



Original software publication

MC-BAM: Moment–curvature analysis for beams with advanced materials

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ABSTRACT

MC-BAM is a software for moment–curvature section analysis of beams. It is developed to facilitate the analysis and structural design of beams and girders with advanced construction materials like ultra-high performance concrete (UHPC), high strength steel rebars and/or large prestressing strands. MC-BAM supports several existing, recently developed, or user-defined constitutive material models that can be applied to rectangular, circular, or other cross-sections such as I- or Pi-girders. The material models and section analysis algorithm are verified, first for conventional materials against commercial software and textbook examples, then for different cross-sections with advanced materials and designs against published experimental results.

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Legal Code License	MIT License
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Software code languages, tools, and services used	MATLAB
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If available Link to developer documentation/manual	
Support email for questions	mmoustafa@unr.edu

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1. Motivation and significance

With the advancement in civil, construction, and material engineering research, higher strength and performance concrete, steel and prestressing strands have been introduced over the years.

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Emerging research demonstrated that such advanced construction materials can provide robust retrofit solutions for our crumbling infrastructure or enable new design opportunities and improve the service life and performance of next generation structural components and infrastructure systems [e.g. [1,2]]. The majority of previous and ongoing research on structural design using advanced construction materials, e.g. ultra-high performance concrete (UHPC) or high strength steel (HSS), used experimental methods and testing, which limit the investigations to few parameters and design alternatives. Robust analytical and computational tools to perform structural analysis and design using advanced materials can improve comprehensive research and further advance this emerging field. However, such tools are not available yet due to the lack of software with implemented advanced materials models. The objective of the development and software presented here is to contribute towards filling the aforementioned gap by providing a tool for the moment–curvature section analysis of beams and girders constructed using such advanced materials.

The developed software supports conventional reinforced concrete and reinforcing steel along with UHPC and HSS as two promising advanced materials. Moreover, prestressed beams and girders that use prestressing strands with common strand sizes (up to 0.6 in. or 1.5 cm) and the new evolving 0.7 in. (1.8 cm) strand size can be analyzed using the software. The significance of supporting UHPC structural design can be justified by the rapidly growing market for UHPC, especially in the field of bridge engineering and accelerated bridge construction. According to a recent market research report [3], the global UHPC market is expected to reach USD 1.9 billion by 2025 compared to about USD 0.9 billion in 2016. Using UHPC for bridge girders as one potential application can increase the unsupported bridge spans or decrease superstructure depth and allow for larger clearance. Combining UHPC with HSS or large prestressing strands can further increase girder spans or reduce steel congestion for better constructability. However, constitutive laws to model UHPC material behavior are still evolving and have not been implemented in commercial analysis and design software such as SAP2000 [4] or open-source and research tools such as OpenSees [5]. In this effort, two constitutive models are selected, modified by the authors, and implemented for UHPC along with a generic user-defined option. Similarly, constitutive laws for modeling different conventional and advanced reinforcing steel and prestressing strands are implemented in the software.

2. Software description

Moment–Curvature for Beams with Advanced Materials (MC-BAM) is a simple computational tool that utilizes a variety of constitutive models with user-defined parameters for different constituent materials of a beam or girder cross-section to determine its full moment–curvature response and capacities. Determining moment and curvature response of cross-sections is an important step in fulfilling strength and serviceability design requirements. The user can choose the constitutive models that best represent the desired construction materials of the beam and input the nominal or expected material properties or simply use default parameters for each model. The software integrates these material properties with the user-defined cross-section geometry, dimensions, and reinforcement details to determine the moment–curvature response and capacity of the beam.

2.1. Software architecture

MC-BAM is coded in MATLAB following procedural programming approach. The code is not based on the finite element method but rather reads the inputs and calculates moments and curvatures based on strain-compatibility and equilibrium of forces, which

stems from the classical Euler–Bernoulli Beam theory as explained in several textbooks, e.g. Moehle [6] section 6.6. For every value of curvature, a neutral axis is assumed. The strains in the concrete, reinforcing steel, or prestressing strands are calculated based on strain-compatibility following the hypothesis of the Euler–Bernoulli theory that plane sections remain plane after bending, i.e. shear deformations at the section level are neglected. These strains are fed to their corresponding material (constitutive model) functions to get the stresses. The stresses are then integrated to get the value of force contribution from each component in compression and tension, i.e. above and below the assumed neutral axis, respectively. If the equilibrium of the compressive and tensile forces is obtained within a specific tolerance, then the moment from all forces is calculated about the neutral axis and summed to get the value of the section moment at that curvature value. Equilibrium of the forces is considered satisfied if the compressive and tensile forces are within 5% of each other for low precision calculation mode, and within 0.5% for high precision calculation mode. If the equilibrium is not satisfied, the depth of neutral axis is changed and another iteration is done for the same value of curvature. To get the full section response, curvature values are increased at a step size of 0.00005 1/in. [\sim 0.00002 1/cm] for low precision mode, and at 0.000005 1/in. [\sim 0.000002 1/cm] for high precision mode once the equilibrium is satisfied, and the whole process is repeated until failure is obtained in one of the materials. The software reports the failed material at which the analysis is completed, e.g. concrete reaches its ultimate compression strain. A flowchart to represent the mentioned steps and general program flow is shown in Fig. 1.

