UNIVERSITY OF NEVADA, RENO

Essays in Economics: Analyzing Homeowners' Willingness-to-Pay for Wildfire Risk Mitigation, and the Overlapping Generations (OG) Model of Interest Rate Dynamics.

A Dissertation submitted in partial fulfillment of the
requirements for the degree Doctor of Philosophy in Economics

by

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Abstract

This dissertation examines the estimation of homeowners' willingness-to-pay for wildfire risk reduction and the determination of interest rates.

The first chapter employs a contingent valuation approach to estimate homeowners' willingness-to-pay (WTP) for wildfire risk reduction in 35 Wildland Urban Interface (WUI) communities in Nevada. Survey respondents were presented with two potential risk reduction programs: A private program focusing on individual home and surrounding vegetation modification, and a public program targeting community-wide risk through fuel management treatments. In this study, we employed a contingent valuation approach to estimate homeowners' willingness-to-pay (WTP) for wildfire risk reduction in 35 WUI communities in Nevada. We presented respondents with two potential risk reduction programs: a private program focusing on individual home and surrounding vegetation modification, and a public program targeting community-wide risk through fuel management treatments. We found significant WTP for private program risk reduction but not for public program risk reduction. Our work also identifies a number of factors that my bias WTP estimates if not considered. These include respondent's previous expenditure on risk mitigation, the possible loss level from fire, concern about the non-financial costs of risk mitigation, and recognition of the communal (i.e. positive externality) benefits of fire risk reduction.

The second chapter presents the Weil (1987) overlapping generations (OG) model, which contains an exogenous probability that the economy's bubble may burst but extended to allow capital accumulation as in Banerjee (2021). Like Weil (1987), we find that the rate of return on the bubble asset must generally be greater than the rate of return on the capital backed asset. Like Banerjee (2021), but in contrast to Weil (1987), a gap between the interest rate paid on the capital backed asset and capital rental rate must occur. Thus, we provide enhanced knowledge of how the rates of return earned on assets relate to the productivity of capital and the capital rental rate.

The third paper extends the OG model of Weil (1987) by adding capital accumulation and stock market clearing as introduced in Banerjee and Pingle (2023). The addition of the stock market clearing eliminates the indeterminacy and inefficiency from Weil's model, resulting in a unique, Pareto efficient equilibrium. Because the bubble may burst in our model, as in the Weil model, we like Weil find that the rate of return on bubbly assets must exceed that on capital backed asset to offset the risk of bubble bursts. In contrast to Weil, we find that the path for the bubble is not influenced by a change in the perception that the bubble will burst. The bubble will form if it can form, and it is the rate of return on the bubble asset that will adjust in response to an increased perception that the bubble will burst.

DEDICATION

" To all who have joined me on my path, offering guidance and support in fulfilling my dreams and achieving my destiny,

and

to the courageous individuals who have transformed their experiences with accidents into stories of hope and resilience."

“When it looks impossible, look deeper. And then fight like you can win.”

- *Rost, Horizon Forbidden West.*

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(with Michael H. Taylor)

Abstract

In this study, we employed a contingent valuation approach to estimate homeowners' willingness-to-pay (WTP) for wildfire risk reduction in 35 WUI communities in Nevada. We presented respondents with two potential risk reduction programs: a private program focusing on individual home and surrounding vegetation modification, and a public program targeting community-wide risk through fuel management treatments. We found significant WTP for private program risk reduction but not for public program risk reduction. Our work also identifies a number of factors that my bias WTP estimates if not considered. These include respondent's previous expenditure on risk mitigation, the possible loss level from fire, concern about the non-financial costs of risk mitigation, and recognition of the communal (i.e. positive externality) benefits of fire risk reduction.

1.1 Introduction

The National Interagency Fire Center (2023) states that wildfires pose a serious risk to human life, property, and natural resources in the United States and have grown to be a serious environmental, economic, and societal problem. Recent climate change has led to increases in temperatures that have caused the frequency and severity of fires to escalate. According to the latest data from the National Interagency Fire Center (2023), as of November 7, 2023, there have been 48,681 fires recorded in 2023, resulting in the burning of 2.54 million acres.

The Wildland-Urban Interface (WUI) communities are particularly vulnerable to wildfires, and they have substantially expanded in recent years, posing increased risks to both the inhabitants and their assets (Hammer, Stwart, & Radeloff, 2008). The Wildland-Urban Interface (WUI), which refers to the area where idle land and human activity meet, has experienced a significant increase in the construction of residential units in close proximity to wildlands. According to Radeloff et al. (2005), approximately 39% of housing units in the United States were located in the Wildland-Urban Interface (WUI) as of 2005. These communities are at risk because they are close to areas of the environment that are prone to wildfires, which makes proactive preventative and mitigation measures even more important.

In addition to the potential threats posed to individual safety and residential properties, wildfires generate substantial economic consequences. The economic consequences of burned or damaged structures, along with the expenses incurred for fire suppression, have a significant impact on both local communities and the overall state.

Rural economies can suffer extensively due to losses in crucial sectors such as ranching, which can amount to tens of millions of dollars annually (GAO, 2004; GAO, 2007).

In addition to the direct losses, measures to reduce the likelihood of fires, safeguard homes and lives, and combat active fires have seen an exponential increase in costs over recent decades (Abbott, Gallipoli, & Violante, 2019). Wildfires incur a significant economic cost. The costs associated with wildfires include various expenses, such as firefighting costs, economic losses in key sectors like ranching, and expenditures on damage prevention and mitigation strategies (Stephens & Ruth, 2005; Calkin & Geoffrey, 2008; Gebert, Calkin, & Jonathan, 2007; Melvin, et al., 2017).

A question that has been of interest is homeowners' willingness-to-pay (WTP) for wildfire risk reduction privately versus paying for risk reduction through a public program. Fried et al. (1999) conducted the first contingent valuation study in Crawford County, Michigan, revealing a higher mean WTP for private initiatives over public ones. Yet, there was still substantial support for public interventions. This preference of private over public programs aligns with findings of later studies by Loomis et al. (2001) and Sánchez et al. (2022), which explored residents' preferences for prescribed burns, mechanical fuel clearing, and herbicide treatments.

A new approach to eliciting WTP is found in choice experiments, which allow for the assessment of multiple attributes of risk mitigation programs. For instance, Sánchez et al. (2022) used a choice experiment to estimate homeowners' WTP for public and private fuel reduction programs in California's WUI areas. This method, which includes “latent class” models and multinomial logit, uncovers how demographic factors influence preferences and mitigation behaviors.

The demographic dimension of WTP has been explored by Loomis et al. (2002), González-Cabán et al (2007) , and González-Cabán and Sánchez (2017), revealing in community responses based on cultural and socioeconomic factors. For instance, Loomis et al. (2002) found uniform support across English and Spanish speaking groups in Florida, while González-Cabán et al (2007) and Loomis et al. (2008) identified distinct patterns of WTP among Native Americans, various ethnic groups in Florida, and Californians. These studies highlight that socioeconomic factors significantly influence homeowners' decisions on participating in fire mitigation programs, as lower-income minority homeowners often require assistance (González-Cabán & Sánchez., 2017).

The integration of Geographic Information System (GIS) data on wildfire hazard and defensible space has been shown to enhance the explanatory power of WTP models significantly, as Kaval and Loomis (2008) demonstrated by incorporating variables like slope and proximity to previous fires. Another layer of complexity is introduced by the moral hazard associated with insurance and risk aversion, with Talberth et al. (2006) finding that risk information plays a key role in decision-making even when insurance is available. The exploration of moral hazard by Talberth et al. (2006), which found that insurance availability does not diminish support for risk-averting activities, provide methodological insights for modern research in this field. Moreover, the role of risk information in shaping homeowners' decisions, as observed by Holmes et al. (2013) and Katuwal et al. (2015), is critical, with higher levels of information leading to more consistent WTP values.

The design of WTP questions is critical, as evidenced in our research, which examines the WTP for wildfire risk reduction in Nevada's WUI. [1] This research builds on previous studies, such as Holmes et al. (2013) and Sánchez et al. (2022) and employs a contingent valuation method to estimate the risk perceptions of homeowners over a ten-year horizon, considering various levels of risk and potential damage to property. In all prior contingent valuation (CV) studies, such as those by Fried et al. (1999), Loomis and González-Cabán (2010), Kaval and Loomis (2008), Walker et al. (2007), and Meldrum et al. (2014), the proposed wildfire risk mitigation projects consistently included activities like mechanical thinning and prescribed burning within the forests located where the survey respondents resided. Our research adds a critical perspective to the literature by analyzing WTP against actual risk levels and potential losses, filling the gap identified in earlier studies that did not offer a detailed spectrum of risk and loss. In contrast to Holmes et al. (2013) and Sánchez et al. (2022), which reported lower ten-year wildfire risks, our study encompasses a more comprehensive risk range, thereby providing a broader understanding of homeowners' investment logic in wildfire risk reduction.

The wildfire risk reduction policies can be valued using either an ex-ante measure or a post-post measure. Our research addresses the question, "Which is the better measurement, ex-ante or ex-post?" Our paper's main contribution lies in the assertion that

[1] Nevada faces frequent and intense wildfires, exacerbated by its dry climate and the proliferation of invasive plants such as cheatgrass, which fuels fires. These conditions often worsen during droughts, leading to more severe fires due to the abundance of dry vegetation. Managing wildfires in Nevada, with its vast stretches of public land, demands a collaborative effort from state and federal agencies, including the Bureau of Land Management (BLM) and the United States Forest Service (USFS), to ensure effective fire prevention and suppression strategies are in place (National Interagency Fire Center , 2018).

ex-ante measures are superior to ex-post measures for valuing wildfire risk reduction policies.

The ex-ante approach is crucial because it aligns with the preventive intent of wildfire policies. It evaluates the efficiency of these policies based on the perceived benefits to homeowners before any damage occurs, thereby offering a measure of value that is both anticipatory and reflective of the homeowners' preferences. This forward-looking perspective allows policymakers to discern which communities are likely to find more economic value in these investments, guiding resource allocation to where the benefits of risk reduction most outweigh the costs. Furthermore, ex-ante valuation underscores the importance of homeowner perceptions and their valuation of safety, which is essential for developing policies that are not only efficient but also align with community values and well-being. By contrast, ex-post estimates, which calculate damages after wildfire events based on simulation studies, provide a reactive measure that may not capture the full spectrum of values and preferences of the homeowners. Therefore, for a comprehensive and proactive policy valuation that accurately reflects homeowner priorities, ex-ante payment measures are the more appropriate choice.

Our work addresses several challenges associated with valuing wildfire risk reduction. One of the primary challenges we address is the potential bias in contingent valuation, where homeowners are asked to value to a good, they may have previously purchased. We explore whether past investments in fire-safe measures affect their willingness to pay for further risk reduction. Moreover, the research addresses with the issue of how to frame the good in the CV questionnaire to yield unbiased responses, considering the good can be delivered in various forms, such as public or private initiatives.

This is crucial since private goods in our study account for hypothetical loss amounts, which could influence the valuation.

Another significant difficulty is accounting for the external benefits (e.g., externalities) of risk reduction programs, which offer spillover advantages to the community at large. This factor is investigated to understand its impact on individual valuations and the necessity to incorporate such benefits in the description of the good.

Lastly, we address the issue of non-monetary costs associated with risk reduction, such as altering vegetation on a property or in a community, which may not be directly quantifiable but can influence homeowners' willingness to pay. Our study's approach is designed to navigate these challenges and provide a robust estimation of homeowners' ex-ante willingness to pay for wildfire risk reduction.

Utilizing a contingent valuation method, we surveyed homeowners on their willingness to support two types of risk reduction measures: a private program that entails modifying their homes and managing surrounding vegetation, and a public program that involves broader community efforts like fuel management. We employ stated preference methods to estimate homeowners' willingness-to-pay for risk reduction because the level of risk reduction associated with a specific fuel treatment project is now known with certainty, either by the agency tasked with project implementation or the property owners who benefit from the risk reduction. By employing an ex-ante approach, we not only estimate homeowners' marginal WTP for risk reduction but also deal with the challenges and potential biases inherent in valuing such risk reduction in WUI. This allows us to understand the economic value that homeowners place on decreasing wildfire risks and to navigate the difficulties of accurately estimating this value.

What do we learn?

Our research suggests that the level of risk reduction significantly influences the WTP for private programs, but surprisingly not for public ones. The average willingness to pay (WTP) for a private program is estimated to be $1,493, whereas the average WTP for a public program is approximately $770. Average WTP for a private program to reduce wildfire is $910 for 1% risk reduction, $1,347 for 2% risk reduction, and $2,217 for 4% risk reduction. We find individuals who spent money to reduce fire risk on their own property were less willing to contribute to public programs and were not willing to pay more for further risk reduction on their own property.

Our findings confirm the need to formulate questions carefully to capture WTP for risk reduction to not introduce bias. First, we find the potential financial loss amount influences the WTP for private risk reduction. Second, we find a higher WTP for both the public and private programs when the expenditure helps reduce the fire risk of others. Third, we find the non-financial costs of homeowners' risk mitigating investments, such as the loss in aesthetic beauty and privacy from changing landscaping and removing large trees, decreases the WTP for wildfire risk reduction. These results present a challenge to using contingent valuation to value a good like wildfire risk reduction. To avoid potential biases, future studies should explicitly include the financial loss amount, present external benefits, and include non-financial costs when defining the hypothetical good.

The structure of this paper is organized methodically. Section 2 introduces our conceptual framework, which incorporates the random utility model devised by Hanemann (1984), serving as the basis for calculating willingness-to-pay (WTP) within a utility-focused framework. We also include hypothesis of this research. Following this, Section 3 outlines the survey methodology, provides details on the study area and the communities involved, and elaborates on the survey execution. Data exploration, including program design variables, is the focus of Section 4, where we delve into the preliminary data analysis. Section 5 advances to the econometric aspect of our study, presenting a modified probit model and detailing the process of maximum likelihood estimation via a double-bounded dichotomous choice approach. The ensuing Section 6 presents the regression findings and discussion. The paper concludes with Section 7, summarizing our findings and their implications.

1.2 The Conceptual Model

Hanemann (1984) model introduced the Random Utility Maximization (RUM) framework, which provides the theoretical foundation for our estimation of the WTP. This model begins with a specification for the utility function for each choice (Phanuef, 2017). We use this model to measure the survey respondent maker's utility related to their answers to a series of Multiple Bounded Dichotomous Choice (MBDC) questions. The utility derived by a participant i, from the counterfactual scenario (1) and the status quo scenario (0) can be denoted as follows:

$$V_{i1}(q^1,\, M_i - B_i, s_i, u_{i1}) = v_{i1}(q^1,\, M_i - B_i, s_i) + u_{i1}$$
$$V_{i0}(q^0,\, M_i, s_i,\, u_{i0}) = v_{i0}(q^0,\, M_i, s_i) + u_{i0} \quad (1)$$

In this context, B_i represents the bid amount presented to participant i, and $v(.)$ is a parametric specification of the observable utility component, which includes income (M). The status quo level of quality is signified by q^0 , and the improved quality level is q^1 . The vector s represents household characteristics, while $u_i(i = 0,1)$ stands for the unobserved random component. The random variables u_{i1} and u_{i0} are assumed to be two independent and identically distributed with zero means each.

When offered an amount of Bid, $\$B_i$, for a given fire risk reduction program, individual will respond "YES" to the bid B_i if and only if $V_{i1} \geq V_{i0}$. This yes answer can be restated as a utility difference $u_i \leq v_{i1}(q^1, M_i - B_i, s_i) - v_{i0}(q^0, M_i, s_i)$, and refuse otherwise. The individual knows for sure which choice maximizes his utility; but, for the econometric investigator, the individual's response is a random variable whose probability distribution is given by $\Pr(Y_i) = Pr[u_i \leq v_{i1}(.) - v_{i0}(.)]$ and $\Pr\ (N_i) = 1 - \Pr(Y_i)$.

The parameters included in $v_{ij}(\,.\,)$[2] are estimated via maximum likelihood estimation. In the section on econometric estimation, we elaborate on the construction of the Maximum Likelihood Estimation (MLE) utilizing the responses to double-bounded dichotomous choice questions. To proceed with estimation, we choose functional forms for $v_{ij}\,(.)$. The simplest approach is to assume that all variables enter linearly:

$$v_{i1} = \alpha + \beta(M_i - B_i) + \gamma q^1 + \delta q^1 \times s_i$$
$$v_{i0} = \beta(M_i) + \gamma q^0 + \delta q^0 \times s_i \qquad (2)$$

Where s_i is a scalar variable measuring a homeowner's characteristic and $(\alpha.\ \beta, \gamma\ \&\ \delta)$ are parameters to be estimated. The value of s_i and the bid amount B_i vary over individuals, while the value of q changes between the two alternatives. Consequently, a

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[2] The utility difference is symbolized as $\Delta v = v_{i1}(q^1, M_i - B_i, s_i) - v_{i0}(q^0, M_i, s_i)$, and the willingness to pay probability as $\Pr(Y_i) = Fu_i(\Delta v)$, where $Fu_i(.)$ denotes the cumulative distribution function (c.d.f.) of u_i. The random variable u_i requires a specific distribution function, often assumed to be Weibull-distributed (Boyle, Welsh, & Bishop, 1988). It is noteworthy that the difference between two Weibull random variables corresponds to a logistic cumulative distribution function. To estimate willingness to pay (WTP), it's useful to consider the cumulative distribution function of the random WTP variable itself, $\phi_{WTP}(B_i)$. This function represents the probability that $WTP \leq B_i$. Hence, $1 - \phi_{WTP}(B_i)$ provides the probability that $(B_i < WTP)$, implying that the respondent will reject the suggested price (B_i) (Kriström, 1990)

person's income doesn't influence his choices or his value for the environmental good, and the marginal utility of money remains constant for all individuals. WTP, in the case of improvement in q, is the reduction in income that leaves the person indifferent between the baseline and improved level of q. This WTP can be defined by

$$WTP_i = \frac{\alpha + (\gamma + \delta s_i)\Delta q}{\beta} - \frac{u_i}{\beta} \qquad (3)^3$$

Estimating the willingness to pay (WTP) for wildfire risk reduction through ex-ante measures presents several difficulties. The following hypotheses identify difficulties and potential biases that come with assessing risk reduction values in Wildland Urban Interface (WUI) regions in Nevada.

Hypothesis 1: *Ex-ante payment for reductions in wildfire risk are the theoretically correct measure for valuing public or private investment to reduce risk compared to the alternative of ex-post estimates of wildfire damage from simulation-based studies.*

We use Contingent Valuation to obtain ex-ante WTP. The reason behind employing state preference methods to estimate homeowners' willingness-to-pay for risk reduction is that the level of risk reduction associated with a specific fuel treatment project is now known with certainty, either by the agency tasked with project implementation or the property owners who benefit from the risk reduction.

Based on the review of literature, a popular approach to eliciting WTP is found in choice experiments, which allow for the assessment of multiple attributes of risk mitigation

[3] The term Δq represents $q^1 - q^0$ and as mentioned earlier, u_i denotes $u_{i1} - u_{i0}$. From the perspective of the analyst, the WTP_i is a random variable, indicating the need for a measure of central tendency to obtain a point estimate of WTP_i. Given the symmetry of its distribution, the median compensating for person i equals $E(WTP_i)$. These discussions and formulations provide a theoretical base to understand the DBDC framework's use in understanding homeowners' preferences in wildfire management strategies.

programs. For instance, Sánchez et al. (2022) used a choice experiment to estimate homeowners' WTP for public and private fuel reduction programs in California's WUI areas. This method, which includes Latent Class models and multinomial logit, uncovers how demographic factors influence preferences and mitigation behaviors.

Hypothesis 2: *Previous Risk Mitigation Expenditures May Influence the Responses to the Hypothetical Contingent Valuation Questions and In Doing So Bias Our Estimates Of WTP.*

Our seeks respondents to reveal their WTP for a good – wildfire risk reduction – that, in many cases, they have already paid for. This previous expenditure may influence the responses to the hypothetical contingent valuation questions. This introduces the potential for bias, but it may not bias the results. Our study design allows us to test whether prior fire-safe investment influences the WTP. We have data on whether the homeowner has previously spent money on fire-safety. Respondents were also questioned on whether they have defensible space around their property.

Hypothesis 3: *The Inclusion of a Hypothetical Loss Amount Could Bias the WTP*

When considering private expenditure for fire risk reduction, it is reasonable to think the potential financial loss a homeowner could influence the WTP. If it does not, then it is more reasonable to apply private WTP for fire risk reduction to broader contexts. That is, while reducing fire risk is through public programs is more general and should not be directly compared to private risk reduction, the two are more similar if a larger potential loss to the homeowner does not impact the homeowner’s private WTP. That said, our study primarily aims to identify factors influencing the WTP, rather than conclude whether the WTP is higher for private risk reduction than public risk reduction.

Hypothesis 4: *External Benefits Have the Spillover Effect on Willingness to Pay (WTP) and Could Lead Potential Bias***.**

We asked homeowners if they valued a program's ability to reduce wildfire risk for the entire community, not just their individual property. If the answer is yes, then the respondent acknowledges there is external spillover benefit to their payment for fire risk reduction. In this case, the estimated WTP often reflects not only the direct benefits they receive but also the indirect benefits that accrue to their neighbors and community at large. Leaving out this indirect impact would underestimate the WTP.

Hypothesis 5: *Omission Of Non-Monetary Costs Could Lead to Potential Bias in the WTP*

Our survey instrument included a "cheap talk" script reminding respondents to consider the opportunity cost of money when answering the contingent valuation questions. The cheap talk script, however, did not discuss the non-financial costs of fire risk mitigating expenditures, such as the loss in aesthetic beauty and privacy from changing landscaping and removing large trees. That is, investment in wildfire risk reduction has both financial and non-financial costs. Non-financial costs are largely related to altering vegetation on a property or in a community.

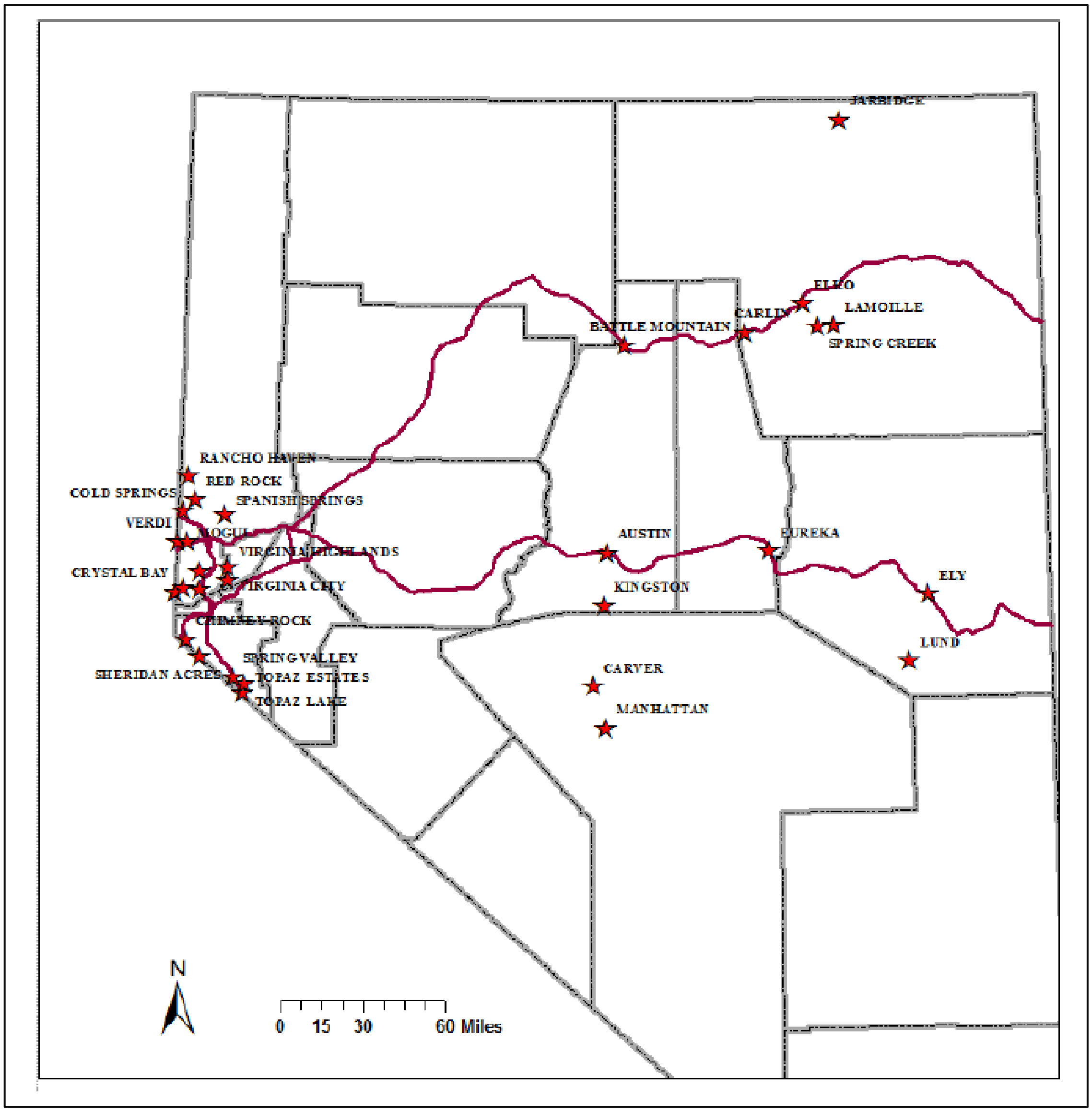
Our research includes a variable constructed to reflect the non-monetary costs of fire-safe investments. If these costs are not explicitly acknowledged and incorporated into the contingent valuation framework, there is a risk that homeowners' subjective perceptions of these costs may introduce bias into their WTP responses.

1.3 Study Area and the Survey Description

As identified in Figure 1, the study area included 35 wildland-urban interface (WUI) communities[4] in Nevada, chosen based on their varied wildfire risk, fuel types, and fuel accumulation, providing a representative cross-section of fire-prone communities across the state. In 2011, a collaborative research endeavor was undertaken by scholars from the University of Nevada, Reno, supported by a grant from the Nevada Agricultural Experiment Station (Rollins & Evans, 2006). As part of this project, a survey was conducted on private residential properties to evaluate the extent of defensible space surrounding residential structures in these communities. The surveyed WUI communities are proximate to public wildlands and are representative of four key terrain types prevalent in the arid West, each with distinct wildfire susceptibility. These include grasslands, pinion pine and juniper woodlands, higher elevation dense pine forests, and sagebrush rangelands.

[4] The communities in the study were selected based on the 2004 research commissioned by the Nevada Fire Safe Council, which highlighted heightened wildfire hazard and ignition risk in these areas (Resource Concepts Incorporated., 2005) Researchers involved in the 2011 study were trained by fuels assessment specialists from the 2004 study in risk assessment methodology and determining compliance with defensible space guidelines.

Figure 1 Survey Location, 35 Wildland Urban Interface Communities in Nevada



The study utilized three main types of information. The first comprised hazard assessments carried out on 8,867 residences across the 35 WUI communities in the summer of 2011. These assessments determined the adequacy of defensible space around each property. The second information type was derived from a mail survey dispatched in autumn 2012 to a representative subset (2,225 houses) of the previously assessed properties. This survey aimed to gather insights into homeowner's perceptions about defensible space, wildfire risk, and their willingness to invest in risk mitigation. The survey employed a stratified sampling method with oversampling from smaller communities to ensure representativeness.

Of the 2225 households surveyed, 737 responded. Unfortunately, some respondents did not complete the whole survey. Of the 737 respondents, 513 completed the private survey, 544 completed the public survey. Because we have some interest in comparing preferences for WTP for reducing wildfire risk privately versus publicly, we used the data from those homeowners who completed both the private and public portions of the survey.[5] Our data cleaning process also identified a few observations clearly did not understand the survey. Dropping observations of these two purposes left us with 471 observations, implying a response rate of 22.2%.

1.4 Data Description

To obtain the survey data, two hypothetical scenarios were presented to respondents in WUI in Nevada communities. One was presented as "*private investment*"[6] program and the other as a "*community program.*"[7] For the private program, respondents were told spending on this program will reduce the chance that fire will reach your home from a

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[5] We run the regressions presented below for private and public cases with the 513 and 544 observations, respectively. We found no significant difference in the results.

[6] Please see the appendix program design section for the detail's description of the private program (private good). Private investment alters the vegetation surrounding the home to provide and maintain defensible space. It also includes educating homeowners.

[7] Please see the appendix program design section for the detail's description of the community program (public good). Public Program includes wildfire suppression costs, pre-fire mitigation strategies, and fuel reduction treatments on public land.

certain amount (which varied across the homeowner respondents) over the years (5- or 10-years program), by creating and maintaining defensible space on your property and modifying your home. For the community program, respondents were told spending on this program reduces the chance that fire will reach your home by a certain amount (which varied across homeowner respondents) every year, by enlisting experts to develop a plan to create and maintain defensible space on lands in and surrounding your community. They were also told this community program would reduce the annual probability to the *entire community* over the program period.

Table 1 provides the description of the bid variables used to estimate the WTP for the wildfire risk reduction. Definitions for several incorporated variables are also provided for reference. The values for bid1through bid4 variables were obtained from the CV questions outlined in the "CV Questions" section A1 of the *Appendix*.

Table 1 Description of the Dependent Variables (Bid and Response Variables)

	Variable	Description
Private Program	bid1	Lowest bid amount for which the respondent said yes
	bid2	Highest bid amount for which the respondent said no
Public Program	bid3	Lowest bid amount for which the respondent said yes
	bid4	Highest bid amount for which the respondent said no

Our questionnaire incorporated Contingent Valuation (CV) questions with diverse bid sets. For the assessment concerning the reduction of private property risk, respondents were presented with six different bid amounts, randomly assigned. These bid values extended from as low as $25 to an upper limit of $16,000. A salient feature of this approach was to ascertain whether respondents were willing to take preventive action even in the absence of a monetary investment, leading to the inclusion of a $0 bid value. In parallel, our "*community program*" risk reduction questionnaires featured a similar design. Here, respondents were presented with a bid set that incorporated a $0 bid value, along with five additional amounts randomly assigned. The range of these amounts spanned from $5 to $650. Our study also included potential hypothetical loss for "private program" but for the "community program" to reduce the fire risk. Table 2 provides the descriptive statistics for the bid and response variables, which represent data on the WTP for private and community programs.

A polychotomous, discrete choice format was employed in the questionnaire design, enabling participants to express their degree of agreement or disagreement to each bid regarding wildfire risk. Respondents could choose among five potential responses: "definitely yes" (DY), "probably yes" (PY), "maybe" (M), "probably no" (PN), or "definitely no" (DN). For the sake of analytical simplicity, we recoded these responses into a binary format: "yes" (Y) and "no" (N). Responses falling into "definitely yes" (DY) and "probably yes" (PY) categories were coded as YES, whereas responses of "maybe," "probably no," and "definitely no" (DN) were coded as NO. Survey variables exhibiting.

missing values necessitated the application of the Multiple Imputation Chained Equation (MICE)[8] technique for imputation (see *Appendix A1* for details) (White , Royston , & Wood

[8] To impute multiple variables using univariate imputation chained equations, an iterative method known as Multiple Imputation Chained Equations (MICE) has been developed. Multiple Imputation relies on the hypothesis that the gaps in data can be accounted for by other, external evidence. There are three distinct missing value notations: MAR, MCAR, and MNAR. MICE simulate by simultaneously executing several unrelated chains. A loop that continues until the chain has settled into a steady state. Imputing missing values when variables are MCAR can introduce noise to the model and lead to incorrect estimates because MICE presume that the data is MAR (Azur, Stuart, Frangakis, & Leaf, 2011). According to the majority of studies (Van Oudshoorn & Oudshoorn, 1999; Rubin, 1987) , M=5 should be enough to draw a reliable conclusion.

, 2011). Moreover, we utilized "factor analysis"[9] to minimize the dimensionality of the extensive range of behavioral questions posed to respondents (Brown , 2014). Table 2 presents the descriptive statistics for the dependent variables.

Table 2 Descriptive Statistics of the Dependent Variables (Bid and Response Variables)

	Variable	Mean	Standard Deviation	Minimum	Maximum
Private Program	bid1	1130.20	1678.27	25	12000
	bid2	1917.09	2411.19	0	16000
Public Program	bid3	53.26	58.37	0	200
	bid4	193.05	229.29	0	650

Table 3 presents a comprehensive list of independent variables in *Table 3*, with each subsection comprising a distinct set of related variables.

[9] In statistics, factor analysis is used to explain the correlation and variability of a set of variables by reducing the number of variables to a smaller set of unobserved variables. Given that factor analysis models the observed variables as linear combinations of the prospective factors plus "error" terms, it can be viewed as a specific example of errors-in-variables models (Wikipedia, n.d.). To make research data more manageable, factor analysis reduces a large number of potential variables to a smaller set of dominant factors. The theory is that deeper factors drive the data's underlying concepts, and one can uncover and work with these instead of the lower-level variables that cascade from them. Factor analysis is also called "dimension reduction." One can reduce data "dimensions" into one or more "super-variables," also known as unobserved variables or latent variables.

