University of Nevada

Reno

Scheduling Production from Underground Mines by Linear Programming

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering

by

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June 1967

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June 1967

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Each mining operation is faced with the problem of scheduling production from its working areas or ore zones unless requirements must be met by mining all ore in the mine. This production scheduling is determined by management and is frequently done on a weekly or monthly basis, whichever is applicable to the particular operation. Management reaches these decisions by balancing the grade and maximum practical production from each ore source against the tonnage and grade to be delivered to the mill.

Often management ignores the fact that production costs vary in different sections of the mine. Only if these variations in cost are taken into account can management hope to determine a production schedule which will result in maximum profit. With the introduction of these and other complications, management is generally inclined to turn the problem over to an organizational division usually known by some such name as "Operation Research Department."

Operations Research is generally described as a "scientifically organized approach to problem solving", and has become increasingly important in the mineral industry within the past few years. Most of the larger mining companies have complete scientific and mathematical facilities at their disposal for the handling of these complex problems. Unfortunately, the smaller mining companies do not have the necessary capital to invest in computers, programmers, or technicians which are often required. These companies can, however, use a systematic mathematical technique such as linear programming to solve many of their problems. Easily accessible digital computers are used for solving a linear program. There are numerous university computer centers as well as company computer centers throughout the country which will run programs for individuals or other companies for a fee.

The staff of a small mining operation usually does not carry anyone who has had much experience in programming or computer work. Therefore, it becomes necessary for one of the members of the staff to take the responsibility of learning about programming if the company wants to partake of the information afforded by a linear program. It is not necessary for this individual to understand all the intricate and minute details of programming and computer functions; however, he should have a basic understanding of the mathematical technique he is using and its limitations and capabilities.

Linear programming is a methodology whereby a linear function is optimized, either minimized or maximized, subject to a set of linear constraints in the form of equalities or inequalities.[1] In our situation, it would be described as a procedure which arrives at the optimum distribution of production with a corresponding maximization of profit. Linear programming is a systematic process of solving simultaneous linear equations when the number of unknowns exceeds the number of equations. Obviously there will be an unlimited number of possible solutions, but only one set of "n" unknowns from a larger set of "m" unknowns will optimize the objective function, i.e., maximum profit.[2] The computational procedure, while always satisfying the equations with "n" unknowns, progressively selects new unknowns to bring into the solution and takes out the less desirable unknowns. Ultimately the optimum set of unknowns is chosen. This type of linear programming described is known as the simplex method and consists of a series of steps whereby one equation—

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1 Bowman and Fetter, <u>Analysis for Production Management</u>, (Homewood, Illinois: Richard D. Irwin, 1961), p. 102.

2 <u>Ibid</u>., p. 86.

the objective function—is optimized while the other equations--the constraints—are being manipulated.[3] The steps in the problem solving procedure are as follows:

1. The problem is defined and all pertinent mathematical relationships are determined.
2. An initial solution is determined.
3. Alternative changes to this solution are considered and evaluated.
4. The alternative with the largest favorable profit which still satisfies the constraints is selected.
5. A new solution is determined by using this most favorable alternative.
6. Steps 3 through 5 are repeated until the best possible solution is reached.
7. The procedure is complete and no better solution exists when step 3 evaluates no alternative choices favorably.[4]

To illustrate a linear programming solution, a problem has been created by using a rather small hypothetical lead, zinc, silver mine of simple dimensions. Although in practice this prototype mine is overly idealized, it is nonetheless, a good basic example upon which more complicated operations can be based.

Let us assume the production from the prototype mine is to be from four stopes on each of three levels while development work is being carried on in ore on the fourth level. Table I lists the necessary data concerning each stope, and therefore the mine in general. This data includes the grade of ore in each stope, the cost per ton of ore produced from each stope, the maximum number of crews that can work in each stope, and the "proven" reserves of each stope. The number of men in each crew is

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3 Bowman and Fetter, <u>Analysis for Production Management</u>, p. 86.

4 <u>Ibid</u>., p. 83.

arbitrary and should be assigned by management. Although it is unlikely that the reserves, as stated in Table I, could be counted as "proven" in the early stages of development in a typical western United States mine, let us assume that on the basis of drill hole data, management is relatively certain about the grade and tonnage of ore in each stope.

It may be beneficial for management to calculate the cost of producing a ton of ore in terms of total costs. That is, the cost should include labor, equipment, development work, etc. This solution, in terms of maximum profit, may then be more significant to management since it has been arrived at by considering nearly all of the costs involved in the operation and thus be in terms of total net profit. The data in Table I must be presented by management and is certain to be available, for it would be necessary for the efficient operation of a mining property.

