel. Aside from increases in lower system river seepage outflow rates, results indicated the 5 cfs ground water withdrawals rate caused very little change compared with the steady state solution for the unstressed system. This result remained basically unchanged in spite of relocating the withdrawal node from 500 feet to a point 1500 feet away from river channel. When the ground water withdrawal rate was raised to 50 cfs considerable distortion of the water surface elevation contours occurred in the vicinity of the sink node. Basically the contour lines reflected the configuration of a cone of depression superimposed on top of the original "unstressed" system gradient. However, as in the previous test problem, the 50 cfs withdrawal did not appreciably alter either the upstream region of the
system or the region across the river channel.

**CONCLUSIONS**

In conclusion it should first be noted that the results reported herein by the model conclude a preliminary comprehensive assessment of model capabilities and limitations. Indeed, much more on analysis of effects of variation in flow elevation, configuration and density on model response, interaction of surface boundary flow conditions and selection of processes using various criteria and also all results considered necessary conclude from this study and are thus considered promising topics for future investigations. These results serve to quantify the feasibility and usefulness of the research which is that, in spite of the enormous physical and mathematical assumptions involved in the theoretical development, as well as measurement errors in collection of data and use of limited model approach described herein is definitely feasible and represents a useful tool for hydrologic investigations. This statement is supported by the following specific observations reached in this study:

1. When calibrated for the Rhodoc River study area the United States successfully reproduced major regime of stream features with the exception of a relatively marked lift in ground water levels during prolonged periods of the calibration.

2. Model predictions, particularly, which were verified by several methods, appeared to be generally good for Rhodoc River System. Also, precipitation, consistency and temperature data were sufficiently accurate for analyses.
CONCLUSIONS

In conclusion it should first be stated that the results reported herein by no means constitute a completely comprehensive assessment of model capabilities and limitations. Indeed such items as analysis of effects of variation in finite element configuration and density on model output, introduction of various boundary flow conditions, and solution of problems using various time step sizes all represent noteworthy omissions from this study and are thus considered promising topics for future investigation. These remarks serve to qualify the fundamental conclusion from this research which is that, in spite of the numerous physical and mathematical assumptions inherent in the theoretical development, as well as measurement errors in collection of field data, the linked model approach described herein is definitely feasible and represents a useful tool for hydrologic investigation. This statement is supported by the following specific conclusions reached in this study:

1. When calibrated for the Truckee River study area the linked model successfully reproduced major aspects of system behavior with the exception of a declining trend in ground water levels during prolonged periods of low streamflow.

2. Model predictive capability, which was evaluated by several methods, appeared to be generally good for Truckee River System. Also predictive accuracy was found to decrease with distance away from river channel.
3. Model results for Truckee River System exhibit a fairly high degree of uniqueness and thus limited extrapolation beyond limits of existing data is justified. However, degree of extrapolation remains a matter of judgment.

4. There is little to be gained, at least in the case of Truckee River System, by attempting to measure a particular model parameter to within less than ±8%. Similarly it is virtually essential that parameter values be estimated within ±28% of true field values.

5. Model output was demonstrated to be most sensitive to changes in channel properties of Manning's n and slope, regardless of type of output considered.

6. The linked model represents a potentially viable method for separation of hydrographs into baseflow and direct runoff components given that time and effort involved are justifiable.

7. Partial theoretical justification was provided for the assumption that baseflow recession is described by an exponential decay function.

8. Analytical relationships were successfully developed which relate aquifer hydraulic conductivity and specific yield to baseflow recession constant for the hypothetical hydrologic system investigated.

9. Potential for modeling response of unconfined aquifer-river systems to both natural and man-induced stress conditions was demonstrated.


APPENDIX I

COMPUTER PROGRAM LISTING
ERSITY OF MINNESOTA 6600 FORTRAN COMPILER KRONOS 2.1. PSF 4.3 77/01/13. 16.

WRITE("E=5, P=100000, R=7,1)

1. OVERLAY(GWHOCH, 0, 0)
2. PROGRAM CONT (INPUT, OUTPUT, PUNCH, TAPE5=INPUT, TAPE6=OUTPUT
3. DIMENSION TITLE (31), G(260, 43), P (400)
4. CDEL(100), P(100), K(100), XFERM(20), YFERM(20)
5. ALPHA(20), SS(20), ODIS(20), VLEAK(20), OME(50), KW(50), CBAND(50)
6. K(50), LQ(50), OBFK(50), OBF(50), IZ(E(20), ND(6, 20), OAR(4, 20)
7. C(4, 20), CFD(4, 20), CA(4, 20), GD(4, 20), GF(4, 20), GA(4, 20), GC(4, 20)
8. E(20), NODE(5), CT(5), QO(100), QO(100), Z(100), ZC(100), V(100)
9. ELEV(100), KXC(100), PRKTR(100), VIF(5), CIF(5), SCF(99), DX(99)
10. FD(99), GOX(99), GWH(99), CVLK(99), AC(100), PA(100), CT(100), PT(100)
11. CP(100), FP(100), TB(100), CPY(100), PPLY(100)
12. COMMON G,B, NRED, NEN, ND, NELS, NWE, NHCS, NOMBD, NSTEP, NPG, NOCH
13. NHCH, NHCH, NHCH, NHCH, NHCH, NHCH, OBFK, OBF, OBF, OBF, OBF
14. OBF, OBF, OBF, OBF, OBF, OBF, OBF, OBF, OBF, OBF
15. CA, CA, CA, CA, CA, CA, CA, CA, CA, CA, CA
16. TPC, TPC, TPC, TPC, TPC, TPC, TPC, TPC, TPC, TPC, TPC
17. TP, TP, TP, TP, TP, TP, TP, TP, TP, TP, TP
18. CP, CP, CP, CP, CP, CP, CP, CP, CP, CP, CP
19. SFC1 = 10, SFC2 = 10
20. WRITE(6, 1)

1 FORMAT (11H1)
2 FORMAT (1X, 9A, 7A10)
3 FORMAT (1X, 26X, 13HINITIAL HEADS/7H NODE, 8X, 4HEA, 6X, 4HEA)
4 FORMAT (1X, 4HEA, 8X, 13HINITIAL VALUES OF DEBTH AND DISCHARGE/1H NO.
5 FORMAT (13X, 9X, 2H70, 13X, 9H4, 3H4C, 8X, 2H70, 13X, 2H70)
6 FORMAT (13X, 7HELEMENT, 7X, 2HGC)
7 FORMAT (13X, 10X, 1PE16.4)
8 FORMAT (10, 1PE16.4)
9 FORMAT (10, 1PE16.4)
10 FORMAT (10, 1PE16.4)
11 FORMAT (10, 1PE16.4)
12 FORMAT (10, 1PE16.4)
13 FORMAT (10, 1PE16.4)
14 FORMAT (10, 1PE16.4)
15 FORMAT (10, 1PE16.4)
16 FORMAT (10, 1PE16.4)
17 FORMAT (10, 1PE16.4)
18 FORMAT (10, 1PE16.4)
19 FORMAT (10, 1PE16.4)
20 FORMAT (10, 1PE16.4)
READ THREE TITLE CARDS

15. READ (5, 2) TITLE
16. IF (EOF(5), NE.0.) GO TO 60
17. WRITE (6, 2) TITLE
18. N = 2
19. GO TO 65
20. 60 N = 3
21. DO 70 I = 1, N
22. READ (5, 2) TITLE
23. IF (EOF(5), NE.0.) STOP
24. 70 WRITE (6, 2) TITLE

READ NUMERICAL DATA

25. CALL OVERLAY (6=GWPOGF, 1, 0)
26. IF (INTL.GT.0) GO TO 110

SOLVE THE STEADY STATE PROBLEM

27. CALL OVERLAY (6=GHGWHCH, 2, 0)

28. 100 DO 100 I = 1, NNDS
29. 100 HC(I) = 0(I)

PRINT INITIAL HEADS, DISCHARGES AND DEPTHS

30. 110 WRITE (6, 3)
31. CALL PRTOTE (HD, NNDS)
32. WRITE (6, 4)
33. CALL PRTOTA (7, 0, NCNDS)
34. IF (INTL.GT.0) GO TO 116
35. WRITE (6, 5)
36. DO 115 I = 1, ND1
37. 115 WRITE (6, 6) I, GNIF(I)
38. 116 IF (ISTCY.GT.0) GO TO 50

COMPUTE HEADS AT CENTER NODES

39. IF (ICONT.GT.0) GO TO 150
40. REWIND 1
41. NCSEL = 20
42. IKAT = 0
43. ISUM = 0
120 ISUM=ISUM+20
46. READ(1) (IZNE(K), (NO(J,K), QAR(J,K), CD(J,K), COFD(J,K), CA(J,K)
1, GOF(J,K), GOF(J,K), GA(J,K), J=1,4), CAC(K), GAC(K), K=1, NOEL)
47. DO 140 I=1, NOEL
48. IKNT=IKNT+1
49. N=4
50. IF (NO(4,I), LE, 0) N=3
51. IZONE=IZNE(I)
52. SUM=0.
53. DC 130 J=1, N
54. NA=NO(J,I)
55. SUM=SUM+QIST(IZONE)*QAR(J,I)-GA(J,I)*B(NA)
56. 140 HC(IKNT)=SUM/GAC(I)
57. IF (ISUM.LT.NELS) GO TO 120
58. 150 CALL OVERLAY(6, GWFOCH, 3, 0)
59. GO TO 50
60. END