Table 3 Descriptions of the independent variables

	Variable	Description
Design variables	*BASELINERISK*	2%, 3%, 6%, 8% and 10% probability of fire will reach the house
	RISKREDUCTION	1%, 2%, 4% reduction in probability of fire
	LOSS	Loss if fire reach the house (private program only)
	YEARSPAN	Years over which change occurs
Attitudes toward wildfire risk reduction program	*LANDMANAGEMENT*	Index score using factor analysis (Land Management)
	COMMWFPLANNING	Index score using factor analysis (Community Wildfire Planning)
	COMMATTACHMENT	Index score using factor analysis (Community Attachment)
	UTILITYAP	Index score using factor analysis (Utility Hit and Aesthetic Privacy)
	GOVTPOLICYFSI	Index score using factor analysis (Govt. Policy Fire Safe Investment)
	HOUSEHOLDFSI	Index score using factor analysis (Household Fire Safe Investment)
	ROGRAMVALUE	Index score using factor analysis (Program Value for Private Program)
	NOAFFORD	=1 if the respondent cannot afford the cost; 0 otherwise
	ALTRUISM	=1 if the respondent like that the program reduces the fire risk to the entire community, 0 otherwise
	OWNPROFERTYDFSFSI	=1 if the respondent prefers to spend money on DFS for their own property, 0 otherwise
Household Risk Preference	*EFFICACY*	Trust in Defensible space
	RTOL	Lifetime risk tolerance
	FIRERISK	Index of subjective chance of fire reaching in the community
Respondent characteristics	*INCOME*	Respondent household income in thousand
	AGE	Respondents age
	AGEHOME	Respondent house age
	INSURANCE_PRI	1= if respondent feels insurance cover all losses from wildfire
	HHVL	Log of house value in dollar
	HHV	Home values in dollar
Biophysical Variables	*WIND*	Avg. Max. Daily Wind Speed (MPH)
	ASPECT	=1 if Property is South Facing
	SLOPE	Slope of Property (%)
	ELEVDIFF	Difference between the Elevation of a Residence and the Avg. Elevation in Community
	LGHTN	Number of Lightning Strikes within 10 Miles

The "*design variables*" category includes variables intentionally altered in the survey to provide insights regarding how various factors impact the respondents' willingness to invest in fire-safe measures.

Each of the version of the questionnaire contained a different baseline risk and a different amount of risk reduction. The design included 5 different baseline risk levels and 4 different levels of risk reduction. Table 4 delineates the distribution of respondents across the various baseline risk and risk reduction scenarios. As shown in the table, the 10% baseline risk was used twice. On question questionnaire, the respondents considered a 1% in reduction of risk from the 10% baseline. On another questionnaire, the respondents considered a 4% reduction from a 10% baseline. The distribution of respondents across the different baseline risk and risk reduction combination explored was roughly uniform, meaning each scenario received roughly equal consideration.

Table 4 Risk Reduction in Different Baseline Risk

	Risk Reduction			
Baseline Risk	1%	2%	4%	Total
2%	89	0	0	89 (18.9)
3%	0	76	0	76 (16.1)
6%	0	0	81	81 (17.1)
8%	0	62	0	62 (13.1)
10%	80	0	83	163 (34.6)
Total	169 (35.88)	138 (29.30)	164 (34.82)	471 (100)

*Parenthesis indicates the percentage of respondent

We created dummy variables[10] (e.g., p106, p62, p86, p31, p109, and p21) to represent each scenario. These dummy variables were used to create the variable *RISKREDUCTION,* which equals the number of percentage points of risk reduction. For example, p106 indicates *RISKREDUCTION*=4 and p86 indicates *RISKREDUCTION*=2.

The variable LOSS indicates the potential loss to the respondent resulting from a wildfire. On the private program questionnaire, respondents faced one of three potential

[10] Please see the dummy variable discussion section in the appendix.

loss levels: $50,000, $100,000, or $200,000. The public program questionnaire did not include a potential loss question. The variation in the potential loss allowed us to evaluate the willingness of the respondents to support risk reduction measures across various cost scenarios.

The variable YEARSPAN indicated the mitigation period was either 5 years, or 10 years, allowing us to determine whether the length of the mitigation period influences the WTP for fire risk reduction.

The second category of independent variables are included to assess respondents' attitudes toward wildfire risk reduction. In some cases, a variety of questions tended to capture a similar factor, so we employed factor analysis[11] to synthesize some independent variables. This statistical technique identifies underlying relationships among a number of variables, so a larger number of variables are reduced to a set of *index variables* [12], each variable with a separate factor score. This process captures the maximum variance in data through minimal variables. The index variables we obtained from our factor analysis are *LANDMANAGEMENT* (land management), *COMMATTACHMENT*[13] (community

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[11] Please see the appendix for the detail's description

[12] Using Factor Analysis, we may generate index variables. In order to acquire a manageable subset of variables (ones that aren't highly correlated with one another) out of a big (and often highly correlated) data collection, and to form indexes with variables that measure similar things, factor analysis is commonly employed (conceptually). Both exploratory and confirmatory factor analysis have been employed. Since there is no prior knowledge of the structure or number of dimensions in our data collection, we have resorted to exploratory factor analysis. We detected 9 factor dimensionalities based on factor loading. Weights and correlations between each variable and the factor are known as "factor loadings." The greater the load, the more significant it is in determining the dimensionality of the component. In the end, we employed confirmatory factor analysis to back up our initial dimensionality assessment from exploratory factor analysis. Hypotheses concerning the underlying structure or number of dimensions can be tested through confirmatory factor analysis. The appendix A1 contains further information regarding the confirmatory and exploratory factor analyses.

[13] Details discussion incorporated in the Appendix, section, respondent behavior towards wildfire risk reduction.

attachment), *COMMWFPLANNING* (community wildfire planning), *GOVTPLOICYFSI* (government policy fire-safe investment), *HOUSEHOLDFSI* (household fire-safe investment), and *UTILITYAP* (utility access premium).

The variable *LANDMANAGEMENT* measures the trust the respondent has in governmental agencies to manage vegetation on public lands. One might expect this to impact the WTP wildfire reduction, especially if it is a public program.

The variable *COMMWFPLANNING*[14] measures the extent to which the respondent has cooperated or coordinate with others on preparing for the threat of a wildfire. Respondents who are proactive in wildfire precautions are more likely to invest in our proposed goods, potentially favoring private solutions over public ones.

The variable *COMMATTACHMENT* encompasses six factors linked to home, neighborhood, and community attachment. Such a connection may drive people to invest in fire safety efforts.

The variable *UTILITYAP* captures the extent to which fire-safe investment decisions may cause decreases in homeowner because of aesthetic concerns or privacy concerns. Such concern may decrease the WTP for wildfire risk reduction.

The variable *GOVTPOLICYFSI* measures the extent to which the respondent believes government's preventative measures against fires are effective. If respondents believe that the government's fire-safe investment policies can reduce wildfire risk, they are more WTP for public fire protection measures but might be less likely to invest privately.

[14] Consisting of three categories of responses: (1) discussions among neighbors regarding coordinated fire risk reduction, (2) neighbors coordinating activities to mitigate fire danger, and (3) future planning for coordinated defensible space. More details provided in the Appendix.

The variable *HOUSEFSI* measures the extent to which the homeowner has maide fire-safe investments. This variable is based on "six self-reported" [15]variables such as fire-resistant roofs and sidings, enclosed or deleted eaves, chimney spark arresters, mesh-covered vents, and under-deck skirting. An affirmative response is more likely from those desiring to protect their homes from wildfires, reflecting their risk aversion.

The variable NOAFFORD is a dummy variable that equals 1 if the respondent does not believe they can afford to pay any cost of fire-risk reduction.

The variable ALTRUISM is a dummy variable that equals 1 if the respondent likes that spending funds for fire risk reduction benefits the broader community. That is, this variable indicates whether there is an externality associated with an individual payment for fire risk reduction.

The variable OWNPROPERTYDFSFSI is a dummy variable that measures whether the respondent has already spent money on providing defensible space around their home.

The third, fourth, and fifth categories of variables are less directly related to fire, risk reduction, but these variables might nonetheless affect the WTP for fire risk reduction. The third category of independent variables, "household risk preferences"[16], aims to elucidate how risk tolerance levels influence the propensity to make fire-safe investments. "Respondent characteristics"[17], the fourth category, consists of variables such as income, age, and house value, which are often associated with potential financial loss in the event

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[15] See the appendix for the detail's description.

[16] See appendix for the detail's description about household risk preference variables where we provided a descriptive analysis.

[17] See *Appendix A1* for the detail's description about the respondent characteristics variables where we provided a descriptive analysis.

of a wildfire. The final category, "biophysical variables[18]", integrates topographical and climatic elements, such as southern exposure, slope, elevation, wind speed, and lightning frequency, to map wildfire susceptibility within the geographical context.

Table 5 displays descriptive statistics for the independent variables, encompassing five distinct categories of dependent variables. These categories include design variables, attitudes towards wildfire risk reduction programs, household risk preferences, respondent characteristics, and biophysical variables.

[18] See *Appendix A1* for the detail's description about biophysical variables where we provided a descriptive analysis.

Table 5 Descriptive Statistics of the Independent Variables.

	Variable	Mean	Standard Deviation	Minimum	Maximum
Design variables	*BASELINERISK*	6.41	3.22	2.00	10.00
	RISKREDUCTION	2.34	1.28	1.00	4.00
	LOSS	115.29	60.49	50.00	200.00
	YEARSPAN	7.28	2.49	5.00	10.00
Attitudes Towards wildfire risk reduction program	*LANDMANAGEMENT*	-0.00	0.96	-1.88	2.77
	COMMWFPLANNING	0.00	0.95	-2.18	2.32
	COMMATTACHMENT	0.00	0.91	-1.72	3.21
	UTILITYAP	-0.00	0.82	-2.11	2.28
	GOVTPOLICYFSI	0.00	0.87	-2.25	1.70
	HOUSEHOLDFSI	-0.00	0.71	-2.46	2.18
	ROGRAMVALUE_PRI	0.00	0.75	-1.76	2.61
	PROGRAMVALUE_PUB	-0.06	0.97	-0.65	2.80
	NOAFFORD_PRI	0.35	0.48	0.00	1.00
	NOAFFORD_PUB	0.32	0.47	0.00	1.00
	ALTRUISM_PRI	0.55	0.50	0.00	1.00
	ALTRUSIM_PUB	0.73	0.45	0.00	1.00
	OWNPROFERTYDFSFSI	.44	.49	0.00	1.00
Household Risk Preference	*EFFICACY*	0.48	0.26	0.00	1.00
	RTOL	0.22	0.03	0.20	0.30
	FIRERISK	0.34	0.25	0.00	1.00
Respondent characteristics	*INCOME*	105.42	70.72	10.00	250.00
	AGE	60.83	10.72	25.00	91.00
	AGEHOME	22.01	9.37	1.00	38.67
	INSURANCE_PRI	0.30	0.46	0.00	1.00
	INSURANCE_PUB	0.24	0.43	0.00	1.00
	HHVL	11.21	1.33	6.23	16.1181
	HHV	158523	477277	525.99	99999.99
Biophysical Variables	*WIND*	31.2017	7.470486	14	46
	ASPECT	.1932059	.3952329	0	1
	SLOPE	6.544246	6.03407	0	31.74
	ELEVDIFF	17.09224	137.2151	-438.1709	697.2383
	LGHTN	969.5223	485.2757	317	2468
N=471					

1.5 Econometric Model and WTP Estimation

Our regression analysis uses a methodological approach known as the double-bounded or interval data model. It allows us to estimate the WTP under the premise of a single valuation[19] function. We employed the doubleb[20] command, a utility by Lopez-Feldman, Alejandro (2012), to conduct direct estimations of β and σ through maximum likelihood, facilitating the calculation of WTP as simply $\tilde{z}'\hat{\beta}$. We estimate the WTP assuming that it can be modelled as the following linear functions,

$$WTP_i = z_i'\beta + u_i \text{ , } where\ \mu_i = z_i'\beta \tag{4}$$

In our investigation of individuals' willingness to pay (WTP), denoted by WTP_i for each i^{th} respondent, we encounter a challenge due to the inherent unobservable nature of WTP. This willingness is presumed to be a function of various explanatory variables, represented by the vector z', and a set of parameters β, encapsulated in the vector (Carson R. , 2000; Carson, et al., 1992; Carson, et al., 2003; Carson, Flores , & Meade , 2001) . The approach used to measure this WTP is contingent valuation (CV), a format that offers a set comprising two bids to each respondent i: an initial or lower bid B_i^I and a follow-up bid B_i^{Fl}.

We classify respondents' reactions into seven distinct WTP intervals, each representing a unique outcome and corresponding range of WTP. This categorization is facilitated by binary-valued indicator variables $d_i^6, d_i^5, d_i^4, d_i^3, d_i^2, d_i^1\ and\ d_i^0$, respectively. The likelihood of these outcomes is symbolized by $\pi_i^6, \pi_i^5, \pi_i^4, \pi_i^3, \pi_i^2, \pi_i^1$ and π_i^0 then faced with a question about paying a predetermined amount B_i the binary response ($Y_i = 0$ for 'no', $Y_i = 1$ for 'yes') aids us in estimating the WTP (Carson, Flores , & Meade , 2001). We define y_i^1 and y_i^2 as dichotomous variables encapsulating the responses to the first and

[19] Cameron & Quiggin (1994), Haab & McConnell (2003) discuss some situation in which the assumption made here might be problematic and suggest alternative estimation methods.
[20] STATA command doubeb created by Lopez-Feldman, Alejandro (2012) to estimate WTP.

second closed questions. The probability of an individual answering "yes" to the first question and "no" to the second, given z_i is expressed as: $pr(y_i^1 = 1, y_i^2 = 0 | z_i) = \pi_i^k(B_i^I, B_i^{Fl})$, [21]with $l = 1\ to\ 5$ and $k = 0\ to\ 6$. Given the assumption that $WTP_i(z_i, u_i) = z_i'\beta + u_i$ and u_i adheres to a normal distribution $N(0, \sigma^2)$, we can assert that the probability of each of the seven outcomes is denoted by the likelihoods $\pi_i^6, \pi_i^5, \pi_i^4, \pi_i^3, \pi_i^2, \pi_i^1$ and π_i^0, (Hanemann, 1989).

$$\pi_i(.) = \begin{cases} \pi_i^{6=Y,Y}(B_i^I, B_i^{F5}) = \phi_{WTP}\left(z_i'\frac{\beta}{\sigma} - \frac{B_i^{F5}}{\sigma}\right) \\ \pi^{1=Y,N}(B_i^I, B_i^{F1}) = \phi_{WTP}\left(z_i'\frac{\beta}{\sigma} - \frac{B_i^I}{\sigma}\right) - \phi_{WTP}\left(z_i'\frac{\beta}{\sigma} - \frac{B_i^{F1}}{\sigma}\right) \\ \pi_i^{2=Y,N}(B_i^{F1}, B_i^{F2}) = \phi_{WTP}\left(z_i'\frac{\beta}{\sigma} - \frac{B_i^{F1}}{\sigma}\right) - \phi_{WTP}\left(z_i'\frac{\beta}{\sigma} - \frac{B_i^{F2}}{\sigma}\right) \\ \pi_i^{3=Y,N}(B_i^{F2}, B_i^{F3}) = \phi_{WTP}\left(z_i'\frac{\beta}{\sigma} - \frac{B_i^{F2}}{\sigma}\right) - \phi_{WTP}\left(z_i'\frac{\beta}{\sigma} - \frac{B_i^{F3}}{\sigma}\right) \\ \pi_i^{4=Y,N}(B_i^{F3}, B_i^{F4}) = \phi_{WTP}\left(z_i'\frac{\beta}{\sigma} - \frac{B_i^{F3}}{\sigma}\right) - \phi_{WTP}\left(z_i'\frac{\beta}{\sigma} - \frac{B_i^{F4}}{\sigma}\right) \\ \pi_i^{5=Y,N}(B_i^{F4}, B_i^{F5}) = \phi_{WTP}\left(z_i'\frac{\beta}{\sigma} - \frac{B_i^{F4}}{\sigma}\right) - \phi_{WTP}\left(z_i'\frac{\beta}{\sigma} - \frac{B_i^{F5}}{\sigma}\right) \\ \pi_i^{0=N}(B_i^I) = 1 - \phi_{WTP}\left(z_i'\frac{\beta}{\sigma} - \frac{B_i^I}{\sigma}\right) \end{cases} \quad (5)$$

WTP is recognized as a random variable with a cumulative distribution function (cdf) define as $\phi_{WTP}(.)$. One way to proceed with the estimations is to construct a likelihood function to directly obtain estimates for β and σ using maximum likelihood estimation. We used following likelihood function estimate β and σ, where,

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[21] $if\ B_i^I = 1\ and\ B_i^{F1} = 0\ then\ pr\left(B_i^I, B_i^{F1}\right) = pr\left\{B_i^I \leq WTP < B_i^{F1}\right\} = pr\left(B_i^I \leq z_i'\beta + u_i < B_i^{F1}\right) = pr\left(B_i^I - X_i'\beta \leq \varepsilon_i < B_i^{F1} - X_i'\beta\right) = pr\left(\frac{B_i^I - X_i'\beta}{\sigma} \leq \frac{\varepsilon_i}{\sigma} < \frac{B_i^{F1} - X_i'\beta}{\sigma}\right) = \phi\left(\frac{B_i^{F1} - X_i'\beta}{\sigma}\right) - \phi\left(\frac{B_i^I - X_i'\beta}{\sigma}\right)$ where the last expression follows from $pr(a \leq X < b) = F(b) - F(a)$. Therefore, using symmetry of the normal distribution we have that:$pr\left(B_i^I, B_i^{F1}\right) = \phi\left(X_i'\frac{\beta}{\sigma} - \frac{B_i^I}{\sigma}\right) - \phi\left(X_i'\frac{\beta}{\sigma} - \frac{B_i^{F1}}{\sigma}\right)$

$d_i^6,\ d_i^5,\ d_i^4,\ d_i^3,\ d_i^2,\ d_i^1\ and\ d_i^0$ are indicator variables that take the value of one or zero depending on the relevant case for each individual, that is to say, a given individual contributes to the logarithm of the likelihood function is only one of its 7 parts. Here we obtain directly $\hat{\beta},\ \hat{\sigma}$ [22] and can estimate WTP using the information obtain from cdf.

$$\ln L^D(B_i;\ \alpha,\beta) = \sum_{i=1}^{n} \left\{ \begin{array}{c} d_i^6 ln\left[\phi_{WTP}\left(z_i'\frac{\beta}{\sigma}-\frac{B_i^{F5}}{\sigma}\right)\right] + d_i^1 ln\begin{bmatrix}\phi_{WTP}\left(z_i'\frac{\beta}{\sigma}-\frac{B_i^{I}}{\sigma}\right)\\ -\phi_{WTP}\left(z_i'\frac{\beta}{\sigma}-\frac{B_i^{F1}}{\sigma}\right)\end{bmatrix} + d_i^2 ln\begin{bmatrix}\phi_{WTP}\left(z_i'\frac{\beta}{\sigma}-\frac{B_i^{F1}}{\sigma}\right)\\ -\phi_{WTP}\left(z_i'\frac{\beta}{\sigma}-\frac{B_i^{F2}}{\sigma}\right)\end{bmatrix} + d_i^3 ln\begin{bmatrix}\phi_{WTP}\left(z_i'\frac{\beta}{\sigma}-\frac{B_i^{F2}}{\sigma}\right)\\ -\phi_{WTP}\left(z_i'\frac{\beta}{\sigma}-\frac{B_i^{F3}}{\sigma}\right)\end{bmatrix} \\ +d_i^4 ln\begin{bmatrix}\phi_{WTP}\left(z_i'\frac{\beta}{\sigma}-\frac{B_i^{F3}}{\sigma}\right)\\ -\phi_{WTP}\left(z_i'\frac{\beta}{\sigma}-\frac{B_i^{F4}}{\sigma}\right)\end{bmatrix} + d_i^5 ln\begin{bmatrix}\phi_{WTP}\left(z_i'\frac{\beta}{\sigma}-\frac{B_i^{F4}}{\sigma}\right)\\ -\phi_{WTP}\left(z_i'\frac{\beta}{\sigma}-\frac{B_i^{F5}}{\sigma}\right)\end{bmatrix} + d_i^0 ln\left[1-\phi_{WTP}\left(z_i'\frac{\beta}{\sigma}-\frac{B_i^{I}}{\sigma}\right)\right] \end{array} \right\} \quad (6)$$

1.6 Regression Results and Discussion

Our research suggests that the level of risk reduction significantly influences the WTP for private programs, but surprisingly not for public ones. The average willingness to pay (WTP) for a private program is estimated to be \$1,493, whereas the average WTP for a public program is approximately \$770. The average WTP for a private program to reduce wildfire risk is \$910 for 1% risk reduction, \$1,347 for 2% risk reduction, and \$2,217 for 4% risk reduction.

These observations lend credibility to the idea that WTP estimates from the private program could be more reliable proxies for the value homeowners place on reducing wildfire risk. A notable example is the work of Fried et al. (1999), as cited by Sánchez et al. (2022), which identified a median willingness-to-pay (WTP) ranging from \$24 to \$75 per household annually for public risk reduction measures, while WTP for private measures on the respondents' properties was significantly higher, ranging from \$200 to \$500. This

[22] To estimate WTP we used STATA command ***doubleb*** instead of probit command, we formed a modified probit model.

disparity between public and private investment preferences is echoed in the research by Loomis and González-Cabán (2010), which concentrated solely on public programs. Their studies across California, Florida, and Montana revealed a mean WTP for prescribed burning ranging from $323 to $460, and for mechanical fuel reduction treatments, the WTP was slightly higher, suggesting that households value both methods of risk mitigation, albeit at different financial levels.

Our research builds upon the foundation laid by these previous works and introduces a nuanced approach that delineates the risk of wildfire damage over the next five/ten years alongside varying degrees of potential property loss. We align our research with the methodology of Holmes et al. (2013) and Sánchez et al. (2022), detailing specific risk levels (from 1% to 5%) and corresponding monetary losses (from $10,000 to $100,000) that a household might incur without new investments in wildfire protection programs. Our research adds a critical perspective to the literature by analyzing WTP against actual risk levels and potential losses, filling the gap identified in earlier studies that either did not directly compare public and private programs or did not offer a detailed spectrum of risk and loss. Unlike Holmes et al. (2013) and Sánchez et al. (2022), which reported lower ten-year wildfire risks, our study encompasses a more comprehensive risk range, thereby providing a broader understanding of homeowners' investment logic in wildfire risk reduction.

Our findings revealed that respondents' thoughts on public land management agencies, utility hits and aesthetic privacy, altruism, fire safe investment had a significant statistical influence in both programs.

When considering private goods, we find the potential financial loss amount significantly impacts the WTP. Specifically, we find the willingness to pay increases by about $12 for each additional thousand dollars of potential loss. This indicates it is important to consider the potential loss in a contingent valuation survey or the results will be biased.

Individuals who chose to invest in their defensible space on their property were less inclined to contribute to the public program. Specifically, we find individuals who spent money on their own property protection (i.e., ***OWNPROPERTYDFSFSI*****=1**) were willing to pay about $86 dollars less on the public program than people who had not spend money on their own property protection n (i.e., ***OWNPROPERTYDFSFSI*****=0**). We do not find evidence that respondent's previous risk mitigating expenditure motivates further WTP for risk reduction through private expenditure.

We also probed the 'indirect costs of defensible space' to ascertain whether externalities influenced their investment decisions. If the respondent likes that their spending or fire risk reduction benefits others (i.e., ***ALTRUISM*****=1*****)***, then they are willing to spend $637 more for private risk reduction and $117 for public risk reduction.

The variable ***UTILITYAP*** reflects the influence of non-monetary costs—such as utility hits and aesthetic privacy concerns—on homeowners' decisions regarding fire-safe investments. Respondents who indicated that they were reluctant to make risk mitigating investments on their property because of concern about the privacy or aesthetic had a higher WTP than respondents who did not.

Our study also incorporated several household risk preference variables such as perception of their own wildfire risk (FIRERISK), the efficacy of fire-safe investments (EFFICACY), and overall lifetime risk tolerance (RTOL). Surprisingly, none of these variables showed a statistically significant influence on the WTP in both programs, suggesting they were not factored into the decision-making process for our proposed programs. Furthermore, the analysis, as presented in Table 6, indicated that none of the respondent characteristics or biophysical variables held any significant sway in our model. No risk preference factors were found to play a significant role in decision-making regarding the proposed programs. Income did not show a direct effect on the WTP for the CV methods. However, even low-income respondents expressed willingness to pay more if they perceived their property was at risk from wildfire.

Table 6 WTP estimation of Public and Private Program

	Private Program		Public Program	
	(1)	(2)	(3)	(4)
VARIABLES	Coefficient (Beta)	Stannard Error	Coefficient (Beta)	Standard Error
Design Variable				
BASELINERISK	-19.48	(39.39)	4.359	(2.744)
RISKREDUCTION	435.7***	(99.25)	-11.02	(6.904)
LOSS	11.53***	(2.169)		
YEARSPAN	-3.777	(48.32)	-1.898	(3.394)
Attitudes Towards Wildfire Risk Reduction Program				
LANDMANAGEMENT	293.4**	(131.8)	27.13***	(8.969)
COMMWFPLANNING	295.0**	(130.2)	-5.789	(9.129)
COMMATTACHMENT	-33.79	(140.6)	-5.887	(9.815)
GOVTPOLICYFSI	-53.63	(143.7)	2.671	(9.856)
UTILITYAP	309.4**	(148.9)	35.93***	(10.55)
HOUSEHOLDFSI	-158.4	(178.4)	15.90	(12.54)
PROGRAMVALUE	-767.1***	(133.2)	-38.36***	(9.753)
NOAFFORD	-1,110***	(268.6)	-65.92***	(19.09)
ALTRUISM	637.1**	(253.9)	116.9***	(20.32)
OWNPROPERTYDFSI	71.13	(250.6)	-84.74***	(18.42)
Household Risk Preference				
FIRE_RISK	758.4	(489.8)	27.24	(34.74)
EFFICACY	-318.0	(499.7)	0.614	(35.16)
RTOL	-1,386	(3,541)	-319.5	(248.1)
Respondent Characteristics				
INCOMETHOUSAND	2.734	(1.951)	0.203	(0.133)
AGEHOME	5.338	(13.26)	-0.636	(0.933)
AGE	-7.825	(11.25)	0.0867	(0.781)
INSUECOVERED	-467.7*	(271.9)	-29.85	(20.57)
HHVL	81.87	(101.2)	13.64*	(7.033)
Biophysical Variables				
WIND	-10.24	(17.10)	-1.571	(1.182)
ASPECT	-22.13	(307.1)	34.07	(21.35)
SLOPE	1.949	(21.66)	-0.886	(1.514)
ELEVDIFF	-0.0432	(0.855)	-0.0111	(0.0632)
LGHTN	-0.0736	(0.279)	-0.0255	(0.0195)
Constant	-834.3	(1,847)	70.23	(130.1)
Sigma				
Constant	2,371***	(99.54)	162.8***	(6.714)
Observations	471	471	471	471

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

1.7 Conclusion

Wildfires are one of the most devastating environmental hazards in United Sates, causing severe social, economic and environmental consequences. Wildland Urban Interface (WUI) communities continue to grow and thus increase the wildfire risk to human lives and property. Increasing wildfire damages in residential communities adjacent to wildlands has emerged as significant policy concern in recent years (Sisante, Taylor, & Rollins, 2019). In response to the threat from wildfire, *wildland-urban interface* (WUI) communities across the United States have enacted policies to reduce the threat to homes and other structures from wildfire. These policies include fuel management treatments on wildlands surrounding WUI communities, as well educational and cost-sharing programs to encourage voluntary investments by homeowners to reduce their wildfire risk.

In this article, we present results from a contingent valuation study that estimates homeowners' willingness-to-pay (WTP) for a reduction in the risk that their home will be destroyed in a wildfire. The results from this study can be used to value actions and policies that reduce homeowners' wildfire risk and identify WUI communities where the benefits of these policies are likely to outweigh the costs.

Wildland-urban-interface (WUI) residents in Nevada were surveyed using a contingent valuation approach to assess their willingness-to-pay (WTP) for reductions in the risk of losing their homes to wildfire. The study uses a unique data set of 471 randomly selected respondents from 35 WUI communities in Nevada. Each respondent was asked about their willingness to support two wildfire risk reduction programs: one program focused on reducing risk by modifying their home and managing vegetation and other flammable material surrounding their home (i.e., *private* program) and one program focused on reducing the wildfire risk to their entire community through targeted fuel management treatments (i.e., *public* program). In addition to the contingent valuation questions, the survey includes information on the respondent's previous wildfire risk mitigation expenditures, attitudes towards wildfire risk reduction programs, risk preferences, and demographic characteristics. WTP was estimated from homeowners' responses to a double bounded dichotomous choice questions using a random utility model (Hanemann, 1984).

We found that more than 75% of respondents expressed positive WTP for the public program and more than 80% expressed positive WTP for the privately program. Respondents indicated that they are willing to pay more for the private program, where wildfire risk mitigation measures are focused on their own property, than for a public program, where risk mitigation efforts would be undertaken at the community level. The average WTP for a private program to reduce wildfire risk is $910 for 1% risk reduction, $1,347 for 2% risk reduction, and $2,217 for 4% risk reduction. While we found positive values for WTP for public program, these results were not statistically significant. Thus, we find people are willing to pay for private programs but not for public programs. This result suggests that the WTP estimates from the private program provide more reliable estimates of homeowners' value of wildfire risk reductions.

Our study employs the contingent valuation method to estimate respondent's WTP for a good – wildfire risk reduction – that, in many cases, they have already invested in. This feature raises concerns that respondent's previous expenditure may influence their responses to the hypothetical contingent valuation questions and, in doing so, bias our estimates of WTP. We find individuals who spent money on their own property had a significantly lower WTP for public programs. However, we do not find evidence that respondent's previous risk mitigating expenditure influences their WTP, which supports our use of the contingent valuation approach in this context.

Our survey instrument included a "cheap talk" script reminding respondents to consider the opportunity cost of money when answering the contingent valuation questions. The cheap talk script, however, did not discuss the non-financial costs of homeowners' risk mitigating investments, such as the loss in aesthetic beauty and privacy from changing landscaping and removing large trees. We find evidence that this omission was a problem in that respondents who indicated that they were reluctant to make risk mitigating investments on their property because of the concern about the privacy or aesthetic had a higher WTP than respondents who did not. This result suggests that future contingent valuation studies should consider expanding their cheap talk script for hypothetical goods whose provision involves non-pecuniary costs.

Respondents who stated that they supported the public program, in part because of the risk reduction benefits to the entire community, had a higher WTP for both the public and private programs. This result presents a challenge to using contingent valuation to value a good, such as wildfire risk reduction, that has external benefits (e.g., spillover benefits). Our result suggests that respondents are likely to consider these external benefits when answering the contingent valuation question even if these external benefits are not explicitly stated in the description of the hypothetical good. To account for this altruistic dimension, our study suggests that future assessments of WTP should explicitly consider these broader community benefits to avoid potential biases and truly capture the comprehensive economic value placed on risk mitigation efforts. Future studies should explicitly present external benefits when defining the hypothetical good to avoid potential biases such as those detected in our study.

A.1 Appendix (Chapter 1)

A.1.1 Recent Wildfire in Nevada

Since 1984, Nevada has endured significant wildfire incidences, with an estimated 5% of its land being ravaged by fire each decade (Allen, Steers, & Dickens, 2011). This situation has been amplified due to the unprecedented drought that Nevada, alongside the rest of the American West, is currently experiencing. According to recent studies, this is considered the worst drought in a span of over 1,200 years. The record-setting dry spell in January and February has sparked fears of imminent large-scale wildfires, which are projected to ignite in Southern Nevada around April and May and gradually move northwards (United States Environmental Protection Agency, 2016).

The deleterious impact of wildfires extends beyond the direct damage caused by the flames. There are profound alterations to the environmental and ecological dynamics of the affected regions, particularly in Nevada's semi-arid rangelands where wildfires frequently occur. Post-fire landscapes undergo substantial changes, most notably in the soil composition, where physical, chemical, and microbial attributes are significantly altered, disrupting the soil health and organic matter cycling. One striking post-fire ecological phenomenon is the invasion of cheatgrass, a non-native annual plant species. Recent studies indicate that cheatgrass rapidly colonizes burned areas, outcompeting native species like the Wyoming sagebrush, which is the prevalent variety in Northern Nevada (Wolterbeek, 2022). Cheatgrass achieves this dominance by monopolizing soil water and nutrients, thereby impeding the regrowth of native plants.

The wildfires also pose significant public health threats. Smoke from the fires is a recognized pollutant, associated with increased hospitalizations and emergency room visits due to chest pain, respiratory, and heart problems (Allen, Steers, & Dickens, 2011). The wildfires in regions like Elko underline the direct threats to Nevada's populous areas. The landscape of Nevada is at a potential tipping point due to the increased fire incidences and prevailing dry conditions. Arid-land plants and animals, already operating close to their tolerance limits, could be pushed over the edge, triggering irreversible landscape transformations. The encroachment of non-native species, like cheatgrass, could outpace native vegetation, exacerbating fire risks due to their greater susceptibility to ignition. Notwithstanding the challenges, recent research from the University of Nevada, Reno points to potential mitigation strategies. One such strategy is targeted livestock grazing, which can effectively control cheatgrass populations. Perryman posits that cheatgrass could serve as a significant grazing resource for cattle during the dormant season, potentially aiding in fire risk reduction (Wolterbeek, 2022).