2.2. Software functionalities

The purpose of MC-BAM is to generate the moment–curvature values for the user-defined beam. The users can define their own concrete or UHPC cross-section geometry and reinforce it with reinforcing steel and/or prestressing strands. For each of the concrete, steel and prestressing strands, MC-BAM provides the user with multiple options to choose their constitutive models to mimic the intended material behavior for design purposes. Once the analysis is complete, the mode of failure is reported and the user can save a report. Details of all the implemented constitutive models and output capabilities of the software are presented in this section.

2.2.1. Constitutive models of materials

Four models are implemented for concrete, three models for reinforcing steel, and two models for prestressing strands as described here.

2.2.1.1. Constitutive models for concrete. (i) UHPC: This is a constitutive model suggested by the authors based on extensive review of existing literature to best represent actual material behavior of UHPC. The model accounts for the sustained tensile strength of UHPC, which is a key feature of UHPC that does not exist in conventional concrete. The lack of this property renders existing concrete constitutive models inappropriate to represent UHPC tensile behavior. The tensile behavior implemented here is based on the direct tension tests comprehensive study conducted by the Federal Highway Administration (FHWA) [9]. For the UHPC compression behavior, an existing model for conventional concrete [10] is slightly modified to better define the UHPC Young's modulus as per Eqs. (1) and (2). The overall suggested constitutive model for UHPC is illustrated in Fig. 2a. The UHPC is said to be failed and moment–curvature analysis is stopped when the strain in the extreme compressive fiber of concrete exceeds $1.4\epsilon_{cu}$. On the other hand, reaching or exceeding the ϵ_{tu} strain in the extreme concrete tensile fiber does not terminate the analysis due to the contribution

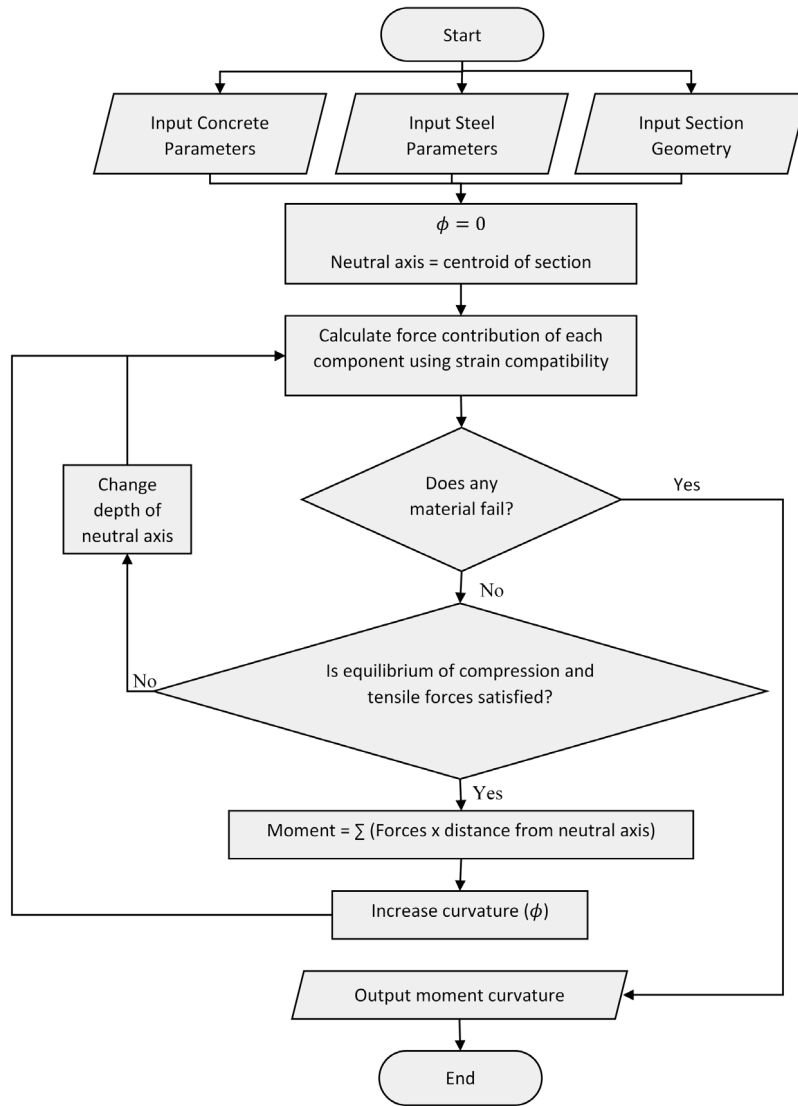


Fig. 1. Flowchart for moment–curvature calculation.