First we must define the problem. The problem is to schedule a month's production from the twelve available stopes in the hypothetical mine so as to meet the mill requirements and predetermined mining procedures while providing maximum profit for the company. Suppose the mill requires 15,000 tons per month including development ore, at grades as follows:

1.0 - 2.0 ounces silver per ton
1.5 - 2.5 per cent lead per ton
1.0 - 2.0 per cent zinc per ton

In order to make the problem interesting, there are any number of other restrictions which may be added. Suppose company mining plans specify that each level is to be mined out in a progressive manner from the top level down, and the stopes in levels I and II must be completely exploited within 60 months and those in level III are to be completed in 72 months. The figures of 60 and 72 months are strictly arbitrary in this example; however, in actuality these limits will be management decisions

TABLE I.—DATA FOR THE PROTOTYPE MINE

LEVEL	STOPE	RESERVES TONS	OZ.Ag./TON	%Pb.	%Zn.	COST $/TON	MAX. PRODUCTION TONS/MONTH	MAXIMUM CREWS	PROFIT $/TON
1	A1	150,000	1.2	2.0	1.5	5.40	5,000	6	6.50
	B1	90,000	0.8	3.0	1.0	7.00	4,000	4	5.93
	C1	50,000	2.5	2.0	0.5	6.00	2,000	3	4.68
	D1	20,000	2.0	4.0	2.0	8.50	1.000	2	11.88
2	A2	180,000	0.7	1.5	3.0	5.00	5,000	6	9.10
	B2	100,000	1.0	1.0	2.5	7.50	5,000	5	4.04
	C2	40,000	2.0	2.0	0.8	8.00	2,000	3	2.90
	D2	10,000	3.0	1.0	1.0	6.00	500	1	3.77
3	A3	100,000	0.6	2.0	0.7	5.00	5,000	5	3.80
	B3	70,000	1.5	2.5	1.0	7.00	2,000	4	5.34
	C3	20,000	2.5	1.8	2.0	7.50	1,000	2	6.93
	D3	10,000	4.0	3.0	3.0	8.00	500	1	13.86
4	Development		2.0	2.0	1.0	7.50	1,000	(4)	7.50

Monthly Production:

Development	1,000 Tons
Stopes	14,000 Tons
Total	15,000 Tons

based on mining rate, ore reserves, and operational confinements of particular areas of the mine—all of which are a portion of the general operating policy.

Assume each stope underlies its corresponding stope number on the level directly above the one in question. That is, stope A2 lies directly under stope A1, and stope A3 lies under stope A2. Because of safety and maintenance reasons, management may want to add the restriction or constraint that no stope is to be completed to the level above until the stope on that level is completed. In other words, a crown pillar is to be left over each stope until the stope above is finished or completed. It is assumed crown pillars to be maintained are as follows:

12,000 tons for each of "A" zone stopes
8,000 tons for each of "B" zone stopes
5,000 tons for each of "C" zone stopes
3,000 tons for each of "D" zone stopes

Of course, a constant work force is desirable; therefore, for solution purposes an arbitrary figure of 40 crews is imposed on the problem. Even though this figure is arbitrary and quite large, it will assure enough available labor to produce the required tonnage. The program will be set up so that the optimum solution will furnish the exact number of crews which are necessary for this operation.

Another constraint which may be placed on the problem is that of putting maximum and minimum limits on the cost of production, such as \$7.50 and \$5.50 per ton. It may not be obvious why a lower limit is placed on the production costs. If production is to be scheduled on a monthly basis, as this problem supposes, and maximum profits are sought, it necessarily follows that ore would be produced from the stopes containing the highest grade ore and the lowest production costs. Needless to say, the mine would be essentially "high-graded". In other words, the mine would soon be stripped of its low cost, high grade ore with only high cost, low grade ore remaining. Depending on the economic conditions at the time, this low grade ore may not be considered economical for mining. Consequently, for conservation purposes, which are becoming increasingly important in the Twentieth Century, the lower limit placed on the production costs will force some of the lower grade ores to be mined. The amount of low grade ore which is to be mined will, of course, depend on the lower limit set on the production costs. Obviously, this will prolong the life of the mine which may be beneficial to the company because of equipment amortization and depletion allowances, and return on their initial capital investment. These cost restrictions, in all probability, will help maintain the monthly profit at a more stable, uniform level which will reduce the complications of monthly budgeting. From this discussion, it is readily seen that determining the lower cost limit is a large share of the whole problem. Management must not treat this restriction lightly and should spend much time and thought in affixing a value to this lower cost limit.