Nevada's recent wildfire scenarios reflect a complex interplay between climate anomalies, ecological changes, and human activities. These fires pose multifaceted challenges, from direct threats to human safety to significant ecological and health impacts. Yet, emerging strategies, such as targeted livestock grazing, may offer feasible mitigation avenues. However, a holistic understanding of these dynamics, underpinned by rigorous scientific inquiry, is crucial for informed policy-making and effective wildfire management.

Figure 2 Nevada Wildfire by County, Source: Nevada Division of Forestry

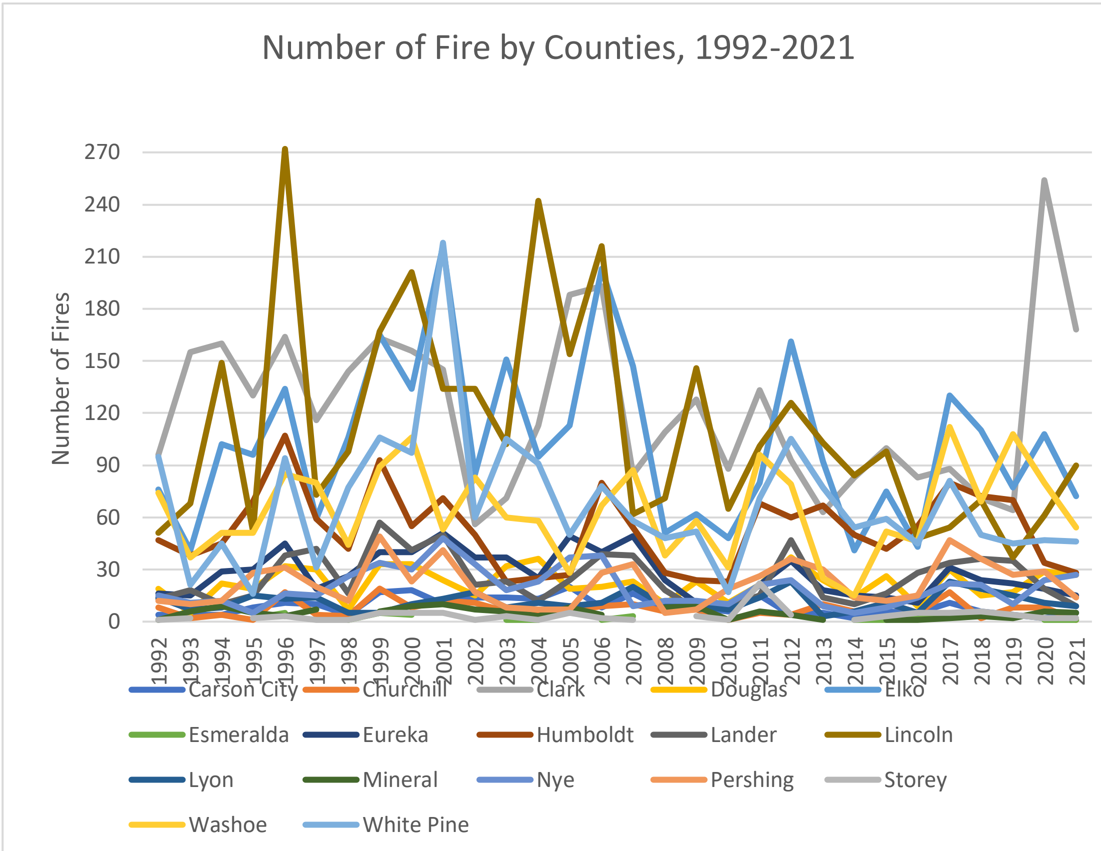
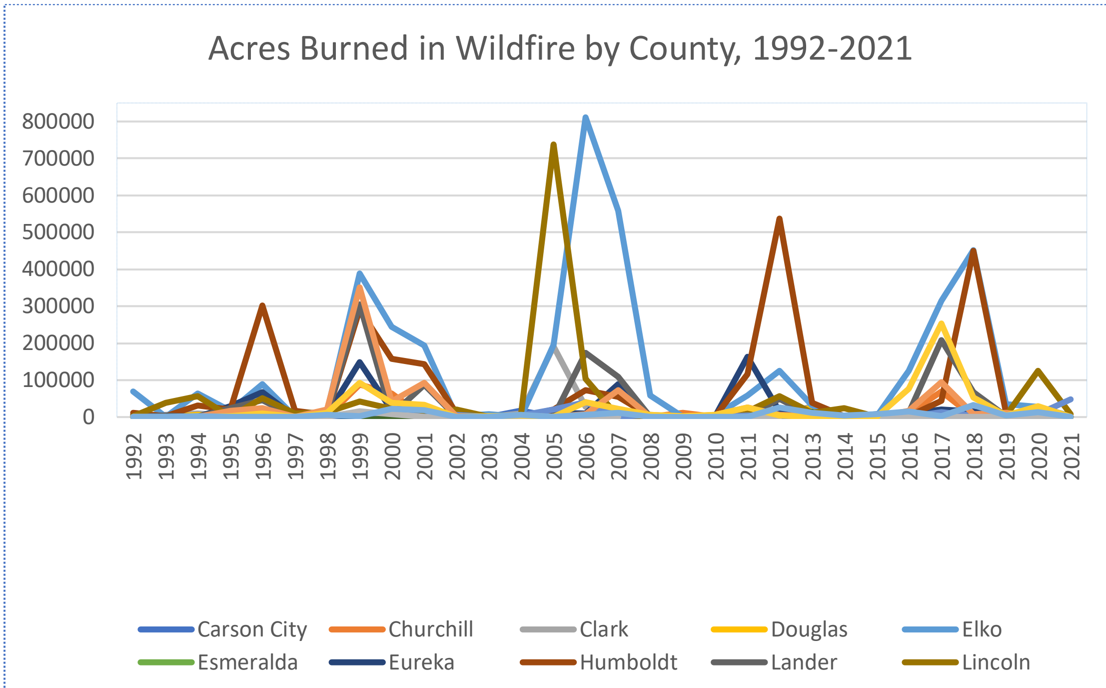


Figure 3 Acres Burned in Wildfire by County, 1992-2021, Source: Nevada Division of Forestry



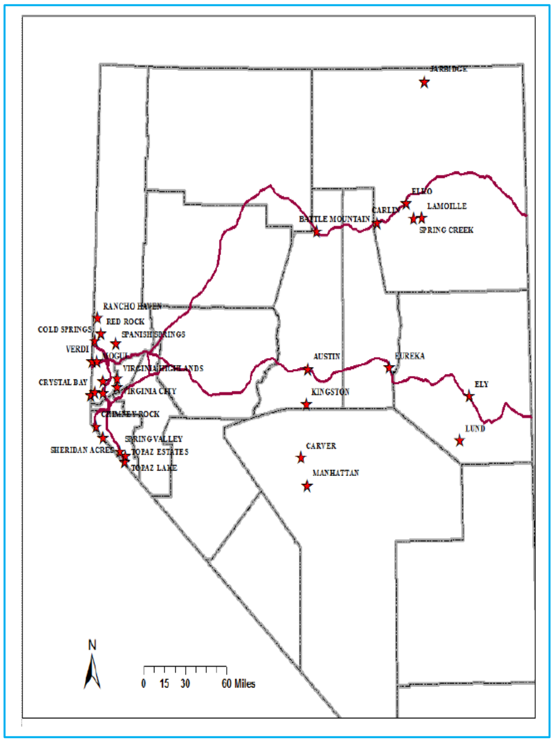


Figure 4 Sample Area, 35 WUI communities

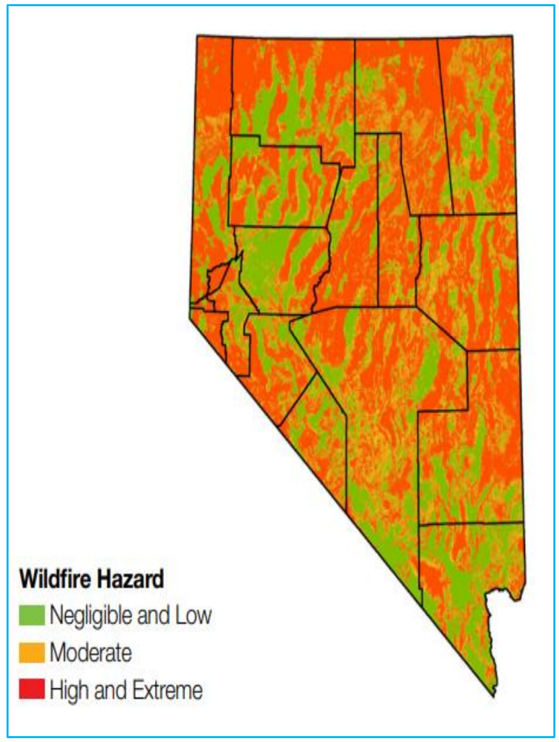


Figure 5 Wildfire Hazard Map, Source: Verisk

Figure-4 and Figure-5 provide a visual overview of the geographical areas considered in our analysis and their corresponding categorizations for wildfire risk, respectively. The risk categories were classified as negligible to low, moderate, and high to extreme. The US Census data indicates that Nevada houses a total of 1,173,800 residential units. However, only a minor fraction, approximately 6% (67,100 units), falls within the high to extreme wildfire risk category. In contrast, a substantial majority, around 84% (990,400 units), is considered to be under negligible or low wildfire risk. A smaller segment, about 10% (116,300 units), is seen as being exposed to a moderate wildfire risk. In terms of acreage affected, the National Interagency Fire Center (2018) reported a total of 1,003,000 acres burnt in Nevada during 2018, of which the Martin (2018) fire accounted for a significant portion, consuming around 439,000 acres. The subsequent table delineates further details, specifying the top five counties facing high to extreme wildfire risk and the corresponding number of housing units within these risk areas.

To summarize, while a considerable part of Nevada's residential units lies within areas of negligible to low wildfire risk, there remains a significant portion exposed to moderate and high to extreme risk categories. This data underscores the necessity for proactive wildfire mitigation strategies, particularly in high-risk counties. Further, the severity of wildfires, as evidenced by the 2018 Martin fire, calls for ongoing efforts to understand these risk patterns better and develop effective risk management practices.

Table 7 Top five counties risk profile, number, and percentage.

Top five counties by number of housing units in high and extreme wildfire risk categories		Top five counties by highest concentration of housing units in high and extreme wildfire risk categories	
County	Number	County	Percentage
Washoe	40,500	Storey	49%
Douglas	7,900	Douglas	33%
Carson City	6,400	Carson City	27%
Elko	4,100	Lincoln	26%
Lyon	2,300	Washoe	22%
Source: Verisk, 2020			

A.1.2 Community wise income and average home values of respondents

Table 8 Community wise income and average home values of respondents

	Community	Number of Respondent	Average of Home Value	Average of income	Average of sqft.
Forested Communities	Upper Tyner	4	381667	212500	2150
	Saddlehorn Tumbleweed	3	418500	208333	3183
	Champagne Burgundy	6	617400	195833	4719
	West Washoe Valley	24	553118	167499	3720
	Tyrolian Village	4	462500	154166	1935
	Galena Forest	29	484868	149499	3111
	Allison Jennifer	8	454367	143749	2675
	Crystal Bay	20	678551	142894	2957
	Chimney Rock	20	461687	129860	2926
	Incline Village	7	547620	93571	2762
Forested Communities Total		**125**	**522818**	**150931**	**3138**
Grassland Communities	Lamoille	14	410905	131249	2993
	Battle Mountain	11	47076	74499	2136
Grassland Communities Total		**25**	**228991**	**105454**	**2636**
Pinyon-Juniper Communities	Virginia Highlands	13	202411	111153	2926
	Rancho Haven	19	123823	81323	1883
	Kingston	13	66786	74166	1454
	Eureka	14	70693	61606	1371
	Manhattan	4	16750	59999	2047
	Austin	10	44383	54499	1431
Pinyon-Juniper Communities Total		**73**	**101063**	**76883**	**1846**
Sagebrush Communities	Red Rock	22	292353	130657	3411
	Spanish Springs	14	171527	129230	2473
	Verdi	23	380203	126447	2998
	Sheridan Acres	10	256667	124687	2665
	Spring Valley	10	285652	112249	3485
	Topaz Lake	16	189165	95999	1732
	Virginia City	12	120572	91666	1808
	Mogul	22	164295	88094	2170
	Carvers	15	50111	84582	1912
	Carlin	12	94545	82499	1585
	Jarbidge	18	48358	79374	1191
	Spring Creek	18	167568	73905	2296
	Cold Springs	15	120000	71785	2275
	Elko	6	82000	70416	1590
	Lund	12	73828	60749	2200
	Ely	8	109667	52856	2366
	Topaz Estates	15	52101	48666	1491
Sagebrush Communities Total		**248**	**164465**	**91981**	**2262**
Grand Total		**471**	**264849**	**105549**	**2454**

A.1.3 Description of Private Risk Mitigation Question

Suppose there is a 6% chance that a wildfire will reach your house in any year for the next 5 years, and that if a fire should reach your house, the loss to you would be $50,000. Suppose you could guarantee to reduce the chance that fire will reach your home from 6% to 2% over the next 5 years, by creating and maintaining defensible space on your property and modifying your home.

To put this into perspective, this translates into reducing the probability that a wildfire will damage your property sometime in the 5-year period from 27% to 10%. Would you spend…

Suppose there is a 6% chance that a wildfire will reach your house in any year for the **next 5 years**, and that if a fire should reach your house, **the loss to you would be $50,000.**

Suppose you could **guarantee** to reduce the chance that fire will reach your home from 6% to 2% over the next 5 years, by creating and maintaining defensible space on your property and modifying your home.

To put this into perspective, this translates into reducing the probability that a wildfire will damage your property **sometime in the 5 year period from 27% to 10%.**

1. Would you spend:	Yes!!	yes	maybe	no	No!!
$ 100 if you were sure it would **reduce fire risk from 6% to 2%?**	❑	❑	❑	❑	❑
$ 400 if you were sure it would **reduce fire risk from 6% to 2%?**	❑	❑	❑	❑	❑
$1,000 if you were sure it would **reduce fire risk from 6% to 2%?**	❑	❑	❑	❑	❑
$2,000 if you were sure it would **reduce fire risk from 6% to 2%?**	❑	❑	❑	❑	❑
$3,000 if you were sure it would **reduce fire risk from 6% to 2%?**	❑	❑	❑	❑	❑
$4,000 if you were sure it would **reduce fire risk from 6% to 2%?**	❑	❑	❑	❑	❑

A.1.4 Description of Public Risk Mitigation Question

Your community faces a 6% probability of a wildfire each year. A community program would enlist fire experts to develop a plan to create and maintain defensible space on lands in and surrounding your community. The plan would be guaranteed to reduce the annual probability that wildfire would impact the community from 6% to 2% for each year over the next 5 years. To put this into perspective, this translates into changing the probability that a wildfire will impact your community sometime in the 5-year period from 27% to 10%.

The lower wildfire risk will benefit you and the entire community. While the plan may not change your particular property or structure at all, it would guarantee the fire risk reduction to your house.

This program would be carried out depending on the result of a vote of homeowners in your community. If a majority votes "yes," then every resident would be assessed an annual fee to support the program. Your annual fee would be used exclusively to create and maintain defensible space that would reduce the risk of wildfire and would not be used for any other purpose If a majority votes "no" then no one would be assessed the fee and the program would not be launched and community-wide probability of a wildfire occurring would remain unchanged, at 6% per year.

How would you vote on a program that would reduce the annual probability of a wildfire impacting your community in the next 5 years from 6% to 2%?

Would you vote "YES" if …

Suppose that…

Your community faces a 6% probability of a wildfire each year. A **community program** would enlist fire experts to develop **a plan to create and maintain defensible space on lands in and surrounding your community.** The plan would be guaranteed to **reduce the annual probability** that wildfire would impact the community **from 6% to 2% for each year over the next 5 years**.

To put this into perspective, this translates into changing the probability that a wildfire will impact your community **sometime in the 5 year period from 27% to 10%**.

The lower wildfire risk will benefit you and the entire community. While the plan may not change your particular property or structure at all, it would guarantee the fire risk reduction to your house.

This program would be carried out depending on the result of a **vote** of homeowners in your community.

If a majority votes "yes," then **every** resident would be assessed an annual fee to support the program. Your annual fee would be used exclusively to create and maintain defensible space that would reduce the risk of wildfire, and would not be used for any other purpose

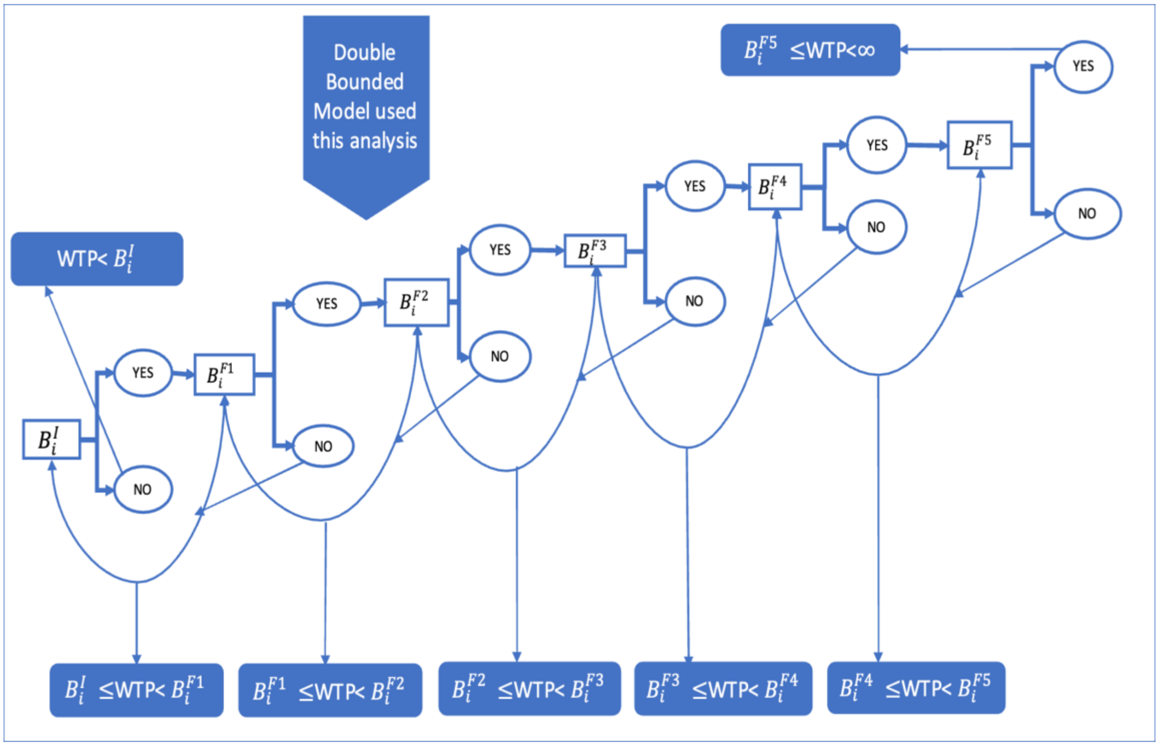
If a majority votes "no" then no one would be assessed the fee and the program would not be launched and community-wide probability of a wildfire occurring would remain unchanged, at 6% per year.

How would you vote on a program that would **reduce** the annual probability of a wildfire impacting your community in the next 5 years **from 6% to 2%**?

2. Would you vote "YES" if …	Yes!!	yes	maybe	no	No!!
The annual cost to you would be nothing.	❑	❑	❑	❑	❑
The annual cost to you would be **$10** per year for the **next 5 years**?	❑	❑	❑	❑	❑
The annual cost to you would be **$30** per year for the **next 5 years**?	❑	❑	❑	❑	❑
The annual cost to you would be **$75** per year for the **next 5 years**?	❑	❑	❑	❑	❑
The annual cost to you would be **$200** per year for the **next 5 years**?	❑	❑	❑	❑	❑
The annual cost to you would be **$650** per year for the **next 5 years**?	❑	❑	❑	❑	❑

A.1.5 Bid, Multiple Bounded Dichotomous Choice

Figure 6 Different Bound of Bid Questions



A.1.6 Frequency Distribution of Bid Response of Public and Private Program

Figure 7 Frequency Distribution of Bid responses

	Private Program		**Public Program**	
	Freq.	Cum.	Freq.	Cum.
NO	78	16.56	68	14.44
YES-NO	62	13.16	51	10.83
	124	26.33	47	9.98
	89	18.90	92	19.53
	44	9.34	87	18.47
	7	1.49	94	19.96
YES-YES	67	14.23	32	6.79
Total	471	100.00	471	100.00

A.1.7 Frequency of Design Variables and Response Rate

Table 9 Frequency of Design Variables and Response Rate

	Risk Reduction						
Baseline Risk	10% to 6%	10% to 9%	2% to 1%	3% to 1%	6% to 2%	8% to 6%	Total
10%	83	80	0	0	0	0	163 (34.61)
2%	0	0	89	0	0	0	89 (18.90)
3%	0	0	0	76	0	0	76 (16.14)
6%	0	0	0	0	81	0	81(17.20)
8%	0	0	0	0	0	62	62 (13.16)
Total	83 (17.62)	80 (16.99)	89(18.90)	76 (16.14)	81 (17.20)	62 (13.16)	471(100)

*Parenthesis indicates the percentage of respondent

A.1.8 Description of the Risk Reduction Baseline Risk

Table 10 Description of the Risk Reduction Baseline Risk

		Private and Public Program			
variable	definition	Mean	Std. Dev.	Min	max
Br10	1=10% probability of fire; 0 otherwise	.3432836	.4751276	0	1
Br2	1=2% probability of fire; 0 otherwise	.1763908	.3814111	0	1
Br3	1=3% probability of fire; 0 otherwise	.1709634	.3767326	0	1
Br6	1=6% probability of fire; 0 otherwise	.1641791	.3706896	0	1
Br8	1=8% probability of fire; 0 otherwise	.1451832	.3525247	0	1
Prd106	1= 10%-6% reduction in probability of fire; 0 otherwise	.1763908	.3814111	0	1
Prd109	1= 10%-9% reduction in probability of fire; 0 otherwise	.1668928	.3731334	0	1
Prd21	1= 2%-1% reduction in probability of fire; 0 otherwise	.1763908	.3814111	0	1
Prd31	1= 3%-1% reduction in probability of fire; 0 otherwise	.1709634	.3767326	0	1
Prd62	1= 6%-2% reduction in probability of fire; 0 otherwise	.1641791	.3706896	0	1
Prd86	1= 8%-6% reduction in probability of fire; 0 otherwise	.1451832	.3525247	0	1
Year 5	1= chance of fire occurring over 5 years, 0=otherwise	.531886	.4993211	0	1
Year10	1= chance of fire occurring over 10 years, 0=otherwise	.468114	.4993211	0	1
Risk_reduction	1%, 2%. 4% reduction in probability of fire	2.33 7856	1.26237	1	4
PRLOSS50K*	1= $50,000 loss from fire; 0 otherwise	.312076	.4636553	0	1
PRLOSS100K*	1= $100,000 loss from fire; 0 otherwise	.358209	.4797996	0	1
PRLOSS200K*	1= $200,000 loss from fire; 0 otherwise	.3297151	.4704289	0	1

* Indicates only private program (not public program)

A.1.9 Program Follow-Up Questions After Contingent Valuation (CV)

Post the initial contingent valuation (CV) query concerning a predetermined bid amount, respondents were engaged with an array of subsequent questions. These were aimed at deciphering the extent of influence the initial inquiry had on their behavioral response and attitudes towards the proposed hypothetical program.

The subsequent inquiries tackled various dimensions, such as respondents' prior experience with wildfires, awareness about characteristics and effectiveness of mitigation measures, trust in the involved agencies, and personal impact of wildfires. Factor analysis was employed to develop a derivative variable for our investigation, referred to as 'PROGRAMVALUE', built from the responses to three distinct follow-up questions. Furthermore, the respondents were queried on their perspective about the insignificance of risk and change in risk (SMALLRISK), as well as whether the risk reduction yielded minimal household value (NOVALUE). A line of questioning was also developed to gauge their beliefs on the feasibility of the work necessary for risk reduction (NOTACHIEVED).

An additional aspect under consideration was the financial accessibility of the hypothetical goods or program to homeowners. Under this vein, a moral hazard evaluation was conducted by inquiring whether respondents' losses would be fully compensated by fire insurance (INSUCOVER).

We also probed into the 'indirect costs of defensible space' to ascertain whether considerations of mitigating externalities influenced their investment decisions. We asked a series of value questions designed to understand if the wider community's reduced risk influenced respondents' valuation of public benefits (ALTRUISM). Our aim was to determine if risk reduction measures from one context could effectively translate into another context. To this end, public CV respondents were questioned if they would prefer to allocate the funds to home defenses (OWNPROPERTY). These structured follow-up queries post-CV form the backbone of our multi-dimensional investigation into wildfire risk mitigation perceptions and willingness to invest in mitigating measures.

Table 11 Program Follow-Up Questions After Contingent Valuation (CV)

		Private Program			Public Program		
Categories	Description	Obs.	Mean	SD.	Obs.	Mean	SD
Follow up questions after CV questions; Program Value ***(PROGRAMVALUE)***							
NOVALUE	=1 if the risk reduction is not valuable to the respondent	459	0.23	0.42	459	0.23	0.42
NOT ACHIEVED	=1 if respondent don't believe program would be achieved	457	0.24	0.43	457	0.24	0.43
SMALLRISK	=1 if risk reduction small matter to the respondent	454	0.19	0.39	459	0.19	0.39
Follow up questions after CV questions							
NOAFFORD	=1 if the respondent cannot afford the cost; 0 otherwise	465	0.35	0.48	457	0.32	0.47
INSUCOVER	1= if respondent feels insurance cover all losses from wildfire	457	0.29	0.45	459	0.73	0.44
ALTRUSIM	1= if the respondent like that the program reduces the fire risk to the entire community	461	0.54	0.50	458	0.23	0.42
OWNPRPERTY	=1 if the respondent prefers to spend money on DFS for their own property, 0 otherwise				461	0.44	0.50

A.1.10 Respondents Behaviors to Reduce Wildfire Risk

Table 12 Respondents Behaviors to Reduce Wildfire Risk (a)

Variable	Description	Obs.	Mean	SD
	Feel About the Community			
ATTATCHCOMMUNITY	1=Respondent feels attached to this community, 0 otherwise	456	0.67	0.47
STRONGCOMMUNITY	1=Respondent feels strongly with this community, 0 otherwise	461	0.63	0.48
COMPARECOMMUNITY	1=Respondent feels this community is incomparable, 0 otherwise	451	0.37	0.48
BESTPLACECOMMUNITY	1=Respondent feels this community is the best place, 0 otherwise	458	0.53	0.50
TIGHTKNITCOMMUNITY	1=Respondent feel the community is tight knit, 0 otherwise	457	0.41	0.49
NEGHBORCOMMUNITY	1=Respondent know well nearest neighbors	457	0.56	0.50
	Community Wildfire Planning			
DISCUSSCOORDINATE	1=Respondent discussed with neighbor coordinate to reduce fire risk	461	0.22	0.42
NEIGHBORCOORDINATE	1=Respondent coordinated actions to reduce fire risk with neighbors	460	0.16	0.37
FUTURECOORDINATE	1=Respondent plan to coordinate DFS with neighbors in future	460	0.18	0.38
GAGREEMENTNEIGHBOR	1= Respondent has general agreement among neighbors about Wildfire Risk Reduction	455	0.41	0.49
AGREEMENTARFRISK	1= Respondent has agreement for taking actions to reduce Fire Risk	457	0.34	0.47
	Government Policy for Fire Safe Investment			
CREATEMAINTAINDFS	1=Respondent agree that homeowners should coordinate creating and Maintaining DFS; 0 otherwise	463	0.63	0.48
INVESEDUCATION	1= Respondents agree that communities should invest more on education; 0 otherwise	466	0.56	0.50
TAXBRREAKDFS	1=Respondent feels people who create and maintain DFS should get tax breaks; 0 otherwise	466	0.52	0.50
LOWINSURANCEDFS	1= Respondents feels People who maintain DFS should have lower insurance rates; 0 otherwise	464	0.83	0.38
LAWMAINTAINDFS	1= Respondent feels people should be required by law to maintain defensible space; 0 otherwise	460	0.33	0.47
GOVTSUBSIDYDFS	1= Respondent feels government should subsidize creation of defensible space through grants; 0 otherwise	463	0.28	0.45
VOLUNTARYPROTEC	1=Respondent feels protect home should be voluntary; 0= otherwise	464	0.64	0.48

Table 13 Respondents Behaviors to Reduce Wildfire Risk (b)

Variable	Description	Obs.	Mean	SD
	Attitudes Public Land Manage Agencies			
SIMILARVALUE	1= Respondent feels to share similar values as the agencies that manage public land near their house; 0 otherwise	469	0.41	0.49
SIMILARGOAL	1= Respondent feels to share similar goals as the agencies that manage public land near their house; 0 otherwise	468	0.39	0.49
SIMILARTHINKING	1= Respondent feels to think similar way as the agencies that manage public land near their house; 0 otherwise	467	0.31	0.46
SAMEPRIORITIES	1= Respondent feels same priorities as the agencies that manage public land near their house; 0 otherwise	466	0.26	0.44
	Household Fire Safe Investment			
FIRERESSISTANTROOF	1= if respondents house has fire resistant roof (tile, cement, asphalt), 0= otherwise	459	0.87	0.33
FIRERESISTANTSIDING	1= if respondents house has fire resistant sidings, 0= otherwise	452	0.39	0.49
EAVESENCLOSED	1= if respondents house has eaves enclosed or eliminated, 0= otherwise	453	0.58	0.49
SPARKCHIMNEYS	1= if respondents house has spark arresters’ chimneys, 0= otherwise	456	0.79	0.41
VENTCOVERESMASH	1= if respondents house has vents covered with mash, 0= otherwise	458	0.81	0.39
SKIRTINGDECKS	1= if respondents house has skirting under decks, 0= otherwise	452	0.61	0.49
	Utility hits and Aesthetic Privacy			
HOUSELESSATTRACTIVE	1= if respondent Concern for loss of house attractiveness, 0 otherwise	454	0.11	0.31
LANDSCAPELESSATTRAT	1= if respondent Concern for loss of landscaping attractiveness, 0 otherwise	452	0.12	0.33
PRIVACYCONCERN	1= if respondent Concern for loss of privacy, 0 otherwise	455	0.12	0.32
WILDLIFEHABITANT	1= if respondent Concern for loss of wildlife habitat, 0 otherwise	455	0.23	0.42

A.1.11 Household Risk Preferences

Within our research parameters, we sought to illuminate the preferences of households concerning risk. This we delineated into three distinct variables: lifetime associated risk, fire safety investment risk, and perceived risk. We incorporated questions mirroring those featured in studies like the University of Michigan's Health and Retirement Survey and the Panel Survey on Income Dynamics. These questions presented homeowners with hypothetical scenarios to either increase or decrease their lifetime income, providing a valuable opportunity to discern the homeowner's coefficient of relative risk preferences (CRRA).

Drawing from the innovative approach of Kimball et al. (2008), we minimized measurement errors across multiple data collection waves. This strategy allowed us to extrapolate reliable estimates of each respondent's coefficient of relative risk tolerance. Utilizing these imputations from Kimball, Sahm, and Shapiro's (2008) work, we were able to ascertain each homeowner's relative risk tolerance (RTOL). This, in turn, provided us with a robust framework to test our hypothesis: that homeowners' attitudes towards financial risk underpin their observed fire-safe investment decisions.

Furthermore, we recognized that homeowners held varied beliefs regarding the efficacy of different risk reduction methods. Consequently, we incorporated measures capturing homeowners' subjective perceptions of their personal wildfire risk, denoted by PERCEIVED RISK (FIRERISK), and their conviction about the effectiveness of fire-safe investments in mitigating this risk, labeled as EFFICACY (Sisante, Taylor, & Rollins, 2019). This comprehensive approach provided us with a multi-dimensional perspective on risk preferences, thereby enriching our understanding of homeowner behavior in the context of fire safety investments.

Table 14 Preferences of households regarding risk.

	Household Risk Preference				
		Mean	SD	Min	Max
EFFICACY	Trust in Defensible space	0.48	0.26	0.00	1.00
RTOL	Lifetime risk tolerance	0.22	0.03	0.20	0.30
FIRERISK	Index of subjective chance of fire reaching in the community	0.34	0.25	0.00	1.00

A.1.12 Respondents Characteristics

Our study took into consideration various respondent characteristics that could influence their wildfire-related financial losses. These variables include income (INCOME), age (AGE), home age (AGEHOME), home value (HHV), and perceptions concerning insurance coverage across different programs.

We anticipated a positive correlation between a homeowner's net equity position - the property value minus any outstanding mortgage balance - and the financial loss due to wildfires. Therefore, the variables INCOME and HOMEVALUE, which are surrogates for the homeowner's net equity position, were predicted to show a positive relationship with financial loss. The age of each residence AGEHOME was another key variable, as it serves as a proxy for the financial cost of fire-safe investments. Our hypothesis was that older homes incur higher investment costs for two reasons: firstly, newer homes are more likely to have fire-safe features pre-installed; secondly, older homes typically have mature landscaping, which can be expensive to modify for fire safety. Consequently, we expect older homes to correlate with higher financial costs and increased loss of privacy due to fire-safe investments. In addition to these characteristics, we factored in the respondents' existing insurance coverage. This allowed us to gauge their understanding of how much of their potential wildfire-related losses would be covered by either private or public programs, thus giving us further insight into their risk perception and mitigation strategies.