1. SUBROUTINE COUPLE(CCH, BSOLVE)
   SOLVES THE COUPLED PROBLEM
2. DIMENSION C(G240, 43), P(40)
   1, DELT(100), HO(140), FC(400), XPERM(20), YPERM(20)
   2, ALPHA(20), SS(20), QDIST(20), VLEAK(20), OWEL(50), KW(50), CBND(50)
   3, KOR(50), LOB(50), CEFK(50), CBFL(50), IZNE(20), NO(4, 20), CA(4, 20)
   4, CO(4, 20), COFD(4, 20), GA(4, 20), GD(4, 20), GOFD(4, 20), GA(4, 20), GAC(20)
   5, GAC(20), NOC=5, CT(5), Q(100), QD(100), ZC(100), ZC(100), ZC(100)
   6, ELEV(100), KNCN(100), PMAH(100), VIF(99), CIF(99), SCF(99), DX(99)
   7, FDY(99), GDY(99), GWIF(99), CVKL(99)
3. COMMON C, B, N, ENE, D, BN, N, NWL, NWL, NDB ND, BSTEP, KPC, NOCH
   1, NCH, NC1, NCH, NC2, NCCH, NC3, NC4, DEI, AP, H, CF, XPERM, YPERM, ALPHA, SS
   2, QDIST, VLEAK, OWEL, KW, QBD, KOR, LOB, CEFK, CBFL, IZNE, NO, CA, CD, COFD
   3, CA, GD, GOFD, GA, CAC, GAC, NODE, GI, ISCE, LCSC, LSC, INITL, ICON
   4, IPNBC, NCND, NCN, OES7NS, IR, DIN, N, SFC1, SFC2, SQTY, AFAC, BFACT
   5, E, Q, CD, O, OQMAX, BUZ, 7, 70, 70, 0, MAX, BNU, V, ELEV, KNCN, PMAH, VIF, QIF
   6, SOR, DX, FEX, GDX, GWF, CVKL
DO 290 IT=1,NUMIT
REMIND 2
CALL OCH
READ (2) ((G(I,J),J=1,IBAND),B(I),I=1,NRED)

COMPLETE ASSEMBLY OF G AND B ARRAYS

IF(NWELS.LE.0) GO TO 158
DO 197 I=1,NWELS
J=KW(I)
B(J)=B(J)+QWEL(I)
IF(NOBN.D.LE.0) GO TO 200
DO 199 I=1,NOBN
K=LOG(I)
B(J)=B(J)+Q8K(I)*QBN(I)
NODE(1)=KCN(J)
GT(1)=7(J)+ELEV(1)
DO 235 I=1,NCM1
J=I+1
NODE(2)=KCN(J)
NODE(3)=J
GT(2)=7(J)+ELEV(J)
DO 230 J=1,2
NA=NODE(J)
NB=NODE(3-J)
N=NODE(J+2)
TMPA=GT(J)
TMFA=GT(3-J)
TEMP=QVLK(I)*PRMTR(N)/12.
TMP=TEMP*3.
205 IF(NA.LE.NFED) GO TO 205
IF(NP.LE.NFED) GC TO 230
G TO 230
205 GC TO 230
205 C(NA,1)=G(NA,1)+TMP
B(NA)=B(NA)+TMF*TMPA+TEMP*TMFB
210 IF(NB.LE.NFED) GO TO 215
B(NA)=P(NA)-TEMP*B(NB)
GC TO 230
210 GC TO 225
210 B(NB)=P(NB)-TEMP*B(NA)
44. 215 N4T=NA
45.  NBT=NB-NA+1
46.  IF(NB.GE.NA) GO TO 220
47.  NAT=NA
48.  NBT=NA-NB+1
49.  220 G(NAT,NBT)=G(NAT,NBT)+TEMP
50.  225 G(NA,1)=G(NA,1)+TEMP
51.  E(NB)=E(NB)+TEMP*(1+MFA+TMPB)
52.  230 CONTINUE
53.  NODE(1)=NODE(2)
54.  NODE(3)=NODE(4)
55.  235 CT(1)=CT(2)

SOLVE MATRIX EQUATION

CALL BSOLVE

CALCULATE GROUNDWATER INFLOW AND CHECK FOR CONVERGENCE

ICNV=0

DO 280 I=1,NOM1
J=I+1
NB=KNC(J)
TMFA=B(NB)-Z(J)-ELEV(J)
280 GWTF(I)=TEMP
   IF(ABS(GWTF(I)-TEMP)*GE.DQMAX) ICNV=1
295 RETURN
END
SUBROUTINE BSOLVE
SOLVE SYMMETRIC MATRIX BY GAUSS ELIMINATION

DIMENSION A(280,43), E(400)
COMMON A,B,N,M

UPPER TRIANGULARIZATION

NM1=N-1
DO 30 K=1,NM1
I=K
DO 20 J=2,M
I=I+1
IF(A(K,J),EQ,0.) GC TO 20
C=A(K,J)/A(K,1)
L=0
DO 10 JJ=J,M
L=L+1
IF(A(K, JJ),NE,0.) A(I,L)=A(I,L)-C*A(K,JJ)
10 CONTINUE
A(K,J)=C
B(I)=B(I)-C*A(K)
20 CONTINUE
B(K)=B(K)/A(K,1)

BACK SUBSTITUTION

B(N)=B(N)/A(N,1)
I=N
40 I=I-1
IF(I.LE.0) GO TO 60
L=I

DO 50 J=2,M
L=L+1
IF(A(I,J),NE,0.) B(I)=B(I)-A(I,J)*B(L)
50 CONTINUE
GO TO 40
60 RETURN
END
SUBROUTINE PRTOTA(VALA, VALP, NO)
PRINT OUT VALUES IN TWO GROUPS OF THREE COLUMNS
DIMENSION VALA(1), VALB(1)

FORMAT LIST
20 FORMAT (1H, 2X, 2(I3, 4X, 1PE11.4, 4X, E11.4, 4X))

IEND = NC/2
ITMP = (NO+1)/2
IF(IEND.EQ.0) GO TO 5
DO I = 1, IEND
   K = ITMP*I
   WRITE(6, 20) I, VALA(I), VALB(I), K, VALA(K), VALB(K)
10   CONTINUE
5   CONTINUE
WRITE(6, 20) ITMP, VALA(ITMP), VALB(ITMP)
10  RETURN
END

SUBROUTINE PRTOTB(VAL, NO)
PRINT OUT VALUES IN THREE GROUPS OF TWO COLUMNS
DIMENSION VAL(1)

FORMAT LIST
30 FORMAT (1H, 2X, 3(I3, 5X, 1PE11.4, 5X))

IEND = NC/3
ITMPA = (NO+1)/3
ITMPB = (NO+2)/3
IF(IEND.EQ.0) GO TO 15
DO 10 I = 1, IEND
   K = ITMPA + I
   L = ITMPB + K
   WRITE(6, 30) I, VAL(I), K, VAL(K), L, VAL(L)
10  CONTINUE
15  CONTINUE
IF(ITMPB.EQ.IEND) GO TO 25
I = IEND + 1
IF(ITMPA.NE.ITMPB) GO TO 20
K = ITMPB + I
WRITE(6, 30) I, VAL(I), K, VAL(K)
20  GO TO 25
WRITE(6, 30) I, VAL(I)
20  RETURN
END
OVERLAY (GWOCCH,1,0)

PROGRAM READER

DIMENSION TITLE(6), G(20,43), R(400), XCOORD(400), YCOORD(400)

1. D E L T ( 1 0 0 ) , H D ( 4 0 0 ) , H C ( 4 0 0 ) , X P E R M ( 2 0 ) , Y P E R M ( 2 0 )

2. A L P H A ( 2 0 ) , S S ( 2 0 ) , Q O I S T ( 2 0 ) , V L E A K ( 2 0 ) , Q W E L ( 5 0 ) , K W ( 5 0 ) , C A N D ( 5 0 )

3. K O B ( 5 0 ) , L G ( 5 0 ) , G B F ( 5 0 ) , G B F L ( 5 0 ) , I Z E ( 2 0 ) , N D ( 4 , 2 0 ) , D A R ( 4 , 2 0 )

4. C O D ( 4 , 2 0 ) , G O F D ( 4 , 2 0 ) , G A T ( 4 , 2 0 ) , G A C ( 2 0 ), N H C ( 2 0 ) , N O D E ( 1 0 0 ) , T C ( 1 0 0 ), P T ( 1 0 0 )

5. N U M ( 1 0 0 ) , X H N O . ( 1 0 0 ) , X H N O . ( 1 0 0 ) , X H N O . ( 1 0 0 ) , X H N O . ( 1 0 0 ) , X H N O . ( 1 0 0 ) , X H N O . ( 1 0 0 )

6. B ( 4 0 0 ) , X C O R D ( A 0 0 ) , Y C O F D ( A G O ) , D E L T ( 1 0 0 ) , H D ( A G O ) , H C ( A G O ) , X P E R M ( 2 0 ) , Y P E R H ( 2 0 )

7. * 5 k b t r l ? 0 . 1 > s g O I S T < 2 0 5 j V L E A K C 2 0 ? , Q W E L ( 5 0 » , K W ( 5 0 > . 0 3 N D ( 5 0 )

8. K O B ( 5 0 ) , L D 8 ( 5 0 ) , C e F K ( S 0 > , Q 8 F L ( 5 0 ) , I 7 N F ( E G ) , N D ( <♦, 2 Q > , Q A R ( 4 . 2 0 )

9. C A ( ^ , 2 D ) , G 0 ( ^ , 2 0 T ) , G C F D ( ^ , 2 0 > , G A ( i * , 2 0 ) , C A C ( 2 0 )

10. G A C ( 2 0 ) , N O D E ( 5 ) , G T ( 5 ) , Q ( 1 0 0 ) , 0 0 ( 1 0 0 ) , Z ( 1 0 0 ) , 7 C ( 1 0 0 ) , V ( 1 0 0 )

11. £ L F V ( 1 0 0 ) , K C N C ( 1 0 0 ) , P R M T R ( 1 0 0 ) , V I F ( g 9 ) . G T F ( 9 9 ) , S C F ( S 9 ) , D X ( 9 9 )