Since linear programming is a mathematical technique, one must be able to express all functions and constraints in a mathematical form either as equalities or inequalities. From Table I, the limitations on stope production rates can be mathematically represented as follows:

(1) $A1 \leq 5000$ tons per month
(2) $B1 \leq 3000$ tons per month
(3) $C1 \leq 2000$ tons per month
(4) $D1 \leq 1000$ tons per month
(5) $A2 \leq 5000$ tons per month
(6) $B2 \leq 4000$ tons per month
(7) $C2 \leq 2000$ tons per month
(8) $D2 \leq 500$ tons per month
(9) $A3 \leq 4000$ tons per month
(10) $B3 \leq 3000$ tons per month
(11) $C3 \leq 1000$ tons per month
(12) $D3 \leq 500$ tons per month

The letter and number represent the tonnage produced from a particular stope. That is, A1 is the monthly production from the A stope on the first level.

As stated previously, the mill requirements are 15,000 tons from development and stope production. From Table I, 1,000 tons of development ore are being produced; consequently, 14,000 tons of ore must be produced from the stopes. The required mill tonnage from the stopes is then expressed by:

$$(13)\quad A1 + B1 + C1 + D1 + A2 + B2 + C2 + D2 + A3 + B3 + C3 + D3 = 14{,}000 \text{ tons.}$$

The mill grade restrictions for silver in ounces is expressed by:

$$(14)\quad 1.2A1 + 0.8B1 + 2.5C1 + 2.0D1 + 0.7A2 + 1.0B2 + 2.0C2 + 3.0D2 + 0.6A3 + 1.5B3 + 2.5C3 + 4.0D3 \leq 2.0(15{,}000) - 2.0(1000) \leq 28{,}000$$

$$(15)\quad 1.2A1 + 0.8B1 + 2.5C1 + 2.0D1 + 0.7A2 + 1.0B2 + 2.0C2 + 3.0D2 + 0.6A3 + 1.5B3 + 2.5C3 + 4.0D3 \geq 1.0(15{,}000) - 2.0(1{,}000) \geq 13{,}000$$

The mill grade restrictions for lead in per cent is expressed by:

$$(16)\quad 2.0A1 + 3.0B1 + 2.0C1 + 4.0D1 + 1.5A2 + 1.0B2 + 2.0C2 + 1.0D2 + 2.0A3 + 2.5B3 + 1.8C3 + 3.0D3 \leq 2.5(15{,}000) - 2.0(1{,}000) \leq 35{,}500$$

$$(17)\quad 2.0A1 + 3.0B1 + 2.0C1 + 4.0D1 + 1.5A2 + 1.0B2 + 2.0C2 + 1.0D2 + 2.0A3 + 2.5B3 + 1.8C3 + 3.0D3 \geq 1.5(15{,}000) - 2.0(1{,}000) \geq 20{,}500$$

The mill grade restrictions for zinc in per cent is expressed by:

$$(18)\quad 1.5A1 + 1.0B1 + 0.5C1 + 2.0D1 + 3.0A2 + 2.5B2 + 0.8C2 + 1.0D2 + 0.7A3 + 1.0B3 + 2.0C3 + 3.0D3 \leq 2.0(15{,}000) - 1.0(1{,}000) \leq 29{,}000$$

$$(19)\quad 1.5A1 + 1.0B1 + 0.5C1 + 2.0D1 + 3.0A2 + 2.5B2 + 0.8C2 + 1.0D2 + 0.7A3 + 1.0B3 + 2.0C3 + 3.0D3 \geq 1.0(15{,}000) - 1.0(1{,}000) \geq 14{,}000$$

The development ore is considered on the right hand side of the equation. The coefficient of each stope designation is simply the grade of ore contained in that stope as shown in Table I.

The production cost restrictions may be represented as follows:

$$(20)\quad 5.40A1 + 7.00B1 + 6.00C1 + 8.50D1 + 5.00A2 + 7.50B2 + 8.00C2 + 6.00D2 + 5.00A3 + 7.00B3 + 7.50C3 + 8.00D3 \leq 7.50(15{,}000) - 7.50(1{,}000) \leq 105{,}000$$

$$(21)\quad 5.40A1 + 7.00B1 + 6.00C1 + 8.50D1 + 5.00A2 + 7.50B2 + 8.00C2 + 6.00D2 + 5.00A3 + 7.00B3 + 7.50C3 + 8.00D3 \geq 5.50(15{,}000) - 7.50(1{,}000) \geq 75{,}000$$

The numerical coefficients are of course the cost per ton of ore produced from each stope as shown in Table I.