Table 15 Description of Respondents Characteristics

Respondent characteristics Distribution					
		Mean	SD	Min	Max
INCOME	Respondent household income in thousand	105.42	70.72	10.00	250.00
AGE	Respondents age	60.83	10.72	25.00	91.00
AGEHOME	Respondent house age	22.01	9.37	1.00	38.67
INSURANCE_PRI	1= if respondent feels insurance cover all losses from wildfire	0.30	0.46	0.00	1.00
INSURANCE_PUB	1= if respondent feels insurance cover all losses from wildfire	0.24	0.43	0.00	1.00
HHVL	Log of house value in dollar	11.21	1.33	6.23	16.1181
HHV	Home values in dollar	158523	477277	525.99	99999.99

A.1.13 Description of Bio-Physical variables

Our study implements five topographical variables from the US Geological Survey's Digital Elevation Model (US Geological Survey, 2023) to appraise the wildfire susceptibility of varied geographical features. Firstly, we include a binary variable, ASECT, that takes the value of 1 for residences with a southern exposure. These areas, characterized by increased wind levels and more flammable vegetation due to enhanced solar heating, present elevated wildfire risks. Secondly, the SLOPE variable is included in our analysis, given its critical role in wildfire propagation. Steeper slopes expedite the uphill spread of wildfires, thus enhancing the hazard.

In the following stratum of our analysis, we evaluate each residence's elevation relative to the community average, captured through the variable ELEVDIFF. As wildfires have a predilection for uphill spread, homes at superior elevations within a community inherently encounter amplified risks. The fourth variable we incorporate is WIND, representing the average maximum wind speed in each community over the preceding five wildfire seasons, sourced from the US Forest Service's National Fire and Aviation Management webpage (2023). This data is crucial, as increased wind speeds facilitate a swifter wildfire spread. Lastly, the variable LGHTH, denoting the number of lightning strikes within a 10-mile radius, is considered, given lightning's potential role in wildfire ignition. Collectively, these biophysical variables furnish a robust portrayal of the wildfire risks pertinent to diverse geographical and climatic contexts.

Table 16 Description of Bio-Physical Variables

Variables	Description	Obs.	Mean	Std. Dev.	Min	Max
ASPECT	=1 if property is south facing	736	0.20	0.40	0	1
ELEVDIFF	Difference between the elevation of a residence and the average elevation in community	736	14.92	131.97	-578	697
SLOPE	Slope of property (percent)	736	6.43	5.81	0	32
FUEL	Average fuel loading in community (tons/acre)	736	3.52	0.93	1	5
BRUSH	=1 if sagebrush rangeland	736	0.53	0.50	0	1
GRS	=1 if grassland	736	0.06	0.24	0	1
PJ	=1 if pinyon-juniper woodland	736	0.16	0.36	0	1
TMBR	Average	736	0.25	0.43	0	1
LGHTN	Number of lightning strikes within 10 miles	736	973.48	489.20	317	2468
WIND	Average maximum daily wind speed (miles per hour)	736	31.11	7.57	14	46

A.1.14 Multiple Imputation Chained Equation (MICE)

In the field of data analysis, the challenge of handling missing data is met with various imputation techniques, among which Multiple Imputation (MI) and its derivatives have gained substantial traction. As noted by Zhong, Hu, and Penn (2018), these methodologies are particularly prevalent in governmental and social science research. A specialized form of MI, known as Multiple Imputation by Chained Equations (MICE), was pioneered by Van Buuren & Oudshoorn (1999) and has been employed to address missing values in datasets while accommodating the diversity of variable types and the constraints of their ranges.

The fundamental tenet of MI posits that missing data can be inferred from the available information—a premise that can be evaluated through the classification of missing data into three categories: Missing at Random (MAR), Missing Completely at Random (MCAR), and Missing Not at Random (MNAR). MAR denotes the condition where missingness is related to observed data, whereas MCAR signifies that the missingness is unrelated to any observed data, and MNAR indicates a dependence on unobserved data (Royston & White, 2011). MICE operate under the assumption of MAR and is sensitive to the introduction of noise and potential biases when applied to MCAR scenarios (Azur, Stuart, Frangakis, & Leaf, 2011).

The execution of MICE involves an iterative process with three distinct steps: Imputation, Analysis, and Pooling, as elucidated by UCLA (2018). The Imputation phase involves a sequence of univariate imputations, utilizing a set of k random variables ordered by the extent of missing data. Starting with the variable with the highest level of missing data, each subsequent variable is imputed by regressing on the full dataset and the imputed values of preceding variables, in a process that typically undergoes 10-20 iterations to generate a single imputed dataset and is repeated to produce multiple datasets (Van Oudshoorn & Oudshoorn, 1999; Royston & White, 2011).

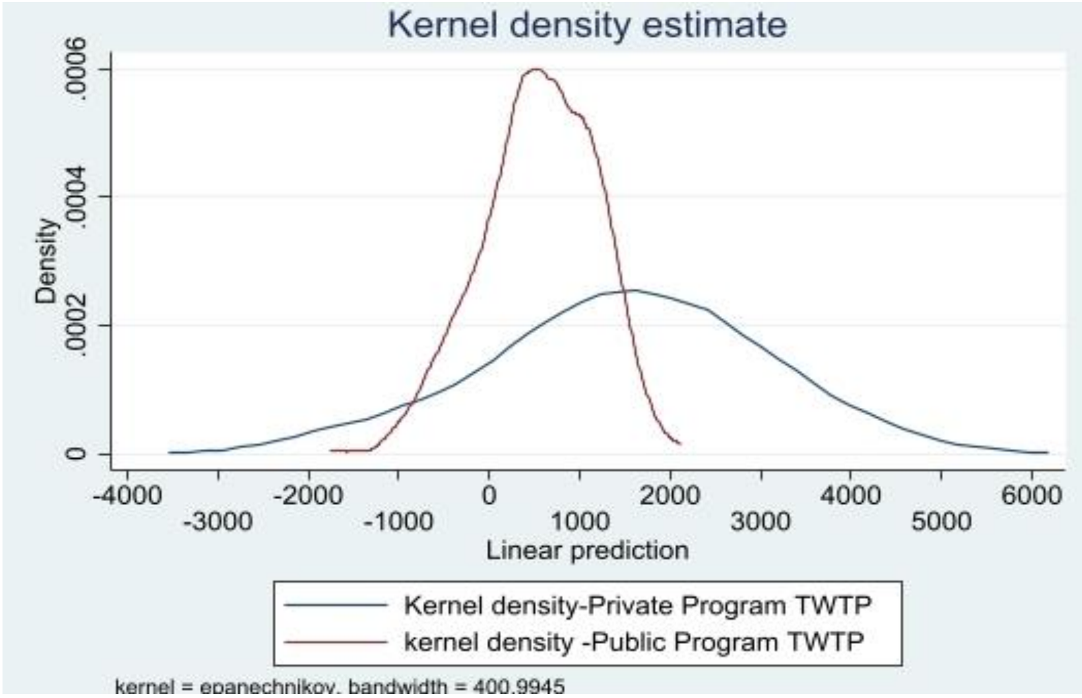
Following the imputation, the Analysis phase involves running the intended econometric model on each of the imputed datasets to obtain estimates of coefficients and standard errors. The final stage, Pooling, aggregates these estimates across all imputed datasets to produce a single set of MI-estimated coefficients, thereby enabling robust statistical inference despite the initial missing data. The application of MICE, in line with the practices of Van Buuren & Oudshoorn (1999) and further clarified by Royston and White (2011), offers a rigorous approach to preparing data for subsequent modeling, ensuring the integrity of the analysis in the presence of missing values.

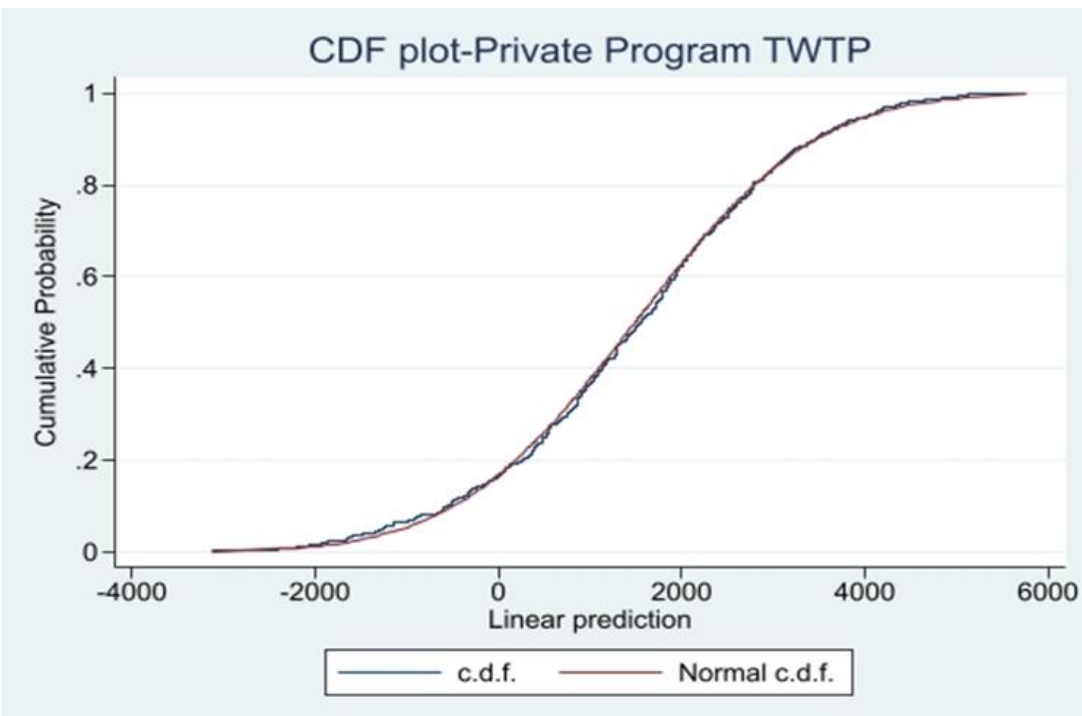
A.1.15 Factor Analysis

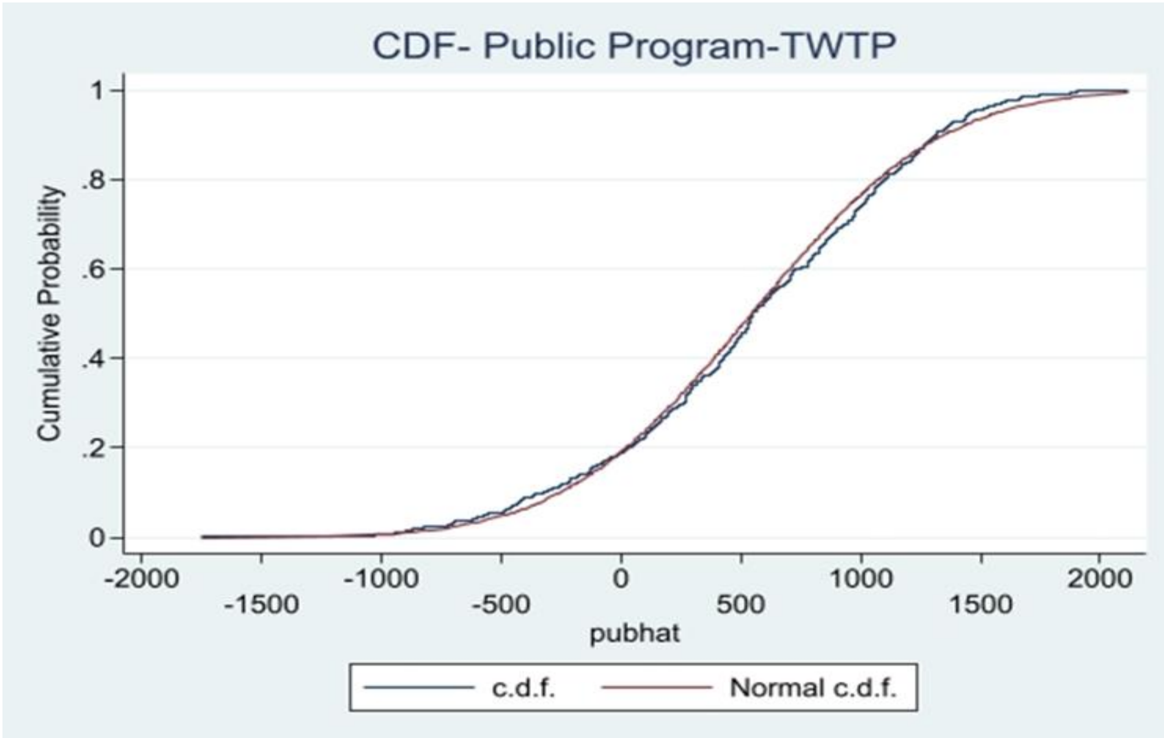
Table 17 Exploratory Factor Analysis

	Variable	Description	Factor Loading
F1	Feel About the Community ***(COMMATTACHMENT)***	Q1) Very attached to the community	0.6920
		Q2) Identify strongly with the community.	0.7527
		Q3) No place comparable to this community	0.5854
		Q4) Best place to live.	0.5855
		Q5) Tight and knit community.	0.4681
		Q6) Know nearest neighbor well	0.3581
F2	Community wildfire planning (***COMMWFPLANNING)***	Q7) Discussed coordinating to reduce WFR.	0.7681
		Q8) coordinated actions to reduce WFR.	0.7755
		Q9) plan to coordinate DFS.	0.7176
		Q10) Is there general agreement about WFR.	0.5896
		Q11) Is there agreement for taking actions	0.5937
F3	Government policy for fire safe investment (***GOVTPOLICYFSI)***	Q12)agree to coordinate creating and Maintaining DFS	0.4349
		Q13)communities should invest more on education.	0.5098
		Q14)create and maintain DFS should get tax breaks.	0.4327
		Q15)maintain DFS should have lower insurance rates.	0.4078
		Q16)required to law to maintain defensible space.	0.5892
		Q17)government should subsidize creation of DFS	0.5437
F4	Attitudes public land manage agencies. (***LANDMANAGEMENT)***	Q18)share similar values as the agencies.	0.8823
		Q19)share similar goals.	0.9008
		Q20)think similar way.	0.9119
		Q21)same priorities as the agencies	0.8222
F5	Household fire safe investment (***HOUSEHOLDFSI)***	Q22)respondents house has fire resistant roof.	0.3082
		Q23)respondents house has fire resistant sidings.	0.3434
		Q24)respondents house has eaves enclosed.	0.3669
		Q25)respondents house has spark arresters' chimneys.	0.4146
		Q26)respondents house has vents covered with mash.	0.3966
		Q27)respondents house has skirting under decks	0.3122
F6	Utility Hits and Aesthetic Privacy ***(UTILITYAP)***	Q28)Concern for loss of house attractiveness	0.8122
		Q29)Concern for loss of landscaping attractiveness	0.8014
		Q30)Concern for loss of privacy	0.5767
F8	Private Program value (***PROGRAMVALUE)***	Q31)program is of no value.	0.6003
		Q32)feels risk reduction wouldn't be achieved.	0.4405
		Q33)reduction in risk is too small	0.5848
F9	Public Program value (***PROGRAMVALUE)***	Q34)program is of no value.	0.5755
		Q35)risk reduction wouldn't be achieved.	0.4540
		Q36)reduction in risk is too small	0.6157

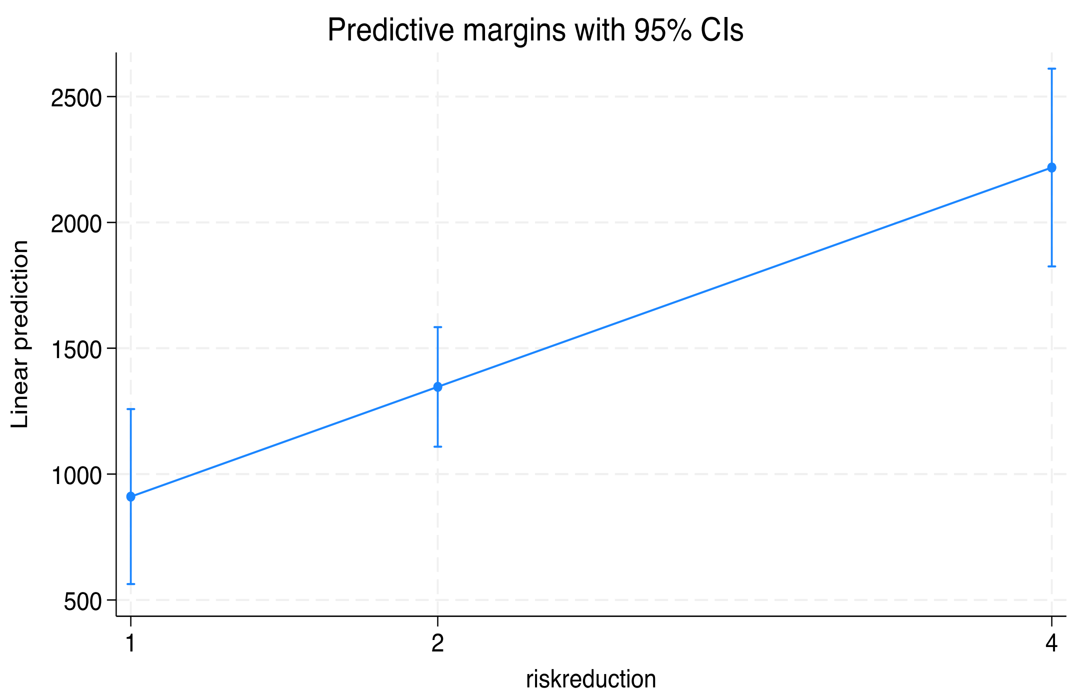
A.1.16 WTP Distribution

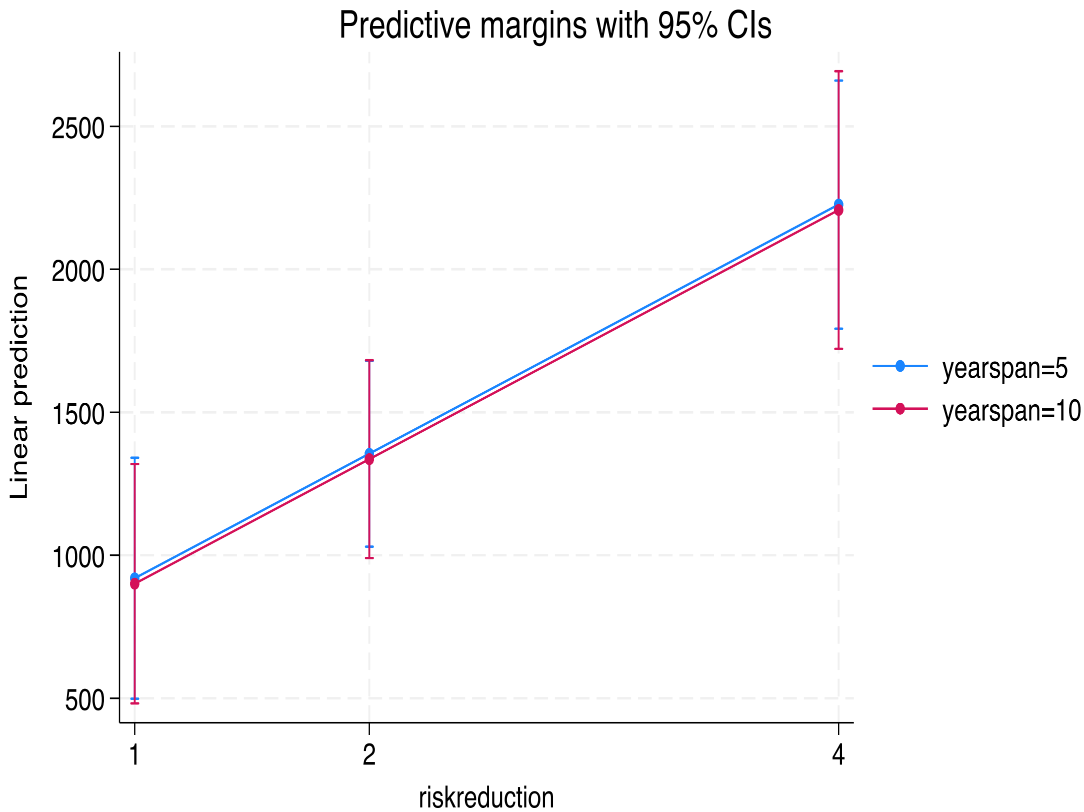






A.1.17 Private and Public WTP, Predictive Margin




Predictive margins with 95% CIs
Linear prediction
2500
2000
1500
1000
500
1
2
4
riskreduction
yearspan=5
yearspan=10

2 Chapter 2 Interest Rate Determination in an Overlapping Generations Model with Capital Accumulation and Bubbles that May Burst

(with Mark Pingle)

Abstract

The Weil (1987) overlapping generations (OG) model, which contains an exogenous probability that the economy's bubble may burst, is extended to allow capital accumulation as in Banerjee (2021). Like Weil (1987), we find that the rate of return on the bubble asset must generally be greater than the rate of return on the capital backed asset. Like Banerjee (2021), but in contrast to Weil (1987), a gap between the interest rate paid on the capital backed asset and capital rental rate must occur. Thus, we provide enhanced knowledge of how the rates of return earned on assets relate to the productivity of capital and the capital rental rate.

2.1 Introduction

Paul Samuelson (1958) developed the overlapping generations (OG) model to obtain a general equilibrium model of interest rate determination. His primary finding was a "biological theory of interest." Samuelson's work led David Gale (1973) to conclude that one cannot expect to understand interest rate determination without considering overlapping generations. In contrast to the work of Fisher (1930), which concluded the interest rate level primarily depended upon the degree to which people are impatient, valuing present consumption more than future consumption, the overlapping generations model relates the interest rate to population growth because the saving of larger young generation can provide a return to the older generation that is earning a return off of its young-aged saving.

Gale's (1973) work shows that the bubbles, inefficiency, and multiple equilibria Sameulson found for his economy depended upon the model parameters be such that people desire to save when young. Since then, scholars have been interested in this "Sameulson Case" because we have observed asset bubbles bursting at different points in time, causing economic turmoil.

Peter Diamond (1965) extended the Samuelson (1958) model by introducing production into the OG framework. Diamond did not recognize the possibility of a bubble, but Jean Tirole (1985) extended Diamond's economy to allow for a bubble. The importance of moving from the pure consumption economies of Samuelson (1958) and Gale (1973) by adding product is it adds the possibility that the productivity of capital can influence the interest rate. As with the Samuelson and Gale pure consumption economies, the path for capital, the capital rental rate, and the interest rate paid on saving in Tirole economy depend upon the size of the bubble that forms. There are multiple equilibrium, an infinite number, and the particular equilibrium path depends upon the size of the initial bubble. Importantly for our work here, the capital rental rate in equilibrium is equal to the interest rate paid on capital.

Philippe Weil (1987) extended Tirole's (1985) model by adding uncertainty. While Tirole (1985) assumed a bubble would not burst once formed, Weil assumed there is a certain exogenous probability that a bubble could burst in any given period. This probability affects the behavior of savers because they must consider the risk that the value of the bubble asset drops to zero if the bubble bursts. The consequence of recognizing this rise is that the rate of return on the bubbly asset must include a risk premium. The risk premium compensates investors for the chance that they could lose their entire investment. The higher the probability of a burst, the higher the risk premium investors must demand in order to hold the bubble asset along with the capital backed asset. A primary finding of Weil is, as the confidence in the bubble asset decreases, the Tirole (1985) Economy where bubbles can form moves toward the Diamond (1965) Economy where they do not form. If confidence is low enough, or we can say if the belief that the bubble will burst is high enough, then no bubble can form.

The contribution of this paper is to extend the Weil (1987) model to allow capital accumulation as in Banerjee (2021). Following Banerjee, we do not restrict the interest rate to equal the rental rate on capital as Weil does, who follows Tirole (1985) and Diamond (1965). Following Banerjee (2021), we add a product market clearing condition to the capital market clearing condition so the interest rate paid on the capital backed asset can be determined separate from the capital rental rate. The model here is relative to Banerjee (2021) because Banerjee follows Tirole (1985) and assumes the consumer has total confidence that any bubble will not burst. Instead, follow Weil (1987) and assume there is a positive, exogenously given probability that the bubble will burst in each period.

What do we learn?

First, as is true for the models of Diamond (1965), Tirole (1985) and Weil (1987), we find three qualitative paths exist from any possible initial condition. These three types of paths depend upon the initial bubble. If the initial bubble is large enough, then the model diverges, and the economy eventually crashes when the young age cannot save enough to cover the bubble required to meet the obligation to old age consumers. That is, paths associated with large initial bubbles are not equilibrium paths. If the bubble is small enough, then the economy converges to the Diamond (1965) no bubble steady state. There are an infinite number of these equilibrium paths. Between these two types of paths, there is path where the initial bubble is "just right" in these sense that the resulting equilibrium path is Pareto optimal and converges to the golden rule steady state, where the bubble is positive. Like Weil (1987), we find that the rate of return on the bubble asset must be greater than the rate of return on the capital backed asset as long as there is some probability that the bubble will burst. The two rates of return will be same when the consumer is certain the bubble will not burst.

Second, in the golden rule steady state, the interest rate paid on savings is equal to the rate of labor growth if the consumer has total confidence that the bubble asset will not burst. That is, interest rate determination entirely conforms to the Samuelson (1958) biological theory of interest. As the confidence in the bubble asset decreases, then the interest rate in this golden rule steady state decreases. Also, when the consumer is not totally confident in the bubble asset, the interest rate in the golden rule steady state is affected by the marginal propensity to save and the elasticity of output with respect to capital relative to the elasticity of output with respect to labor. An increase in the marginal propensity to save decreases the interest rate. Alternatively, the interest rate increases when the elasticity of output with respect to capital increases relative to the elasticity of output with respect to labor. For the interest rate to be positive, the rate of labor growth n must be positive and not too small, and confidence in the bubble asset also cannot be too small.

However, when we compare the interest rate to the capital rental rate, we find that our results differ from Weil (1987). As in Banerjee (2021), we find that capital accumulation causes a gap to occur between the interest rate and capital rental rate. We find interest rate and capital rental rate are equal in the special case where capital entirely depreciations from one period to the next and in the special case where there is total confidence in the bubble asset. Otherwise, capital rental rate is always greater than interest rate. The gap between interest rate and capital rental rate is increases when the depreciation rate is smaller, when the elasticity of output with respect to capital is smaller (which implies elasticity of output with respect to labor is bigger), when the confidence in the bubble asset decreases, and when the marginal propensity to save increases.

The paper unfolds as follows. In section 2, we present a modified version of the Weil (1987) economy. In section 3, we derive the dynamic equations that describe the equilibrium path for the economy, and we present the solutions for the model's endogenous variables. In section 4, we presented a steady state analysis, and we show that the steady state for this model is stable. Section 5 concludes.

2.2 The Model

The generation t consumer obtains utility $U(c_t^y, c_{t+1}^o)$ from young age consumption c_t^y and old age consumption c_{t+1}^o. The consumer receives the wage w_t in young age, but no wage income in old age. To obtain income in old age, the consumer must save. To save, the young age consumer has two options: Place saving into a capital asset or a bubble asset. Saving x_t placed in the capital asset earns interest rate i_{t+1} from period t to period $t+1$. Following Weil (1987, p. 5), the young consumer can also save by buying m_t units of the bubble asset in period t at the price p_t. This implies the real bubble given in terms of good t is

(1) $b_t = p_t m_t$,

the saving level of the generation t consumer is

(2) $s_t = x_t + b_t$

and young age consumption is

(3) $c_t^y = w_t - x_t - p_t m_t$

The level of old age consumption depends upon whether or not the bubble bursts. Following Weil (1987), we assume the bubble does not burst with probability q. If the bubble does not burst, the consumer sells the m_t units of bubble asset in period $t+1$ at the price p_{t+1}. Therefore, if the bubble does not burst, the old age consumption level is

(4) $c^{o+}_{t+1} = x_t\,[1 + i_{t+1}] + p_{t+1}m_t$,

whereas if the bubble bursts the old age consumption level is

(5) $c^{o-}_{t+1} = \; x_t\,[1 + i_{t+1}]$.

The generation t consumer maximizes expected utility $V = qU\left(c^y_t, c^{o+}_{t+1}\right) + [1 - q]U\left(c^y_t, c^{o-}_{t+1}\right)$ by optimally choosing the bubbly asset saving level m_t and the capital asset saving level x_t. Optimization yields the first order conditions

(6) $U_{c^y} = q\,\frac{p_{t+1}}{p_t}U_{c^{o+}}$

and

(7) $[1 + i_{t+1}][q\,U_{c^{o+}} + [1 - q_t\,]U_{c^{o-}}] = U_{c^y}$.

Together, conditions (1)-(7) determine b_t, s_t, c^y_t, c^{o+}_{t+1}, c^{o-}_{t+1}, m_t, and x_t as they depend upon $w_t,\; p_t$, p_{t+1}, i_{t+1}, and q.

Following Weil (1987), we can obtain specific consumer demand functions by using the log linear utility function $U\left(c^y_t, c^o_{t+1}\right) = ln\left(c^y_t\right) + \beta ln(c^o_{t+1})$, where β is the discount factor the generation t consumer applies to old age consumption. Letting $h = \frac{\beta}{1+\beta}$ denote the marginal propensity to save, and letting $z_t = \frac{1-q}{1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]}$ denote the share of saving devoted to the capital asset, we can express the solutions for the seven variables as:

(8) $s_t = \mathrm{h}w_t$,

(9) $b_t = [1 - z_t]\mathrm{h}w_t$,

(10) $\quad x_t = z_t h w_t$,

(11) $\quad c^y_t = \frac{hw_t}{\beta}$,

(12) $c_{t+1}^{o+} = hw_t q \frac{p_{t+1}}{p_t}$,

(13) $c_{t+1}^{o-} = [1 + i_{t+1}]z_t hw_t$,

and

(14) $m_t = \frac{[1-z_t]}{p_t}\mathrm{h}w_t$.

The population in period t includes L_t young age consumers and L_{t-1} old age consumers, and the population of generation t consumers grow at rate n, so $L_t = [1+n]L_{t-1}$. Total period t production depends upon labor and capital according to $Y_t = F(K_t, L_t)$. Defining capital and output per young age consumer as $k_t = K_t/L_t$ and $y_t = Y_t/L_t$, the assumption that production exhibits diminishing returns to the inputs and constant returns to scale implies production per young consumer can be presented as $y_t = f(k_t)$, with $f'(k_t) > 0$, and $f''(k_t) < 0$. Given that capital depreciates at rate δ, firm profit maximization implies the "net" rental rate paid on capital is $r_t = f'(k_t) - \delta$, and the wage rate paid on labor is $w_t = y_t - [r_t + \delta]k_t$. Following Diamond (1965), using the particular production function $f(k_t) = Ak_t^{\alpha}$, the producer conditions become

(15) $y_t = Ak_t^{\alpha}$, $0 < \alpha < 1$,

(16) $r_t + \delta = \alpha A k_t^{\alpha-1}$,

and

(17) $w_t = [1-\alpha]Ak_t^{\alpha}$.

Following Weil (1987), we assume the money supply is constant and equal to one unit of money. Consequently, the bubble asset market is in equilibrium when $L_t m_t = 1$. Thus, in the next period, $L_{t+1}m_{t+1} = 1$. Using these two conditions, we find $L_t m_t = L_{t+1}m_{t+1}$, which further implies $p_t m_t = [1+n]\frac{p_t}{p_{t+1}} p_{t+1} m_{t+1}$. Using the bubble definition (1), we then obtain

(18) $b_{t+1} = \frac{1}{1+n}\frac{p_{t+1}}{p_t} b_t$.

Investment accumulates as capital according to $K_{t+1} = K_t + I_t - \delta K_t$, where I_t is the investment level and δ is the depreciation rate. The capital market is in equilibrium when the investment level I_t is equal to the saving level $L_t x_t$ the consumer places in the capital asset. Thus, $K_{t+1} - [1-\delta]K_t = L_t x_t$, which implies

(19) $x_t = [1+n]k_{t+1} - [1-\delta]k_t$

Weil (1987) followed Tirole (1985) and Diamond (1965) in that (1) he did not include a product market clearing condition, (2) he restricted the interest rate i_t to equal the rental rate on capital r_t, and (3) he did assumed capital is consumed rather than accumulates. However, Banerjee (2021) shows that when capital accumulates capital market clearing does not always imply product market clearing as long as i_t is restricted to equal r_t. As long as capital old age consumers consume capital each period as part of the return promised on saving, the product market clears when the capital market clears, implying the models of Weil, Tirole, and Diamond is sound. However, if we desire a model where capital is allowed to accumulate, Banerjee shows capital market clearing implies product market clearing only in two special cases i_t is restricted to equal r_t: (1) when capital totally depreciates (i.e., $\delta = 1$); and (2) when the economy is in the golden rule so $r = n$). For both the product market and capital market to clear when capital accumulations, the interest rate i_t must be allowed to differ from the capital rental rate r_t. We desire to allow capital accumulation, so we follow Banerjee (2021) and not only add a product market clearing condition but also allow the interest rate to deviate from the capital rental rate. The proper product market clearing condition is

(20) $y_t = c_t^y + \frac{c_t^o}{1+n} + [1+n]k_{t+1} - [1-\delta]k_t.$

As the economy enters period t, the variables k_t, b_t, and p_t are predetermined. Equations (8)-(20) determine the paths of the variables b_{t+1}, s_t, c_t^y, c_{t+1}^{o+}, c_{t+1}^{o-}, m_t, x_t, y_t, r_t, w_t, p_{t+1}, k_{t+1}, and i_{t+1} as they depend upon q, n, δ, A, h and α.