12. C P ( 1 0 0 ) , F P ( 1 0 0 ) , T B ( 1 0 0 ) , C F Y ( 1 0 0 ) , F P Y ( 1 0 0 )


16. E Q U I V A L E N C E ( X C C R C ( 1 ) , G ( 1 , 1 ) f , ( Y C C R D ( 1 ) , G ( l , 3 ) ? , ( F t X P, * F C 2)

17. T I T L E ( 1 ) , A L P H A ( 1 )

F O R M A T L I S T

1. FORMAT ( 1 H , 8 X, I 4 , 4 X, I 4 , 4 X, 1 P E 1 1, 4)

2. FORMAT ( 1 X, 1 7 , 3 I B)

3. FORMAT ( 1 X, 3 7 , 9 1 0)

4. FORMAT ( 1 X, 1 7 , 3 F 0 . 0)

5. FORMAT ( 1 X, 2 8 , 4 9 6)

6. FORMAT ( 1 X, 1 2 , 9 5 6)

7. FORMAT ( 1 X, 1 4 , 1 5 6)

8. FORMAT ( 1 X, 1 4 , 2 4 6)

9. FORMAT ( 1 X, 1 4 , 4 4 6)

10. FORMAT ( 1 X, 1 4 , 7 4 6)

11. FORMAT ( 1 X, 1 4 , 9 4 6)

12. FORMAT ( 1 X, 1 4 , 1 2 4)

13. FORMAT ( 1 X, 1 4 , 1 5 4)

14. FORMAT ( 1 X, 1 4 , 1 7 4)

15. FORMAT ( 1 X, 1 4 , 1 9 4)

16. FORMAT ( 1 X, 1 4 , 2 1 4)
FORMAT (///1H0,20X,4HPARAMETERS DEFINED ON CHANNEL NODES, BY ZON
1E/1H ,6H NO.,3X,2HNB,4X,2HHE,7X,2HCF,11X,2HPA,11X,2HPP,11X,2PTB/)
18.  19 FORMAT (///1H0,27X,17HNOODAL COORDINATES/1H ,2X,4HNODE,6X,5HXCORD
1,10X,5HYCCORD,7X,4HNODE,6X,5HXCORD,10X,5HYCCORD/)
19.  21 FORMAT (///1H0,33X,16PARAMETERS BY ZONE/5H ZONE,4X,5HPERM,3X,
15HPERM,3X,5HALPHA,7X,2HSS,9X,5HDIST,6X,5HVLEAK,9X,2HHR/)
20.  22 FORMAT (///1H0,10X,24HSPECIFIED BOUNDARY HEADS/14X,4HNODE,8X
1,4HHRCH/)
21.  25 FCPMAT (///1H0,13X,19HPOINT SOURCES (+VE)/1H ,10X,3HNC.,4X,4HNODE
1,6X,8HRCHARGE/)
22.  26 FORMAT (///1H0,5X,3HSPECIFIED BOUNDARY RECHARGES (+VE)/1H ,10X
1,18HBOUNDARY,7X,4HUNIT/1H ,3X,3HNODE,5X,6HNODE A,4X,6HNODE
1E B,6X,8HRCHARGE/)
23.  27 FCPMAT (1X,17,1E8,6F8.0)
24.  28 FORMAT (1X,17,218,7F8.0)
25.  29 FORMAT (1H ,15,5X,15,6X,14,5X,1PE11.4)
26.  30 FORMAT (1H ,15,218,512X,1PE11.4)
27.  31 FORMAT (///1H0,11X,47HPARAMETERS DEFINED ON CHANNEL ELEMENTS, BY
2ZONE/1H ,6H NO.,3X,2HNB,4X,2HHE,7X,1HND,12X,2HQA,10X,3HVEE,10X
3,4HVCIV/)
28.  33 FORMAT (1H ,13,2X,21PE11.4,2X),OPF8.2,2X,3,1PE11.4,2X),OPF8.2)
29.  36 FORMAT (1H0,35HSCALE CHANGE FOR NOODAL COORDINATES/1)

READ JOB SPECIFICATION

READ(5,2) NNDs,NNCS,NELS,NN7NS,NNZNS,NCZNS,NNWELS,NNHDS,NNBND
1,NSTEPS,1NPC,NUNIT,NIT
11,NNDS=NN1DS-NNDS
12.  32 NN1=NNDS-1
33.  33 READ(5,2) NNCH,NNCH,NNCH,NNCH,NNCC,NNCC
34.  34 READ(5,2) ICNT,ICNT,ICNT,ICNT,ICNT
35.  35 READ(5,5) WF,RF,DZMAX,DQMAX,GRVTY,TIME
36. WRITE(F,17) NNDs,NNCS,NELS,NN7NS,NNZNS,NCZNS,NNWELS,NNHDS,NNBND
1,NSTEPS,1NPC,NUNIT,NIT,WF,RF,DZMAX,DQMAX,GRVTY,TIME
READ NODAL COORDINATES AND INITIAL VALUES

DO 47 J=1,NNDS
  READ (5,4) I,XCORD(I),YCORD(I),HD(I)
  XCORD(I) = XCORD(I)/7.0
47 CONTINUE

WRITE(6,19)
CALL PRPTOTA(XCORD,YCORD,NNDS)

CONVERT SCALE
IF (IUNIT.LE.0) GO TO 49
WRITE(6,36)
READ(5,3) TITLE
WRITE(6,3) TITLE
READ(5,5) CONST
DO 48 I=1,NNDS
  XCORD(I)=XCORD(I)*CONST
  YCORD(I)=YCORD(I)*CONST
48 CONTINUE

READ CHANNEL NODES AND THEIR ELEVATIONS
WRITE(6,9)
DO 50 I=1,KCOUNT
  READ(5,27) J,KCOND(J),ELEV(J)
50 WRITE(6,1) J,KCOND(J),ELEV(J)

COMPUTE NODAL SPACING ARRAY
DO 51 I=1,NQM1
  J=KCOND(I)
  K=KCOND(I+1)
  TMP=XCORD(K)-XCORD(J)
  TEMP=YCORD(K)-YCORD(J)
  CX(I)=SQRT(TM*TM*TEMP*TEMP)
51 CONTINUE

READ TIME STEPS
IF(ISTCY.GT.0) GO TO 52
READ(5,5) (DELT(I),I=1,NSTEPS)
WRITE(6,13)
CALL PRFGE(DECY,NSTEPS)

READ MEDIA PROPERTIES
WRITE(6,21)
DO 63 J=1,NNZNS
READ(5,4) I,XPERM(I),YPERM(I),ALPHA(I),SS(I),QDIST(I),VLEAK(I),HR.
WRITE(6,33) I,XPERM(I),YPERM(I),ALPHA(I),SS(I),QDIST(I),VLEAK(I)
1 HR
DO 53 QDIST(I)=QDIST(I)*VLEAK(I)*HR
WRITE(6,18)
DO 54 J=1,NNNZNS
READ(5,28) K,NB,NE,TMPA,TMPB,TMPC,TMPC,TMPE
WRITE(6,30) K,NB,NE,TMPA,TMPB,TMPC,TMPC,TMPE
53 READ AND PRINT PARAMETERS DEFINED ON NODES; COMPUTE ARRAYS
BASED ON THESE PARAMETERS
WRITE(6,16)
DO 59 J = 1,NCNZNS
READ (5,23) K, NB, NE, TMPA, TMPB, TMPC, TMPC, TMPE
WRITE (6,30) K, NB, NE, TMPA, TMPB, TMPC, TMPC, TMPE
59 READ AND PRINT PARAMETERS DEFINED ON ELEMENTS; COMPUTE ARRAYS
BASED ON THESE PARAMETERS
WRITE(6,31)
55 READ AND PRINT PARAMETERS DEFINED ON ELEMENTS; COMPUTE ARRAYS
BASED ON THESE PARAMETERS
WRITE(6,30)
55 CALL SUBROUTINE GEES TO SET UP MATRIX EQUATION
REWIND 1

CALL GES

CHECK FOR PROBLEM TERMINATION

IF (ISTCP.LE.0) GO TO 56
WRITE (6,16)
STOP

INITIALIZE B VECTOR

DO 57 I=NHED,ANDS
B(I)=0.