The mathematical equations expressing retreat from the three levels within the time limit of 60 and 72 months are as follows:

$$(22)\quad A1 \geq 150{,}000 - 5{,}000(60)$$
$$(23)\quad B1 \geq 90{,}000 - 4{,}000(60)$$
$$(24)\quad C1 \geq 50{,}000 - 2{,}000(60)$$
$$(25)\quad D1 \geq 20{,}000 - 1{,}000(60)$$
$$(26)\quad A2 \geq 180{,}000 - 5{,}000(60)$$
$$(27)\quad B2 \geq 100{,}000 - 5{,}000(60)$$
$$(28)\quad C2 \geq 40{,}000 - 2{,}000(60)$$
$$(29)\quad D2 \geq 10{,}000 - 500(60)$$
$$(30)\quad A3 \geq 100{,}000 - 5{,}000(72)$$
$$(31)\quad B3 \geq 70{,}000 - 2{,}000(72)$$
$$(32)\quad C3 \geq 20{,}000 - 1{,}000(72)$$
$$(33)\quad D3 \geq 10{,}000 - 500(72)$$

These equations state that the production from any one stope must be greater than, or equal to, the remaining reserves in that stope, minus the maximum production rate per month, times the number of months allowable for production in each stope. In other words, in order for a stope to be completed within the time limit designated; the product of the time limit in months, times the maximum production per month possible in each stope, must at least be equal to, or exceed, the total amount of remaining reserves in the stope. Obviously the production rate can never be negative; therefore, if the right hand side of the equation is less than zero, it should be replaced by zero. These equations will tend to force production from various stopes in later months.

In order to leave crown pillars above each stope and to force one level to completion before the one directly below it, the following equations are employed:

$$
\begin{aligned}
&(34) \quad 150{,}000A2 - 168{,}000A1 \leq 0\\
&(35) \quad 180{,}000A3 - 88{,}000A2 \leq 0\\
&(36) \quad 90{,}000B2 - 92{,}000B1 \leq 0\\
&(37) \quad 100{,}000B3 - 62{,}000B2 \leq 0\\
&(38) \quad 50{,}000C2 - 35{,}000C1 \leq 0\\
&(39) \quad 40{,}000C3 - 15{,}000C2 \leq 0\\
&(40) \quad 20{,}000D2 - 7{,}000D1 \leq 0\\
&(41) \quad 10{,}000D3 - 7{,}000D2 \leq 0
\end{aligned}
$$

Actually, a so-called "if" statement would be the easiest method of handling this pillar constraint. In other words, "if" 'x' tons are produced from stope A2, "then" at least 'x' tons must be produced from stope A1 or possibly 'x' tons plus an additional amount of 'y' tons. Unfortunately, in linear programming such "if" statements cannot be handled. Therefore, it becomes necessary to put the constraints in the form of mathematical inequalities once again. One must make certain the time required to mine out an upper stope is less than, or equal to, the time required to mine out the stope directly below, exclusive of the pillar which is to be left. Looking at constraint (35) in particular, the expressions are arrived at as follows: (see Table I)

$$\frac{180{,}000}{A2} \leq \frac{(100{,}000 - 12{,}000)}{A3}$$

After rearranging and simplifying:

$$180{,}000A3 - 88{,}000A2 \leq 0$$

Consequently, in order for the expression to be true, more tonnage must be produced from stope A2 than from A3 which is what is desired. One may note that an equation such as (34) may remain correct mathematically since some tonnage may be extracted from stope A2 even though none is taken from A1. Although this is true, it should be noted that with the production from A2, the coefficient of A1 will steadily decrease while that of A2 remains the same--providing no ore is extracted from stope A1. Consequently, within a very few months equation (34) will take on the form of equation (35) and will force the production from stope A1 to be greater than that of stope A2.

The only remaining constraint which must be written in mathematical terms is that governing the number of crews. Previously an arbitrary figure of 40 crews was given which was considered excessive. Therefore, the constraint must be written so that the optimum number of crews required to produce the necessary ore is obtained in the solution. The coefficient preceding each stope designation is obtained by dividing the maximum number of crews in each stope by the maximum production from that stope, thus resulting in crews per ton produced. The crew constraint is then as follows:

$$(42)\quad \begin{aligned} &(0.0012)A1 + (0.001)B1 + (0.0015)C1 + (0.002)D1 + \\ &(0.0012)A2 + (0.001)B2 + (0.0015)C2 + (0.002)D2 + \\ &(0.001)A3 + (0.002)B3 + (0.002)C3 + (0.002)D3 \leq 40 \end{aligned}$$

Now that all of the restrictions or constraints have been written for the problem, there is only one function left. This last equation is the optimization criteria, or, in this case, the maximum profit equation. Naturally this equation must be expressed completely in terms of profit. Therefore, the numerical coefficients are simply the difference between the gross value of a ton of ore and its cost of production. For example, stope A1 contains ore whose grade is as follows:

1.2 ounces silver per ton
2.0 per cent lead
1.5 per cent zinc

In figuring these profits, July, 1966, values of these metals were used and were as follows:

Silver (Ag)----$1.29 per ounce
Lead (Pb)----$0.15 per pound
Zinc (Zn)----$0.145 per pound

Therefore, the value per ton of ore computations take this form:

$$2\% \text{ Pb} \times 2000\text{Lb.} \times \frac{\$0.15}{\text{Lb.}} = \$6.00$$

$$1.5\% \text{ Zn} \times 2000\text{Lb.} \times \frac{\$0.145}{\text{Lb.}} = \$4.35$$

$$1.2 \text{ oz. Ag} \times \frac{\$1.29}{\text{Oz.}} = \$1.55$$

Profit per ton of ore from stope A1 is then:

$$\$6.00 + \$4.35 + \$1.55 - \$5.40 = \$6.50$$

The maximum profit equation may then be written:

$$\text{(P)} \quad 6.50A1 + 5.93B1 + 4.68C1 + 11.88D1 + 9.10A2 + 4.04B2 + 2.90C2 + 3.77D2 + 3.80A3 + 5.34B3 + 6.93C3 + 13.86D3 = Z = \text{TOTAL PROFIT}$$

One will note that each of the mathematical expressions is in the first power or the desired linear form. Since the simplex method of linear programming requires the equations to be expressed as equalities rather than inequalities, an additional variable called a "slack variable", must be introduced into the inequality expressions.[5] For instance, equation (12) becomes $D3 + Y = 500$ instead of $D3 \leq 500$. This "slack variable", Y, represents some available source such as idle time or additional production which may be utilized. For example a maximum of 500 tons of ore may be extracted from stope D3, but if less than 500 tons is extracted in a month, the available tonnage not being produced is represented by the slack variable Y.

If an inequality takes the form of "greater than or equal to"($\geq$),

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5 Bowman and Fetter, Analysis for Production Management, p. 84.

a negative slack must be added to the equation. For instance, equation (22) Al $\geq$ 150,000 - 5,000(60) becomes Al - Y = 150,000 - 5,000(60). This slack variable still represents the difference between limiting and actual production. After the addition of these slack variables, one suddenly finds that there is an enormous amount of unknown variables in the problem.

The programmer need not concern himself with adding these slack variables manually to the equations. A computer such as the IBM Model 7044 handles this problem quite easily. When writing the input matrix, the programmer need only place the correct mathematical signs with the corresponding equations. For example, a negative sign is associated with those inequalities containing a "greater than or equal to" sign; a positive sign is associated with those inequalities containing a "less than or equal to" sign; and a zero is placed next to equations containing an equal sign. From these mathematical signs, the computer then selects the correct slack variable to be used and proceeds toward the soltion. Since these slack variables have a zero profit associated with them, it is undesirable to have them appear in the solution so the computer arbitrarily associates a large cost with each, and thus keeps them from appearing in the final solution. It is possible, however, for one of these added variables to appear in the optimum solution. If this happens, it most probably indicates that a restriction was incompatible or illogical with the total system. Then it is necessary to isolate this restriction and alter or eliminate it with regard to the total system. Thus the computer handles the slack variables quite easily, and they represent no problem for the programmer.

Now that the equations are all developed, it is advisable to compile

TABLE II.—BRIEF MATRIX

CONSTRAINT	A1	B1	C1	D1	A2	B2	C2	D2	A3	B3	C3	D3	SIGN	RHS
PROFT	6.50	5.93	4.68	11.88	9.10	4.04	2.90	3.77	3.80	5.34	6.93	13.86	=	0000
Con 01	1												≤	5000
Con 02		1											≤	3000
Con 03			1										≤	2000
Con 04				1									≤	1000
Con 05					1								≤	5000
Con 06						1							≤	4000
Con 07							1						≤	2000
Con 08								1					≤	500
Con 09									1				≤	4000
Con 10										1			≤	3000
Con 11											1		≤	1000
Con 12												1	≤	500
Con 13	1	1	1	1	1	1	1	1	1	1	1	1	=	14000
Con 14	1.2	0.8	2.5	2.0	0.7	1.0	2.0	3.0	0.6	1.5	2.5	4.0	≤	28000
Con 15	1.2	0.8	2.5	2.0	0.7	1.0	2.0	3.0	0.6	1.5	2.5	4.0	≥	13000
Con 16	2.0	3.0	2.0	4.0	1.5	1.0	2.0	1.0	2.0	2.5	1.8	3.0	≤	35500
Con 17	2.0	3.0	2.0	4.0	1.5	1.0	2.0	1.0	2.0	2.5	1.8	3.0	≥	20500
Con 18	1.5	1.0	0.5	2.0	3.0	2.5	0.8	1.0	0.7	1.0	2.0	3.0	≤	29000
Con 19	1.5	1.0	0.5	2.0	3.0	2.5	0.8	1.0	0.7	1.0	2.0	3.0	≥	14000
Con 20	5.40	7.00	6.00	8.50	5.00	7.50	8.00	6.00	5.00	7.00	7.50	8.00	≤	105000
Con 21	5.40	7.00	6.00	8.50	5.00	7.50	8.00	6.00	5.00	7.00	7.50	8.00	≥	75000
Con 22	1												≥	0000
Con 23		1											≥	0000
Con 24			1										≥	0000
Con 25				1									≥	0000