2.3 The Dynamic Equilibrium Path

Adding b_t to both sides of the capital market clearing condition (19), we obtain $x_t + b_t = [1+n]k_{t+1} - [1-\delta]k_t + b_t$. Using conditions (8)-(10) we can replace $x_t + b_t$ with the savings variable to obtain $s_t = [1+n]k_{t+1} - [1-\delta]k_t + b_t$. This condition indicates the saving of the young generation consumer is used to finance both the capital accumulation of the firm and the period t bubble. Using the savings function (8), we obtain $\mathrm{h}w_t = [1+n]k_{t+1} - [1-\delta]k_t + b_t$. Finally, using condition (17) to eliminate the wage w_t and rearranging, we find,

(21) $k_{t+1} = \mathrm{h}\left[\frac{1-\alpha}{1+n}\right]Ak_t^\alpha + \left[\frac{1-\delta}{1+n}\right]k_t - \frac{1}{1+n}b_t.$

Condition (21) determines k_{t+1} from k_t and b_t.

In the Appendix we show, the price ratio $\frac{p_{t+1}}{p_t}$ is given by

(22) $\frac{p_{t+1}}{p_t} = \frac{[1+n]\alpha Ak_{t+1}^\alpha}{q[h[1-\alpha]Ak_t^\alpha]-b_t},$

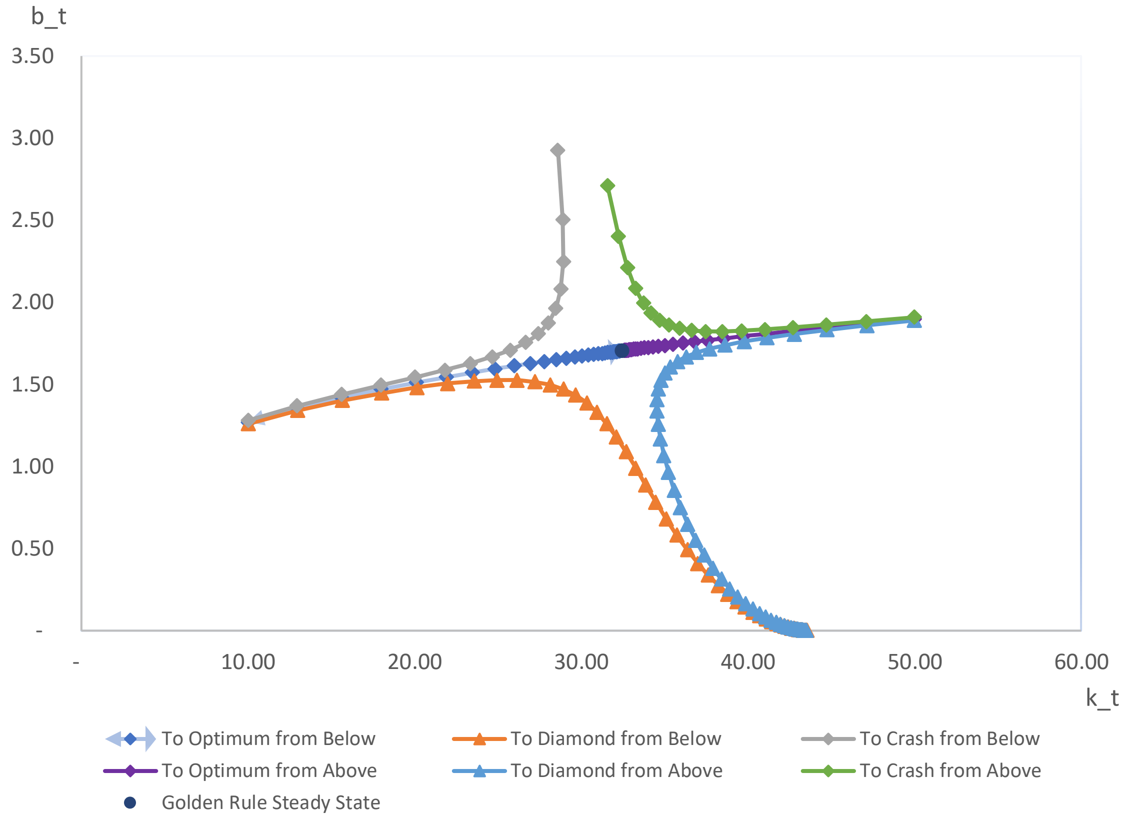
and we show the gross rate of return $1 + i_{t+1}$ on the capital asset is given by

(23) $1 + i_{t+1} = \frac{[1+n]\alpha Ak_{t+1}^\alpha}{q[h[1-\alpha]Ak_t^\alpha]-b_t}\left[1 - \frac{[1-q_t]h[1-\alpha]Ak_t^\alpha}{[1+n]k_{t+1}-[1-\delta]k_t}\right].$

Together, conditions (18), (21), (22), and (23) are a reduced form version of the model, providing the paths followed by b_{t+1}, k_{t+1}, p_{t+1}, and i_{t+1} as they depend upon the predetermined variables b_t, k_t, p_t and the exogenous variables q, n, δ, A, h and α. By plotting the paths of these four variables, we can examine how the economy unfolds from different initial conditions for particular parameter values.

Figure 8 presents six different paths followed by the two core variables of the model: The capital stock level k_t and the bubble level b_t. Three of the paths initiate from a capital level below the Diamond (1965) steady state capital level, and three paths initiate from above. From either initial condition, three qualitative possibilities exist, and we present examples: (1) the path is Pareto optimal and converges to the steady state with a positive bubble, (2) the path converges to the Diamond (1965) no bubble steady state, or (3) the model diverges, and the economy eventually crashes when the young age cannot save enough to cover the bubble required to meet the obligation to old age consumers.

Figure 8 Capital Bubble Paths



For the cases where the initial capital stock is below the Diamond steady state capital level, Figure 9 presents the paths of the gross rate of return $1 + i_{t+1}$ on the capital asset and the gross rate of return $\frac{p_{t+1}}{p_t}$ on the bubble asset. A number of facts are of interest. First, the rate of return on the bubble asset is always greater than that for the capital backed asset. This must be true for people to be willing to hold the bubble asset that may become valueless along with the capital backed asset that will surely maintain its value. Second, if the economy diverges and eventually crashes, the gross rates of return for each asset increase at an increasing rate en route to the crash. Third, if the economy does not crash, then the gross rates of return converge to positive values. In one special case where the economy does not crash, the economy follows the Pareto optimal path and converges to the golden rule allocation with a positive bubble. In all of the other cases where the economy does not crash, the economy follows a suboptimal path to the Diamond no bubble steady state. Compared to the Pareto optimal path, the rates of return on any suboptimal path for both the bubble asset and capital backed asset are lower, for a given capital level.

Figure 9 Gross Rate of Return – Capital Asset and Bubble Asset

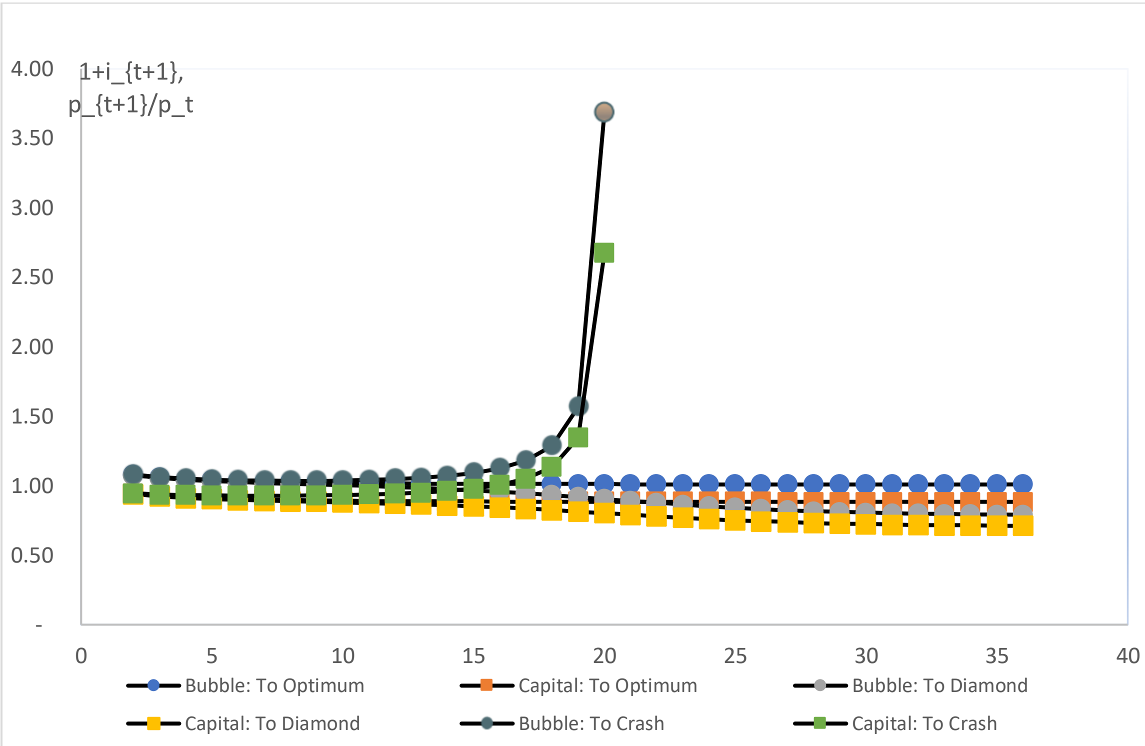


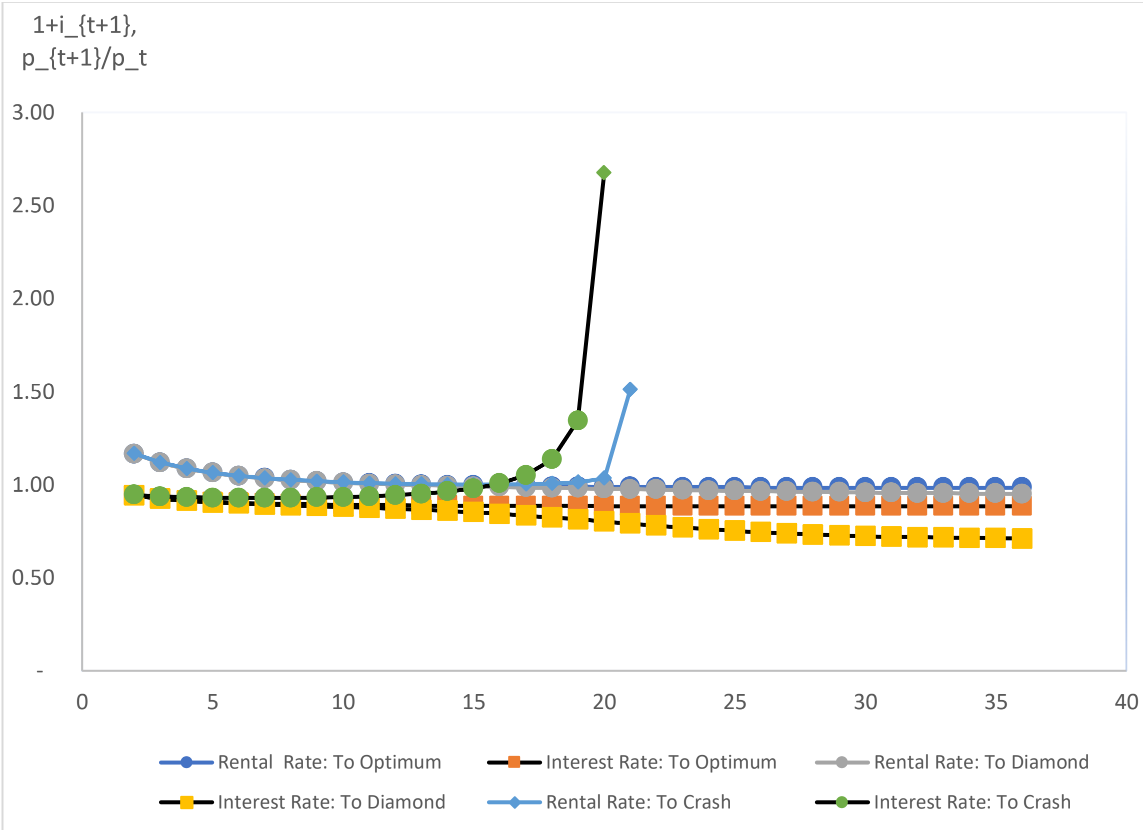
Figure 10 shows the relationship between the gross interest rate $1 + i_{t+1}$ and the gross capital rental rate $1 + r_{t+1}$ under the assumption $0 < \delta < 1$. In the Appendix, we show

(24) $$1 + i_{t+1} = \frac{\delta + r_{t+1}}{1 - \frac{[1-\delta]k_t}{[1+n]k_{t+1}}}.$$

From this condition, we know the complete depreciation case is a special case in that $\delta = 1$ implies $i_t = r_t$ in each time period t. A second special case is the Golden Rule steady state. In a steady state, $k_t = k_{t+1} = k$, so condition (24) becomes $1 + i = [1 + n]\frac{\delta + r}{n + \delta}$. In the Appendix, we show the Golden Rule steady state is associated with $r = n$. When $r = n$, the condition $1 + i = [1 + n]\frac{\delta + r}{n + \delta}$ reduces to $i = n$. Thus, $i = r$ will hold for all time periods if the economy begins and remains in the Golden Rule steady state. Except for these two special cases, the interest rate level will deviate from the capital rental rate.

Figure 10 shows the relationship between the interest rate and capital rental rate when the initial capital level is below the Diamond Steady State capital level, for three cases. In one case, the economy sub optimally converges to the Diamond Steady State. In the Appendix, we show $r < n$ in this steady state. Condition (24) therefore indicates the economy converges to a state where the interest rate is less than the capital rental rate (i.e., $i < r$). In a second case, the economy optimally converges to the Golden Rule Steady State. Along this path, $i_t < r_t$ holds for each time period t, but the interest rate and capital rental rate are each converging to the labor growth rate n. In the third case, the economy eventually implodes because the bubble bursts. Along this path the interest rate is less than the capital rental rate, but both diverge upward as the economy approaches collapse.

Figure 10 Relationship between Interest Rate and Capital Rental Rate



As the probability q that the bubble does not burst increases toward 1, the rate of return on the bubbly asset approaches the rate of return on the capital backed asset. The greater confidence in the bubble, allows for a larger initial bubble. Given an increase in the confidence level q, some initial bubbles that had previously been associate with paths leading a bubble burst and disequilibrium become feasible, and the largest of these new feasible bubbles is the new optimal initial bubble. In any given time period, the increase in the probability q increases the level of capital employed, decreases the bubble, decreases the rental rate on capital, decreases the real interest rate paid on savings, and increases the gap between the capital rental rate and real interest rate.

2.4 The Steady State Analysis

In the Appendix, it is shown the model has two steady states. In one steady state, the Diamond (1965) no bubble steady state, we find

(25) $\bar{b} = 0$ and $\bar{k} = \left[\frac{h[1-\alpha]A}{n+\delta}\right]^{\frac{1}{1-\alpha}}$

The gross rate of return on saving placed in capital is $1 + \bar{\imath} = \frac{\alpha[1+n]}{h[1-\alpha]}$, while the gross rate of return on saving placed in the bubbly asset is $\frac{p_{t+1}}{p_t} = \frac{\alpha[1+n]}{qh[1-\alpha]}$. The capital rental rate is $\bar{r} = \frac{\alpha[\delta+n]}{\mathrm{h}[1-\alpha]} - \delta$.

Given these results, we find the difference between two rates of return is $\frac{p_{t+1}}{p_t} - [1+i] = \frac{[1-q]\alpha[1+n]}{qh[1-\alpha]} \geq 0$. As noted in the discussion surrounding Figure 2, the rate of return on the bubble asset is greater as long as there is some probability that the bubble will burst (i.e., $0 \leq q < 1$). This difference will be greater when the labor growth rate n is bigger, when the elasticity of output with respect to capital α is bigger, when the marginal propensity to save h is smaller. The two rates of return will be same when q equals one, or when it is certain the bubble will not burst.

Another question of interest is, "When will the rate of return on saving i that flows into capital be positive?" The answer is when $i = \frac{\alpha[1+n]}{h[1-\alpha]} - 1 > 0$. In the Appendix, we show $\mathrm{q} > \frac{\alpha}{h[1-\alpha]}$ must hold in order for bubbles to form. If $\frac{\alpha}{h[1-\alpha]} \geq 1$, then no bubble can form even when $\mathrm{q} = 1$, meaning consumers have full confidence that a bubble will not burst. Following Diamond (1965), Tirole (1985), and Weil (1987), we assume $\frac{\alpha}{h[1-\alpha]} < 1$

so bubbles can form in our model if consumers have enough confidence that a bubble will not burst. Using some algebraic transformation, we can rewrite the steady state interest level as $i = \frac{\alpha}{h[1-\alpha]}\left[1 + n - \frac{h[1-\alpha]}{\alpha}\right]$, which is positive when $n > \frac{h[1-\alpha]}{\alpha} - 1$. The assumption $\frac{\alpha}{h[1-\alpha]} < 1$ implies $\frac{h[1-\alpha]}{\alpha} - 1 > 0$. Thus, we learn that the population growth rate n must be positive and large enough in order to have a positive real rate of return earned on saving flowing into capital in this Diamond $b = 0$ steady state.

Noting that $h = \frac{\beta}{1+\beta}$, we can restate the real rate of return earned on capital as $i = \frac{\alpha[1+\beta][1+n]}{[1-\alpha]\beta} - 1$. Examining this condition, we find that this rate of return will be higher when the elasticity of output with respect to capital α is higher, when the generation t consumer discounts old age consumption more significantly relative to young age consumption (i.e., when β is smaller), and when the rate of population growth n is higher.

Note that the degree of confidence q in the bubble does not influence either the interest rate i nor the rental rate of capital r. As noted above the confidence level q affects the rate of return on the bubbly asset such that the degree to which the rate of return on the bubbly asset is greater than the rate of return on the capital asset increases as q decreases from 1. When q decreases to the level $\frac{\alpha}{h[1-\alpha]}$, the rate of return on bubbly asset increases to $\frac{p_{t+1}}{p_t} = 1 + n$. In this case, the maximum difference between the two gross rate of return would be $\frac{p_{t+1}}{p_t} - [1 + i] = \left[1 - \frac{\alpha}{h[1-\alpha]}\right][1 + n]$.

Comparing the capital rental rate r to the interest rate i earned on saving placed into capital, we find $r - i = \frac{[1-\delta][h[1-\alpha]-\alpha]}{h[1-\alpha]}$. The assumption $\frac{\alpha}{h[1-\alpha]} < 1$ implies

$h[1-\alpha]-\alpha>0$, which implies $r>i$ will generally hold. The difference $h[1-\alpha]-\alpha$ is the measure of the degree to which bubble can form. Thus, the difference between r and i will be greater when $h[1-\alpha]-\alpha$ is greater. Also, the difference between r and i will be greater when the elasticity of output with respect to capital α is smaller (implying the elasticity of output with respect to labor $1-\alpha$ is bigger) and when the marginal propensity to save h is bigger. One special case is the complete depreciation case where $\delta=1$. In that case, $r=i$ in the Diamond steady state. Thus, we learn the difference between r and i depends upon the degree to which bubble can form and the depreciation rate.

In the Appendix, we show the other steady state is the golden rule steady state where,

(26) $b=[qh[1-\alpha]-\alpha]A^{\frac{1}{1-\alpha}}\left[\frac{[[1-q]h[1-\alpha]+\alpha]}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$ and $k=\left[\frac{[[1-q]h[1-\alpha]+\alpha]A}{n+\delta}\right]^{\frac{1}{1-\alpha}}$

The gross rate of return on saving placed in capital is $1+i=[1+n]\left[1-\frac{1}{1+\frac{\alpha}{[1-q]\,h[1-\alpha]}}\right]$, while the gross rate of return for saving placed in the bubbly asset is $\frac{p_{t+1}}{p_t}=1+\text{n}$ The capital rental rate is $r=\frac{\alpha[n+\delta]}{[1-q]h[1-\alpha]+\alpha}-\delta$.

Given these results, we find the difference between two rates of return is $\frac{p_{t+1}}{p_t}-[1+i]=[1+\text{n}]\left[\frac{[1-q]h[1-\alpha]}{[1-q]h[1-\alpha]+\alpha}\right]>0$. As in the Diamond steady state, we find the rate of return on the bubble asset is greater as long as there is some probability that the bubble will burst (i.e., $0\leq q<1$). The two rates of return will be same when q equals one, or when it is certain the bubble will not burst.

In this steady state, the rate of return on saving will be positive when $i = [1+n]\left[1-\frac{1}{1+\frac{\alpha}{[1-q]\,h[1-\alpha]}}\right]-1>0$. In the appendix we show $\frac{1-q}{n}<\frac{\alpha}{h[1-\alpha]}<q\leq 1$ must hold in order for this inequality to hold. From this condition, we learn, a positive interest rate requires that the rate of labor growth n be positive and not too small. Also, the confidence level q cannot be too small.

Notice, the golden rule steady state interest rate level depends upon the confidence level q, while the diamond steady state interest rate level does not. As q decreases from 1 to 0, i decreases from n to $[1+n]\left[\frac{\alpha}{h[1-\alpha]+\alpha}\right]-1$. As with Diamond steady state interest rate level, the Golden rule steady state level increases with increases in the labor growth level n, increases in the elasticity of output with respect to capital α, and with decreases in the marginal propensity to save h.

In the Appendix, we also find the difference between capital rental rate r and the interest rate i is $r-i=\frac{[1-\delta]}{1+\frac{\alpha}{[1-q]h[1-\alpha]}}$. This condition implies $r>i$ will generally hold. There are two special cases where $r=i$ will hold. One is the complete depreciation case where $\delta = 1$, and the other is the case $q=1$ where consumers are entirely confident the bubble will not burst. As the fraction $\frac{\alpha}{h[1-\alpha]}$ decreases, bubble can more readily form, and as this fraction decreases the difference $r-i$ increases. Thus, we learn the difference between r and i in the golden rule steady state depends upon the degree to which bubbles can form, the degree rate of depreciation and the confidence that bubble will not burst.

2.5 Conclusion

As is true for the models of Diamond (1965), Tirole (1985) and Weil (1987), we find three qualitative paths exist from any possible initial condition. These three types of paths depend upon the initial bubble. If the initial bubble is larger enough, then the model diverges, and the economy eventually crashes when the young age cannot save enough to cover the bubble required to meet the obligation to old age consumers. That is, paths associated with large initial bubbles are not equilibrium paths. If the bubble is small enough, then the economy converges to the Diamond (1965) no bubble steady state. There are an infinite number of these equilibrium paths. Between these two types of paths, there is path where the initial bubble is "just right" in these sense that the resulting equilibrium path is Pareto optimal and converges to the golden rule steady state, where the bubble is positive. Like Weil (1987), we find that the rate of return on the bubble asset must be greater than the rate of return on the capital backed asset as long as there is some probability that the bubble will burst. The two rates of return will be same when the consumer is certain the bubble will not burst.

In the golden rule steady state, the interest rate i paid on savings is equal to the rate of labor growth n if the consumer has total confidence that the bubble asset will not burst. That is, interest rate determination entirely conforms to the Samuelson (1958) biological theory of interest. As the confidence in the bubble asset decreases, then the interest rate in this golden rule steady state decreases, so that we have $i < n$ and an increase gap between the two. Also, when the consumer is not totally confident in the bubble asset, the interest rate in the golden rule steady state is affected by the marginal propensity to save and the elasticity of output with respect to capital relative to the elasticity of output with respect to labor. An increase in the marginal propensity to save decreases the interest rate. Alternatively, the interest rate increases when the elasticity of output with respect to capital increases relative to the elasticity of output with respect to labor. For the interest rate to be positive, the rate of labor growth n must be positive and not too small, and confidence in the bubble asset also cannot be too small.

However, when we compare the interest rate i to the capital rental rate r, we find that our results differ from Weil (1987). As was true for Banerjee (2021), we find that capital accumulation causes a gap to occur between the interest rate and capital rental rate. We find $i = r$ in the special case where capital entirely depreciations from one period to the next and in the special case where there is total confidence in the bubble asset. Otherwise, $r > i$ will hold. The gap between r and i increases when the depreciation rate is smaller, when the elasticity of output with respect to capital is smaller (which implies elasticity of output with respect to labor is bigger), when the confidence in the bubble asset decreases, and when the marginal propensity to save increases.

In the diamond no bubble steady state, the interest rate $\bar{\imath}$ paid on savings is less than the rate of labor growth n. The consumer confidence in the bubble asset only affects the rate of return on the bubbly asset. It doesn't affect the interest rate $\bar{\imath}$, the capital rental rate $\bar{r}$, nor the capital level $\bar{k}$. In general, the capital rental rate will be greater than the interest rate. The exception occurs when capital totally depreciates from one period to the next. In that case, the two are equal.

In the diamond no bubble steady state, we find the difference between two rates of return is $\frac{p_{t+1}}{p_t} - [1 + i] = \frac{[1-q]\alpha[1+n]}{qh[1-\alpha]} \geq 0$. As noted in the discussion surrounding Figure

2, the rate of return on the bubble asset is greater as long as there is some probability that the bubble will burst (i.e., $0 \leq q < 1$). This difference will be greater when the labor growth rate n is bigger, when the elasticity of output with respect to capital α is bigger, when the marginal propensity to save h is smaller. The two rates of return will be same when q equals one, or when it is certain the bubble will not burst.

A.2 Appendix (Chapter 2)

A.2.1 Proof that $\frac{p_{t+1}}{p_t} = \frac{[1+n]\alpha Ak_{t+1}^{\alpha}}{q[h[1-\alpha]Ak_t^{\alpha}]-b_t}$ and $1 + i_{t+1} = \frac{[1+n]\alpha Ak_{t+1}^{\alpha}}{q[h[1-\alpha]Ak_t^{\alpha}]-b_t}\left[1 - \frac{[1-q_t]h[1-\alpha]Ak_t^{\alpha}}{[1+n]k_{t+1}-[1-\delta]k_t}\right]$

Starting with (19), we have $x_t = [1+n]k_{t+1} - [1-\delta]k_t$. Using capital supply condition (10), we obtain $z_t hw_t = [1+n]k_{t+1} - [1-\delta]k_t$. Using wage condition (17), we obtain $z_t h[1-\alpha]Ak_t^{\alpha} = [1+n]k_{t+1} - [1-\delta]k_t$. Using the definition of z_t, we obtain $\left[\frac{1-q}{1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]}\right]h[1-\alpha]Ak_t^{\alpha} = [1+n]k_{t+1} - [1-\delta]k_t$. Solving for $1+i_{t+1}$, we obtain

(A1) $1 + i_{t+1} = \frac{p_{t+1}}{p_t}\left[1 - \frac{[1-q]h[1-\alpha]Ak_t^{\alpha}}{[1+n]k_{t+1}-[1-\delta]k_t}\right]$

Next, using with the capital market clearing condition (19), we have

$x_t = [1+n]k_{t+1} - [1-\delta]k_t$. Add $c_t^y + \frac{c_t^o}{1+n} - y_t$ to both sides.

$$x_t + c_t^y + \frac{c_t^o}{1+n} - y_t = c_t^y + \frac{c_t^o}{1+n} + [1+n]k_{t+1} - [1-\delta]k_t - y_t$$

Given that the product market clearing condition (20) implies $0 = c_t^y + \frac{c_t^o}{1+n} + [1+n]k_{t+1} - [1-\delta]k_t - y_t$, we know the product market clears if and only if $x_t + c_t^y + \frac{c_t^o}{1+n} - y_t = 0$

Using (10) and (11), (15), (16), (17), we obtain $z_t hw_t + \frac{hw_t}{\beta} + \frac{c_t^o}{1+n} - [w_t + [r_t+\delta]k_t] = 0$. Rearranging, we obtain $z_t hw_t + \frac{1}{1+\beta}w_t - \frac{1+\beta}{1+\beta}w_t + \frac{c_t^o}{1+n} - [r_t+\delta]k_t = 0$, which implies $z_t hw_t. - hw_t + \frac{c_t^o}{1+n} - [r_t + \delta]k_t = 0$. Using bubble demand condition (9), this becomes $-b_t + \frac{c_t^o}{1+n} - [r_t + \delta]k_t = 0$, which implies $\frac{c_t^o}{1+n} - b_t = [r_t + \delta]k_t$. Moving forward one period, we then obtain

(A2) $\frac{c_{t+1}^o}{1+n} = [r_{t+1} + \delta]k_{t+1} + b_{t+1}$

Next, using (12), we know $c_{t+1}^{o+} = hw_t q \frac{p_{t+1}}{p_t}$. Using (10) to eliminate w_t, we obtain $c_{t+1}^{o+} = \frac{x_t}{z_t} q \frac{p_{t+1}}{p_t}$. Using the capital market clearing condition (19), we obtain $c_{t+1}^o = \frac{[1+n]k_{t+1} - [1-\delta]k_t}{z_t} q \frac{p_{t+1}}{p_t}$. Using the definition of z_t, we obtain $c_{t+1}^o = [[1+n]k_{t+1} - [1-\delta]k_t] q \frac{p_{t+1}}{p_t} \left[\frac{1 - \frac{p_t}{p_{t+1}}[1+i_{t+1}]}{1-q}\right]$, which we can rewrite as

(A3) $\frac{c_{t+1}^o}{1+n} = \left[\frac{[1+n]k_{t+1} - [1-\delta]k_t}{1+n}\right]\left[\frac{q\frac{p_{t+1}}{p_t} - q\,[1+i_{t+1}]}{1-q_t}\right]$

Using (A2) and (A3)

$$\left[\frac{[1+n]k_{t+1} - [1-\delta]k_t}{1+n}\right]\left[\frac{q\frac{p_{t+1}}{p_t} - q\,[1+i_{t+1}]}{1-q}\right] = [r_{t+1} + \delta]k_{t+1} + b_{t+1}$$

$$\frac{p_{t+1}}{p_t} - [1+i_{t+1}] = \left[\frac{1-q}{q}\right]\left[[r_{t+1} + \delta]k_{t+1} + b_{t+1}\right]\left[\frac{1+n}{[1+n]k_{t+1} - [1-\delta]k_t}\right]$$

Using the bubble dynamic (18), we obtain

$$\frac{p_{t+1}}{p_t} - [1 + i_{t+1}]$$

$$= \left[\frac{1-q}{q}\right]\left[[r_{t+1}+\delta]k_{t+1} + \frac{1}{1+n}\frac{p_{t+1}}{p_t}b_t\right]\left[\frac{1+n}{[1+n]k_{t+1}-[1-\delta]k_t}\right]$$

(A4) $1 + i_{t+1} = \frac{p_{t+1}}{p_t} - \left[\frac{1-q}{q}\right]\left[[r_{t+1}+\delta]k_{t+1} + \frac{1}{1+n}\frac{p_{t+1}}{p_t}b_t\right]\left[\frac{1+n}{[1+n]k_{t+1}-[1-\delta]k_t}\right]$

We can now equate the two solutions for $1 + i_{t+1}$ in (A1) and (A4), we obtain

$$\frac{p_{t+1}}{p_t}\left[1 - \frac{[1-q]h[1-\alpha]Ak_t^{\alpha}}{[1+n]k_{t+1}-[1-\delta]k_t}\right]$$

$$= \frac{p_{t+1}}{p_t} - \left[\frac{1-q}{q}\right]\left[[r_{t+1}+\delta]k_{t+1} + \frac{1}{1+n}\frac{p_{t+1}}{p_t}b_t\right]\left[\frac{1+n}{[1+n]k_{t+1}-[1-\delta]k_t}\right]$$

$$\frac{p_{t+1}}{p_t}\left[\frac{[1-q]h[1-\alpha]Ak_t^{\alpha}}{[1+n]k_{t+1}-[1-\delta]k_t}\right] = \left[\frac{1-q}{q}\right]\left[[r_{t+1}+\delta]k_{t+1} + \frac{1}{1+n}\frac{p_{t+1}}{p_t}b_t\right]\left[\frac{1+n}{[1+n]k_{t+1}-[1-\delta]k_t}\right]$$

$$q\frac{p_{t+1}}{p_t}[h[1-\alpha]Ak_t^{\alpha}] = \left[[r_{t+1}+\delta]k_{t+1} + \frac{1}{1+n}\frac{p_{t+1}}{p_t}b_t\right][1+n]$$

$$q\frac{p_{t+1}}{p_t}[h[1-\alpha]Ak_t^{\alpha}] = [1+n][r_{t+1}+\delta]k_{t+1} + \frac{p_{t+1}}{p_t}b_t$$

$$\frac{p_{t+1}}{p_t}[q[h[1-\alpha]Ak_t^{\alpha}] - b_t] = [1+n][r_{t+1}+\delta]k_{t+1}$$

$$\frac{p_{t+1}}{p_t} = \frac{[1+n][r_{t+1}+\delta]k_{t+1}}{q[h[1-\alpha]Ak_t^{\alpha}] - b_t}$$

Using (16) to replace $r_{t+1} + \delta$, we obtain

(A5) $\frac{p_{t+1}}{p_t} = \frac{[1+n]\alpha Ak_{t+1}^{\alpha}}{q[h[1-\alpha]Ak_t^{\alpha}]-b_t}$.