READ PUMPING RATES FOR WELLS

IF(NWELS.LE.0) GO TO 59
WRITE (6,25)
DO 58 I=1,NWELS
READ (5,27) J,K,GWEL(J)
WRITE (6,1) J,K,GWEL(J)
IF(K.GT.NRED) GWEL(J)=0.
KH(J)=K
CONTINUE

READ KNOWN HEAD BOUNDRY CONDITIONS AND STORE IN B VECTOR

IF (NHES.LE.0) GO TO 65
WRITE (6,22)
DO 60 I=1,NHDS
READ (5,4) K,B(K)
WRITE (6,7) K,B(K)
CONTINUE

READ KNOWN BOUNDARY FLOW RATES

IF (NQBSD.LE.0) GO TO 68
WRITE (6,26)
DO 67 I=1,NQBSD
READ (5,28) J,K,L,QNBD(J)
WRITE (6,29) J,K,L,QNBD(J)
TEMP=SQR ((XQRD(K)-XQRD(L))**2+(YQRD(K)-YQRD(L))**2)/2.
TMP=TEMP
TMPA=TMP
IF(K.GT.NRED) TMP=0.
IF (L.GT.NRED) TMPA=0.
KQB(J)=K
LGB(J)=L
OPK(J)=TMPA
GFJ(J)=TMPA

INITIALIZE GROUNDWATER INFLOW VECTOR

DO 69 I=1,NDM1
GWIF(I)=0.
C

READ INITIAL HEADS, DISCHARGES AND DEPTHS
IF (ICCNT.LE.0) GO TO 70
READ(5,6) (HD(I),I=1,NNDS)
READ(5,6) (HC(I),I=1,NELS)
70 READ(5,5) (Z(I),I=1,NCND)
AFACT=0.
IF (INTLT.LE.0) GO TO 75
READ(5,5) (Q(I),I=1,NCND)
GC TO 85
75 READ(5,5) TEMP
IF (TEMP.LE.0.) GO TO 80
80 TMP=7(1)
TMPA=AC(1)+TMP**PA(1)+TB(1)*TMP
TMPC=CP(1)+TMP**PP(1)+TB(1)
TMPB=TMPA/TMPC
TMPC=EXP2
AFACT=TMPA*TMPB**TMPC
CONTINUE
END
SUBROUTINE GEES
SUBROUTINE TO CALCULATE MATRIX ELEMENTS

1. DIMENSION G(280,43), E(400), XCOORD(400), YCOORD(400)
2. ALPHA(20), SS(20), CDIST(20), VLEAK(20), YHEL(50), KW(50), QBL(50)
3. KOR(50), LGA(50), QBF(50), QBF(50), IZNE(20), ND(4,20), QAR(4,20)
4. CO(4,20), CQFD(4,20), CA(4,20), GD(4,20), QCFD(4,20), GA(4,20), CAC(20)
5. GAC(20), NODE(5), CT(15)

3. DIMENSION XX(5), YY(5), GTO(5), CTO(5), CT(5)
4. COMMON G, B, NED, IBAND, NOS, NELE, NWE, NHEL, NOMB, ISTEPS, APD, NCH
5. NHCH, NHCH, NHCH, NHCH, NHCH, NHCH, NHCH, NHCH, NHCH, NHCH
6. (G(1,1), G(1,1), G(1,1), G(1,1), G(1,1), G(1,1), G(1,1), G(1,1), G(1,1), G(1,1))
7. (CT(1), CT(1), CT(1), CT(1), CT(1), CT(1), CT(1), CT(1), CT(1), CT(1))

FORMAT LIST

1. FORMAT (42H0NEGATIVE OR ZERO AREA FOR ELEMENT NUMBER ,14)
2. FORMAT (///1H0, 24X, 12HELEMENT DATA/9H ELEMENT, 2X, 6HNODE 1, 12X, 6HNODE 2, 2X, 6HNODE 3, 2X, 6HNODE 4, 3X, 4HZONE, 4X, 9HTHICKNESS/) 
3. FORMAT (1X, I7, 5E6.0)
4. FORMAT (1H0, 36HMAXIMUM MATRIX BAND WIDTH (IBAND) GREATER THAN DIMENSION LIMITS (MAXBW) FOR ELEMENT NUMBER ,14)
5. FORMAT (92H0MAXIMUM MATRIX BAND WIDTH (IBAND) GREATER THAN DIMENSION LIMITS (MAXBW) FOR ELEMENT NUMBER ,14)
6. FORMAT (1H0, 36HMAXIMUM MATRIX BAND WIDTH (IBAND) OF 14, 24H CORRESPONDS TO ELEMENT,14)

*** BEGIN OUTER TRIANGULAR AND/OR QUAD ELEMENT LOOP

ISTOP=0
MAXBW=43
IBAND=0
KNT=0
NOEL=20
IF(NELS.LT.NOEL) NOEL=NELS
WRITE(6,2)
DO 200 I=1, NELS
KNT=KNT+1

READ ELEMENT DATA
READ(5,3) IEL, (NODE(J), J=1,4), IZNE(KNT),THICK
IF(IZNE(KNT).LE.0) IZNE(KNT)=1
IZONE=IZNE(KNT)
WRITE(6,4) IEL, (NODE(J), J=1,4), IZONE,THICK
N=4
IF(NODE(4).LE.0) N=3
NP1=N+1

DETERMINE MAXIMUM BAND WIDTH (IBAND)

MIND=NRED
MAXNO=0
DO 9 J=1,N
NUDE=NODE(J)
IF(NUDE.GT.NRED) GO TO 9
IF (NUDE.GT.MAXNO) MAXNO=NUDE
IF (NUDE.LT.MINNO) MINNO=NUDE
9 CONTINUE
MIND=MAXNO-MINNO
MAXND=0
DO 10 J=1,N
NUDE=NOOE(J) IF(NUDE.GT.NRED) GO TO 9
MAXND=NUDE
9 CONTINUE
MAXND=MAXND-MIND
IF (MIND.LE.IBAND) GO TO 10
IBANC=MIND
KEYEL=IEL
IF(IBAND.LT.MAXBW) GO TO 10
WRITE(6,5) IEL
ISTOP=1

INITIALIZE

10 XX(4)=0.
YY(4)=0.
GAC(KNT)=0.
CAC(KNT)=0.
DO 11 K=1,4
NO(K,KNT)=0.
GO(K,KNT)=0.
GQFD(K,KNT)=0.
GA(K,KNT)=0.
CO(K,KNT)=0.
CCFD(K,KNT)=0.
CA(K,KNT)=0.
CAR(K,KNT)=0.
CT(K)=0.
GTD(K)=0.
CT(K)=0.
11 GTD(K)=0.
DETERMINE LOCAL COORDINATES

DO 14 J = 1 , N
   KA = NODE(J)
   XX(J) = XCORC(KA)
   YY(J) = YCORC(KA)
   IF (AMS(ANGLE), LE.0.) GO TO 16
   ANGLE = AMS(ANGLE)
   CSANG = COS(ANGLE)
   DO 15 J = 1 , N
      TEMP = XX(J)
      TMP = YY(J)
      XX(J) = TEMP*CSANG + TMP*SNANG
      YY(J) = TEMP*SNANG + TMP*CSANG
   15
   XX(NP1) = XX(1)
   YY(NP1) = YY(1)
   XF = (XX(1) + XX(2) + XX(3) + XX(NP1))/N
   YF = (YY(1) + YY(2) + YY(3) + YY(NP1))/N

*** BEGIN INNER TRIANGULAR ELEMENT LOOP

DO 100 J = 1 , N
   ND(J,KNT) = NODE(J)
   XNA = XX(J)
   YNA = YY(J)
   XNB = XX(J+1)
   YNB = YY(J+1)
   XK = XK + THCK
   YK = YK + THCK
   IF (THICK.LE.0.) GO TO 23
   XKT = XK
   YKT = YK
   DETERMINE TRANSMISSIVITY TERMS

23
DETERMINE X AND Y INCREMENTS, ELEMENT AREA, AND TEMPORARY VARIABLES INVOLVING THEM

23 BJ = YNP - YCN
24 BK = YCN - YNA
25 EL = YNA - YNP
26 CJ = XCN - XNB
27 CK = XNA - XCN
28 GL = XNB - XNA
29 AREA = 2.0 * (XNA * BJ + XNB * BK + XCN * BL)
30 IF (AREA .GT. 0.0) GO TO 22
31 WRITE (6, 1) IEL
32 ISTOP = 1
33 IF (ISTOP .GT. 0) GO TO 200
34 XKT = XKT / AREA
35 YKT = YKT / AREA
36 TPXL = XKT * PL
37 TPYL = YKT * CL
38 TPXJ = XKT * BJ
39 TPYJ = YKT * CJ

COMPUTE COMPONENTS OF MATRIX AND VECTOR ENTRIES

15 TMP = 0.0
16 TEMP = 0.0
17 TMFB = AREA / 43.
18 IF (VLEAK (IZONE) .LE. 0.) GO TO 26
19 TEM = VLEAK (IZONE) * TMFB
20 TMP = 2.0 * TEMP
21 CAC (J, KNT) = AREA / 12.
22 TMFA = SC * TMP
23 TMFB = 2.0 * TMFA
24 CAC (KNT) = CAC (KNT) + TMPE
25 CT (J) = CT (J) + TMFA
26 CT (J + 1) = TMFA
27 CTO (J) = CTO (J) + TMPB
28 CTO (J + 1) = TMPB
29 CCER (J, KNT) = TMFA
30 GAC (KNT) = GAC (KNT) + TPXL * BL + TPYL * CL + TMP
31 GT (J) = GT (J) + TPXJ * BL + TPYJ * CL + TEMP
32 GT (J + 1) = TPXL * BK + TPYL * CK + TEMP
33 GTD (J) = GTD (J) + TPXJ * BK + TPYJ * CK + TEMP
34 GTD (J + 1) = XKT * BK + YKT * CL + CK * CK + TEMP
35 GOFD (J, KNT) = TPXJ * BK + TPYJ * CK + TEMP
36 100 CONTINUE
OVERLAY(GWH0CH, 2, 0)

PROGRAM STADY

DIMENSION G(200, 40), E(400), XPERM(20), YPERM(20)

ALPHA(20), SS(20), COIST(20), VLEAK(20), QNEL(50), KW(50), GBNO(50)

GAC(20), NODE(5), GT(5), 0(100), 00(100), 04(20), QF(4, 20)

GA(J, K), GOFO(J, K), GA(J, K), J=1, 4, CAC(K), GAC(K), K=1, NOEL

IF(KNT.LT.NOEL) GC TC 200

WRITE COMPONENTS ON TAPE 1

WRITE (1) (TNE(K), NE(J, K), CA(J, K), CA(J, K), CA(J, K), GA(J, K)