TABLE II.—CONTINUED

CONSTRAINT	A1	B1	C1	D1	A2	B2	C2	D2	A3	B3	C3	D3	SIGN	RHS
Con 26					1								≥	0000
Con 27						1							≥	0000
Con 28							1						≥	0000
Con 29								1					≥	0000
Con 30									1				≥	0000
Con 31										1			≥	0000
Con 32											1		≥	0000
Con 33												1	≥	0000
Con 34	-168000				150000								≤	0000
Con 35					-88000				180000				≤	0000
Con 36		-92000				90000							≤	0000
Con 37						-62000				100000			≤	0000
Con 38			-35000				50000						≤	0000
Con 39							-15000				40000		≤	0000
Con 40				-7000				20000					≤	0000
Con 41								-7000				10000	≤	0000
Con 42	.0012	.001	.0015	.002	.0012	.001	.0015	.002	.001	.002	.002	.002	≤	40

the constraints into a table which resembles a brief matrix in form. Table II expresses the equations for this particular problem in the desired form. The matrix itself can be written quite easily from this table. The exact form of the matrix will of course depend upon the type of computer which is being used.

The optimum solution to this particular example is given in Table III. In other words, this is the most profitable distribution of production which will satisfy all the constraints imposed on the problem. If this solution is followed, it will result in a maximum net profit to the organization.

Another part of the output is contained in Table IV. It should be noted that the values shown in this table are only one-hundredth of their actual value. When writing the matrix, it became necessary to divide all numbers by 100 in order to comply with space limitations in the matrix. It will be noted that the values of B(I) are just the right hand side values of the equations which define our problem. The values of PI(I) are marginal values. This table then gives us a relationship between the actual values in the problem and associated marginal values. For each unit increase in B(I), the value of the objective function, "Z", will increase by the amount PI(I).

From Table IV, it is seen that only constraints 04, 05, and 13, would have any appreciable effect on the objective function if the right hand side values associated with these constraints should vary for any reason. It should be noted that constraint 04 is associated with production from stope D1; constraint 05 is associated with production from stope A2; and constraint 13 is associated with the mill requirements. If then, for any reason, management should want an extra profit for a

TABLE III.—OPTIMUM SOLUTION

STOPE NO.	PRODUCTION TONS/MONTH	OPTIMUM NUMBER OF CREWS
A1	4464.28	
B1	2940.71	
C1	0.00	
D1	999.99	
A2	4999.99	
B2	0.00	
C2	0.00	
D2	349.99	
A3	0.00	
B3	0.00	
C3	0.00	
D3	244.99	
Total	14000.00	22.5

TABLE IV.—MARGINAL VALUES

ROW (I)	PI(I)	B(I)
Con 01		50.00
Con 02		30.00
Con 03		20.00
Con 04	7.13684999	10.00
Con 05	3.67892857	50.00
Con 06		40.00
Con 07		20.00
Con 08		5.00
Con 09		40.00
Con 10		30.00
Con 11		10.00
Con 12		5.00
Con 13	5.92999999	140.00
Con 14		280.00
Con 15		130.00
Con 16		355.00
Con 17		205.00
Con 18		290.00
Con 19		140.00
Con 20		1050.00
Con 21		750.00
Con 22		
Con 23		
Con 24		
Con 25		
Con 26		
Con 27	2.25580000	
Con 28		
Con 29		
Con 30		
Con 31		
Con 32		
Con 33		
Con 34	0.00000339	
Con 35		
Con 36		
Con 37	0,00000590	
Con 38		
Con 39	0.00002500	
Con 40	0.00016955	
Con 41	0.00079300	
Con 42		.40

given month, they simply have to increase the right hand side values of one of these constraints. In this case, management would obtain additional profit by increasing the producing capacity of stopes D1 or A2 or increasing the tonnage sent to the mill. This is assuming of course, that the mill is capable of handling the extra tonnage. Each of these specific marginal values are based on the assumption that all other variables remain constant.