Using (A5) to replace $\frac{p_{t+1}}{p_t}$ in (A1), we obtain

(A6) $1 + i_{t+1} = \frac{[1+n]\alpha Ak_{t+1}^{\alpha}}{q[h[1-\alpha]Ak_t^{\alpha}]-b_t}\left[1 - \frac{[1-q_t]h[1-\alpha]Ak_t^{\alpha}}{[1+n]k_{t+1}-[1-\delta]k_t}\right]$.

Conditions (A5) and (A6) are the desired conditions, so the proof is complete.

A.2.2 Proof that $1+i_{t+1}=\frac{\delta+r_{t+1}}{1-\frac{[1-\delta]k_t}{[1+n]k_{t+1}}}$

From condition (23), we have $1+i_{t+1}=\frac{[1+n]\alpha Ak_{t+1}^{\alpha}}{q[h[1-\alpha]Ak_t^{\alpha}]-b_t}\left[1-\frac{[1-q_t]h[1-\alpha]Ak_t^{\alpha}}{[1+n]k_{t+1}-[1-\delta]k_t}\right]$. From condition (21), we know $k_{t+1}=\mathrm{h}\left[\frac{1-\alpha}{1+n}\right]Ak_t^{\alpha}+\left[\frac{1-\delta}{1+n}\right]k_t-\frac{1}{1+n}b_t$. Rearranging, we find $[1+n]k_{t+1}-[1-\delta]k_t=\mathrm{h}[1-\alpha]Ak_t^{\alpha}-b_t$. Using this condition to replace $[1+n]k_{t+1}-[1-\delta]k_t$ in condition (23), we obtain $1+i_{t+1}=\frac{[1+n]\alpha Ak_{t+1}^{\alpha}}{q[h[1-\alpha]Ak_t^{\alpha}]-b_t}\left[1-\frac{[1-q_t]h[1-\alpha]Ak_t^{\alpha}}{\mathrm{h}[1-\alpha]Ak_t^{\alpha}-b_t}\right]$. Finding a common denominator we obtain $1+i_{t+1}=\frac{[1+n]\alpha Ak_{t+1}^{\alpha}}{q[h[1-\alpha]Ak_t^{\alpha}]-b_t}\left[\frac{\mathrm{h}[1-\alpha]Ak_t^{\alpha}-b_t-[1-q]h[1-\alpha]Ak_t^{\alpha}}{\mathrm{h}[1-\alpha]Ak_t^{\alpha}-b_t}\right]$, which implies we find $1+i_{t+1}\frac{[1+n]\alpha Ak_{t+1}^{\alpha}}{q[h[1-\alpha]Ak_t^{\alpha}]-b_t}\left[\frac{\mathrm{h}[1-\alpha]Ak_t^{\alpha}-b_t+qh[1-\alpha]Ak_t^{\alpha}-h[1-\alpha]Ak_t^{\alpha}}{\mathrm{h}[1-\alpha]Ak_t^{\alpha}-b_t}\right]$, which implies $1+i_{t+1}=\frac{[1+n]\alpha Ak_{t+1}^{\alpha}}{q[h[1-\alpha]Ak_t^{\alpha}]-b_t}\left[\frac{qh[1-\alpha]Ak_t^{\alpha}-b_t}{\mathrm{h}[1-\alpha]Ak_t^{\alpha}-b_t}\right]$, which implies $1+i_{t+1}=\frac{[1+n]\alpha Ak_{t+1}^{\alpha}}{\mathrm{h}[1-\alpha]Ak_t^{\alpha}-b_t}$. Using $[1+n]k_{t+1}-[1-\delta]k_t=\mathrm{h}[1-\alpha]Ak_t^{\alpha}-b_t$, we then find $1+i_{t+1}=\frac{[1+n]\alpha Ak_{t+1}^{\alpha}}{[1+n]k_{t+1}-[1-\delta]k_t}$. Dividing numerator and denominator of the right side by k_{t+1},we obtain $1+i_{t+1}=\frac{\alpha Ak_{t+1}^{\alpha-1}}{1-\frac{[1-\delta]k_t}{[1+n]k_{t+1}}}$.

Finally, using (16), we obtain $1+i_{t+1}=\frac{\delta+r_{t+1}}{1-\frac{[1-\delta]k_t}{[1+n]k_{t+1}}}$.

A.2.3 Proof that the Diamond Steady State yields the conditions $b=0$, $k=\left[\left[\frac{h[1-\alpha]A}{n+\delta}\right]\right]^{\frac{1}{1-\alpha}}$, where $\frac{p_{t+1}}{p_t}=\frac{\alpha[1+n]}{qh[1-\alpha]}$ and $1+i=\frac{\alpha[1+n]}{h[1-\alpha]}$ and $r=\frac{\alpha[\delta+n]}{\mathrm{h}[1-\alpha]}-\delta$.

From (21), we have $k_{t+1} = \mathrm{h}\left[\frac{1-\alpha}{1+n}\right] Ak_t^{\alpha} + \left[\frac{1-\delta}{1+n}\right] k_t - \frac{1}{1+n} b_t$. Setting $k_{t+1} = k_t = k$ and $b_t = b = 0$, we obtain $k = \mathrm{h}\left[\frac{1-\alpha}{1+n}\right] Ak^{\alpha} + \left[\frac{1-\delta}{1+n}\right] k$. Dividing by k, we obtain $1 = \mathrm{h}\left[\frac{1-\alpha}{1+n}\right] Ak^{\alpha-1} + \left[\frac{1-\delta}{1+n}\right]$. Solving for k, we obtain $\boldsymbol{k = \left[\frac{\mathbf{h}[1-\alpha]A}{n+\delta}\right]^{\frac{1}{1-\alpha}}}$.

Next, from (A5), we know $\frac{p_{t+1}}{p_t} = \frac{[1+n]\alpha A k_{t+1}^{\alpha}}{q[h[1-\alpha]Ak_t^{\alpha}]-b_t}$. Setting $k_{t+1} = k_t = k$, $q_{t+1} = q_t = q$ and $b_t = b = 0$, we obtain $\frac{p_{t+1}}{p_t} = \frac{[1+n]\alpha A k^{\alpha}}{q[h[1-\alpha]Ak^{\alpha}]}$, which implies $\boldsymbol{\frac{p_{t+1}}{p_t} = \frac{\alpha[1+n]}{qh[1-\alpha]}}$.

Next, from (A6), we know $1 + i_{t+1} = \frac{[1+n]\alpha A k_{t+1}^{\alpha}}{q_t[h[1-\alpha]Ak_t^{\alpha}]-b_t}\left[1 - \frac{[1-q_t]h[1-\alpha]Ak_t^{\alpha}}{[1+n]k_{t+1}-[1-\delta]k_t}\right]$. Setting $k_{t+1} = k_t = k$, $q_{t+1} = q_t = q$ and $b_t = b = 0$, we obtain $1 + i_{t+1} = \frac{[1+n]\alpha A k^{\alpha}}{q[h[1-\alpha]Ak^{\alpha}]}\left[1 - \frac{[1-q]h[1-\alpha]Ak^{\alpha}}{[1+n]k-[1-\delta]k}\right]$, which becomes $1 + i_{t+1} = \frac{[1+n]\alpha}{qh[1-\alpha]}\left[1 - \frac{[1-q]h[1-\alpha]Ak^{\alpha-1}}{n+\delta}\right]$. Recognizing $k = \left[\frac{\mathrm{h}[1-\alpha]A}{n+\delta}\right]^{\frac{1}{1-\alpha}}$ in steady state we obtain $1 + i_{t+1} = \frac{[1+n]\alpha}{qh[1-\alpha]}\left[1 - \frac{[1-q]h[1-\alpha]A\left[\frac{\mathrm{h}[1-\alpha]A}{n+\delta}\right]^{\frac{\alpha-1}{1-\alpha}}}{n+\delta}\right]$, which becomes $1 + i_{t+1} = \frac{[1+n]\alpha}{qh[1-\alpha]}\left[1 - \frac{[1-q]h[1-\alpha]A[n+\delta]}{[n+\delta][\mathrm{h}[1-\alpha]A]}\right]$, which becomes $1 + i_{t+1} = \frac{[1+n]\alpha}{qh[1-\alpha]}\left[1 - [1-q]\right]$, which becomes $\boldsymbol{1 + i = \frac{\alpha[1+n]}{h[1-\alpha]}}$

Next, from (16), we know $r = \alpha A k^{\alpha-1} - \delta$. Recognizing, $k = \left[\frac{\mathrm{h}[1-\alpha]A}{n+\delta}\right]^{\frac{1}{1-\alpha}}$ in the Diamond steady state, we obtain $r = \alpha A\left[\left[\frac{\mathrm{h}[1-\alpha]A}{n+\delta}\right]^{\frac{1}{1-\alpha}}\right]^{\alpha-1} - \delta$, which implies $\boldsymbol{r = \frac{\alpha[n+\delta]}{h[1-\alpha]} - \delta}$.

A.2.4 Proof that a second steady state, the Golden Rule Steady State, yields the conditions $\frac{p_{t+1}}{p_t} = 1 + n$, $k = \left[\frac{[[1-q]h[1-\alpha]+\alpha]A}{n+\delta}\right]^{\frac{1}{1-\alpha}}$, $b = [qh[1-\alpha] - \alpha]A^{\frac{1}{1-\alpha}}\left[\frac{[[1-q]h[1-\alpha]+\alpha]}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$, $1 + i = [1+n]\left[1 - \frac{1}{1+\frac{\alpha}{[1-q]\,h[1-\alpha]}}\right]$, $r = \frac{\alpha[n+\delta]}{[1-q]h[1-\alpha]+\alpha} - \delta$.

From (18), we have $b_{t+1} = \frac{1}{1+n}\frac{p_{t+1}}{p_t} b_t$. Setting $b_t = b$, we learn $b \neq 0$ implies,

(A7) $\frac{p_{t+1}}{p_t} = 1 + n.$

From (A5) in this Appendix, we know $\frac{p_{t+1}}{p_t} = \frac{[1+n]\alpha A k_{t+1}^{\alpha}}{q_t[h[1-\alpha]Ak_t^{\alpha}]-b_t}$. Setting $k_{t+1} = k_t = k$, $b_{t+1} = b_t = b$, and $q_{t+1} = q_t = q$, this becomes $\frac{p_{t+1}}{p_t} = \frac{[1+n]\alpha A k^{\alpha}}{q[h[1-\alpha]Ak^{\alpha}]-b}$. Using the result (A7) to eliminate $\frac{p_{t+1}}{p_t}$, we find $1 + n = \frac{[1+n]\alpha A k^{\alpha}}{q[h[1-\alpha]Ak^{\alpha}]-b}$. This implies $1 = \frac{\alpha A k^{\alpha}}{q[h[1-\alpha]Ak^{\alpha}]-b}$, or $q[h[1-\alpha]Ak^{\alpha}] - b = \alpha A k^{\alpha}$, which implies $b = qh[1-\alpha]Ak^{\alpha} - \alpha A k^{\alpha}$, which implies

(A8) $b = [qh[1-\alpha] - \alpha]Ak^{\alpha}$.

Using (21), we have $k_{t+1} = \mathrm{h}\left[\frac{1-\alpha}{1+n}\right]Ak_t^{\alpha} + \left[\frac{1-\delta}{1+n}\right]k_t - \frac{1}{1+n}b_t$. Setting $k_{t+1} = k_t = k$, setting $b_{t+1} = b_t = b$, and using (A8) to eliminate b, we obtain $k = \mathrm{h}\left[\frac{1-\alpha}{1+n}\right]Ak^{\alpha} + \left[\frac{1-\delta}{1+n}\right]k - \frac{1}{1+n}[qh[1-\alpha] - \alpha]Ak^{\alpha}$. This implies

$[1+n]k = \mathrm{h}[1-\alpha]Ak^{\alpha} + [1-\delta]k - [qh[1-\alpha] - \alpha]Ak^{\alpha}$, which implies

$[1+n]k = \left[[1-\mathrm{q}]h[1-\alpha]+\alpha\right]Ak^{\alpha} + [1-\delta]k$. Dividing by k, we obtain $1+n = \left[[1-\mathrm{q}]h[1-\alpha]+\alpha\right]Ak^{\alpha-1} + [1-\delta]$, which implies $n+\delta = \left[[1-\mathrm{q}]h[1-\alpha]+\alpha\right]Ak^{\alpha-1}$, which implies

(A9) $\boldsymbol{k} = \left[\frac{[[\mathbf{1}-\mathbf{q}]\boldsymbol{h}[\mathbf{1}-\boldsymbol{\alpha}]+\boldsymbol{\alpha}]\boldsymbol{A}}{\boldsymbol{n}+\boldsymbol{\delta}}\right]^{\frac{\mathbf{1}}{\mathbf{1}-\boldsymbol{\alpha}}}$.

Using (A8), we have $b = [qh[1-\alpha]-\alpha]Ak^{\alpha}$. Using (A9) to eliminate k, we obtain $b = [qh[1-\alpha]-\alpha]A\left[\frac{[[1-\mathrm{q}]h[1-\alpha]+\alpha]A}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$, which is which is equal to

(A10) $b = [qh[1-\alpha]-\alpha]A^{\frac{1}{1-\alpha}}\left[\frac{[[1-\mathrm{q}]h[1-\alpha]+\alpha]}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$

Next, from (16), we know $r = \alpha Ak^{\alpha-1} - \delta$ in the steady state. Using (A9) to eliminate k, we obtain $r = \alpha A\left[\left[\frac{A[h[1-\alpha][1-q]+\alpha]}{n+\delta}\right]^{\frac{1}{1-\alpha}}\right]^{\alpha-1} - \delta$, which implies

(A11) $r = \frac{\alpha[n+\delta]}{h[1-\alpha][1-q]+\alpha} - \delta$

Next, from (A6), we know $1+i_{t+1} = \frac{[1+n]\alpha Ak_{t+1}^{\alpha}}{q_t[h[1-\alpha]Ak_t^{\alpha}]-b_t}\left[1-\frac{[1-q_t]h[1-\alpha]Ak_t^{\alpha}}{[1+n]k_{t+1}-[1-\delta]k_t}\right]$. Setting $k_{t+1} = k_t = k$, setting $q_{t+1} = q_t = q$ and using (A8) to set $b_t = b = [qh[1-\alpha]-\alpha]Ak^{\alpha}$, we obtain $1+i = \frac{[1+n]\alpha Ak^{\alpha}}{q[h[1-\alpha]Ak^{\alpha}]-[qh[1-\alpha]-\alpha]Ak^{\alpha}}\left[1-\frac{[1-q]h[1-\alpha]Ak^{\alpha}}{[1+n]k-[1-\delta]k}\right]$, which becomes $1+i = \frac{[1+n]\alpha}{qh[1-\alpha]-[qh[1-\alpha]-\alpha]}\left[1-\frac{[1-q]h[1-\alpha]Ak^{\alpha}}{[n+\delta]k}\right]$, which becomes $1+i = [1+n]\left[1-\frac{[1-q]h[1-\alpha]Ak^{\alpha-1}}{[n+\delta]}\right]$.

Using (A9) to eliminate k, we obtain $1+i = [1+n]\left[1-\frac{[1-q]h[1-\alpha]A\left[\left[\frac{[[1-\mathrm{q}]h[1-\alpha]+\alpha]A}{n+\delta}\right]^{\frac{1}{1-\alpha}}\right]^{\alpha-1}}{[n+\delta]}\right]$,

which becomes $1+i = [1+n]\left[1-\frac{[1-q]h[1-\alpha]}{[[1-\mathrm{q}]h[1-\alpha]+\alpha]}\right]$, which we can rewrite as

(A12) $\mathbf{1} + \boldsymbol{i} = [\mathbf{1} + \boldsymbol{n}]\left[\mathbf{1} - \frac{\mathbf{1}}{\mathbf{1} + \frac{\boldsymbol{\alpha}}{[\mathbf{1}-\mathbf{q}]\,\boldsymbol{h}[\mathbf{1}-\boldsymbol{\alpha}]}}\right].$

A.2.5 Proof that to obtain the case of interest to Diamond (1965), $q > \frac{\alpha}{h[1-\alpha]}$ must hold.

The case of interest to Diamond (1965) was the case where the golden rule capital stock is less than the no bubble steady state capital stock. For our framework, the steady state capital calculations in this appendix would imply

$$\left[\frac{[[1-q]h[1-\alpha]+\alpha]A}{n+\delta}\right]^{\frac{1}{1-\alpha}} < \left[\left[\frac{h[1-\alpha]A}{n+\delta}\right]\right]^{\frac{1}{1-\alpha}}$$

$$\left[[1-q]h[1-\alpha]+\alpha\right]A < h[1-\alpha]A$$

$$[1-q]h[1-\alpha]+\alpha < h[1-\alpha]$$

$$\frac{[1-q]h[1-\alpha]+\alpha}{h[1-\alpha]} < 1$$

$$1 - q + \frac{\alpha}{h[1-\alpha]} < 1$$

$$\frac{\alpha}{h[1-\alpha]} < q$$

$$q > \frac{\alpha}{h[1-\alpha]}$$

Since $q \leq 1$ must hold, we know $\frac{\alpha}{h[1-\alpha]} < 1$ must hold for $q > \frac{\alpha}{h[1-\alpha]}$ to hold.

A.2.6 Proof that $r > i$ must hold in Diamond No Bubble Steady State as long as $0 \leq \delta < 1$.

From the proof above for the Diamond Steady State, we know $1 + i = \frac{\alpha[1+n]}{h[1-\alpha]}$ and $r = \frac{\alpha[\delta+n]}{h[1-\alpha]} - \delta$. Thus,

$$r - i = \frac{\alpha[\,\delta + n] - \delta\mathrm{h}[1-\alpha]}{\mathrm{h}[1-\alpha]} - \frac{\alpha[1+n] - h[1-\alpha]}{h[1-\alpha]}$$

$$r - i = \frac{\alpha\,\delta + \alpha n - \delta\mathrm{h}[1-\alpha] - \alpha - \alpha n + h[1-\alpha]}{h[1-\alpha]}$$

$$r - i = \frac{\alpha[\,\delta - 1] + [1-\delta]h[1-\alpha]}{h[1-\alpha]}$$

$$r - i = \frac{[\,1-\delta][h[1-\alpha] - \alpha]}{h[1-\alpha]}$$

$$r - i = \frac{[\,1-\delta][h[1-\alpha] - \alpha]}{h[1-\alpha]}$$

To have our case of interest, where bubbles are possible, $\mathrm{q} > \frac{\alpha}{h[1-\alpha]}$ must hold. Since we restrict $0 \leq \mathrm{q} \leq 1$, $\alpha < h[1-\alpha]$ must also hold. Consequently, we find $r > i$ must hold when $0 \leq \delta < 1$.

A.2.7 Proof that to have a positive interest rate, $\frac{1-q}{n} < \frac{\alpha}{h[1-\alpha]} < q$ must hold.

For our framework, the steady state gross interest rate on capital calculations in this appendix would imply $i = [1+n]\left[1 - \frac{[1-q]h[1-\alpha]}{[[1-\mathrm{q}]h[1-\alpha]+\alpha]}\right] - 1 > 0$. In order to hold positive interest rate

$$[1+n]\left[1 - \frac{[1-q]h[1-\alpha]}{[[1-\mathrm{q}]h[1-\alpha]+\alpha]}\right] - 1 > 0$$

$$[1+n]\left[1 - \frac{[1-q]h[1-\alpha]}{[[1-\mathrm{q}]h[1-\alpha]+\alpha]}\right] > 1$$

$$1-\frac{[1-q]h[1-\alpha]}{\left[[1-\mathrm{q}]h[1-\alpha]+\alpha\right]}>\frac{1}{1+n}$$

$$1-\frac{1}{1+n}>\frac{[1-q]h[1-\alpha]}{\left[[1-\mathrm{q}]h[1-\alpha]+\alpha\right]}$$

$$\frac{n}{1+n}>\frac{[1-q]h[1-\alpha]}{[1-\mathrm{q}]h[1-\alpha]+\alpha}$$

$$\frac{1+n}{n}<\frac{[1-\mathrm{q}]h[1-\alpha]+\alpha}{[1-q]h[1-\alpha]}$$

$$\frac{1}{n}+1<1+\frac{\alpha}{[1-q]h[1-\alpha]}$$

$$\frac{1}{n}<\frac{\alpha}{[1-q]h[1-\alpha]}$$

$$\frac{1-q}{n}<\frac{\alpha}{h[1-\alpha]}$$

To have case of interest to Diamond, we know $\mathrm{q}>\frac{\alpha}{h[1-\alpha]}$. Therefore, $\frac{1-q}{n}<\frac{\alpha}{h[1-\alpha]}<q\leq 1$.

A.2.8 Proof that $r-i=\frac{[1-\delta]}{1+\frac{\alpha}{[1-q]h[1-\alpha]}}$ in the golden rule steady state

From the proof above for the golden rule Steady State, we know $1+i=[1+n]\left[1-\frac{[1-q]h[1-\alpha]}{\left[[1-\mathrm{q}]h[1-\alpha]+\alpha\right]}\right]$ and $\boldsymbol{r}=\frac{\boldsymbol{\alpha}[\boldsymbol{n}+\boldsymbol{\delta}]}{[\mathbf{1}-\boldsymbol{q}]\boldsymbol{h}[\mathbf{1}-\boldsymbol{\alpha}]+\boldsymbol{\alpha}}-\boldsymbol{\delta}$. Thus,

$$r-i=\frac{\alpha[n+\delta]}{h[1-\alpha][1-q]+\alpha}-\delta-[1+n]\left[1-\frac{[1-q]h[1-\alpha]}{\left[[1-\mathrm{q}]h[1-\alpha]+\alpha\right]}\right]+1$$

$$r-i=\frac{\alpha[n+\delta]}{h[1-\alpha][1-q]+\alpha}-\delta\;-[1+n]+\frac{[1+n][1-q]h[1-\alpha]}{\left[[1-\mathrm{q}]h[1-\alpha]+\alpha\right]}+1$$

$$r - i = \frac{\alpha[n+\delta]}{h[1-\alpha][1-q]+\alpha} - \delta - 1 - n + \frac{[1+n][1-q]h[1-\alpha]}{\left[[1-q]h[1-\alpha]+\alpha\right]} + 1$$

$$r - i = \frac{\alpha[n+\delta]}{h[1-\alpha][1-q]+\alpha} - \delta - n + \frac{[1+n][1-q]h[1-\alpha]}{[1-q]h[1-\alpha]+\alpha}$$

$$r - i = \frac{\alpha[n+\delta]}{[1-q]h[1-\alpha]+\alpha} - \delta\frac{[1-q]h[1-\alpha]+\alpha}{[1-q]h[1-\alpha]+\alpha} - n\frac{[1-q]h[1-\alpha]+\alpha}{[1-q]h[1-\alpha]+\alpha} + \frac{[1+n][1-q]h[1-\alpha]}{[1-q]h[1-\alpha]+\alpha}$$

$$r - i = \frac{\alpha[n+\delta] - \delta\left[[1-q]h[1-\alpha]+\alpha\right] - n\left[[1-q]h[1-\alpha]+\alpha\right] + [1+n][1-q]h[1-\alpha]}{[1-q]h[1-\alpha]+\alpha}$$

$$r - i = \frac{\alpha[n+\delta] - \delta[1-q]h[1-\alpha] - \delta\alpha - n[1-q]h[1-\alpha] - n\alpha + [1-q]h[1-\alpha] + n[1-q]h[1-\alpha]}{[1-q]h[1-\alpha]+\alpha}$$

$$r - i = \frac{n\alpha + \alpha\delta - \delta[1-q]h[1-\alpha] - \delta\alpha - n\alpha + [1-q]h[1-\alpha]}{[1-q]h[1-\alpha]+\alpha}$$

$$r - i = \frac{[1-q]h[1-\alpha] - \delta[1-q]h[1-\alpha]}{[1-q]h[1-\alpha]+\alpha}$$

$$r - i = \frac{[1-q]h[1-\alpha][1-\delta]}{[1-q]h[1-\alpha]+\alpha}$$

$$r - i = \frac{\frac{[1-q]h[1-\alpha][1-\delta]}{[1-q]h[1-\alpha]}}{\frac{[1-q]h[1-\alpha]+\alpha}{[1-q]h[1-\alpha]}}$$

$$r - i = \frac{[1-\delta]}{1 + \frac{\alpha}{[1-q]h[1-\alpha]}}$$

If $q = 1$ then $r - i = 1 - \delta$. If $q = 1$ and $\delta = 0$, then $r - i = 1$ implies $r = 1 + i$

If $q = 0$, $r - i = \frac{[1-\delta]}{1+\frac{\alpha}{h[1-\alpha]}} > 0$

3 Chapter 3 Enhancing the Overlapping Generation Model: Capital Accumulation, Stock Market Clearing and Bubble Uncertainty

(with Mark Pingle)

Abstract

This paper extends the Overlapping Generations model of Weil (1987) by adding capital accumulation and stock market clearing as introduced in Banerjee and Pingle (2023). The addition of the stock market clearing eliminates the indeterminacy and inefficiency from Weil's model, resulting in a unique, Pareto efficient equilibrium. Because the bubble may burst in our model, as in the Weil model, we like Weil find that the rate of return on bubbly assets must exceed that on capital backed asset to offset the risk that the bubble bursts. In contrast to Weil, we find that the path for the bubble is not influenced by a change in the perception that the bubble will burst. Rather, an increased perception that the bubble will burst causes an increase in the rate of return on the bubble asset, so people will be willing to hold it.

3.1 Introduction

Irving Fisher is recognized for initiating a meticulous examination of the factors influencing the determination of interest rates. In his seminal work entitled, "The loanable funds theory of interest as Determined by Impatience to Spend Income and Opportunity to Invest It," Fisher (1930) provides a framework for understanding how the interest rate is determined by the interaction of savings and borrowing, underpinned by people's preferences for current versus future consumption, and adjusted for the effects of inflation, and the presence of investment prospects. Fisher's basic finding was that the interest rate tended to be positive because people are impatient, preferring consumption in the present rather than the future.

Paul Samuelson (1958) developed an interest rate theory by developing the overlapping generations model. His model is important because it represents the first general equilibrium model of interest rate determination, which he referred to as the "biological theory of interest." This theory relates the interest rate to the human life cycle, and the interactions between different generations within an economy. Samuelson's (1958) biological theory of interest concludes that the interest rate is fundamentally influenced by population growth. Prior models, such as Fisher's (1930) framework, did not consider this demographic effect on interest rate determination.

David Gale's (1973) theory of overlapping generations (OG) built upon the framework initially introduced by Paul Samuelson (1958). Gale (1973) extended the model to consider that individuals might receive income in both periods of their life, not just when they are young. This lead Gale (1973) to two cases. The "Classical case" occurs if people desire to consume more than their income when young, implying they want to borrow or dissave. The "Samuelson case" occurs if people desire to consume less than their income when young, implying they want to save. Importantly, Gale (1973) shows that the inefficiency identified by Samuelson (1958) can only occur in the Samuelson case, and the Samuelson case can only occur when the person has a low enough level of old age income.

Peter Diamond's (1965) contribution expanded the Samuelson (1958) model by introducing production into the OG framework. The previous models were pure consumption economies, where the income of consumers were unexplained endowments. In this expanded framework, firms employ capital and labor to produce goods. Young individuals receive wage income from the firm income by supplying labor to the firm, and they save for old age. This young-age saving flows into capital, which is then used by firms in the next period to produce output. Individuals work and save when they are young, and then retire and live off their savings when they are old.

Jean Tirole (1985) extended Diamond's Overlapping Generations (OG) model by incorporating what is commonly referred to as a "bubbly" asset. The addition of this non-fundamental asset added a new layer of complexity. The presence of a bubble affects the capital accumulation process because the bubble assets offer a savings alternative to the capital-backed asset. The rate of return on the bubbly asset is dependent upon the purchase of the bubble asset by the next generation, rather than being dependent upon the productivity of capital. Tirole's (1985) demonstrates model shows that bubble formation can change the path of the economy.

Significant for our work, the equilibrium for the economy is indeterminate when a bubble can form in the Tirole's (1985) economy. When a bubble can form, there are an infinite number of equilibrium paths, where each path is associated with a different initial bubble. One of these paths, the one with the highest possible initial bubble, is Pareto efficient. However, there is nothing in the model that selects the Pareto efficient equilibrium from among the set of equilibrium paths. Consequently, inefficiency, along with indeterminacy is associated with Tirole's (1985) bubbly economy.

Philippe Weil (1987) extended Tirole's (1985) model by adding uncertainty. While Tirole (1985) assumed a bubble would not burst once formed, Weil assumed there is a certain exogenous probability that a bubble could burst in any given period. This probability affects the behavior of savers because they must consider the risk that the value of the bubble asset drops to zero if the bubble bursts. The consequence of recognizing this rise is that the rate of return on the bubbly asset must include a risk premium. The risk premium compensates investors for the chance that they could lose their entire investment. The higher the probability of a burst, the higher the risk premium investors must demand in order to hold the bubble asset along with the capital backed asset. A primary finding of Weil (1987) is, as the confidence in the bubble asset decreases, the Tirole (1985) Economy where bubbles can form moves toward the Diamond (1965) Economy where they do not form. If confidence is low enough, or we can say if the belief that the bubble will burst is high enough, then no bubble can form.

Banerjee and Pingle (2023) modify the Tirole (1985) model by allowing capital to accumulate. Banerjee shows that when capital accumulates, rather than being consumed, the OG model with production fundamentally changes. In particular, if the interest rate paid on saving is restricted to equal capital rental rate, as in Diamond (1965), Tirole (1985) , and Weil (1987), then capital market clearing does not necessarily imply product market clearing. He shows, if we desire a model where capital accumulates, then the interest rate must be allowed to differ from the capital rental rate in order for both the product market and capital market to clear.

One contribution of this paper is to extend the Weil (1987) model to allow capital accumulation as in Banerjee and Pingle (2023). However, the second and more significant contribution is we extend the Weil (1987) economy by adding the stock market clearing condition following Banerjee and Pingle (2023). Our model is unique relative to Banerjee and Pingle (2023) in that they assume the consumer has total confidence that any bubble will not burst, whereas we follow Weil (1987) and assume there is a positive, exogenously given probability that the bubble will burst in each period.

What do we learn?

First, consistent with finding of Banerjee and Pingle (2023) for the Tirole (1985) economy, we find that adding a stock market clearing condition eliminates the indeterminacy and inefficiency present in the Weil (1987) economy. We follow Diamond (1965), Tirole (1985) and Weil (1987) and impose a restriction on consumer preferences and production such that bubble can form. Unlike Weil (1987), we do not find multiple equilibria for the economy, where each equilibrium path depends upon the size of the initial bubble. Rather, we find there is a unique equilibrium path for our economy, and this equilibrium path is Pareto efficient.

Second, consistent with Weil (1987), we find that the rate of return paid on the bubble asset must be higher than the rate of return paid on the capital backed asset for young savers to be willing to be able to hold both assets. The return must be higher on the bubble asset to compensate for the risk associated with the possibility that the bubble may burst, which means the saver not only earns no return but also loses the original amount saved. We find that the gap between the rate of return on the bubble asset and capital back asset is directly related to the confidence savers have that the bubble will not burst. If there is total confidence the bubble will not burst, then the two assets earn the same rate of return. As the confidence decreases, the gap between the two rates of return increases.

Importantly, we also find that the path for the bubble in the economy does NOT depend upon the probability that the bubble will burst. Rather than decreasing the size of the bubble, an increase in the probability that the bubble will burst increases the equilibrium gap between the rate of return on the bubble asset and the rate of return on the capital backed asset. The bubble must maintain its size for markets to clear.

The paper unfolds as follows. In section 2, we present a modified version of the Weil (1987) economy. In section 3, we derive the basic dynamic equation that describes the equilibrium path for the economy, and we present the solutions for the model's endogenous variables. In section 4, we presented a steady state analysis, and we show that the steady state for this model is stable. In section 5, we present the dynamics of the economy. Section 6 concludes.

3.2 The Model

The model, in period $t = \{1,2, \ldots\}$, includes L_t consumers of generation t, and L_{t-1} consumers of generation $t-1$. The population of consumers grows at the rate n, so $L_t = [1+n]L_{t-1}$. Except for the consumers of generation 0, all consumers are identical. Each generation t consumer is born at the beginning of period t, is young during period t, is old during period $t+1$, and dies at the end of period $t+1$. Each receives utility $U\left(c_t^y, c_{t+1}^o\right)$

from young age consumption c_t^y and old age consumption c_{t+1}^o. Each of the L_0 consumers in the initial generation 0 receives utility that is increasing in the consumption level c_1^o.