1, GC(J, K), GOFO(J, K), GA(J, K), J=1, 4, CAC(K), GAC(K), K=1, NOEL

KNT=0

CONTINUE

IBAND=IBAND+1

WRITE(6,6) IBAND, KEYEL

RETURN

END

COMMON G, B, NPEE, IBANC, NNC, NELS, NWELS, NHCS, NOPNO, NSTEPS, NPC, NOCH

NWCH, NHC, NBC, NCC, NCH, DELT, WF, HD, HC, XPF, YPERM, ALPHASS

ODIST, VLEAK, QNEL, KW, QBND, QKB, LOO, QBFK, QBLF, IT NL, NO, QA, JD, COE, CO

IBP(NCH), NNC, NCH, NOE, IT, SFCT, SPC1, SPC2, SFV, CY, FACT, BFACT

RF, G, CO, CB, DMAX, BU2, Z, Z0, ZE, DMLX, BU1, V, ELEV, KCH, PRMTR, VIF, DIF

SOFT, DX, FOX, GOX, GCHF, CVLK, AC, PA, CT, PT, CP, PP, TF, CPY, PPY

EXTERNAL OCHST, BSCLV

FORMAT LIST
STEADY STATE SOLUTION OBTAINED AFTER 13,114 ITERATIONS

PARTIALLY ASSEMBLE G MATRIX AND B VECTOR

DC 105 I=1,NP
DC 100 J=1,IP
100 G(I,J)=0.
105 B(I)=0.
REVIEW 1
12.
NCEL=20
13.
IKNT=0
14.
ISUM=0
15.
110 ISUM=ISUM+20
16.
IF(ISUM.GT.NELS) NCEL=NELS-ISUM+20
17.
READ(1) (I7NE(K), (N0(J,K), CAP(J,K), CC(J,K), COFC(J,K), C(J,K),CA(J,K), CA(K,K), K=1,NELS)
18.
DO 190 I=1,NELS
19.
IKNT=IKNT+1
20.
N=4
21.
IF(N0(4,I).LE.0) N=3
22.
NP=NP+1
23.
IZONE=I7NE(I)
24.
BCN=0.
25.
NODE(1)=N0(1,I)
26.
NODE(2)=N0(2,I)
27.
NODE(3)=N0(3,I)
28.
NODE(4)=N0(4,I)
29.
NODE(NF1)=NODE(1)
30.
DO 160 J = 1, N
31.
NA=N0E(J)
32.
NB=NODE(J+1)
33.
QC=ODST(IZONE)*QAR(J,J)
34.
BCN=BCN+QC
35.
TEMP=GA(J,J)
36.
TMP=GOFO(J,J)
37.
IF(NA.LE.NP) GO TO 130
38.
BCN=BCN-TEMP*NA
39.
IF(NA.LE.NP) GO TO 135
40.
GO TO 160
41.
130 GT(J)=TEMP
42.
G(NA,1)=G(NA,1)+GC(J,J)
43.
B(NA)=E(NA)+QQ
44.
IF(NP.LE.NP) GO TO 140
45.
B(NA)=B(NA)-TMP*BNB
46.
GO TO 160
47.
135 B(NB)=B(NB)-TMP*B(NA)
GO TO 155
NAT=NA
NBT=NR-NA+1
IF(NB.GE.NA) GO TO 150
NAT=NB
NBT=NA-NB+1
150 G(NAT,NBT)=G(NAT,NBT)+TEMP
155 B(NB)=B(NB)+DD
160 CONTINUE

ELIMINATE CENTRAL NODE POINT OF EACH OUTER TRIANGULAR OR QUADRILLATERAL ELEMENT FROM MATRIX EQUATION

GC=GC(I)
GO 140 J=1,N
NA=NODE(J)
IF(NA.GT.NRED) GO TO 180
TEMP=GT(J)/GC
B(NA)=B(NA)-BCN*TEMP
DO 170 K=1,N
IF(NA.GT.NB.OR.NA.GT.NRED) GO TO 170
NBT=NR-NA+1
G(NA,NBT)=G(NA,NBT)-CT(K)*TEMP
170 CONTINUE
180 CONTINUE
190 CONTINUE
200 CONTINUE
IF(ISUM.LT.NELS) GO TO 110
REWIND 2
WRITE(2) ((G(I,J),J=1,IBAND),B(I),I=1,NRED)
210 BEGIN ITERATIVE SOLUTION
CALL COUPLE(OCHST,BSLCLVE)
WRITE(6,1) IT
END
SUBROUTINE OCHST
SOLVES EQUATIONS FOR STEADY STATE OPEN CHANNEL FLOW

DIMENSION A(280,43),E(400)
1. DELT(100),HO(400),HC(400),XPERM(20),YPERM(20)
2. ALPH(20),SS(20),QDST(20),VLEAK(20),QWEL(5),GW(50),CBND(50)
3. CT(50),LOI(50),OFK(50),OFE(50),DFE(20),GD(4,20),DE(4,20),GOFO(4,20),GA(4,20),CAC(20)
4. QAC(20),COND(5),GT(5),OQ(100),BQ(100),QY(100),Y(100)
5. ELEV(100),KND(100),PRMT(100),VIF(99),CIF(99),SCF(99),CX(99)
6. FDX(99),GDX(99),GWIF(99),CVLK(99),CA(100),PA(100),CT(100),PT(100)
7. CP(100),PP(100),TB(100),CPY(100),PPY(100),FI(100),G1(100)
8. COMMON A,B,NRED,IPANE,ICOM,NELS,NWDS,NOBBK,NSTEPS,NPO,NOCH
9. NWCH,NWCH,NWCH,NWCH,NWCH,NWCH,NWCH,NWCH,NWCH,NWCH
10. QDST,VLEAK,QWEL,KND,LOI,OFK,OFE,DFE,DE,GOFO,GA,CAC,COND
11. CT,LOI,OFK,LOI,OFK,DFE,DE,GOFO,GA,CAC,COND,GT,IST,IST,IST
12. IF,IF,IF,IF,IF,IF,IF,IF,IF,IF,IF
13. TP,TP,TP,TP,TP,TP,TP,TP,TP,TP,TP
14. IF,IF,IF,IF,IF,IF,IF,IF,IF,IF,IF
15. TP,TP,TP,TP,TP,TP,TP,TP,TP,TP,TP
16. TP,TP,TP,TP,TP,TP,TP,TP,TP,TP,TP
17. TP,TP,TP,TP,TP,TP,TP,TP,TP,TP,TP
18. TP,TP,TP,TP,TP,TP,TP,TP,TP,TP,TP
19. TP,TP,TP,TP,TP,TP,TP,TP,TP,TP,TP
20. TP,TP,TP,TP,TP,TP,TP,TP,TP,TP,TP
21. TP,TP,TP,TP,TP,TP,TP,TP,TP,TP,TP
22. TP,TP,TP,TP,TP,TP,TP,TP,TP,TP,TP
23. TP,TP,TP,TP,TP,TP,TP,TP,TP,TP,TP
24. TP,TP,TP,TP,TP,TP,TP,TP,TP,TP,TP
25. TP,TP,TP,TP,TP,TP,TP,TP,TP,TP,TP

BEGIN STEADY FLOW COMPUTATIONS

DO 65 T=2,NNDS

65 O1(T)=T*(OIF(J)+GWIF(J))

DO 155 M=1,NIT

BEGIN STEADY FLOW COMPUTATIONS

TMP=Y(1)

10. UAF=CA(1)*TMP**PA(1)+TB(1)*TMP
11. TUP=CT(1)*TMP**PF(1)+TR(1)
12. PUP=CP(1)*TMP**PF(1)+TR(1)
13. PRMT(1)=PUP
14. RUP=AUP/PUP
15. TEMP=Q(1)
16. SFUP=TEMP*UP*(UAF*AUP*(RUP**REXP))
17. SFUP=SFUP*(SCF1*TUP-SCF2*TUP*CPY(1))**TMP**PPY(1))**AUP
18. V(1)=TEMP/UUP
19. TMP=Y(2)
20. AMD=CA(2)*TMP**PA(2)+TB(2)*TMP
21. TMD=CT(2)*TMP**PF(2)+TB(2)
22. FMD=CP(2)*TMP**PF(2)+TB(2)
23. PRMT(2)=FMD
24. RMD=AMD/RMD
25. TEMP=Q(2)
36. SFMD=TEMP*TEMP/(AMC*AMO*(RMD**R.EXP))
37. SFYM=SFMC*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
38. V(2)=TEMP/AMO
39. AUP=AUP*G
40. AMO=AMO*G

**COMPUTE COEFFICIENTS AND REDUCE EQUATIONS FOR UPPER BOUNDARY POINT**

41. TMPA=GDX(1)*(AUP+AMO)
42. TMPA=GDX(1)*(3.*AUP+AMO)
43. TMPA=2.*AUP+AMO
44. B1=TMPD+TMP*SFYUP
45. C1=TMPD+TMPA*SFYMD
46. CI=(V(1)-2.*V(2)*C(2)-(V(2))^2-0.1+TMPO**SOF(1)-TMAP*(SFMD
1-SFYMD*Y(2))-TMPC*(SFUP-SFYUP*Y(1)+VIF(1))
47. T1TP=CI/B1
48. G1TP=CI/B1
49. F1TP=CI/B1

**COMPUTE COEFFICIENTS FOR INTERIOR NODES**

1. DO 120 I=2,ND1
2. J=I-1
3. K=I+1
4. TMP=YN(K)
5. A0N=CA(K)*(TMP**PA(K))+TB(K)*TMP
6. TDN=CT(K)*(TMP**PT(K))+TB(K)
7. TDN=CP(K)*(TMP**PP(K))+TB(K)
8. PMTR(K)=PDN
9. RDN=ADN/PDN
10. TEM=Q(K)
11. SFMD=ADN*ADN*RDN**R.EXP
12. SFYMD=TEMP*TEMP/SFMD
13. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
14. SFYMD=SFMD*(SFCC1**TMC-SFC2*CPY(2)**TMF**PPY(2))/AMO
15. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
16. SFYMD=SFMD*(SFCC1**TMC-SFC2*CPY(2)**TMF**PPY(2))/AMO
17. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
18. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
19. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
20. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
21. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
22. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
23. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
24. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
25. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
26. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
27. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
28. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
29. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
30. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
31. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
32. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
33. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
34. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
35. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
36. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
37. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
38. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
39. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
40. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
41. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
42. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
43. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
44. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
45. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
46. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
47. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
48. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
49. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
50. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
51. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
52. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
53. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
54. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
55. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
56. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
57. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
58. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
59. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO
60. SFYMD=SFMD*(SFCC1**TMC-SFC2**CPY(2)**TMF**PPY(2))/AMO