Other information which may be gained from a linear program is the range through which the right hand side values may vary. Table V is a compilation of the output specifically relating to this problem. Once again, the values are only one-hundredth of their actual value. Principally, this means that if all of the right hand side values remain constant but one, and if the variable right hand side value stays within the designated interval, then the basis of the problem remains unchanged. The value of the objective function may change slightly, but the basis will not be altered. For example, constraint 04, which is associated with production from stope D1, may range between a minimum value of 962.83 tons and a maximum value of 1428.57 tons. At the minimum value, constraint 02 will come into play which will prevent production from falling below the designated minimum value. Likewise, constraint 08 will keep production from exceeding the upper range of production. Therefore, constraint 02 and constraint 08 act as controls on the range through which production may vary from this particular stope. In effect these two constraints prevent the basis of our problem from being changed. Of course, there are no limitations set forth for these stopes which are to produce no ore for this particular time period.

More useful information which may be obtained from the linear program is shown in Table VI. The stopes which are not shown have no significance to the problem at this stage of the operation. This output is similar to that shown and described in Table V, except that in this case we are dealing with ranges in profit per ton produced. As long as the profit values remain within the designated limits, the limiting constraints of the incoming vector will not come into play, and the basis of this problem will remain unchanged.

Essentially then the program gives management ranges of profit and right hand side values to work within. As long as values remain within these limits, all the constraints imposed upon the general problem will be satisfied and the basis of the problem will remain unchanged. Consequently, if more or less ore is produced from a given stope or the profit per ton produced in a given stope varies, management will be able to determine whether or not the basic program would be valid for the next time period simply by checking to see if the new value is within the designated limits. Management will also be able to get an idea of how any new values will effect the objective function, optimum profit.

From Table IV, management will readily be able to determine which stopes to increase production from, if an emergency should arise and the company should suddenly need extra profit from the operation. The quantity which may be taken from one of these stopes is by no means infinite; but rather, it will be determined by the right hand side ranges from Table V. Therefore, only a finite amount is obtainable at any one time.

The important thing is that a linear program can furnish a large amount of extremely useful information in excess of the optimum production figures for each stope. Since it is virtually impossible to produce

TABLE V.—RIGHT HAND SIDE RANGES

ROW NAME	CURRENT RHS VALUE	P(I) VALUE	MIN. VALUE	MAX. VALUE	OUTGOING VECTOR AT MIN.	AT MAX.
Con 01	50.0000		44.642857	Unbounded	Con 01	
Con 02	30.0000		29.407143	Unbounded	Con 02	
Con 03	20.0000				Con 03	
Con 04	10.0000	7.1368500	9.628302	14.285714	Con 02	Con 08
Con 05	50.0000	3.6789285	49.686792	55.223358	Con 02	Con 18
Con 06	40.0000			Unbounded	Con 06	
Con 07	20.0000			Unbounded	Con 07	
Con 08	5.0000		3.500000	Unbounded	Con 08	
Con 09	40.0000			Unbounded	Con 09	
Con 10	30.0000			Unbounded	Con 10	
Con 11	10.0000			Unbounded	Con 11	
Con 12	5.0000		2.450000	Unbounded	Con 12	
Con 13	140.0000	5.9299999	129.63980	140.59286	Con 21	Con 02
Con 14	280.0000		152.39714	Unbounded	Con 14	
Con 15	130.0000		Unbounded	152.39714		Con 15
Con 16	355.0000		303.35714	Unbounded	Con 16	
Con 17	205.0000		Unbounded	303.35714		Con 17
Con 18	290.0000		277.22143	Unbounded	Con 18	
Con 19	140.0000		Unbounded	277.22143		Con 19
Con 20	1050.0000		822.52143	Unbounded	Con 20	
Con 21	750.0000		Unbounded	822.52143		Con 21
Con 42	0.4000		0.174878	Unbounded	Con 42	

TABLE VI.—PROFIT RANGES

STOPE	SUGGESTED PRODUCTION	PROFIT $/TON	LIMIT #1	LIMIT #2	INCOMING VECTOR AT LIMIT #1	INCOMING VECTOR AT LIMIT #2
A1	4464.2857	6.50	2.3796	6.5000	Con 05	Con 34
B1	2940.7142	5.93	5.9300	5.9300	Con 34	Con 37
D1	999.9999	11.88	4.7431	Infinite	Con 04	Unbounded
A2	4999.9999	9.10	5.4210	Infinite	Con 05	Unbounded
B2		4.04	Infinite	6.2958	Unbounded	Con 27
D2	349.9999	3.77	0.3790	Infinite	Con 40	Unbounded
B3		5.34	5.3400	8.9783	Con 37	Con 27
C3		6.93	5.9300	14.0100	Con 39	C2
D3	245.0000	13.86	9.0157	Infinite	Con 40	Unbounded

the exact amount from each stope suggested by the program output, it is extremely important for management to know which stopes may vary in production more than others and the effect this variation will have in terms of company profits.