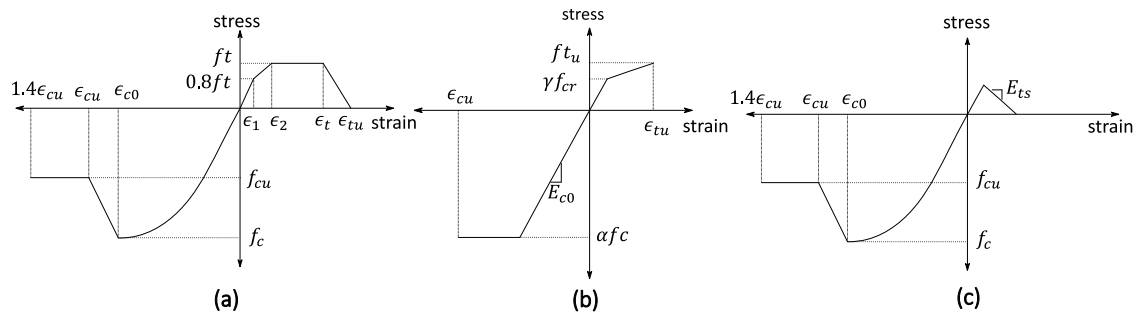


Fig. 2. Constitutive model for (a) UHPC; (b) Idealized UHPC; and (c) Conventional concrete.

from the reinforcing or prestressing steel that usually dictates the section failure in tension.

$$\sigma_c = E_{c0} * \epsilon * \left(1 - \frac{1}{n+1} * \frac{\epsilon^n}{\epsilon_{c0}^n} \right) \quad (1)$$

$$E_{c0} = \frac{n+1}{n} * \frac{f_c}{\epsilon_{c0}} \quad (2)$$

$$\epsilon_1 = 0.8 * \frac{f_t}{E_{c0}} \quad (3)$$

$$\epsilon_2 = \epsilon_1 + 0.4(\epsilon_t - \epsilon_1) \quad (4)$$

(ii) Idealized UHPC: This is the second model for defining UHPC behavior which is based on a bilinear idealized behavior in both tension and compression. This model accounts for strain hardening in tension, which is not provided by the first model, and is based on several FHWA studies as defined according to Fig. 2b.

(iii) Conventional Concrete: The conventional concrete constitutive model is adopted from Yassin [10] with the factors considered for unconfined concrete with monotonic loading. Except for

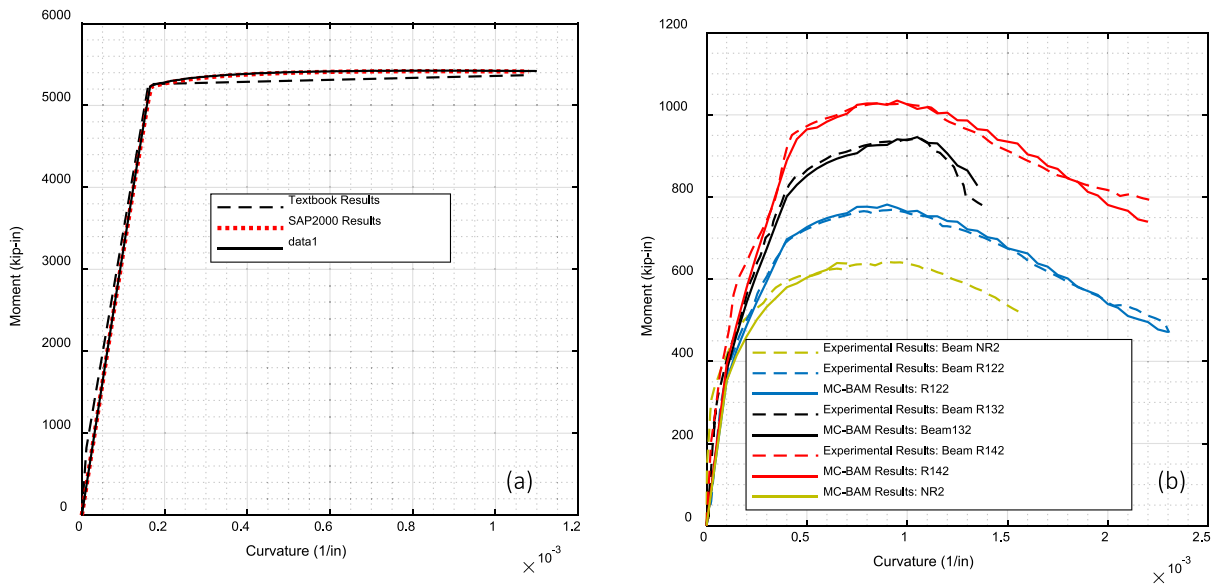


Fig. 3. MC-BAM moment–curvature relationships as compared to verification cases: (a) Moehle [6] example 6.1; and (b) experimental results for four different UHPC beams as conducted and reported by Yang et al. [7].