157
TMPO = TMP + ADN
A1 = TMPO + TMP & * SFUP
C1 = ADN + AUPEMP * SFUP
C1 = TMPC + TMFD * SFYN
1 * TMPO + SFU (J) + TMPE + SFU (I) - TMPA + SFUP - SFUP + Y (J) - TMPC - SFMD - SFYMD
2 * SFUP - SFYUP * Y (J) - TMPE * SFDN - SFYDN * Y (K) + VIF (J) + VIF (I)

REDUCE TRIDIAGONAL MATRIX FOR INTERIOR NODES

W = B1 - A1 * F1 TP
F1 TP = C1 / W
F1 (I) = F1 TP
G1 TP = (C1 - A1 * G1 TP) / W
AUP = AMD
AND = ADN
IUP = TMF
TMF = TMFD
SFUP = SFMD
SFYUP = SFYMD
SFYMO = SFYON

120 TMPA = TMPB

COMPUTE COEFFICIENTS AND REDUCE EQUATIONS FOR LOWER BOUNDARY POINT

TMF = SORT (SFUP * FACT)
DY = 2. * (Q (NND) / TMF - 1.) / SFYSV
TMP = DY + Y (NND)

BACK SOLVE FOR Unknowns

IND = 0
Y (NND) = Y (NND) + RF * DY
IF (ABS (DY) . GT. CYMX) IND = 1
K = NNDS
DO 130 I = 1, NDM
K = K - 1
TMP = G1 (K) - F1 (K) * TMP
DY = TMP - Y (K)
Y (K) = Y (K) * RF * DY
IF (ABS (DY) . GT. CYMX) IND = 1
CONTINUE

for interior nodes
CHECK FOR CONVERGENCE

IF (IND.LE.0 .AND. AFACT.LE.0.) GO TO 160

IF (AFACT.LE.0.) GC TC 155

TMP=Y(1)

AUP=CA(1)*TMP**PA(1)+TB(1)*TMP

FUP=CP(1)*TNP**FP(1)+TB(1)

R'P=AUP/PUP

tehp =

THPA=REXP/2.

TMF=TFFP*AFACT/(AlJF*RUP**TMP4>

Q(1)=T EP

JL+CIF<J>fGWIF (J)

CONTINUE

GO TO 160

RETURN

END

OVERLAY(GWHOCH, 3, 0)

PROGRAM UNSTDY

DIMENSION G(200), E(400)

1, DDEL, I, HD, XPEEP, YPERM

2, KCR(50), LDP(50), QBFK(50), GFL(50), IZNE(20), NC(4, 20), QAR(4, 20)

3, CC(4, 20), COFD(4, 20), CA(4, 20), GO(4, 20), GOFD(4, 20), GA(4, 20), LAC(20)

4, ELEV(100), KCND(100), PKHR(100), VIF(50), CIF(99), SCF(99), DX(99)

5, CP(100), PP(100), PIB(100), CPY(100), PIPY(100)

CCMON C, BNPEO, IEANC, NEND, NELS, NWEIS, NHES, NREO, NAPS, NAP, NCH

1, NCH, NCHC, NCHB, NCOK, N2NB, N2B, WP, HD, XPEEP, YPERM, ALPHA

2, DDEL, VLEAK, WNL, KB, QBND, KOB, LDP, QBFK, QBF, IZNE, NC, QAR, LO, DIF

3, CA, GO, SOF, GA, CAC, NAC, NODE, GT, ISTO, ISTO, JNT, INTL, LCON

4, ENCH, NEND, NORS, NATO, NHE2NS, IT, SUMIT, IT, SFC1, SFC2, GRVTY, AFACT, EFACT

5, RE, CO, CB, DMAX, B2, 7, TC, ZC, DMAX, E1, V, GL, KSAD, PKHR, VIF, QIF

6, SOF, DX, FOX, GXK, GWIF, CMLK, AC, PA, CT, PT, CPC, TP, TD, CPY, CPY, TIME

EXTERNAL UCHUNS, ESOLVE
FORMAT LIST

1. FORMAT (A1,1PE12.5,5E13.5)
2. FORMAT (1X,17,E18)
3. FORMAT (A1,F7.3,9F6.3)
4. FORMAT (1X,17,E13.5)
5. FORMAT (A1,F7.1,9F8.1)
6. FORMAT (27H1BEGINENSION ON TIME STEP NO.,I4/6H AT ,1PE11.4,12H TIM
1E UNITS)
7. FORMAT (///1H0,13X,3HCOMPUTED VALUES OF HYDRAULIC HEAD/1H
1,6H NODE,8X,4HHEAD,8X,4HNODE,8X,4HHEAD,8X,4HNODE,8X,4HHEAD/)
8. FORMAT (///1H0,13H OUTPUT FOR TIME STEP NO.,I3,4H AT ,1PE11.4
1,11H TIME UNITS/11X,23HSOLUTION OBTAINED AFTER,I3,11H ITPETIONS)
9. FORMAT (///1H0,12X,36H NEW VALUES OF POINT SOURCES OR LINKS/1H
1,15X,26H NEW TOTAL NC. OF POINTS = ,1X/1H,18X,3HNC.,3X,4HNODE,6X
2,8HRECHARGE/)
10. FORMAT (1H,17X,2(I3,4X),1PE11.4)
11. FORMAT (1X,17,18,8F8.0)
12. FORMAT (///1H0,11X,23HSURFACE WATER VARIABLES/4X,4HNODE,8X,1HZ
1,12X,1HP,12X,1HVP/)
13. FORMAT (1H,16,3(1X,F10.3))

14. FORMAT (///42H GROUNDWATER INFLOW TO CHANNEL PER ELEMENT/13X
1,7HELEMENT,7X,2HOC/)
15. FORMAT (///30H DISTSIRUER Source SOURCES OR SINKS
1/IH,37X,4HUNIT/IH,18X,4ZONE,13X,8HRECHARGE/)
16. FORMAT (1H,16,7X,11X,1PE11.4)
17. FORMAT (1X,17,11X,1PE11.4)
18. FORMAT (///1H0,19X,22HN Ew INFLOWS TO CHANNEL/1H,13X,2HNODE,4X,2HNE
1,7X,2HOC,10X,3HVMX/)
19. FORMAT (///1H0,18X,24H NEW VALUES OF KNOWN HEAD/1H,18X,4HNODE
1,14X,4HHEAD/)
20. FORMAT (///1H0,16X,28H NEW VALUES OF BOUNDARY FLOWS/1H,3EX,4HUNIT
1/IH,18X,3HNO,13X,8HRECHARGE/)
21. FORMAT (9X,17,16,2(2X,1PE11.4))
22. FORMAT (///1H0,15X,31HNEw UPSTREAM BOUNDARY CONDITION/1H0,21X
1,7HZ(1) = ,1PE11.4)
23. FORMAT (///1H0,15X,31HNEw UPSTREAM BOUNDARY CONDITION/1H0,21X
1,7HQ(1) = ,1PE11.4)
INITIALIZE

DO 70 I = 1, NCH
ZC(I) = Z(I)
70 DDQ(I) = Q(I)

DO 72 I = 1, NDM1
CIF(I) = 6. * DIF(I)

CT = DELT(I)
TF = WF / CT

IF = NPO
ISTEP = 1
***BEGIN TIME STEPS***

DO 75 I = 1, NDM1
FQ = Q(I) + TF * T
WRITE (6, 6) ISTEP, T

DEFINE NEW DISTRIBUTED SOURCES AND SINKS

IF (NOCH .NE. ISTEP) GO TO 77
READ (5, 2) M, NQCH
WRITE (6, 35) M, NQCH

DO 76 J = 1, M
READ (5, 7) J, QDIST(J)
WRITE (6, 35) J, QDIST(J)

DEFINE NEW POINT SOURCES OR SINKS (WELLS)

IF (NWCH .NE. ISTEP) GO TO 80
READ (5, 2) NWELS, N, NWCH
WRITE (6, 23) NWELS

DO 78 J = 1, N
READ (5, 27) J, KW(J), QWEL(J)
WRITE (6, 24) J, KW(J), QWEL(J)

DEFINE NEW HEADS

IF (NCH .NE. ISTEP) GO TO 82
READ (5, 2) N, NHCH
WRITE (6, 39)

DO 81 I = 1, N
READ (5, 4) J, B(J)
WRITE (6, 36) J, B(J)
DEFINE NEW BOUNDARY FLOWS

IF (N0CP.NE.ISTEP) GO TO 85
READ(5,2) N,NBCCH
WRITE(6,40)
DO 84 I=1,N
READ(5,4) J,GBND(J)
WRITE(6,35) J,GBND(J)

DEFINE NEW INFLOWS

IF(NCOCH.NE.ISTEP) GO TO 90
READ(5,2) NCEZNS,NCOCH
WRITE(6,37)
DO 87 K=1,NCEZNS
READ(5,27) NB,NE,TMPA,TMPB
WRITE(6,41) NB,NE,TMPA,TMPB
TMPA=6.*TMFA
TMPB=.5*TMFB
DO 87 I=NB,NE