For the generation t consumer, young age consumption is financed by income obtained from work, but the consumer retires at the end of period t so old age consumption must be financed by young age saving. One unit of labor is supplied in exchange for the real wage w_t, so the young age budget constraint is

(27) $c_t^y = w_t - s_t,$

where s_t is the saving level. The consumer saves for old age by purchasing financial assets. Specifically, θ_t units of a capital backed asset are purchased, and m_t units of a bubbly asset. In terms of "good t," the price of the capital backed asset is v_t, and the price of the bubbly asset is p_t. This implies the consumer's real saving in the form of capital backed asset is

(28) $x_t = v_t \theta_t,$

real saving in the form of bubbly asset is

(29) $b_t = p_t m_t,$

and total real saving is

(30) $s_t = x_t + b_t.$

The consumer finances old age consumption by selling the assets purchased when young. The capital backed asset is not risky, so the consumer will receive $v_{t+1}\theta_t$ from its sale. The bubble asset is risky. Specifically, there is a $1 - q$ probability that the bubble will burst, meaning the price of the bubbly asset goes to zero. Alternatively, the old age consumer is able to sell the bubble asset at the price $p_{t+1} > 0$ with probability q. This implies the old age budget constraint is

(31) $$c_{t+1}^{o^+} = v_{t+1}\theta_t + p_{t+1}m_t$$

with probability q and

(32) $$c_{t+1}^{o^-} = v_{t+1}\theta_t$$

with probability $1 - q$.

The generation t consumer chooses asset purchase levels θ_t and m_t to maximize the expected utility of consumption $qU(w_t - v_t\theta_t - p_tm_t, v_{t+1}\theta_t + p_{t+1}m_t) + [1 - q]U(w_t - v_t\theta_t - p_tm_t, v_{t+1}\theta_t)$. This optimization yields the additional first order conditions

(33) $$\frac{qU_{c_t^y}(w_t - v_t\theta_t - p_tm_t,\ v_{t+1}\theta_t + p_{t+1}m_t) + [1-q]U_{c_t^y}(w_t - v_t\theta_t - p_tm_t, v_{t+1}\theta_t)}{qU_{c_t^{o^+}}(w_t - v_t\theta_t - p_tm_t, v_{t+1}\theta_t + p_{t+1}m_t) + [1-q]U_{c_t^{o^-}}(w_t - v_t\theta_t - p_tm_t, v_{t+1}\theta_t)} = \frac{v_{t+1}}{v_t}$$

and

(34) $$\frac{qU_{c_t^y}(w_t - v_t\theta_t - p_tm_t, v_{t+1}\theta_t + p_{t+1}m_t) + [1-q]U_{c_t^y}(w_t - v_t\theta_t - p_tm_t, v_{t+1}\theta_t)}{U_{c_t^{o^+}}(w_t - v_t\theta_t - p_tm_t, v_{t+1}\theta_t + p_{t+1}m_t)} = q\frac{p_{t+1}}{p_t}.$$

Together, conditions (27)-(34) determine the consumer's optimal choices for θ_t, m_t, x_t, b_t, s_t, c_t^y, $c_{t+1}^{o^+}$, and $c_{t+1}^{o^-}$ as they depend upon the gross rate of return $\frac{v_{t+1}}{v_t}$ on the riskless capital backed asset, the expected gross rate of return $q\frac{p_{t+1}}{p_t}$ on the risky bubble asset, and the real wage w_t. Following Weil (1987), we use the log linear utility function $U\left(c_t^y, c_{t+1}^o\right) = ln\left(c_t^y\right) + \beta ln(c_{t+1}^o)$ to allow us to obtain particular results, where β is the discount factor that applies to old age consumption. In the Appendix, following Weil, we show that if we define $h = \frac{\beta}{1+\beta}$, which turns out to be the marginal propensity to save, and define $z_t = \frac{1-q}{1-\frac{p_t}{p_{t+1}}\frac{v_{t+1}}{v_t}}$, which turns out to be the share of saving devoted to the capital asset, we find consumer optimization implies

(35) $x_t = z_t h w_t,$

(36) $b_t = [1 - z_t]\mathrm{h}w_t,$

(37) $s_t = \mathrm{h}w_t,$

(38) $c_t^y = [1 - h]w_t,$

(39) $c_{t+1}^{o+} = hw_t q \frac{p_{t+1}}{p_t},$

(40) $c_{t+1}^{o-} = \frac{v_{t+1}}{v_t} z_t h w_t,$

(41) $m_t = \frac{[1-z_t]}{p_t}\mathrm{h}w_t,$

and

(42) $\theta_t = \frac{z_t}{v_t} h w_t.$

Consumers in the initial old generation are the initial owners of the firm, and they also introduce the bubble asset. Each old age consumer owns θ_0 shares of stock, so the total initial supply of stock is $L_0\theta_0$ shares. Each old age consumer is endowed with m_0 units of the bubble asset, so the total initial supply of the bubble asset is $L_0 m_0$ units. Given the initial prices v_1 and p_1, the consumption level of each initial old age consumer is

(43) $c_1^o = v_1\theta_0 + p_1 m_0.$

Total period t production depends upon labor and capital according to $Y_t = F(K_t, L_t)$. Defining capital and output per generation t consumer as $k_t = K_t/L_t$ and $y_t = Y_t/L_t$, the assumption that production exhibits diminishing returns to the inputs and constant returns to scale implies production per young consumer can be presented as

(44) $y_t = f(k_t)$ with $f'(k_t) > 0$, and $f''(k_t) < 0$.

Assuming capital depreciates[23] at rate δ, firm profit maximization implies the "net" rental rate paid on capital is

(45) $r_t = f'(k_t) - \delta \qquad 0 \leq \delta \leq 1$,

and the wage rate paid on labor is

(46) $w_t = y_t - [r_t + \delta]k_t$.

Following Diamond (1965) and Weil (1987), we use the particular production function $f(k_t) = Ak_t^\alpha$ and find producer optimization implies output and factor prices depend upon capital employed:

(47) $y_t = Ak_t^\alpha \qquad 0 < \alpha < 1,$

(48) $r_t = \alpha Ak_t^{\alpha-1} - \delta \qquad 0 < \delta < 1,$

(49) $w_t = [1 - \alpha]Ak_t^\alpha$.

Period t investment I_t accumulates as capital[24] according to $K_{t+1} = K_t + I_t - \delta K_t$. where I_t is the investment level. The capital market is in equilibrium when the real investment level I_t is equal to the real saving $L_t x_t$ that the consumer places into the capital asset. That is, the capital market clears when $K_{t+1} - [1 - \delta]K_t = L_t x_t$. Using the

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[23]By recognizing depreciation, we extend the work Banerjee and Pingle (2023). Their work follows Diamond (1965), Tirole (1985), and Weil (1987) who all assume zero depreciation.

[24] Diamond (1965), Tirole (1985), and Weil (1987) all assume capital is consumed after production. We follow Banerjee and Pingle (2023) by assuming capital accumulates rather than being consumed.

population growth condition $L_{t+1} = [1+n]L_t$ and the definition $k_t = K_t/L_t$, we can write the capital market clearing condition as

(50) $x_t = [1+n]k_{t+1} - [1-\delta]k_t.$

The product market clears[25] when $Y_t = C_t^y + C_t^o + K_{t+1} - [1-\delta]K_t$. That is, output supplied equals aggregate demand, where aggregate demand includes consumption demand and investment demand. Using the population growth condition $L_{t+1} = [1+n]L_t$ and the definition $k_t = K_t/L_t$, we can write the product market clearing condition as

(51) $y_t = c_t^y + \frac{c_t^o}{1+n} + [1+n]k_{t+1} - [1-\delta]k_t.$

The economy we have presented so far is the Weil (1987) economy with two modifications. One modification is we provide a specific explanation of how the consumer earns the gross rate of return $1 + i_{t+1}$ on capital backed saving. The consumer buys ownership in the firm (i.e., saves) at the price v_t when young and then sells that ownership (i.e., receives the gross return on saving) at the price v_{t+1} when old. Thus, the gross rate of return earned is $1 + i_{t+1} = \frac{v_{t+1}}{v_t}$. The second modification is we assume capital accumulates rather than being consumed each period, and this is a more significant modification. As Banerjee and Pingle (2023) show, this assumption implies the product market and capital market cannot each clear unless the rate of return earned on saving deviates from the capital rental rate. This explains why the rate of return earned on saving may deviate from the capital rental rate.

These two modifications do not change the fundamental character of the model. Absent any further conditions, this model still has multiple equilibrium paths associated with different initial bubbles. Moreover, only one of these equilibrium paths is Pareto efficient.

Following Banerjee and Pingle (2023), the final modification we now make is the most significant. We add a stock market clearing condition for the capital backed asset:

(52) $\theta_t = [1+n]\theta_{t+1}$.

Weil (1987) imposes the capital market clearing condition (50), but following Tirole (1985) and Diamond (1965) he does not provide the financial market equilibrium condition (52) that captures the transfer of firm ownership from generation t to generation $t+1$.

With probability q, money that has value in period t will have value in period $t+1$. Thus, with probability q we have bubble asset market clearing condition

(53) $m_{t+1} = \frac{1}{1+n} m_t$,

while with probability $1-q$ the money becomes worthless and $m_{t+1} = 0$.

The bubble asset equilibrium (53) is significant because, as we show in the Appendix and as Weil shows, condition (53) implies the bubble follows the path $b_{t+1} = \left[\frac{\frac{p_{t+1}}{p_t}}{1+n}\right] b_t$. As we show below, adding the stock market clearing condition (52) is even more significant in that it eliminates the indeterminacy of the model. When we include condition (52) in the model, there is only one equilibrium path for the economy, and it is the Pareto efficient path that converges to the Golden Rule steady state.

To summarize our complete model, the economy enters period t with the variables k_t, $m_{t-1}, \theta_{t-1}, v_t$, and p_t predetermined. Equations (27)-(34), (44-46), (50-52) determine the paths of the variables $\theta_t, m_t, x_t, b_t, s_t, c_t^y,\ c_{t+1}^{o+},\ c_{t+1}^{o-},\ y_t, r_t, w_t, k_{t+1}, v_{t+1}$, and p_{t+1} as they depends upon the exogenous variables q, n, δ, A, h and α. Equation (43) determines the consumption level c_1^o of old consumers in the initial time period.

3.3 The Basic Dynamic Equation and Endogenous Variable Solutions

In the Appendix, we show the economy can be reduced to the basic dynamic equation.

(54) $$k_{t+1} = \left[\frac{1+f'(k_t)-\delta}{1+n}\right] k_t.$$

Using the Cobb-Douglas production function (47), condition (54) becomes

(55) $$k_{t+1} = \left[\frac{1+\alpha A k_t^{\alpha-1}-\delta}{1+n}\right] k_t.$$

Using the solution (55), in the Appendix we find following solutions for other key endogenous variables:

(56) $$x_t = \alpha A k_t^{\alpha},$$

(57) $$s_t = \mathrm{h}[1-\alpha] A k_t^{\alpha},$$

(58) $$b_t = [\mathrm{h}[1-\alpha]-\alpha] A k_t^{\alpha},$$

(59) $$\frac{v_{t+1}}{v_t} = [1+n]\left[\frac{1+\alpha A k_t^{\alpha-1}-\delta}{1+n}\right]^{\boldsymbol{\alpha}}$$

(60) $$\frac{p_{t+1}}{p_t} = \frac{1+n}{q}$$

(61) $$z_t = \frac{\alpha}{h[1-\alpha]}.$$

Following Diamond (1965) and Tirole (1985) we are interested in the possibility that a positive bubble can arise. From the solution (58), we find $b_t > 0$ implies $\mathrm{h}[1-\alpha]-\alpha > 0$, which implies

(62) $$\mathrm{h}[1-\alpha] > \alpha.$$

For a bubble to form, condition (62) implies the consumer must place sufficient weight on old age consumption, and the elasticity of output with respect to capital must be low enough relative to the elasticity of output with respect to labor. To allow the possibilities of bubbles, we assume (62) holds.

Conditions (56) and (57) indicate the quantity $z_t = \frac{\alpha}{h[1-\alpha]}$ given as condition (61) is the proportion of saving allocated to the capital backed asset. Conditions (57) and (58) indicate the remaining fraction $\frac{h[1-\alpha]-\alpha}{h[1-\alpha]}$ of saving is allocated to the bubbly asset. Condition (61) indicates this proportional allocation of saving to the two assets remains fixed over time as the economy evolves. The portion allocated to the bubble asset is larger when the bubble when the consumer places more weight on old age consumption relative to young age consumption and when the elasticity of output with respect to capital is small relative to the elasticity of output with respect to labor.

Before we examine the dynamics in more detail, we examine the steady state. We show there is a unique steady state, and we show this steady state is stable. This knowledge helps us understand the dynamics.

3.4 The Steady State

In the Appendix, we show that the unique steady state capital level $\hat{k}$ is determined by the condition $f'(\hat{k}) = n + \delta$. For our Diamond production function (47), this steady state is

(63) $$\hat{k} = \left[\frac{\alpha A}{n+\delta}\right]^{\frac{1}{1-\alpha}}.$$

In the Appendix, we show $k = \left[\frac{\alpha A}{n+\delta}\right]^{\frac{1}{1-\alpha}}$ is also a golden rule level of capital that maximizes steady state utility of the representative consumer. Using this steady state solution and other equations in the model, in the Appendix we derive the following steady state values:

(64) $\hat{r} = n$

(65) $\hat{y} = A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}},$

(66) $\hat{w} = [1-\alpha]A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}},$

(67) $\widehat{\frac{v_{t+1}}{v_t}} = 1 + n,$

(68) $\hat{x} = \alpha A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}},$

(69) $\hat{b} = [\mathrm{h}[1-\alpha] - \alpha]A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}},$

(70) $\hat{s} = \mathrm{h}[1-\alpha]A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}},$

(71) $\hat{c}^{y} = [1-h][1-\alpha]A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}},$

(72) $\hat{c}^{o+} = [1+n]h[1-\alpha]A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$, and

(73) $\hat{c}^{o-} = [1+n]\alpha A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$

In the Appendix, we show that the steady state for this model is stable. Thus, if the economy is not in the steady state where the variable values are those shown in conditions (64)-(73), then the economy is moving toward the steady state where those values hold.

Examining the steady state values in conditions (64)-(73), we see that q never appears. This implies, under our restriction (62) that allows bubbles to form, the degree of confidence in the bubble does not influence where the economy ultimately settles.

However, from condition (60), we see that the degree of confidence in the bubble asset q does affect the rate of return on the bubble asset. In the steady state, but also under all no-steady state conditions, the gross rate of return on the bubble asset is the constant $\frac{p_{t+1}}{p_t} = \frac{1+n}{q}$. In the steady state, the gross rate of return on the capital backed asset is $\frac{\widehat{v_{t+1}}}{v_t} = 1 + n$. Comparing these two, we find that as long as the confidence in the bubble asset is not complete (i.e., $q < 1$) the rate of return on the bubbly asset will be higher than the rate of return on the capital backed asset. The difference in the return is the extra compensation necessary to get the consumers to hold an asset with greater risk.

3.5 The Dynamics of the Economy

Figure 11 presents two different unique equilibrium paths for the capital stock variable k_t and bubble variable b_t for two different initial conditions. The solution (54) gives equilibrium path for k_t. The solution (58) gives equilibrium path for b_t. When the initial condition is $k_t = 10$, the equilibrium path converges to the steady state from below. When the initial condition is $k_t = 50$, the equilibrium path converges to the steady state from above. What we observe is that k_t and b_t are directly related. When the capital stock increases from its initial low level, the bubble increases. When the capital stock decreases from its initial high level, the bubble decreases. In either case, the capital stock level converges to its golden rule level, and the economy is Pareto efficient as it proceeds along its path.

Figure 11 Capital Bubble Path

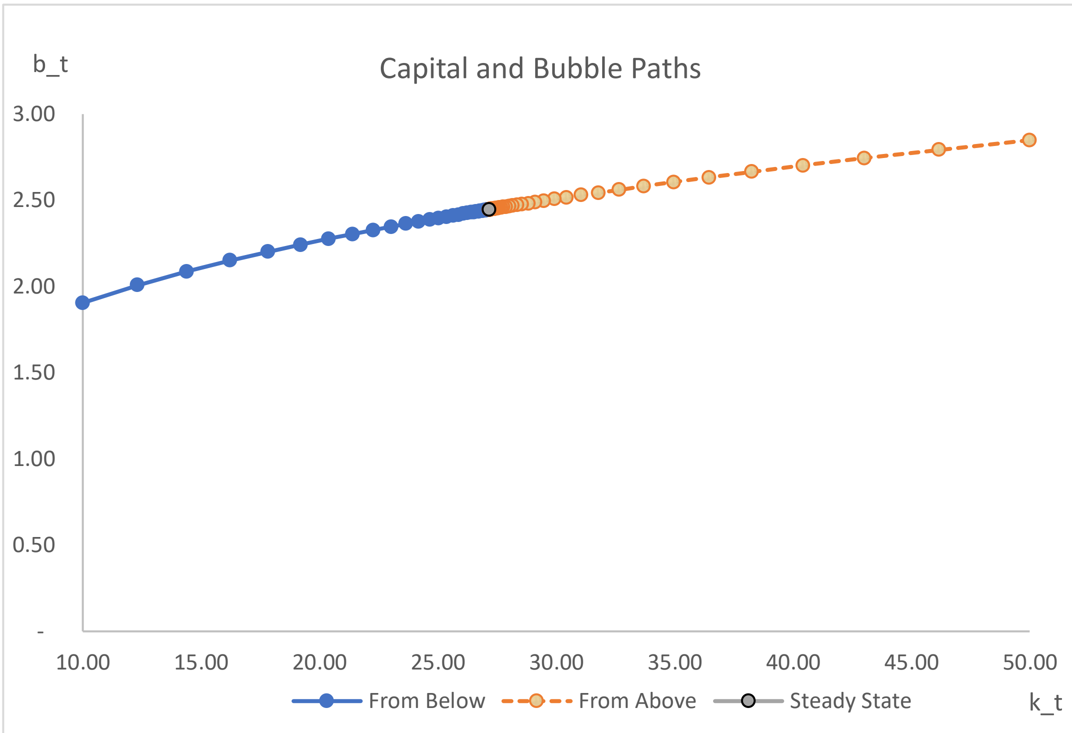


Figure 12 presents the model's saving level and the two components of saving when the economy initiates from a low capital level. As capital increases over time, we find that the total savings level increases, as do the two components of saving. As noted above the shares of saving constituted by the capital backed asset and bubbly asset remain constant over time, with the share of the capital backed asset being equal to $z_t = \frac{\alpha}{h[1-\alpha]}$ and the share of the bubbly asset equal to $\frac{h[1-\alpha]-\alpha}{h[1-\alpha]}$. Condition (62) indicates $\mathrm{h}[1-\alpha] > \alpha$ must hold for a bubble to exist, so we see in Figure 2 and in the model conditions that a bubble exists to the extent that it can exist.

Figure 12 Saving and Its Components

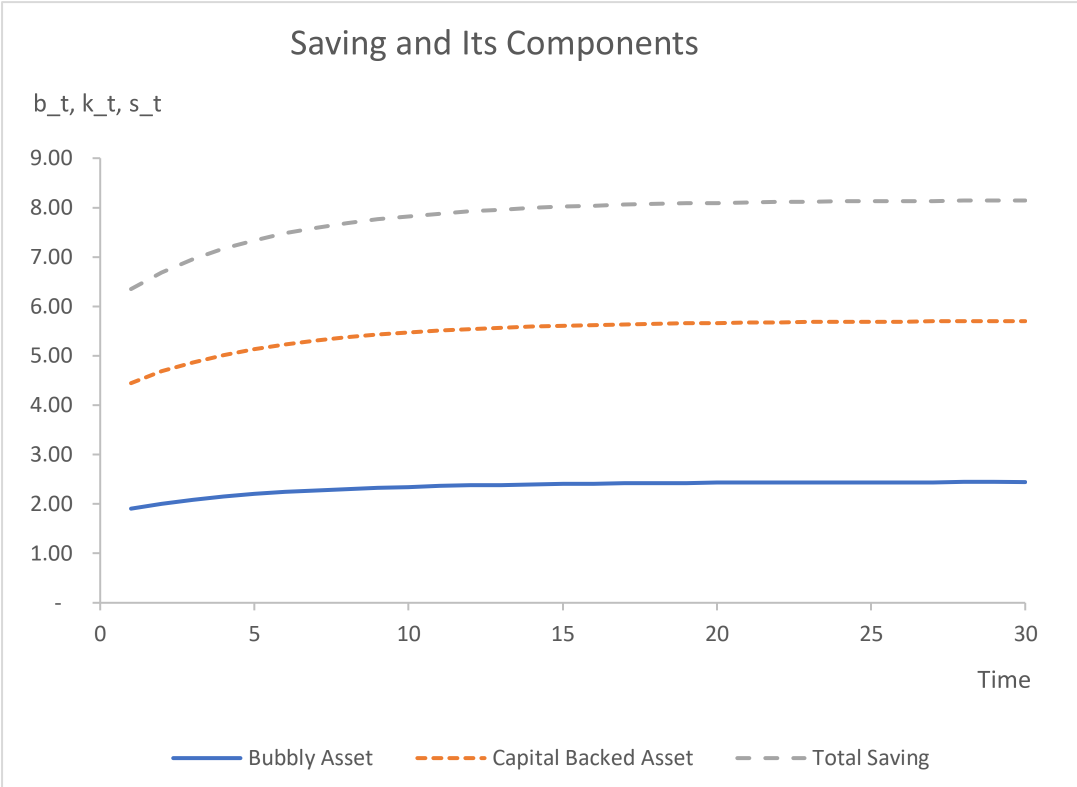
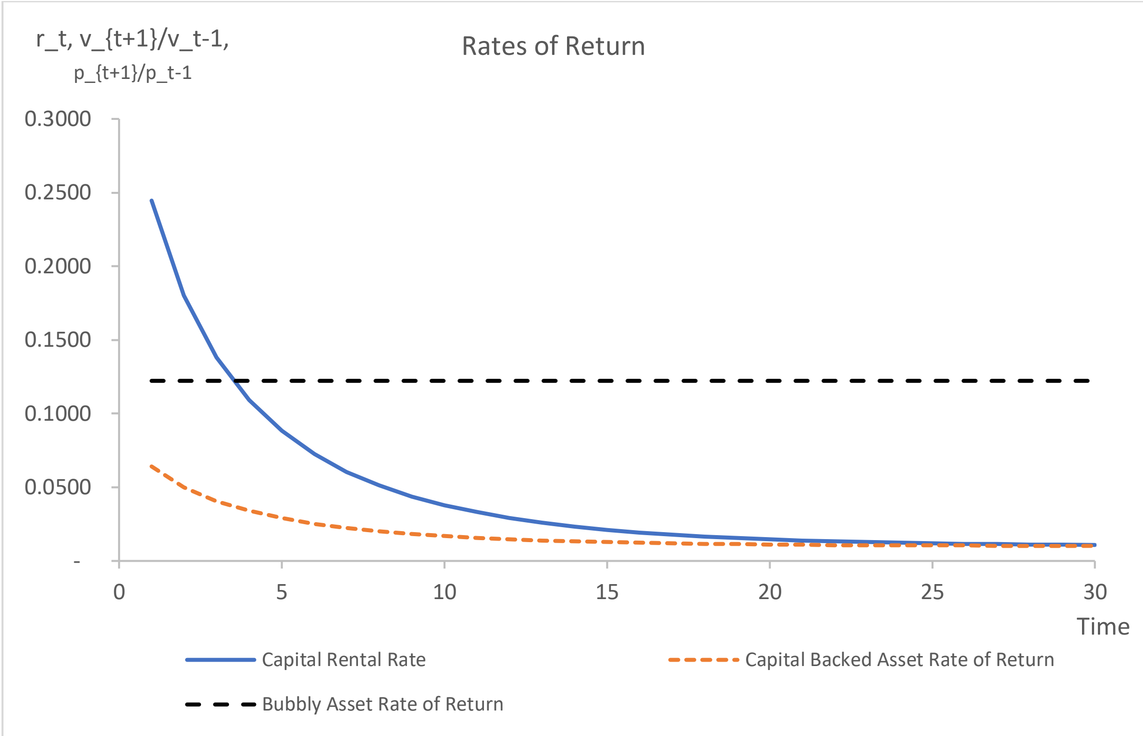


Figure 13 presents the paths followed by the model's rates of return when the initial capital stock level is low. Intuitively, the capital rental rate r_t decreases as the capital stock k_t increases to its steady state value, because there are diminishing returns to capital. For both product market and capital market to clear, the interest rate $i_t = \frac{v_t}{v_{t-1}} - 1$ paid on saving that flows into the capital backed asset must be below the capital rental rate as shown. The capital rental rate decreases faster than this interest rate so that each converges in Figure 3 to the rate of labor growth $n = 0.01$.

Figure 13 Rates of Return



Surprisingly, the rate of return on the bubbly asset $\frac{p_t}{p_{t-1}} - 1 = \frac{1+n}{q} - 1$ remains constant over time. It is the only rate of return influenced by the uncertainty introduced into the model. As long as $q < 1$, so there is some probability the bubble will burst from one period to the next, the rate of return on the bubble asset will exceed the rate of labor growth n. So, as the model converges, the difference in the rate of return on the bubbly asset and capital backed asset is entirely explained by the uncertainty. If there is no uncertainty, meaning the decision maker is entirely confident the bubble will not burst, then the rate of return on the bubbly asset will be equal n, and the other two rates of return would converge to that level over time.

3.6 Conclusion

This work takes the model of Weil (1987) and modifies in three ways. First, in Weil (1987), capital doesn't accumulate but is consumed after it is used in production. Here, we assume capital accumulates, which implies the savings of young consumers finances only investment not the entire capital stock. Second, we assume capital depreciates, allowing us to examine the effect of depreciation. Third, following Banerjee and Pingle (2023), we model transfer of firm ownership from old to young.

Modeling the transfer of ownership is significant because it eliminates the indeterminacy and inefficiency in the Weil (1987) model. There is a unique equilibrium path in our model, and it is Pareto efficient.

Because we follow Weil (1987) and include the possibility that the bubble can burst, there must be a gap between the return on the bubbly asset and the return on the capital backed asset. For log linear utility function we use, we find the rate on the bubbly asset is constant and equal to $\frac{p_t}{p_{t-1}} - 1 = \frac{1+n}{q} - 1$. If the bubbly asset is perceived to survive certainty ($q = 1$), then the rate of return on the bubbly asset equal to labor growth rate n. The rate of return is higher than n when the likelihood q that the bubble will survive is smaller.

Consistent with Banerjee and Pingle (2023), we find the allowing capital to accumulate places a wedge between capital rental rate and interest rate paid on savings that is following into capital. As the level of capital increases towards a steady state value, the capital rental rate and interest rate each decease, and each converge to steady state rate n. Thus, consistent with Samuelson (1958), our model produces a biological theory of interest. That is, the interest rate level for the economy long term is equal to the population growth rate.

Examining the asset portfolio of young age savers, we find the percentage of savings allocated to bubbly asset versus capital backed asset remain constant over time. The share allocated to the bubble asset increases when bubbles are more capable of forming. Put differently, a bubble exists to the extent it can exist. As the level of capital increases towards a steady state value, the level of saving increases with allocations to bubble asset and capital backed asset increasing accordingly.

Because the degree of uncertainty does not affect capital, it implies the degree of uncertainty also does not affect the level of output, the capital rental rate, the real wage rate, nor the real interest rate paid on saving that flows into capital. We also find uncertainty does not affect young age consumption, saving, nor old age consumption. However, these results may depend upon our use of the log linear utility function which is special because saving is independent of the rate of interest. Further exploration is needed to discover whether different utility functions would produce an impact of uncertainty on consumption choices.

In our model, an increase in uncertainty does not reduce the size of the steady state bubble as it does in the Weil (1987) model. Rather, it just increases the rate of return that is paid on the bubble so the bubble size can remain the same. If this result generalizes, it is important because it would indicate the forces that push the economy into an equilibrium are prone to create bubbles if bubbles can form. It would also indicate that increases in uncertainty do not reduce bubbles, but rather just increase the rates of return offered on bubble assets.

A.3 Appendix (Chapter 3)

A.3.1 Derivation of Optimal Consumer Choices

Our log linear utility function as a function of the choice variables θ_t and m_t can be written as

$$U = ln(w_t - v_t\theta_t - p_t m_t) + \beta[qln(v_{t+1}\theta_t + p_{t+1}m_t) + [1-q]ln(v_{t+1}\theta_t)]$$

Differentiating with respect to m_t, we obtain $U_{m_t} = \frac{1}{w_t - v_t\theta_t - p_t m_t}[-p_t] + \frac{\beta q}{v_{t+1}\theta_t + p_{t+1}m_t}[p_{t+1}]$. Setting the derivative equal to zero, we obtain $\frac{1}{w_t - v_t\theta_t - p_t m_t} = \frac{1}{v_{t+1}\theta_t + p_{t+1}m_t}\beta\left[q\frac{p_{t+1}}{p_t}\right]$, which implies $\frac{1}{w_t - v_t\theta_t - p_t m_t} = \frac{1}{\frac{v_{t+1}}{v_t}v_t\theta_t + p_{t+1}m_t}\beta\left[q\frac{p_{t+1}}{p_t}\right]$. Using condition (28) and defining $1 + i_{t+1} = \frac{v_{t+1}}{v_t}$ as the gross return earned on the capital asset, we can rewrite this condition as

(A1) $\frac{1}{w_t - x_t - p_t m_t} = \frac{1}{[1+i_{t+1}]x_t + p_{t+1}m_t}\beta\left[q\frac{p_{t+1}}{p_t}\right]$.

Differentiating with respect to θ_t, we obtain $U_{\theta_t} = \frac{1}{w_t - v_t\theta_t - p_t m_t}[-v_t] + \frac{\beta q}{v_{t+1}\theta_t + p_{t+1}m_t}[v_{t+1}] + \frac{\beta[1-q]}{v_{t+1}\theta_t}[v_{t+1}]$. Setting the derivative equal to zero, we obtain $\frac{1}{w_t - v_t\theta_t - p_t m_t} = \left[\frac{q}{v_{t+1}\theta_t + p_{t+1}m_t} + \frac{[1-q]}{v_{t+1}\theta_t}\right]\beta\left[\frac{v_{t+1}}{v_t}\right]$, which implies $\frac{1}{w_t - v_t\theta_t - p_t m_t} = \left[\frac{q}{\frac{v_{t+1}}{v_t}v_t\theta_t + p_{t+1}m_t} + \frac{[1-q]}{\frac{v_{t+1}}{v_t}v_t\theta_t}\right]\beta\left[\frac{v_{t+1}}{v_t}\right]$. Using condition (28) and defining $1 + i_{t+1} = \frac{v_{t+1}}{v_t}$ as the gross return earned on the capital asset, we can rewrite this condition as

(A2) $\frac{1}{w_t - x_t - p_t m_t} = \left[\frac{q}{[1+i_{t+1}]x_t + p_{t+1}m_t} + \frac{[1-q]}{[1+i_{t+1}]x_t}\right]\beta[1+i_{t+1}]$

Conditions (A1) and (A2) are the particular first order conditions that arise from the log linear utility function. Condition (A2) is condition (33) for the log linear utility function. Condition (A1) is condition (34) for the log linear utility function. Together, conditions (27)-(32), (A1) and (A2) determine the optimal levels for the variables x_t, b_t, s_t, c_t^y, c_{t+1}^{o+}, c_{t+1}^{o-}, m_t, and θ_t for the log linear utility function.