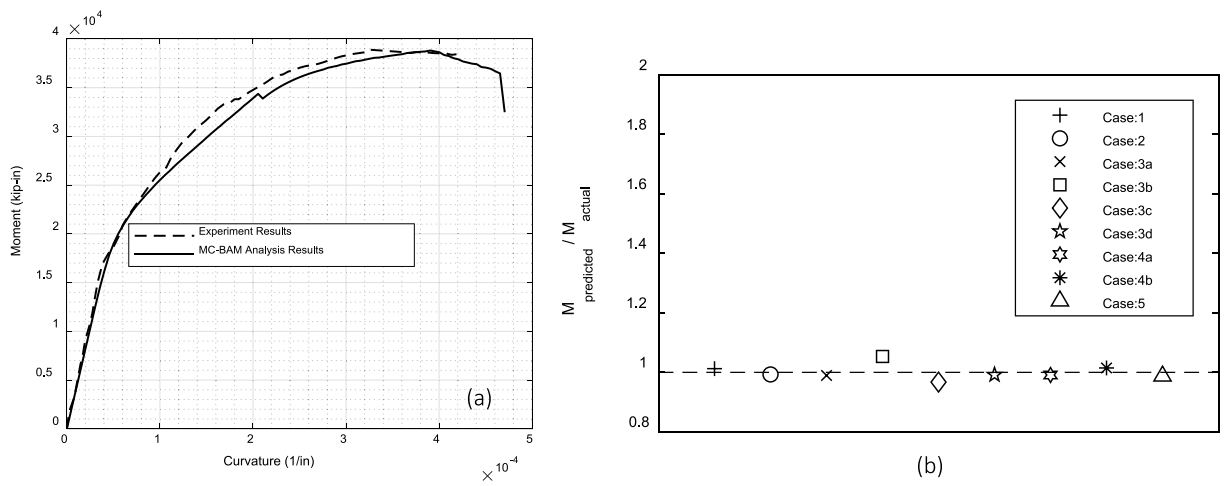


Fig. 4. (a) MC-BAM moment–curvature comparison against experimental results for the FHWA UHPC I-Girder [8]; (b) variation between MC-BAM predicted moment capacity and actual values for different verification cases summarized in Table 1.