TMP=TMFA*CX(I)
WFI(I)=TMP*TMP8
GMFI(I)=TMP

DEFINE NEW UPSTREAM BOUNDARY CONDITIONS

IF(NCBCH.NE.ISTEP) GO TO 100
READ(5,42) TEMF,IEC,NCBCH
FPMAT(I,F7.0,218)
BU1=0.
BU2=1.
ZB=0.
OB=0.
IF(LOC.LE.0) GO TO 92
BU2=0.
ZB=TEMF
WRITE(6,43) ZB
GO TO 100
BU1=0.
OB=TEMF
WRITE(6,44) OB

PARTIALLY ASSEMBLE G MATRIX AND F VECTOR
100 CC 105  I=1,NRFD
105 CC 102  J=1,1BAND
102 G(I,J)=0.
105 G(I)=0.
REMNC 1
NOEL=20
IKNT=0
ISUM=0

110 TSUM=TSUM+20
IF(ISUM. GT.NELS) NOEL=NELS-TSUM+20
READ(1) (IZNE(K),(NO(J,K),QAR(J,K),C0(J,K),COF(J,K),CA(J,K),CA(J,K),J=1,4),CAC(K),GAC(K),K=1,NOEL)
DO 190 I=1,NOEL
DO 190 I=1,NOEL
IKNT=IKNT+1
N=4
DO 190 I=1,NOEL
N=4
IF(IZNE(I).LE.0) N=3
NP=N+1
IZONE=IZNE(I)
CLL=GAC(I)*TF
BCN=CL*HC(IKNT)
NODE(I)=NC(I)
NODE(2)=NO(2)
NODE(3)=NO(3)
NODE(4)=NO(4)
NODE(NF1)=NONE(I)
DO 160 J=1,N
NA=NODE(J)
NA=NODE(J)
GC=DDIST(IZONE)*QAR(J,I)
CJL=CA(J,I)*TF
CJJ=CD(J,I)*TF
CJK=COF(J,I)*TF
BCN=BCN+CJL*HD(NA)+QC
TEMP=CJL+GA(J,I)
TMP=CJK+COF(J,I)
IF(NA.LE.NRFD) GO TO 130
BCN=BCN-TEMP*2(NA)
IF(NA.LE.NRFD) B(NB)=E(NB)+CJK*HD(NA)+QQ-TMP*8(NA)
GO TO 160
CT(J)=TEMP
G(NA,1)=G(NA,1)+CJJ+CA(J,I)
J=6(NB)=E(NB)+CJK*HD(NA)+CJL*HC(IKNT)+QQ
IF(NA.LE.NRFD) GO TO 140
E(NA)=E(NA)-TMP*B(NB)
GO TO 160
140 NAT=NA
39. NBT=NB-NA+1
40. IF(NB*GE.NF) GO TO 150
41. NAT=NB
42. NBT=NB-NA+1
43. 150 G(NAT,NBT)=G(NAT,NBT)+TMP
44. G(NB)=G(NB)+CJ+HC(NA)+OD
45. 160 CONTINUE

CCC ELIMINATE CENTRAL NODE POINT OF EACH OUTER TRIANGULAR OR
QUADRILATERAL ELEMENT FROM MATRIX EQUATION
CCC

+6. GC=CL+GAC(I)
+7. DO 180 J=1,N
+8. NA=NODE(J)
+9. IF(NA.GT.NRED) GC TO 180

30. TEMP=GT(J)/GC
31. B(NA)=P(NA)-GCN*TEMP
32. DO 170 K=1,N
33. NP=NODE(K)
34. IF(NA.GE.NB.OR.NB.GE.NRED) GO TO 170
35. NC=NB-NA+1
36. G(NA,NC)=G(NA,NC)+GT(K)*TEMP
37. 170 CONTINUE
38. CONTINUE
39. CONTINUE
40. CONTINUE
41. IF(ISUM.LT.NELS) GC TO 110
42. REWIND 2 ((G(I,J),J=1,IBAND),B(I),I=1,NRED)
43. BEGIN ITERATIVE SOLUTION
44. DO 196 I=1,NCNDS
45. J=KCNO(I)
46. IF(J.GE.NRED) GO TO 196
47. B(J)=HD(J)
48. 196 CONTINUE
49. CALL COUPLE(OCHUN,RSOLVE)
50. PRINT HYDRAULIC HEADS
51. NF=NP-1
52. IF(NF.GE.0) GO TO 310
53. NF=NP
54. TEMP=TIME+DT/WF
55. WRITE(6,20) ISTEP,TEMP,IT
56. WRITE(6,14)
57. CALL PRINTB(B,KNDS)
PRINT RESULTS FROM THE SAINT-VENANT EQUATIONS

WRITE(6,28)
CO 300 I=1,NCND
300 WRITE(6,29) KND(I),Z(I),Q(I),V(I)

PRINT GROUNDWATER INFLOW

WRITE(6,30)
DO 305 I=1,NDM1
308 WRITE(6,31) I,GWIF(I)
310 IF(ISTEP.GE.NSTEPS .AND.IPNCH.LE.0) GO TO 340

COMPUTE HEADS AT CENTER NODES

REWIND 1
NOEL=20
IKNT=0
TSUM=0
315 ISUM=TSUM+20
188. IF(ISUM.GT.NELS) NOEL=NELS-ISUM+20
189. READ(I),I7NE(K),(ND(J,K),GA(J,K),CL(J,K),COFD(J,K),CA(J,K)
1,GO(J,K),GCFD(J,K),GA(J,K),J=1,4),CAC(K),GAC(K),K=1,NOEL
L90. DO 319 I=1,NOEL
L91. IKNT=IKNT+1
L92. N=4
L93. IF(ND(I)>0) N=3
L94. I7ONE=I7NE(I)
L95. CSUM=0.
L96. DO 317 J=1,N
L97. NA=ND(J,I)
L98. CSUM=CSUM+CA(J,I)*B(NA)-HC(NA))
200. 317 SUM=SUM-CA(J,I)*B(NA)+CDIST(IZNE)*(GA(J,I)
201. CSUM=CSUM*CAC(1)*KC(IKNT)
202. TMP=(SUM+CSUM+TF)/(GAC(I)+TFGAC(I))
203. HC(IKNT)=WF*(TMP-HC(IKNT))+HC(IKNT)
204. 319 CONTINUE
205. IF(ISUM.LT.NELS) GO TO 315

DEFINE HEADS FROM LAST TIME STEP

DEF I=1,NCND
320 HD(I)=WF*(B(I)-HC(I))+HD(I)
DEFINE Z AND Q FROM LAST TIME STEP

DO 325 I=1,NCNDS
Z(I)=WF*(Z(I)-Z0(I))+Z0(I)
Q(I)=WF*(Q(I)-Q0(I))+Q0(I)
J=J+1
325 Q(I)=Q(I)

COMPUTE TIME STEP VARIABLES

IF(ISTEP.GE.NSTEPS) GO TO 330
ISTEP=ISTEP+1
TIME=TIME+DT
CT=DELFT(ISTEP)
IF(WF/CT GO TO 74

FUNCH DEPENDENT VARIABLES

IF(FUNCH.LE.0) GO TO 340
TEMP=1/N
DO 332 I=1,N
NE=6*I
NB=NE-5
WRITE(CU,TEMP,(C(J),J=NI,NE)
NB=NE+1
IF(NB.GT.NNDS) GO TO 333
WRITE(CU,TEMP,(C(J),J=NB,NNDS)
333 TEMP=1/H
N=NELS/6
DO 334 I=1,N
NE=6*I
NB=NE-5
WRITE(CU,TEMP,(C(J),J=NI,NE)
NB=NE+1
IF(NB.GT.NELS) GO TO 335
WRITE(CU,TEMP,(C(J),J=NB,NELS)
335 TEMP=1/H
N=NCNDS/10
DO 336 I=1,N
NB=NE-9
WRITE(CU,TEMP,(C(J),J=NB,NE)
NB=NE+1
IF(NB.GT.NC) GO TO 337
WRITE(CU,TEMP,(C(J),J=NB,NCNDS)
336 TEMP=1/H
337 TEMP=1/H
DO 338 I=1,N
NE=10*I
NB=NE-9
IF(NP.GT. NCNDS) GC TC 340
WRITE(5,5) TFMP, (C(J), J=NB,NE)
CONTINUE
END

SUBROUTINE CCHUMS
SOLVES THE SAINT-VENANT EQUATIONS WITH LATERAL INFLOW

1. DIMENSION A(240,43), E(400)
2. C
3. COMMON A,B,NRED,IBANC,IOUM,NELS,RELS,MHDS,NCBND,STEPS,APO,NCCH
4. C
5. C

COMPUTE AND REDUCE COEFFICIENT MATRICES

THF=Y(1)
AUF=CA(1) * (THF**PA(1)) * TR(1)*TMF
TUP=CT(1) * (THF**PT(1)) * TR(1)
PUP=CP(1) * (THF**PF(1)) * TR(1)
PRTR=TR(1)=FUP
RUF=AUF/PUP
TEMP=0(1)
SFUP=TEMP*THF/(AUF*AUP*(RUF**EXP))
SF2UP=2.*SFUP/THF
SFYUP=-SFUP*(SFC1*TUP=SFC2*RUF*CPY(1)*TMF**PPY(1))/AUF
V(1)=TEMP/AUP

1. C
2. C
3. C
4. C
5. C
6. C
7. C
8. C
9. C
10. C
11. C
12. C
\[
\begin{align*}
AHD &= C(t)^2 \times (T_{MP}^2 + T_{QQ}^2) + T_{MQ}^2 \\
T_{MQ} &= C(t)^2 \times (T_{MP}^2 + T_{QQ}^2) + T_{MQ}^2 \\
PHR &= C^2 \times (T_{MP}^2 + T_{QQ}^2) + T_{MQ}^2 \\
PRHTR &= PRC \\
RHC &= AHC / PHC \\
TEHP &= Q(t)^2 \\
SFMC &= TEMP / (AHC \times AMD / ARMD) \\
SFCD &= 2 \times SFMC / TEMP \\
SFYMD &= -SFMC \times (SFC1 \times TMD - SFC2 \times RMD \times CPY(t)^2 \times TMP \times PPY(t)^2) / AMD \\
V(t)^2 &= TEHP / AMD \\
AUP &= AUP^*G \\
AMD &= AMD^*G
\end{align*}
\]