Starting with (A1), we find $\frac{1}{w_t - x_t - p_t m_t} = \frac{\beta}{[1+i_{t+1}]x_t + p_{t+1}m_t}\left[q\ \frac{p_{t+1}}{p_t}\right]$, which implies we can rewrite (A2) as $\beta[1+i_{t+1}]\left[\frac{q}{[1+i_{t+1}]x_t + p_{t+1}m_t} + \frac{[1-q]}{[1+i_{t+1}]x_t}\right] = \frac{\beta}{[1+i_{t+1}]x_t + p_{t+1}m_t}\left[q\ \frac{p_{t+1}}{p_t}\right]$. Using condition (31), we can write $\frac{q[1+i_{t+1}]}{c_{t+1}^{o+}} + \frac{[1-q][1+i_{t+1}]}{x_t[1+i_{t+1}]} = q\ \frac{p_{t+1}}{p_t}\frac{1}{c_{t+1}^{o+}}$, which implies $\frac{[1-q]}{x_t} = q\ \frac{p_{t+1}}{p_t}\frac{1}{c_{t+1}^{o+}} - \frac{q[1+i_{t+1}]}{c_{t+1}^{o+}}$, or $\frac{[1-q]}{x_t} = \frac{1}{c_{t+1}^{o+}}\left[q\ \frac{p_{t+1}}{p_t} - q[1+i_{t+1}]\right]$, which implies

(A3) $c_{t+1}^{o+} = \frac{q}{1-q}\left[\frac{p_{t+1}}{p_t} - [1+i_{t+1}]\right]x_t.$

Starting again with (A1), we find $[1+i_{t+1}]x_t + p_{t+1}m_t = \beta q\ \frac{p_{t+1}}{p_t}[w_t - x_t - p_t m_t]$, which implies $p_t m_t + \beta q p_t m_t = \beta q\ w_t - \beta q x_t - [1+i_{t+1}]\frac{p_t}{p_{t+1}}x_t$, which implies $[1+\beta q]p_t m_t = \beta q\ w_t - x_t\left[\beta q + [1+i_{t+1}]\frac{p_t}{p_{t+1}}\right]$, which implies

(A4) $\quad p_t m_t = \left[\frac{\beta q}{1+\beta q}\right] w_t - \left[\frac{\beta q + [1+i_{t+1}]\frac{p_t}{p_{t+1}}}{1+\beta q}\right] x_t.$

Now, starting with (A2) and using (28) and (31), we find $\beta[1+i_{t+1}]\left[\frac{q}{c^{o+}_{t+1}} + \frac{[1-q]}{[1+i_{t+1}]x_t}\right] = \frac{1}{w_t - x_t - p_t m_t}$, which using (A3) then implies $\beta[1+i_{t+1}]\left[\frac{q}{\frac{q}{1-q}\left[\frac{p_{t+1}}{p_t} - [1+i_{t+1}]\right]x_t} + \frac{[1-q]}{[1+i_{t+1}]x_t}\right] = \frac{1}{w_t - x_t - p_t m_t}$. We can rewrite this as $\beta[1+i_{t+1}][w_t - x_t - p_t m_t]\left[\frac{q}{\frac{q}{1-q}\left[\frac{p_{t+1}}{p_t} - [1+i_{t+1}]\right]} + \frac{[1-q]}{[1+i_{t+1}]}\right] = x_t$, which using (A4) becomes $\beta[1+i_{t+1}]\left[w_t - x_t - \left[\left[\frac{\beta q}{1+\beta q}\right] w_t - \left[\frac{\beta q + [1+i_{t+1}]\frac{p_t}{p_{t+1}}}{1+\beta q}\right] x_t\right]\right]\left[\frac{q}{\frac{q}{1-q}\left[\frac{p_{t+1}}{p_t} - [1+i_{t+1}]\right]} + \frac{[1-q]}{[1+i_{t+1}]}\right] = x_t$. We now reduce and rearrange this equation as follows:

$$\beta[1+i_{t+1}]\left[w_t - x_t - \left[\frac{\beta q}{1+\beta q}\right] w_t + \left[\frac{\beta q + [1+i_{t+1}]\frac{p_t}{p_{t+1}}}{1+\beta q}\right] x_t\right]\left[\frac{q}{\left[\frac{q p_{t+1} - q p_t[1+i_{t+1}]}{p_t[1-q]}\right]} + \frac{[1-q]}{[1+i_{t+1}]}\right] = x_t$$

$$\left[w_t - x_t - \left[\frac{\beta q_t}{1+\beta q_t}\right] w_t + \left[\frac{\beta q_t + [1+i_{t+1}]\frac{p_t}{p_{t+1}}}{1+\beta q_t}\right] x_t\right]\left[\frac{p_t[1-q]}{p_{t+1} - p_t[1+i_{t+1}]} + \frac{[1-q]}{[1+i_{t+1}]}\right] = x_t \frac{1}{\beta[1+i_{t+1}]}$$

$$\left[w_t\left[1 - \left[\frac{\beta q}{1+\beta q}\right]\right] + \left[\frac{\beta q + [1+i_{t+1}]\frac{p_t}{p_{t+1}}}{1+\beta q} - 1\right] x_t\right]\left[\frac{p_t[1-q][1+i_{t+1}] + [1-q][p_{t+1} - p_t[1+i_{t+1}]]}{[p_{t+1} - p_t[1+i_{t+1}]][1+i_{t+1}]}\right] = x_t \frac{1}{\beta[1+i_{t+1}]}$$

$$\left[\left[\frac{1}{1+\beta q}\right] w_t + \left[\frac{[1+i_{t+1}]\frac{p_t}{p_{t+1}}-1}{1+\beta q}\right] x_t\right]\left[\frac{[1-q\,]p_{t+1}}{p_{t+1}-p_t[1+i_{t+1}]}\right] = x_t\,\frac{1}{\beta}$$

$$\left[w_t + \left[\,[1+i_{t+1}]\,\frac{p_t}{p_{t+1}} - 1\right] x_t\right]\left[\frac{[1-q\,]}{1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]}\right] = x_t\,\frac{1+\beta q}{\beta}\,.$$

Introducing $z_t = \frac{[1-q\,]}{1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]}$, we then obtain $\left[w_t + \left[\,[1+i_{t+1}]\,\frac{p_t}{p_{t+1}} - 1\right] x_t\right] z_t = x_t\,\frac{1+\beta q}{\beta}$, which implies $z_t w_t = x_t\,\frac{1+\beta q}{\beta} - z_t\left[\,[1+i_{t+1}]\,\frac{p_t}{p_{t+1}} - 1\right] x_t$, which implies $z_t w_t = x_t\,\frac{1+\beta q}{\beta} + z_t\left[\,1-[1+i_{t+1}]\,\frac{p_t}{p_{t+1}}\right]\frac{[1-q\,]}{[1-q\,]}\,x_t$, which implies $z_t w_t = x_t\,\frac{1+\beta q}{\beta} + z_t\,\frac{[1-q\,]}{z_t}\,x_t$, which implies $z_t w_t = x_t\,\frac{1+\beta q}{\beta} + [1-q\,]x_t$, which implies $z_t w_t = \left[\frac{1+\beta q}{\beta} + [1-q]\right] x_t$, which implies $z_t w_t = \left[\frac{1+\beta q+\beta[1-q\,]}{\beta}\right] x_t$, which implies $z_t w_t = \left[\frac{1+\beta}{\beta}\right] x_t$, which implies

(A5) $x_t = z_t h w_t$,

where $h = \frac{\beta}{1+\beta}$.

Next, beginning with condition (A4) and using our solution (A5), we find $p_t m_t = \left[\frac{\beta q}{1+\beta q}\right]\, w_t - \left[\frac{\beta q + [1+i_{t+1}]\frac{p_t}{p_{t+1}}}{1+\beta q}\right] z_t h w_t$. Using the definition $z_t = \frac{[1-q\,]}{1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]}$, we then have $p_t m_t = \left[\frac{\beta q}{1+\beta q}\right] w_t - \left[\frac{\beta q + [1+i_{t+1}]\frac{p_t}{p_{t+1}}}{1+\beta q}\right]\left[\frac{[1-q]}{1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]}\right]\left[\frac{\beta}{1+\beta}\right] w_t$. We now reduce

and rearrange this condition as follows:

$$p_t m_t = \left[\left[\frac{\beta q}{1+\beta q}\right] - \left[\frac{\beta q + [1+i_{t+1}]\frac{p_t}{p_{t+1}}}{1+\beta q}\right]\left[\frac{[1-q]}{1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]}\right]\left[\frac{\beta}{1+\beta}\right]\right] w_t$$

$$p_t m_t = \left[\frac{\beta q\left[1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]\right][1+\beta] - \beta\left[\beta q + \frac{p_t}{p_{t+1}}[1+i_{t+1}]\right][1-q]}{[1+\beta q]\left[1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]\right][1+\beta]}\right] w_t$$

$$p_t m_t = \left[\frac{\beta q\left[1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]\right] + \beta\beta q\left[1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]\right] - \beta\beta q[1-q] - \beta\frac{p_t}{p_{t+1}}[1+i_{t+1}][1-q]}{[1+\beta q]\left[1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]\right][1+\beta]}\right] w_t$$

$$p_t m_t = \left[\frac{\beta q - \beta q\frac{p_t}{p_{t+1}}[1+i_{t+1}] + \beta\beta q - \beta\beta q\frac{p_t}{p_{t+1}}[1+i_{t+1}] - \beta\beta q[1-q] - \beta\frac{p_t}{p_{t+1}}[1+i_{t+1}] + \beta q\frac{p_t}{p_{t+1}}[1+i_{t+1}]}{[1+\beta q]\left[1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]\right][1+\beta]}\right] w_t$$

$$p_t m_t = \left[\frac{\beta q - \beta\beta q\frac{p_t}{p_{t+1}}[1+i_{t+1}] + \beta\beta q q - \beta\frac{p_t}{p_{t+1}}[1+i_{t+1}]}{[1+\beta q]\left[1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]\right][1+\beta]}\right] w_t$$

$$p_t m_t = \left[\frac{\beta q[1+\beta q] - \beta[1+\beta q]\frac{p_t}{p_{t+1}}[1+i_{t+1}]}{[1+\beta q]\left[1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]\right][1+\beta]}\right] w_t$$

$$p_t m_t = \left[\frac{\beta q - \beta\frac{p_t}{p_{t+1}}[1+i_{t+1}]}{\left[1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]\right][1+\beta]}\right] w_t$$

$$p_t m_t = \left[\frac{q - \frac{p_t}{p_{t+1}}[1+i_{t+1}]}{\left[1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]\right]}\right]\left[\frac{\beta}{1+\beta}\right] w_t$$

$$p_t m_t = \left[\frac{q - \frac{p_t}{p_{t+1}}[1+i_{t+1}]}{\left[1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]\right]}\right]\left[\frac{\beta}{1+\beta}\right] w_t.$$

We then reintroduce $z_t = \frac{[1-q\,]}{1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]}$ to obtain $p_t m_t = z_t \left[\frac{q-\frac{p_t}{p_{t+1}}[1+i_{t+1}]}{[1-q_t]}\right] \mathrm{h} w_t$, which implies $p_t m_t = z_t \left[\frac{q-1+1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]}{[1-q]}\right] \mathrm{h} w_t$, which implies $p_t m_t = z_t \left[\frac{1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]}{[1-q_t]} - \frac{1-q}{[1-q]}\right] \mathrm{h} w_t$. Again, using $z_t = \frac{[1-q\,]}{1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]}$, we find $p_t m_t = z_t \left[\frac{1}{z_t} - 1\right] \mathrm{h} w_t$, which implies, $p_t m_t = [1 - z_t]\mathrm{h} w_t$. Using (29), we then obtain

(A6) $b_t = [1 - z_t]\mathrm{h} w_t$

and using (29)

(A7) $m_t = \frac{[1-z_t]\mathrm{h}}{p_t} w_t$.

Next, starting with condition (A3) and using (A5), we obtain $c_{t+1}^{o+} = \frac{q}{1-q}\left[\frac{p_{t+1}}{p_t} - [1 + i_{t+1}]\right] z_t h w_t$, which implies $c_{t+1}^{o+} = q\,\frac{p_{t+1}}{p_t}\left[\frac{1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]}{1-q}\right] z_t h w_t$. Using the definition $z_t = \frac{[1-q\,]}{1-\frac{p_t}{p_{t+1}}[1+i_{t+1}]}$, we find $c_{t+1}^{o+} = q\,\frac{p_{t+1}}{p_t}\left[\frac{1}{z_t}\right] z_t h w_t$, which implies

(A8) $c_{t+1}^{o+} = q\,\frac{p_{t+1}}{p_t} h w_t$.

Next, starting with condition (A1), and then using conditions (27)-(31), we obtain $\frac{c_{t+1}^{o+}}{c_t^{y}} = \beta\left[q\,\frac{p_{t+1}}{p_t}\right]$. Using the solution (A8), when then obtain $\frac{h w_t\left[q\,\frac{p_{t+1}}{p_t}\right]}{c_t^{y}} = \beta\left[q\,\frac{p_{t+1}}{p_t}\right]$, which implies $c_t^{y} = \frac{h w_t}{\beta}$, which implies

(A9) $c_t^{y} = [1 - h] w_t$

A.3.2 Derivation of $\frac{\mathbf{b_{t+1}}}{\mathbf{b_t}} = \frac{\mathbf{b_{t+1}}}{\mathbf{b_t}} = \frac{\mathbf{1}}{\mathbf{1+n}}\frac{\mathbf{p_{t+1}}}{\mathbf{p_t}}$.

Starting with equilibrium condition (53) for the bubble asset, we have $m_t = [1+n]m_{t+1}$. Multiplying through by p_t, we obtain $p_t m_t = [1+n]p_t m_{t+1}$, which implies $p_t m_t = [1+n]\frac{p_t}{p_{t+1}}p_{t+1}m_{t+1}$. Using condition (29)(9), we find $b_t = [1+n]\frac{p_t}{p_{t+1}}b_{t+1}$ or

(A10) $\frac{b_{t+1}}{b_t} = \frac{1}{1+n}\frac{p_{t+1}}{p_t}$.

A.3.3 Derivation of $\mathbf{x_{t+1}} = \left[\frac{\frac{\mathbf{v_{t+1}}}{\mathbf{v_t}}}{\mathbf{1+n}}\right]\mathbf{x_t}$

Starting with stock market equilibrium condition for the capital backed asset (52), we have $\theta_t = [1+n]\theta_{t+1}$. Multiplying through by v_t, we obtain $v_t\theta_t = [1+n]v_t\theta_{t+1}$, which implies $v_t\theta_t = [1+n]\frac{v_t}{v_{t+1}}v_{t+1}\theta_{t+1}$. Using the capital financing condition (28)(50), we obtain $x_t = [1+n]\frac{v_t}{v_{t+1}}x_{t+1}$, which implies

(A11) $x_{t+1} = \left[\frac{\frac{v_{t+1}}{v_t}}{1+n}\right]x_t$.

A.3.4 Derivation of $\boldsymbol{k_{t+1}} = \left[\frac{\mathbf{1}+\boldsymbol{f'(k_t)-\delta}}{\mathbf{1}+\boldsymbol{n}}\right]\boldsymbol{k_t}$

Beginning with condition (A11)(52), we obtain $x_t = \frac{v_t}{v_{t+1}}[1+n]x_{t+1}$. Using the capital market clearing condition (50), we obtain $[1+n]k_{t+1} - [1-\delta]k_t = \frac{v_t}{v_{t+1}}[1+n]x_{t+1}$. Using the product market clearing condition (51), we obtain $y_t - c_t^y - \frac{c_t^o}{1+n} = \frac{v_t}{v_{t+1}}[1+n]x_{t+1}$. Using (46), we obtain $w_t + [r_t+\delta]k_t - c_t^y - \frac{c_t^o}{1+n} = \frac{v_t}{v_{t+1}}[1+n]x_{t+1}$. Using condition (27), we obtain $s_t + [r_t+\delta]k_t - \frac{c_t^o}{1+n} = \frac{v_t}{v_{t+1}}[1+n]x_{t+1}$. Using condition (30), we obtain $x_t + b_t + [r_t+\delta]k_t - \frac{c_t^o}{1+n} = \frac{v_t}{v_{t+1}}[1+n]x_{t+1}$. Using (A11), we obtain $\frac{c_t^o}{1+n} = [r_t+\delta]k_t + b_t$. Assuming the bubble has not yet burst, condition (31) implies $\frac{v_t\theta_{t-1} + p_t m_{t-1}}{1+n} = [r_t+\delta]k_t + b_t$, which implies $\frac{\frac{v_t}{v_{t-1}}v_{t-1}\theta_{t-1} + \frac{p_t}{p_{t-1}}p_{t-1}m_{t-1}}{1+n} = [r_t+\delta]k_t + b_t$ Using conditions (28) and (29), we then obtain $\frac{\frac{v_t}{v_{t-1}}x_{t-1} + \frac{p_t}{p_{t-1}}b_{t-1}}{1+n} = [r_t+\delta]k_t + b_t$, which implies $\frac{\frac{v_t}{v_{t-1}}x_{t-1}}{1+n} = [r_t+\delta]k_t + b_t - \frac{\frac{p_t}{p_{t-1}}b_{t-1}}{1+n}$. Imposing 0 then implies $\frac{\frac{v_t}{v_{t-1}}}{1+n}x_{t-1} = [r_t+\delta]k_t$. Using condition (A11), we obtain $x_t = [r_t+\delta]k_t$. Using capital market clearing condition (50) we obtain $[1+n]k_{t+1} - [1-\delta]k_t = [r_t+\delta]k_t$, which implies $[1+n]k_{t+1} - k_t + \delta k_t = r_t k_t + \delta k_t$, which implies $[1+n]k_{t+1} = [1+r_t]k_t$, which implies $k_{t+1} = \left[\frac{1+r_t}{1+n}\right]k_t$. Using (45), we obtain

(A12) $k_{t+1} = \left[\frac{1+f'(k_t)-\delta}{1+n}\right]k_t$.

A.3.5 Derivation of non-steady state values $x_t = \alpha A k_t^\alpha$, $s_t = h[1-\alpha]Ak_t^\alpha$, $b_t = [h[1-\alpha]-\alpha]Ak_t^\alpha$, $\frac{v_{t+1}}{v_t} = [1+n]\left[\frac{1+\alpha A k_t^{\alpha-1}-\delta}{1+n}\right]^\alpha$, $\frac{p_{t+1}}{p_t} = \frac{1+n}{q}$, and $z_t = \frac{\alpha}{h[1-\alpha]}$.

Starting with the capital market clearing condition (50), we have $x_t = [1+n]k_{t+1} - [1-\delta]k_t$. Using the solution (55), we find $x_t = [1+n]\left[\frac{1-\delta+\alpha A k_t^{\alpha-1}}{1+n}\right]k_t - [1-\delta]k_t$, which implies $x_t = [1-\delta+\alpha A k_t^{\alpha-1}]k_t - [1-\delta]k_t$, which implies $x_t = \alpha A k_t^{\alpha-1}k_t$, which implies $x_t = \alpha A k_t^\alpha$.

Starting with condition (37), $s_t = \mathrm{h}w_t$. Using the condition (49) we obtained $s_t = \mathrm{h}[1-\alpha]Ak_t^\alpha$.

Starting with condition (30), we have $b_t = s_t - x_t$. Using the solution (56) and (57) we obtained $b_t = \mathrm{h}[1-\alpha]Ak_t^\alpha - \alpha A k_t^\alpha$, which implies $b_t = [\mathrm{h}[1-\alpha]-\alpha]Ak_t^\alpha$.

Starting with condition (A11)0, we find $x_{t+1} = \left[\frac{\frac{v_{t+1}}{v_t}}{1+n}\right]x_t$, which implies $\frac{v_{t+1}}{v_t} = [1+n]\frac{x_{t+1}}{x_t}$. Using the solution (56), we obtain $\frac{v_{t+1}}{v_t} = [1+n]\frac{\alpha A k_{t+1}^\alpha}{\alpha A k_t^\alpha}$, which implies $\frac{v_{t+1}}{v_t} = [1+n]\left[\frac{k_{t+1}}{k_t}\right]^\alpha$. Using (55), we obtain $\frac{v_{t+1}}{v_t} = [1+n]\left[\frac{\left[\frac{1+\alpha A k_t^{\alpha-1}-\delta}{1+n}\right]k_t}{k_t}\right]^\alpha$, which reduces to $\frac{v_{t+1}}{v_t} = [1+n]\left[\frac{1+\alpha A k_t^{\alpha-1}-\delta}{1+n}\right]^\alpha$.

Starting with the product market clearing condition (51), we have $y_t = c_t^y + \frac{c_t^o}{1+n} + [1+n]k_{t+1} - [1-\delta]k_t$. Using optimal consumption values (35), (38) and (39) assuming the bubble does not burst, we find $y_t = [1-h]w_t + \frac{hw_t q\frac{p_{t+1},}{p_t}}{1+n} + x_t$. Using solutions (47), (49), and (56), we obtain $Ak_t^\alpha = [1-h][1-\alpha]Ak_t^\alpha + \frac{h[1-\alpha]Ak_t^\alpha q\frac{p_{t+1},}{p_t}}{1+n} +$

$\alpha A k_t^{\alpha}$, which implies $1 = [1-h][1-\alpha] + \frac{h[1-\alpha]q\frac{p_{t+1}}{p_t}}{1+n} + \alpha$, which implies $1 - \alpha - [1-h][1-\alpha] = \frac{h[1-\alpha]q\frac{p_{t+1}}{p_t}}{1+n}$, which implies $h[1-\alpha] = \frac{h[1-\alpha]q\frac{p_{t+1}}{p_t}}{1+n}$, which implies $1 = \frac{q\frac{p_{t+1}}{p_t}}{1+n}$, which implies $\frac{p_{t+1}}{p_t} = \frac{1+n}{q}$.

Starting with condition (35), $x_t = z_t h w_t$. Using condition (49) and the solution (56) we obtain $z_t = \frac{\alpha A k_t^{\alpha}}{h[1-\alpha]A k_t^{\alpha}}$, which implies $z_t = \frac{\alpha}{h[1-\alpha]}$.

A.3.6 Derivation of the Steady State Values

Setting $k_{t+1} = k_t = k$ in (A12), we obtain $k = \left[\frac{1+f'(k)-\delta}{1+n}\right]k$. For $\hat{k} > 0$, this implies $1 = \left[\frac{1+f'(\hat{k})-\delta}{1+n}\right]$ or $1 + n = 1 + f'(\hat{k}) - \delta$ or $n = f'(\hat{k}) - \delta$, which implies

(A13) $f'(\hat{k}) = n + \delta$.

Using the Diamond production function $f(k_t) = A k_t^{\alpha}$, we find $f'(k_t) = \alpha A k_t^{\alpha-1}$. Therefore, (A13) becomes $\alpha A \hat{k}_t^{\alpha-1} = n + \delta$, which implies $\hat{k}^{\alpha-1} = \frac{n+\delta}{\alpha A}$, which can rewrite as

(A14) $\hat{k} = \left[\frac{\alpha A}{n+\delta}\right]^{\frac{1}{1-\alpha}}$.

Using (45), we know the steady state capital rental rate is defined by $\hat{r} = f'(\hat{k}) - \delta$, which using (A13) becomes $\hat{r} = n + \delta - \delta$, or

(A15) $\hat{r} = n$.

Using (44), we know the steady state output level is given by $\hat{y} = f(\hat{k})$. Our Cobb Douglas production function (47) indicates the steady state output level is $\hat{y} = A\hat{k}^{\alpha}$.

Using the solution (A14), this becomes $\hat{y} = A\left[\left[\frac{\alpha A}{n+\delta}\right]^{\frac{1}{1-\alpha}}\right]^{\alpha}$, which becomes $\hat{y} =$

$A\left[\frac{\alpha A}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$, which becomes

(A16) $\hat{y} = A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$.

Using (46), (45) and (44), we know the steady state wage level is given by $\hat{w} = f(\hat{k}) - f'(\hat{k})\hat{k}$. For our Cobb Douglas production function, this becomes

(A17) $\hat{w} = [1-\alpha]A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$.

Using condition (59), we know the steady state gross return on capital is $\frac{v_{t+1}}{v_t} =$

$[1+n]\left[\frac{1+\alpha A\, k^{\alpha-1}-\delta}{1+n}\right]^{\boldsymbol{\alpha}}$. Using the solution (A14), we find $\frac{v_{t+1}}{v_t} = [1+n]\left[\frac{1+\alpha A\left[\frac{n+\delta}{\alpha A}\right]-\delta}{1+n}\right]^{\boldsymbol{\alpha}}$,

which implies $\frac{v_{t+1}}{v_t} = [1+n]\left[\frac{1+n+\delta-\delta}{1+n}\right]^{\alpha}$, which implies $\frac{v_{t+1}}{v_t} = 1+n.$

From the production function (47), we know in the steady state $Ak^{\alpha} = \hat{y}$, which using the solution (63) becomes $Ak^{\alpha} = A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$. Thus, using condition (56), we find $\hat{x} = \alpha A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$. Using condition (58), we find $\hat{b} = [\mathrm{h}[1-\alpha] - \alpha]A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$. Using condition (57), we find $\hat{s} = \mathrm{h}[1-\alpha]A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$.

Using condition (38), we know the steady state young age consumption level $\hat{c}^y = [1-h]\hat{w}$, which using the solution (66) implies $\hat{c}^y = [1-h][1-\alpha]A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$.

Condition (39) and the steady state solutions (60) and (66) indicate

$$c_{t+1}^{o+} = hw_t q \frac{p_{t+1}}{p_t}$$

$$c_{t+1}^{o+} = hw_t q \frac{1+n}{q}$$

$$c_{t+1}^{o+} = hw_t q \frac{1+n}{q}$$

$\hat{c}^{o+} = [1+n]h[1-\alpha]\, A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$. Conditions (40), (61), (66), and (67) imply $\hat{c}^{o-} = [1+n]\frac{\alpha}{h[1-\alpha]}h[1-\alpha]A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$, which implies $\hat{c}^{o-} = [1+n]\alpha A^{\frac{1}{1-\alpha}}\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$.

A.3.7 Proof that Steady State $\hat{k}$ is Stable if $-f''(\hat{k})\hat{k} < 1+n$, and Proof that the Steady State $\hat{k}$ is Stable for the Diamond Production Function

To examine the stability, we differentiate condition (A12) with respect to k_t, and evaluate the result at the steady state. Differentiating we obtain $\frac{\partial k_{t+1}}{\partial k_t} = \frac{\partial}{\partial k_t}\left[\left[\frac{1+f'(k_t)-\delta}{1+n}\right]k_t\right]$, which implies $\frac{\partial k_{t+1}}{\partial k_t} = k_t \frac{\partial}{\partial k_t}\left[\left[\frac{1+f'(k_t)-\delta}{1+n}\right]\right] + \left[\frac{1+f'(k_t)-\delta}{1+n}\right]\frac{\partial}{\partial k_t}[k_t]$, which implies $\frac{\partial k_{t+1}}{\partial k_t} = k_t \frac{f''(k_t)}{1+n} + \frac{1+f'(k_t)-\delta}{1+n}$, which implies $\frac{\partial k_{t+1}}{\partial k_t} = \frac{f''(k_t)k_t + 1 + f'(k_t)-\delta}{1+n}$. At the steady state $k = \hat{k}$. Condition (A13) implies $f'(\hat{k}) = n+\delta$. Thus, we find $\left.\frac{\partial k_{t+1}}{\partial k_t}\right|_{k=\hat{k}} = \frac{f''(\hat{k})\hat{k} + 1 + n}{1+n} = 1 + \frac{f''(\hat{k})\hat{k}}{1+n}$. The steady state is locally stable if and only if $0 < \left.\frac{\partial k_{t+1}}{\partial k_t}\right|_{k=\hat{k}} < 1$. Therefore, our steady state locally stable if and only if $0 < 1 + \frac{f''(\hat{k})\hat{k}}{1+n} < 1$, or $-1 < \frac{f''(\hat{k})\hat{k}}{1+n} < 0$. Given the assumption $f''(\hat{k}) < 0$, the right side of the inequality always holds. Left side of the inequality holds if $-1 < \frac{f''(\hat{k})\hat{k}}{1+n}$, or if $-1 - n < f''(\hat{k})\hat{k}$ or if $-f''(\hat{k})\hat{k} < 1 + n$. For the diamond production function $f(k_t) = Ak_t^{\alpha}$, $f'(k_t) = \alpha Ak_t^{\alpha-1}$ and $f''(k_t) = \alpha[\alpha - 1]Ak_t^{\alpha-2}$. This implies, at the steady state, $\frac{f''(k_t)k_t}{1+n} = \frac{\alpha[\alpha-1]Ak_t^{\alpha-2}\,k_t}{1+n} = \frac{\alpha[\alpha-1]Ak_t^{\alpha-1}}{1+n} = \frac{\alpha[\alpha-1]A\left[\frac{\alpha A}{n+\delta}\right]^{\frac{\alpha-1}{1-\alpha}}}{1+n} = \frac{\alpha[\alpha-1]A\left[\frac{n+\delta}{\alpha A}\right]}{1+n} = \frac{[\alpha-1][n+\delta]}{1+n}$. Since $-1 < -\frac{[1-\alpha][n+\delta]}{1+n} < 0$, we find $-1 < \frac{f''(\hat{k})\hat{k}}{1+n} < 0$. Thus, we have shown that steady state is stable for the Diamond production function.

A.3.8 Deriving the Golden Rule

The economy wide budget constraint for a given time period is the product market clearing condition (51), which in the steady state is $y = c^y + \frac{c^0}{1+n} + [1+n]k - [1-\delta]k$, which implies $y = c^y + \frac{c^0}{1+n} + [n+\delta]k$. Using (47), the consumption possibility constraint becomes $Ak^{\alpha} = c^y + \frac{c^0}{1+n} + [n+\delta]k$. The golden rule is obtained by maximizing the Weil Utility function $U(c^y, c^o) = ln(c^y) + \beta ln(c^o)$ subject to the subject possibility constraint. To do so, we construct the Lagrandian $L = ln(c^y) + \beta ln(c^o) + \lambda\left[Ak^{\alpha} - c^y - \left[\frac{1}{1+n}\right] c^o - [n+\delta]k\right]$.

Optimization yields the first order conditions $L_{c^y} = \frac{1}{c^y} - \lambda = 0$, $L_{c^o} = \frac{\beta}{c^o} - \lambda \frac{1}{1+n} = 0$, $L_k = \alpha A k^{\alpha-1} - [n+\delta] = 0$, and $L_\lambda = Ak^\alpha - c^y - \left[\frac{1}{1+n}\right] c^o - [n+\delta]k = 0$. The capital first order condition implies $\alpha A k^{\alpha-1} - [n+\delta] = 0$, which implies $\alpha A k^{\alpha-1} = n + \delta$, which implies $k^{\alpha-1} = \frac{n+\delta}{\alpha A}$. That is, the golden rule capital stock level is $k = \left[\frac{\alpha A}{n+\delta}\right]^{\frac{1}{1-\alpha}}$.

Using the two consumption first order conditions, we find $\frac{\frac{1}{c^y}}{\frac{\beta}{c^o}} = \frac{\lambda}{\lambda\frac{1}{1+n}}$, which implies $\frac{c^o}{\beta c^y} = 1+n$, which implies $c^o = \beta[1+n]c^y$. Using the consumption possibilities constraint, we obtain $Ak^\alpha - [n+\delta]k = c^y + \left[\frac{1}{1+n}\right] c^o$, which using our previous result becomes $Ak^\alpha - [n+\delta]k = c^y + \left[\frac{1}{1+n}\right]\beta[1+n]c^y$, which implies $Ak^\alpha - [n+\delta]k = [1+\beta]c^y$, which implies $c^y = \frac{1}{1+\beta}[Ak^\alpha - [n+\delta]k]$. Using the golden rule capital stock solution we obtain $c^y = \frac{1}{1+\beta}\left[A\left[\frac{\alpha A}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} - [n+\delta]\left[\frac{\alpha A}{n+\delta}\right]^{\frac{1}{1-\alpha}}\right]$. It is useful to transform this result as follows:

$$c^y = \frac{1}{1+\beta}\left[A\left[\frac{\alpha A}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} - [n+\delta]\left[\frac{\alpha A}{n+\delta}\right]^{\frac{1}{1-\alpha}}\right],$$

$$c^y = [1-h]A^{\frac{1}{1-\alpha}}\left[\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} - [n+\delta]^{\frac{1-\alpha}{1-\alpha}}\left[\frac{1}{n+\delta}\right]^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}\right]$$

$$c^y = [1-h]A^{\frac{1}{1-\alpha}}\left[\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} - [n+\delta]^{\frac{1-\alpha}{1-\alpha}}[n+\delta]^{\frac{-1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}\right]$$

$$c^y = [1-h]A^{\frac{1}{1-\alpha}}\left[\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} - [n+\delta]^{\frac{-\alpha}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}\right]$$

$$c^y = [1-h]A^{\frac{1}{1-\alpha}}\left[\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} - \left[\frac{1}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}\right]$$

$$c^y = [1-h]A^{\frac{1}{1-\alpha}}\left[\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} - \left[\frac{1}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} \frac{\alpha}{\alpha^{\frac{1-\alpha}{1-\alpha}}}\right]$$

$$c^y = [1-h]A^{\frac{1}{1-\alpha}}\left[\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} - \left[\frac{1}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha-1}{1-\alpha}}\alpha\right]$$

$$c^y = [1-h]A^{\frac{1}{1-\alpha}}\left[\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} - \left[\frac{1}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}}\alpha\right]$$

$$c^y = [1-h]A^{\frac{1}{1-\alpha}}\left[\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} - \left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} \alpha\right]$$

$$c^y = [1-h][1-\alpha]A^{\frac{1}{1-\alpha}} \left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$$

Using this solution for c^y and $c^o = \beta[1+n]c^y$ from above, we obtain $c^o = \beta[1+n][1-h][1-\alpha]A^{\frac{1}{1-\alpha}} \left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$, which becomes $c^o = h[1+n][1-\alpha]A^{\frac{1}{1-\alpha}} \left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$.

Using $y = c^y + \frac{c^0}{1+n} + [n+\delta]k$, we find $y = [1-h][1-\alpha]A^{\frac{1}{1-\alpha}} \left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} + h[1-\alpha]A^{\frac{1}{1-\alpha}} \left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} + [n+\delta]\left[\frac{\alpha A}{n+\delta}\right]^{\frac{1}{1-\alpha}}$, which implies $y = [1-\alpha]A^{\frac{1}{1-\alpha}} \left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} + [n+\delta]\left[\frac{\alpha A}{n+\delta}\right]^{\frac{1}{1-\alpha}}$, $y = [1-\alpha]A^{\frac{1}{1-\alpha}} \left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} + \alpha\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha-1}{1-\alpha}} A^{\frac{1}{1-\alpha}} \left[\frac{\alpha}{n+\delta}\right]^{\frac{1}{1-\alpha}}$, $y = [1-\alpha]A^{\frac{1}{1-\alpha}} \left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} + \alpha\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}}$, $y = A^{\frac{1}{1-\alpha}} \left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$, $y = A^{\frac{1}{1-\alpha}}A^{\frac{\alpha-1}{1-\alpha}}A\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$, $y = A^{\frac{\alpha}{1-\alpha}}A\left[\frac{\alpha}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$, $y = A\left[\frac{\alpha A}{n+\delta}\right]^{\frac{\alpha}{1-\alpha}}$, $y = Ak^{\alpha}$. Thus, we have checked that the golden rule solutions satisfy the product market clearing condition.

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