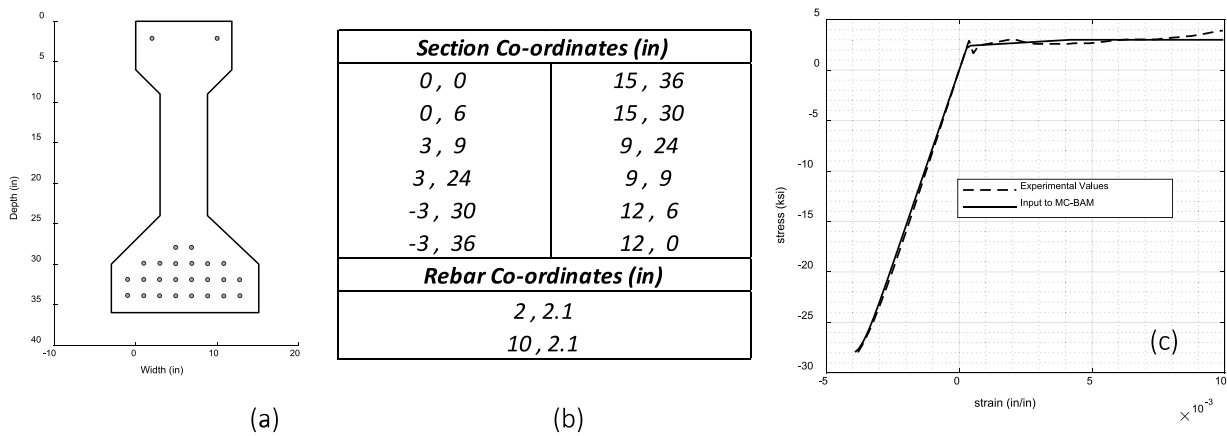
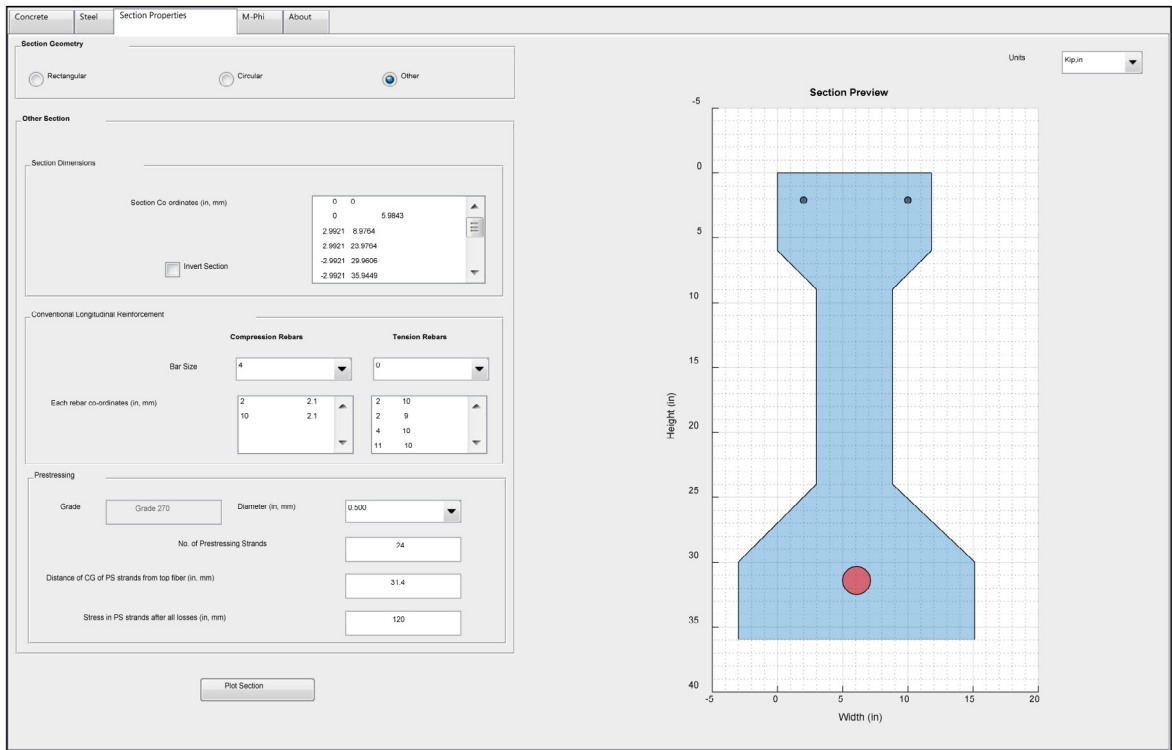
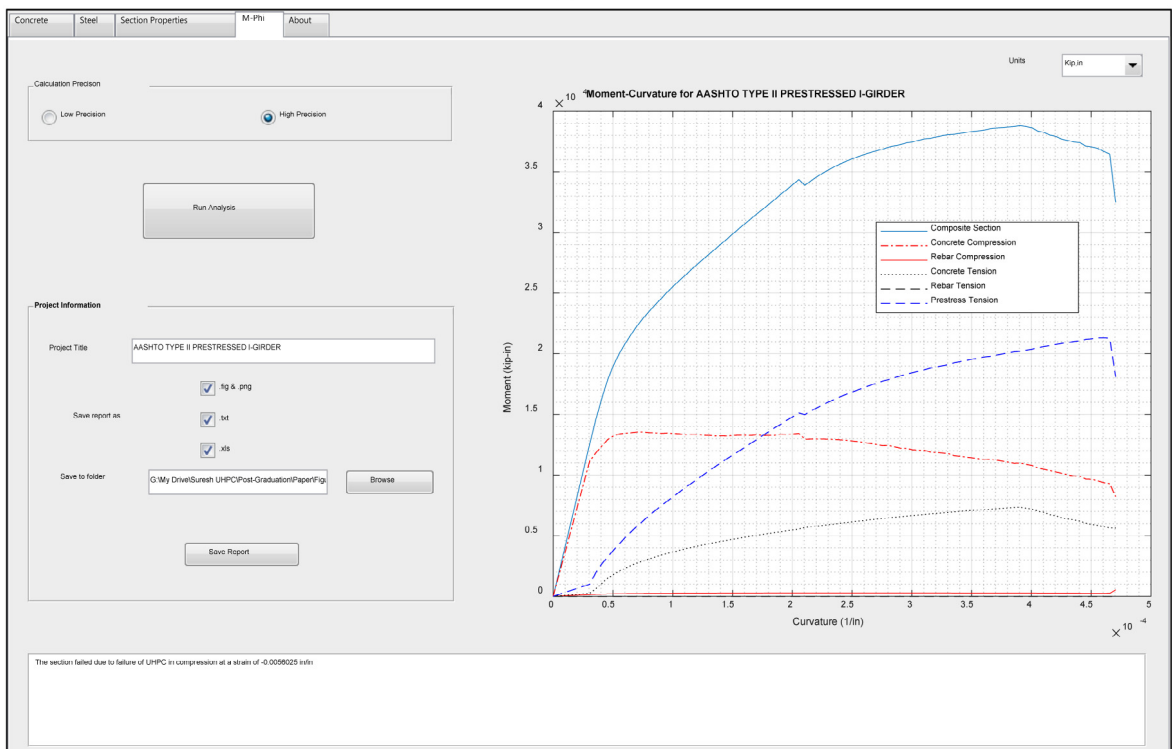


Fig. 5. (a) Actual I-girder section; (b) section and reinforcing bars co-ordinates; and (c) stress–strain relationship comparison of reported UHPC values [8] and input used for MC-BAM software.