**Compute Coefficients and Reduce Equations for Upper Boundary Point**

\[
\begin{align*}
CUP &= \text{SQRT}(AUP / TUP) \\
CMD &= \text{SQRT}(AMD / THD) \\
TMP &= V(1) \times CUP \\
TEMP &= V(2) \times CMD \\
TMPF &= 3 \times TUP \\
TMPH &= 2 \times TMD \\
TMPB &= (12 \times TUP \times TMPG) \times TMP + (TMPF + TMPH) \times TEMP \\
TMPF &= (TMPF + TMPH) \times TMP + (2 \times TUP \times TMPG) \times TEMP \\
TEMP &= 2 \times THF + TEMP \\
TMPA &= GDX(1) \times (AUP + AMD) \\
TMPC &= GDX(1) \times (3 \times AMD) \\
TMPD &= 2 \times AUP + AMD \\
TD &= -1 \times FD(1) \\
B1 &= TMP / TMPB \\
C1 &= TMP / TMPD \\
R2 &= 2 \times FD(1) \\
C2 &= FD(1) \\
TMP &= 3 \times TUP \\
TMPG &= 3 \times TMD \\
TMPH &= 2 \times TMPD \\
TMPB &= (12 \times TUP + TMPG) \times THP + (TMPF + TMPH) \times TEMP \\
TMPF &= (TMPF + TMPH) \times TMP + (2 \times TUP + TMPG) \times TEMP \\
TMPF &= 3 \times TUP \\
TMPH &= 3 \times TMD \\
TMPB &= (12 \times TUP + TMPG) \times (THF + TMPH) \times TEMP \\
TEMP &= -1 \times CVLK(1) \\
THPE &= THP \times THPE \\
I = KCND(1) \\
J = KCND(2) \\
HOLP &= R(I) - ELEV(1) - Y(1) \\
CMD &= R(J) - ELEV(2) - Y(2)
\end{align*}
\]
\[
\begin{align*}
D1 &= B2*C0(1) + C2*C0(2) + B1*Y0(1) + C1*Y0(2) + TD*SCF(1) + TMPC*(SFUP + SFYUP) \\
&+ TMPC*SFYUP*Y(1) + TMPC*(SFUP + SFYUP)*Y(2) + VIF(1) - TEMP*Q1F(1)/6 + TMPC*HCU \\
&+ TMPC*SFYUP*Y(1) + TMPC*(SFUP + SFYUP)*Y(2) + VIF(1) - TEMP*Q1F(1)/6 + TMPC*HCU \\
B1 &= B1 + TMPC + TMPC + SFYUP \\
C1 &= C1 + TMPC + TMPC + SFYUP \\
TMPC &= 2 + CUP + CMD \\
B2 &= B2 + 2 + V(2) - V(1) + TMPC*SF0UP \\
C2 &= C2 + V(2) - V(1) + TMPC*SF0UP \\
&\quad + TMPC*SF0UP \\
B3 &= B3 + P01 \\
B4 &= B4 + 8U2 \\
D2 &= YB + 0B \\
W &= B1 + B4 - B2 + B3 \\
\text{F1IP} &= A4*C1/W \\
\text{F1}(1) &= \text{F1IP} \\
\text{F2TP} &= B4 + C2/W \\
\text{F2}(1) &= \text{F2TP} \\
\text{F3TP} &= -B3 + C1/W \\
\text{F3}(1) &= \text{F3TP} \\
\text{F4TP} &= B3 + C2/W \\
\text{F4}(1) &= \text{F4TP} \\
\text{G1TP} &= (A4*D1 - B2*D2)/W \\
\text{G1}(1) &= \text{G1TP} \\
\text{G2TP} &= (R1*D2 - B3*D1)/W \\
\text{G2}(1) &= \text{G2TP}
\end{align*}
\]

**COMPUTE COEFFICIENTS FOR INTERIOR NODES**

\[
\begin{align*}
A1 &= \text{FDX}(1)*(TP*TMPC) \\
\text{TMPC} &= \text{CVLK}(1)*(PUP + PMC) \\
A2 &= 6. \\
C2 &= 6. \\
D0 &= I = 2, NDM1 \\
J &= I + 1 \\
K &= I + 1 \\
\text{TMP} &= Y(K) \\
ADN &= CA(K)*(TMPC**PA(K)) + TB(K)*TMP \\
TDN &= CT(K)*(TMPC**PT(K)) + TB(K) \\
PDN &= CP(K)*(TMPC**PP(K)) + TB(K) \\
PRMTR(K) &= PDN \\
RCN &= ADN/PRMTR \\
\text{TEMP} &= D(K) \\
\text{SFVN} &= \text{ADN}*(RD*REXP) \\
\text{RSF} &= \text{TEMP}**\text{SFVN} \\
\text{SFON} &= 2.1*\text{SFON}/\text{TEMP} \\
\text{SFYSVN} &= (\text{SFSC1} + \text{RDN} + \text{SFSC2} + \text{CFY}(K) + \text{TMPC**PPY}(K))/ADN \\
\text{SFYDN} &= \text{SFON} + \text{SFYSVN} \\
V(K) &= \text{TEMP}/\text{ADN}
\end{align*}
\]
REDUCE BI-TRIDIAGONAL MATRIX FOR INTERIOR NODES

**Variables:**
- $A_i$: Coefficients for the bi-tridiagonal matrix
- $B_i$: Diagonal elements
- $C_i$: Off-diagonal elements to the right
- $D_i$: Off-diagonal elements to the left
- $E_i$: Rightmost elements of the diagonal
- $F_i$: Rightmost elements of the off-diagonal

**Example Code:**

```c
101.  TMP = 3.0 * TD
102.  B1 = FDX(J) + (TMP + TUP) + FDX(I) * (TMP + TON)
103.  TMP = 2.0 * PNC
104.  TEMP = CVLK(J) * (TEMP + PUP) + CVLK(I) * (TEMP + PON)
105.  CI = FDX(I) * (TMO + TON)
106.  TMPH = CVLK(I) * (PNC + PON)
107.  L = KCND(K)
108.  HOND = B(K) - ELEV(K) - Y(K)
109.  C1 = B1 + YO(I) + TMPF*HDPF + B1*YO(I) + TEMP*HMOQ + C1*YO(K) + TMPH*HCDN + QDF(I)
110.  + C1*GDF(I)
111.  ADN = ADN*6
112.  TMDF = GDX(J) + (TKP + AUP) + GDX(I) * (TMF + ADN)
113.  TMP = 2.0 * AMG
114.  TMPD = TMP + AUP
115.  TMPE = TMP + ACN
116.  E3 = -ADN + AUP + TMPC*SYMD
117.  C3 = TMPE + TMP3*SFYDN
118.  A4 = FDX(J)
119.  B4 = 2.0 * FDX(J) + FDX(I)
120.  C4 = FDX(I)
121.  E2 = 4.0*OO(J) + B4*OO(I) + C4*DD(K) + TMPD*SGF(J) + TMPE*SGF(I) + TMPA*(SFUP + 1)
122.  SFUP = Y(J) + TMPC*(SFPC*SFYMD + Y(I)) + TMPB*(SFDP + SFYDN + Y(K))
123.  2*VIF(I) + VIF(J)
124.  A4 = A4 - VFI(I) - 2.0 * V(J) + TPA*SFUP
125.  B4 = B4 + V(K) - V(J) + TMPC*SFQMD
126.  C4 = C4 + V(I) + 2.0 * V(K) + TMPE*SFCDN
```

**Notes:**
- The code snippet represents a step in a numerical algorithm, likely for solving a system of equations.
- The variables are used to calculate intermediate values in the computation process.
- The operations include additions, multiplications, and comparisons.
- The logic involves handling data from different parts of a matrix or tensor.
GTP = (BET1 * TEMP - BET2 * TEMP) / W

GTP = (BET1 * TEMP - BET3 * THP) / W

CUP = AMP

AMD = ACN

TUP = TMC

THC = TON

PM = POM

HUP = HCMO

HMD = HODN

SFUP = SFMD

SFMD = SFON

SFON = SFQO

SFQO = SFYK

SFYM = SFYK

TMBF = TMBH

140

TPA = TMB

CUP = SORT (AUP / TUP)

CUP = SORT (AMD / TMD)

TEMP = V (TMD) - CMD

THCS = V (TMD) - CUP

TMPF = 3. * TMD

THF = 3. * TUP

TMBF = 12. * TMPF * TMPG

TMBF = 2. * TMD * TMPG

TMBF = 2. * TMBF

TMBF = 2. * TMD

TMBF = 2. * TMPF

TMBF = 2. * TMBF

TMBF = 2. * TMBF

TMBF = 2. * TMD

TMBF = 2. * TMD

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TMBF = 2. * TMBF

TMBF = 2. * TMD

TMBF = 2. * TMBF

TMBF = 2. * TMBF

TMBF = 2. * TMBF
BEGINNING ON TIME STEP NO. 1 AT -9 TIME UNITS:

NEW UPSTREAM BOUNDARY CONDITION

\[ q(1) = 1.8140 \times 10^4 \]

OUTPUT FOR TIME STEP NO. 1 AT 1.00E+03 TIME UNITS

SOLUTION OBTAINED AFTER 3 ITERATIONS

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### Surface Water Variables

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### Groundwater Inflow to Channel per Element (G)

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