This program is designed to establish the distribution of monthly production from a mine and to insure maximum profit. It may be used for several months in succession before a new portion of the program need be written. For example, as the reserves and the grade constants change during the course of mining, only a few new input cards need be inserted into the program. This is true for all the constants which appear in the program.

Of course, if the general corporate or company policy changes regarding such changes as the number of crews, production requirements, or mining procedures, it becomes necessary to insert or delete equations within the program. If, for example, one of the old stopes is completed or a new stope comes into production, or a production limitation is imposed upon a particular level or stope, then new equations will have to be added to the program to handle these new problems. There is only one requirement concerning these constraints; that being that the equations must be expressed in linear form. It is possible that a new constraint may conflict drastically with one of the original constraints. Generally, this can only be determined after the program has been processed. If a conflict exists, management must decide which constraint is more costly to violate or which will accomplish the company's goals more satisfactorily.

Although this program was designed on a monthly basis, it is possible to program a digital computer so that it cycles the linear program and alters the various constraints representing such factors as ore reserve by using the results of a previous solution. Consequently, it is possible to get an idea of future production on a long range basis. However, this cycling process cannot handle any changes that may occur in new producing areas, reserve tonnages, or ore grade changes. This can be a very useful tool for management if used for only a few months in succession. Small mines in particular have many pertinent changes occurring within a few months, thus causing the usefulness of the cycling process to be limited. Management should also realize that the solution regarding a particular monthly production may not be the best solution for use in solving long range or future production schedules. However, if no major changes occur in the mining procedure or restrictions, there is no reason why one monthly production schedule could not be followed for three or four months in succession.

The usefulness and efficiency of linear programming in solving a mining problem of this nature is obvious. Not only is an enormous amount of time saved, but the possibility of making human errors is also greatly reduced. Only by a systematic mathematical approach, such as linear programming, can a problem of this complexity be solved in a reasonable amount of time. Once the program has been written, it can usually be run periodically, with only minor additions or deletions to the basic program itself.

The cost of running such a program is indeed minimal when compared to the amount of information it supplies the company. This program was run on an IBM 7044 Digital Computer and the total costs were as follows:

Description	Hours/ Quantity	Rate per Hour Hour/Unit	Amount
7044 Computer	0.14	190.00	$26.60
7044 Computer Operator	0.14	4.00	0.56
Printed Lines	21.00	0.001/L	0.03
Key Punch with Operator	1.08	2.75	2.97
Verifier with Operator	0.83	2.75	47.50
Total			$79.95

The bulk of the cost for running this program is the consulting programmer's fee. The detailed matrix for this problem was written in a form required by the Model 7044 computer before the consulting programmer received it. It was then his responsibility to have the necessary data cards punched, verified, and fed into the computer. A programmer's fee may be more or less than the one stated depending on where the program is processed, and the time consumed by unexpected problems which may occur. The type of computer used will also have a bearing on the cost since the running time of some computers is more expensive than others. This problem took 2.65 minutes to run on the IBM Model 7044.

Because of the keen competition in the mineral industry at the present time, the linear programming technique should be especially appealing to the smaller mining operations. With a minimum amount of expense, management is able to determine whether or not a particular operation will be able to compete economically within the industry. The small operation should also be encouraged by the fact that it is not necessary to have a computer technician on the staff. A mining engineer should be capable of writing a linear program with only a small amount of study being required. It is becoming increasingly important for the small mining company to produce ore which meets the necessary requirements while, at the same time, profit is being maximized. Linear programming is a procedure which can accomplish this efficiently and economically.

BIBLIOGRAPHY

Bowman, Edward H. and Robert B. Fetter, "Linear Programming," in Analysis for Production Management. Homewood, Illinois: Richard D. Irwin, 1961. 26pp.

Dotson, Jay C. and William C. Peters, ed. Short Course and Symposium on Computers and Computer Applications in Mining and Exploration. 3 vols. Tucson, Arizona: University of Arizona Press, 1965.

Madge, D.N. "Mathematical Programming A Tool for Production Scheduling and Product Distribution in the Mineral Industry," New York: Society of Mining Engineers of A.I.M.E., Preprint No. 66AR23. 1966. 10pp.

Weiss, A. "Computer Techniques Spur Operation's Research in Mining," Mining Engineering, XVIII (Feb. 1966), 126-28.