(a)



(b)

Fig. 6. Screenshots of MC-BAM software illustrative example: (a) section input; and (b) analysis results and reporting options.

the ascending branch of compression defined by Eqs. (5) and (6), the stress–strain relationship is assumed to be piecewise linear as shown in Fig. 2c. Similar to UHPC, the concrete is considered to be failed in compression and the moment–curvature analysis is stopped when the strain in the extreme compression fiber reaches $1.4\epsilon_{cu}$.

$$\sigma_c = E_{c0} * \epsilon * \left(1 - \frac{1}{2} * \frac{\epsilon}{\epsilon_{c0}}\right) \quad (5)$$

$$E_{c0} = 2 * \frac{f_c}{\epsilon_{c0}} \quad (6)$$

(iv) Custom Concrete: Under Custom Concrete, the user can define the stress–strain properties of the concrete or UHPC using strain and stress co-ordinates to provide more modeling options beyond what is readily implemented in the other three modeling options (UHPC, Idealized UHPC, and Conventional Concrete). MC-BAM assumes linear variation between the stress–strain co-ordinates.

2.2.1.2. Reinforcing steel. (i) Steel having yield plateau (Park Model): This model is similar to the Park Model as implemented in the commercial software SAP2000 [4], which is characterized by Eqs. (7)–(9) in the strain hardening region. This model best represent A615 and A706 Grade 60 steel. Same material behavior is assumed in both tension and compression.

$$\sigma = f_y \left(\frac{m(\epsilon - \epsilon_{sh}) + 2}{60(\epsilon - \epsilon_{sh}) + 2} + \frac{(\epsilon - \epsilon_{sh})(60 - m)}{2(30r + 1)^2} \right) \quad (7)$$

$$r = \epsilon_u - \epsilon_{sh} \quad (8)$$

$$m = \frac{(f_u - f_y)(30r + 1)^2 - 60r - 1}{15r^2} \quad (9)$$

(ii) Steel without yield plateau: This model is a simplified approximation suggested by the authors, as per Eqs. (10)–(12) for the strain hardening region, to represent typical reinforcing bars stress–strain behavior without a yield plateau. This model can reasonably capture both Grade 60 and Grade 80 behavior for reinforcing bars without a well-defined yield plateau. As before, same behavior is assumed in both tension and compression.

$$\sigma = \left(E_s \left(\epsilon - \frac{(\epsilon - \epsilon_l)^{n+1}}{n+1} (\epsilon_p - \epsilon_l)^n \right) - \sigma_l \right) ES_{factor} + \sigma_l \quad (10)$$

$$\sigma_l = E_s \left(\epsilon_p - \frac{(\epsilon_p - \epsilon_l)^{n+1}}{n+1} (\epsilon_p - \epsilon_l)^n \right) \quad (11)$$

$$ES_{factor} = \frac{\sigma_p - \sigma_l}{\sigma_1 - \sigma_l} \quad (12)$$

(iii) HSS Grade 100: This model is another unique feature of MC-BAM given that it is lacking in currently available software and tools. The constitutive model for HSS Grade 100 bars is based on the American Concrete Institute ACI ITG-6R-10 [11] and characterized by Eqs. (13)–(15).

$$\epsilon \leq 0.0024: f_s = 29000\epsilon(\text{ksi}) \quad (13)$$

$$0.0024 < \epsilon \leq 0.02: f_s = 170 - \frac{0.43}{\epsilon + 0.0019}(\text{ksi}) \quad (14)$$

$$0.024 < \epsilon \leq 0.06: f_s = 150(\text{ksi}) \quad (15)$$

2.2.1.3. Prestressing strands. (i) Power Equation: The prestressing strands are represented by the power equation (Equation 16) as proposed by Skogman et al. [12].

$$\sigma = \epsilon E_{ps} \left[Q + \frac{1 - Q}{\left(1 + \left(\frac{\epsilon E_{ps}}{K * f_y}\right)^R\right)^{\frac{1}{R}}} \right] \quad (16)$$

(ii) PCI Design Handbook Equation: Another popular model for prestressing strands is what given by the PCI Design Handbook [13] and represented by Eqs. (17)–(18) and (19)–(20) for the 250 ksi (~ 1725 MPa) and 270 ksi (~ 1860 MPa) prestressing strands, respectively.

For 250 ksi strand:

$$\epsilon \leq 0.0076: f_s = 28500\epsilon(\text{ksi}) \quad (17)$$

$$\epsilon > 0.0076: f_s = 250 - \frac{0.04}{\epsilon - 0.0064}(\text{ksi}) \quad (18)$$

For 270 ksi strand:

$$\epsilon \leq 0.0086: f_s = 28500\epsilon(\text{ksi}) \quad (19)$$

$$\epsilon > 0.0086: f_s = 270 - \frac{0.04}{\epsilon - 0.007}(\text{ksi}) \quad (20)$$

2.2.2. Saving the report

To facilitate further analysis and manipulation of the moment–curvature analysis data, MC-BAM provides the user with the functionality to export and save the analysis report. After the complete analysis of the section until failure, the users can choose to save a report that includes the stress–strain values used to characterize each material, the moment contribution from each section constituent, and the total section's moment for the range of curvature until failure. It is noted that the reported stress–strain values represent the full range over which the constitutive model is defined and not only the stress–strain ranges experienced through failure. The users can export the analysis results in .txt and/or .xls files. The users can also save the stress–strain plots of the constituent materials, the cross-section geometry, and the resulting moment–curvature plot in a MATLAB .fig format and .jpg format.

3. Verification

The objective of this section is to validate and verify the various proposed constitutive laws for material modeling, the constitutive models implementation, and the overall moment–curvature analysis algorithm using MC-BAM. Several verification examples are presented. First, a beam with conventional concrete and reinforcing steel (Example 6.1 from Moehle [6]) is analyzed using MC-BAM and the results are verified against SAP2000 and textbook analytical solution as shown in Fig. 3a. Next, experimental results for four different UHPC beams reported by Yang et al. [7] are successfully reproduced with acceptable accuracy as shown in Fig. 3b. A third verification example focus on prestressed UHPC girders, which compared the MC-BAM results against AASHTO Type-II Prestressed UHPC I-Girder experimental results from a FHWA study reported by Graybeal [8]. The I-girder moment–curvature response can be reproduced using MC-BAM as demonstrated in Fig. 4a. The numerical data for the maximum moment and corresponding curvature value for the aforementioned verification cases are summarized in Table 1. The table also presents two more verification cases that cover a more complicated cross-section example for the FHWA UHPC Pi-girder [14] and beams with HSS from the ACI document [11]. The variation or difference (%) between the different experimental results, designated as actual values, and the corresponding MC-BAM results, designated as predicted values, are also summarized in Table 1 and visually presented in Fig. 4b. It is noted that all verification cases used the actual material

properties and section geometry as per the best available reported data in the corresponding literature, and few assumptions based on engineering judgments were made wherever necessary. Overall, the figures and results in Table 1 verify that modeling laws and implementation are acceptable and confirm that the software has a satisfactory performance.

4. Illustrative example

The details of input parameters used for the analysis of the AASHTO Type-II prestressed UHPC I-girder tested by the FHWA [8], which is previously discussed in the verification section, is presented here to illustrate the use of the MC-BAM software. The tested I-girder used UHPC and 0.5 in., 270 ksi low-relaxation prestressing strands. The geometry of the actual section is shown in Fig. 5a, the input co-ordinates for the section and equivalent prestressing strands are summarized in Fig. 5b, and a screenshot of the resulting defined section in MC-BAM is shown in Fig. 6a. The stress–strain relationship of UHPC from the experiment is reproduced using the “UHPC” constitutive model and a comparison between the reported and input values is shown in Fig. 5c. The MC-BAM supports only one layer of prestressing strands. Thus, the equivalent section of all 24 prestressing strands, originally arranged in 4 layers in the bottom bulb of the I-section, is defined at their centroid (geometric center) as shown in Fig. 6a. The two prestressing strands in the top bulb are approximated using two #4 reinforcing bars with their stress–strain properties defined through the “steel without yield plateau” to match those of prestressing strands. Full details of all input parameters for the concrete, steel, and section tabs of the software are presented in Table 2. A screenshot of the analysis tab and resulting analysis results including the identified mode of failure in the textbox is shown in Fig. 6b. It is noted that the MC-BAM obtained moment–curvature response is compared to the experimental results as previously shown in Fig. 4a above.

5. Impact

Applications and markets for advanced construction materials such as high strength steel and ultra-high performance concrete are rapidly growing. However, readily available computational tools that help structural designers explore new design opportunities using such materials is lacking. The authors in this effort selected, and modified as necessary, theoretical constitutive models to represent UHPC and implemented it for the first time in a simple computational tool with focus only on beams and girders. Meanwhile, approximation of material behavior is widely accepted in the structural design process. Yet, very close estimates of UHPC beams and girders moment capacities can be obtained using MC-BAM and its material models for different design cases as presented in the verification section of this paper. The user-friendly software can provide a simple but effective tool to better understand the behavior of UHPC structural components and explore new design possibilities for beams and girders.

Research on the structural behavior of UHPC components is only emerging, let alone the design optimization of UHPC structural components when combined with HSS and large prestressing strands for longer spans or shallower sections. Therefore, the MC-BAM software provides a feasible and convenient way of exploring numerous design alternatives without conducting expensive or time-consuming tests. This process can also inform future experimental tests where only the few most promising design alternatives can be tested and verified.

The moment–curvature analysis helps engineers understand the flexural and deformation capacity of the beam which is needed

for satisfying strength and serviceability design requirements. Current design codes and standards are not suited for advanced materials such as UHPC and HSS strength, and the design guidelines documents are evolving. The availability of computational tools can help expedite the codification process of design using advanced materials.

The use of this software can impact several construction industries. In the field of bridge engineering as one example, incorporating UHPC, HSS reinforcing bars, and large prestressing strands can significantly increase the length of possible prefabricated girders or reduce the girders cross-section for a given span. The first case can lead to less number of supporting piers, while targeting the second design objective can lead to lighter girders and feasible site construction handling and lifting. In either case, accelerated bridge construction can benefit from the new generation of structural components designed and built using advanced materials.

The MC-BAM software can be used for design optimization but from a different perspective, which is section topology optimization. Although a formal optimization framework is not implemented in this software, simple and large number of user-defined section geometry can be analyzed. The inexpensive trials can provide the foundational work for future topology optimization frameworks for sections with advanced materials. The flexibility in material and geometry definitions in MC-BAM also help accommodate users custom needs in adhering to depth or width requirements or acceptable designs limited by construction materials availability.

6. Conclusions

The introduction and implementation of various constitutive models for three emerging construction materials namely: UHPC, HSS, and large prestressing strands is provided in this paper. The validity of the proposed material models and efficiency of the moment–curvature analysis algorithm as implemented in MC-BAM is verified through successfully reproducing published experimental and analytical results. The software can be used for section analysis of beams and girders designed using conventional construction materials as well as advanced materials and has been verified for different designs and configurations. The MC-BAM can be used with confidence to inform experimental research, understand structural response of components made of advanced materials, optimize reinforcing steel and prestressing details, and in turn, explore new design opportunities for different applications such as prefabricated bridge girders or parking garage structures.

Conflict of interest

The authors declare that there is no conflict of interest in this paper.

Appendix. Notations

E_{c0}	Initial modulus of elasticity of concrete
E_{ps}	Modulus of elasticity of prestressing strands
E_s	Modulus of elasticity of steel rebar
f_c	Compressive strength of concrete
f_s	Stress
f_u	Ultimate stress
f_y	Yield stress
n	Power term
Q, K, R	Power equation coefficients
ϵ	Strain

Table 1
Summary of verification results.

Case no.	Description	Curvature at maximum moment (1/in)			Maximum moment (kip-in)		
		Actual	Predicted	% difference	Actual	Predicted	% difference
1	Textbook [6] example 6.1	0.001080	0.000820	24.07	5,370	5,426	1.04
2	AASHTO Type-II Prestressed I-Girder [8]	0.000324	0.000391	20.53	38,907	38,818	0.23
3	Yang et al. (2010) [7]						
3a	Beam NR2	0.000903	0.000650	28.02	642	639	0.38
3b	Beam R122	0.000700	0.000900	28.57	735	782	6.36
3c	Beam R132	0.001070	0.001050	1.87	942	946	0.43
3d	Beam R142	0.000870	0.000950	9.20	1,029	1,035	0.54
4	ACI ITG-6R-10 [11]						
4a	Appendix A Example 6.1(a)	0.000437	0.000450	2.97	2,109	2,090	0.89
4b	Appendix A Example 6.1(b)	0.000548	0.000550	0.38	1,765	1,791	1.48
5	FHWA Pi-Girder Prototype [14]		0.000245		37,500	36,047	3.88

Table 2
Summary of illustrative example input parameters.

Concrete tab		Steel tab		Section properties tab	
Constitutive model	UHPC	Compression rebar		Section geometry	Other
Comp. strength, f_c (ksi)	−28	Constitutive model	w/o yield plat.	Section co-ordinates (in)	Fig. 8(b)
strain at f_c , ϵ_{pc0}	−0.004	Initial elastic tangent, E_s (ksi)	28500	Comp. rebars: Bar Size	4
ult. comp. strength, f_{cu} (ksi)	0	Stress at the end of elastic region (ksi)	220	Comp. rebars: co-ordinates	2, 2.1, 10, 2.1
Strain at f_{cu} , ϵ_{pcu}	−0.004	Strain at the start of end plateau	0.02	Tension rebars: bar size	0
ten. strength, f_t (ksi)	3	Stress at the end plateau (ksi)	264.6	Prestressing	
ten. strain end plateau, ϵ_{pst}	0.01	Ultimate strain	0.04	Diameter (in)	0.50
ten. strain at failure, ϵ_{pstu}	0.01	Transition factor, n	0.1	No. of prestressing strands	24
Power term, n	10	Prestressing strands		Dist. of PS from top fiber (in)	31.4
		Model	PCI D.H.	Stress in PS after losses (ksi)	120
		Grade	Grade 270		

ϵ_{c0}	Strain at maximum strength, f_c
ϵ_l	Strain at the end of linear region
ϵ_p	Strain at the beginning of plastic plateau
ϵ_{sh}	Strain at onset of strain hardening
ϵ_u	Ultimate strain
σ	Stress
σ_c	Compressive stress
σ_l	Stress at the end of linear region
σ_p	Stress at plastic plateau
σ_y	Yield stress

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